

Nonlocal origin and correlations in the Johnson noise at nonuniform temperature

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The conventional description of thermal noise in conductors (the Nyquist scenario) assumes that this noise has local origin and, therefore, the noise in the emf (electromotive force) in a conductor with nonuniform temperature can be described as a sum of contributions from uncorrelated regions, each in local thermal equilibrium. Here we propose an alternative scenario for the propagation of thermal noise in a conductor and suggest experiments that could distinguish between different scenarios. To explore the proposed scenario, we introduce a workable 1D model for a gas of particles that undergo stochastic collisions with the lattice and exert distance-dependent forces on each other. We define current, voltage, and emf in a manner appropriate to a wire with limited number of particles. For uniform temperature and within appropriate length and temperature ranges, we verify that our simulations comply with Nyquist's result. Our numerical simulations indicate that (1) thermal noise in a resistor at uniform local temperature is influenced not only by its own temperature but also by the temperatures of other resistors in the circuit; (2) for sufficiently long circuits, the deviation from the Nyquist prediction is inversely proportional to the distance between the centers of the resistors; and (3) if the resistors differ in temperature, their emf can be correlated, even if they are detached. Analytic results can be obtained in the limit of strong interparticle interaction. The long-range repulsion between charges in electrically connected resistors may have conceptual and technological impact on nanodevices.

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I. INTRODUCTION

The electromotive effect of the thermal agitation of the carriers of electricity in conductors was predicted by Einstein [1], measured by Johnson [2], and explained by Nyquist [3] by invoking the second law of thermodynamics and detailed balance. This effect is called “Johnson noise” and will be spelled out below.

Noise sets a limitation to every measurement, unless it can be performed at sufficiently low temperatures. On the other hand, in recent decades, noise has become a measuring tool [4], and the assumption that thermal noise depends only on the resistance and the temperature allows its use as an ideal thermometer. Due to its wide range of applicability, Johnson noise thermometry is the tool of choice for many applications and characterization of materials, but also for fundamental questions, such as redefinition of the Kelvin [5,6] or observation of fractional charges in the quantum Hall effect [7,8]. A particular application of noise thermometry is the design and characterization of measuring devices such as bolometers;

due to its outstanding properties (e.g., almost perfect lattice, decoupling of heat and charge currents, low electronic heat capacity), graphene [9–14] is the ideal material for this purpose.

It is customary to describe Johnson noise in the frequency domain, but for our purposes it will be clearer to describe it in the time domain: when a current I is driven along a resistor with resistance R at temperature T and the voltage is measured during a lapse of time Δt , the time-average voltage drop $V(t_i) = (1/\Delta t) \int_{t_i}^{t_i+\Delta t} \tilde{V}(t) dt$, where $\tilde{V}(t)$ is the instantaneous voltage drop across the resistor, is not just IR , but due to the Johnson noise, it is $V(t_i) = IR + \Delta V$, where the ensemble average of the random variable ΔV is 0 and its variance is [15]

$$\langle (\Delta V)^2 \rangle = 2k_B T R / \Delta t, \quad (1)$$

where k_B is the Boltzmann constant. For very small values of T or Δt there are deviations from (1), that are beyond the scope of this study. Likewise, if a voltage V is applied to the resistor, the electric current will have a time-average $V/R + \Delta I$, where the ensemble average of ΔI is zero and its variance is

$$\langle (\Delta I)^2 \rangle = 2k_B T / R \Delta t. \quad (2)$$

The derivation of these results is discussed in Appendix A.

In this study, we will consider circuits without power sources, and in the following we will drop the Δ from ΔV and ΔI . Equations (1) and (2) are applicable to particular

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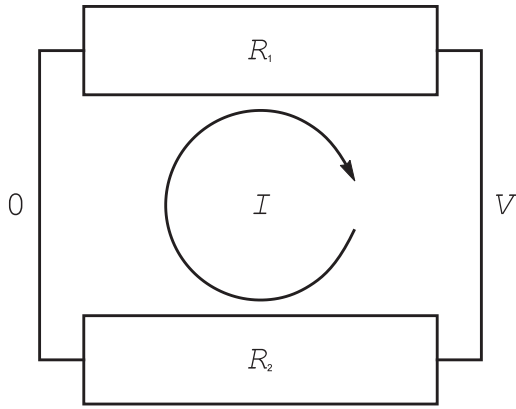


FIG. 1. A circuit with two resistors. There are no power supplies in the circuit, but due to thermal noise, there is a voltage drop (defined as positive at the right end) and a current (positive if clockwise).

situations in which either the current or the voltage is fixed and does not fluctuate; in the general case (e.g., in a segment of a wire), the Johnson noise is described by means of the emf (electromotive force) $\varepsilon = V + IR$. Its variance is

$$S(\varepsilon) := \langle \varepsilon^2 \rangle - \langle \varepsilon \rangle^2 = 2k_B T R / \Delta t. \quad (3)$$

The physical agent responsible for this noise is a fluctuating electric field in addition to that applied by power sources. The question we intend to address is: Where does this field originate? The naive answer is that this field is a local effect, induced by thermal motion of the charged material that makes up the resistor.

This local-origin view was present in Nyquist's description. Reference [3] regards a resistor as a power source, with the power originating inside it, as expressed in passages such as "The electromotive force due to thermal agitation in conductor I..." or "...the member in which the electromotive force is generated." We will call this view the "Nyquist scenario" (NS).

II. MOTIVATION FOR AN ALTERNATIVE SCENARIO

A. Heuristic arguments

We know that an electric signal propagates along a wire or, in general, through a conductor. From a detailed perspective, this propagation could be described as a plasma-electromagnetic wave, subject to the condition that no current crosses the boundary of the conductor. We conjecture that the electric field generated by thermal agitation is not an exception, and can therefore propagate along an electric circuit. Accordingly, we envisage the following scenario: thermal motions *inside all the conducting material that makes up the circuit* create or modulate plasma-electromagnetic waves that pervade the entire circuit and give rise to the voltages or currents in the Johnson noise.

As a simple example, we may consider a closed circuit with two resistors, R_1 and R_2 , as sketched in Fig. 1. We can regard the circuit as a pair of resistors in parallel, with equivalent resistance $R_1 R_2 / (R_1 + R_2)$, with no net current flowing from one end of the equivalent resistor to the other. Elaborating on (1), it follows that the time-average voltage across the ends of

the equivalent resistor during a lapse of time Δt is

$$V = \eta_V \sqrt{2k_B T R_1 R_2 / (R_1 + R_2) \Delta t}, \quad (4)$$

where η_V is a random variable with average 0, variance 1, and normal distribution.

We can also regard the circuit as a pair of resistors in series, with an equivalent resistance $R_1 + R_2$ and no voltage drop across the equivalent resistor. Therefore it follows from (2) that the time-average current around the pair of resistors during the lapse of time Δt is

$$I = \eta_I \sqrt{2k_B T / (R_1 + R_2) \Delta t}, \quad (5)$$

where η_I is a random variable with average 0, variance 1, and normal distribution. During Δt , the electric field performs work $VI\Delta t$ on the charges in R_1 and $-VI\Delta t$ on the charges in R_2 , with $VI\Delta t = 2\eta_V \eta_I k_B T \sqrt{R_1 R_2} / (R_1 + R_2)$.

We note that in this example each resistor has to "know" what is the resistance of the other resistor. The reason it "knows" is that signal propagation is built-in into the NS through the assumption that the current is uniform along the circuit. In this example, our conjecture is that if a resistor is able to "know" the resistance of its companion, it should also be able to "know" its temperature.

The difference between two scenarios is physically irrelevant unless they lead to different falsifiable predictions. We therefore require experiments that can confirm or reject the local or the nonlocal scenario. Since Nyquist's result is based on thermodynamics and detailed balance, any admissible scenario is expected to lead to the same result in situations with uniform temperature; only when more than one temperature is involved, their predictions can differ.

For example, if two resistors are connected and the variance of the emf due to Johnson noise in one of the resistors depends on the temperature of its neighbor, then the NS should be rejected.

As another example, let us consider two initially separate resistors, R_1 and R_2 , with volumes V_1 and V_2 at different temperatures, T_1 and T_2 , as sketched in Fig. 2. To fix ideas, let us assume that $R_1 \ll R_2$, $V_1 \ll V_2$, $T_1 \gg T_2$, and $R_1 T_1 \gg R_2 T_2$. Let the variance of the voltage across R_1 be measured as reviewed, e.g., in Ref. [4]. At some moment, the two resistors are connected.

Let τ_{th} be the time required by the resistors in Fig. 2 to approach thermal equilibrium. Then, the effective temperatures of the resistors after, say, $0.1\tau_{th}$ will still be close to the temperatures they had before making contact and, according to NS, R_1 would essentially be an unloaded power source with a voltage variance similar to that before contact was made. On the other hand, according to the scenario suggested here, we may expect that the electromagnetic waves have already propagated to the entire circuit and, since $V_1 \ll V_2$, they would be close to the case of temperature T_2 and the Johnson noise would decrease significantly.

Many variations of this experiment can be conceived. Instead of separate or joined resistors, electrical contact could be controlled by a MOSFET or HEMT transistor. A

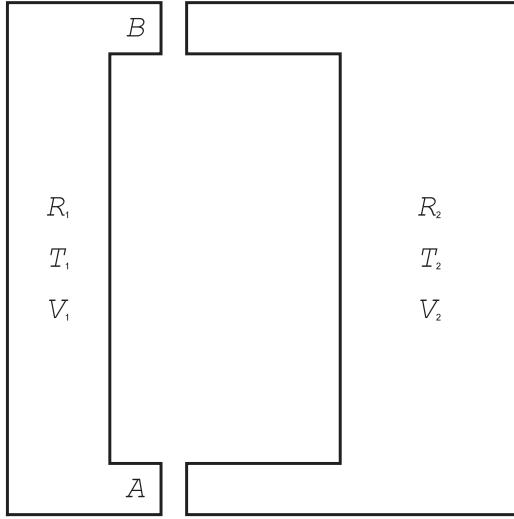


FIG. 2. Experiment intended to determine which is the pertinent scenario. Two resistors are initially separated and their temperatures are different. The variance of the voltage between A and B is measured before and after the resistors touch each other and form a closed circuit.

steady-state version can be obtained if the resistors are permanently connected and a part of each of them is in thermal contact with a heat bath. By periodically connecting and disconnecting the resistors, a lock-in analysis could measure the time lag between the electrical connection and the change in the voltage variance. A situation in which the noise received by an active amplifier (designed to operate as a “cold resistor”) is modulated by switching the connection to a distant resistor was reported in Ref. [16].

Nonuniform temperature can not only take the form of a function of position, but may also arise as a difference between the temperature T_e of the electrons and the temperature T_{ph} of the lattice. It is well established [17,18] that when current is driven along a resistor and the electron-phonon coupling is weak, then $T_e \gg T_{ph}$, and the temperature in Eqs. (1) and (2) should be replaced with T_e . Obviously, in this case, the Johnson noise cannot be attributed to “thermal agitation in the conductor,” because there is no such thermal agitation. In this context, our claim can be stated as follows: the strong repulsion between electrons, which nearly imposes electroneutrality, effectively acts as a long-range interaction along the circuit that diverts the electrons from their local equilibrium.

B. Available experimental result

Strictly speaking, since NS is based on equilibrium thermodynamic arguments, it does not predict anything in the case of nonuniform temperature. However, it can be naturally extended to the case of a wire with steady temperature $T(x)$, $0 \leq x \leq L$. From (1), denoting by ρ the resistance per length and assuming that the Johnson noise is a local effect, the variance of the emf (and of the voltage, if there is no current) between x and $x + dx$ would be $2k_B T(x) \rho(x) dx / \Delta t$. Assuming also that the fluctuating voltages in different segments are

uncorrelated, we obtain

$$(\Delta t / 2k_B) \langle V^2 \rangle = \int_0^L T(x) \rho(x) dx. \quad (6)$$

Denoting longitudinal average by an overline [e.g., $\bar{T} = L^{-1} \int_0^L T(x) dx$], substituting in (6) $T(x) = \bar{T} + [T(x) - \bar{T}]$ and $\rho(x) = \bar{\rho} + [\rho(x) - \bar{\rho}]$, and integrating each term separately, (6) becomes

$$(\Delta t / 2k_B) \langle V^2 \rangle = \bar{T} \bar{R} + \overline{L(T - \bar{T})(\rho - \bar{\rho})}, \quad (7)$$

where R is the resistance of the wire.

Monnet, Ciliberto and Bellon (MCB) [19] measured the variance of the voltage between the ends of a wire with nonuniform temperature. In the MCB experiment, the wire was made of a metallic alloy, ρ increased linearly with temperature, implying $\overline{(T - \bar{T})(\rho - \bar{\rho})} > 0$, and therefore the variance of the voltage was expected to be greater than $2k_B R \bar{T} / \Delta t$ or, translated into MCB notation, the result expected from this extended NS was $T^{\text{fluc}} > T^{\text{avg}}$. However, as observed for the larger temperature differences in Fig. 7 of MCB, they found that $T^{\text{fluc}} < T^{\text{avg}}$. $T^{\text{avg}} - T^{\text{fluc}}$ is just about twice the estimated experimental error, but remains positive throughout the region where the experiment was most sensitive to this difference. A possible explanation could be a negative correlation between different segments, but measurements are available for the entire wire only, and not for segments of it.

Another explanation is that $\overline{(T - \bar{T})(\rho - \bar{\rho})}$ was small (translates into about three times the estimated error in T^{avg} or T^{fluc}) and the measurement was unable to detect it. We suggest an alternative explanation: the Johnson noise is nonlocal, and $T(x)$ has to be replaced with a smeared value. Smeared values are closer than the local values to the longitudinal averages, and therefore the effective value of $\overline{(T - \bar{T})(\rho - \bar{\rho})}$ was hindered. The configuration considered by MCB, in which cold and hot short segments alternated, could have facilitated this smearing effect.

In the MCB experiment, $|T - \bar{T}| \lesssim 0.13 \bar{T}$ and $|\rho - \bar{\rho}| \lesssim 0.2 \bar{\rho}$, and therefore $\overline{L(T - \bar{T})(\rho - \bar{\rho})}$ is smaller than $\bar{T} \bar{R}$ by two orders of magnitude. $\overline{(T - \bar{T})(\rho - \bar{\rho})}$ would increase quadratically with the difference in temperature between the hot and the cold reservoir. $|\rho - \bar{\rho}|$ would also be significantly larger for an intrinsic semiconductor with energy gap of the order of $k_B \bar{T}$ or for a conductor with very nonuniform cross section.

An additional experiment that could have tested the locality of Johnson noise was reported in [20,21]. Two resistors held at different temperatures were connected through a capacitor. Indeed, the variance of the voltage in the hot (cold) resistor was smaller (larger) than predicted by Eq. (1) (as follows from Eq. (16) in Ref. [21]). However, this result does not necessarily contradict the NS, because the current along the resistors did not vanish; in this case Eq. (1) should not be applied to the voltages, but rather to the electromotive forces (emf), that were not evaluated.

Further experimental motivation is provided by the successful use of the Sukhorukov-Loss results, to be described in the following, to a wide range of measurements.

C. Multiterminal conductors

These are conductors with an arbitrary number of terminals, at which the temperature and either the current or the electric potential are fixed, whereas the rest of their surface is impermeable to charges and heat. Sukhorukov and Loss (SL) [22] treated this system following a Boltzmann-Langevin approach. Their starting equation (2.1) accounts for the interaction between electrons as a local scattering agent, whereas the long-range influence of the Coulomb interaction is not considered. SL predicted that the strength of the noise at a given terminal depends on the *temperatures at the other terminals*, as expressed by their Eqs. (3.16) and (6.2) (where Π plays the role of a local effective temperature).

In the case of junctions (that is, circuits with a place where the temperature jumps and with negligible resistance in the rest of the circuit), it is found [23,24] that the variance of the current can be described by a term like (2), with T replaced by the average of the temperatures at both sides of the junction, and an additional term that increases with the temperature jump; other possibilities are analyzed in Ref. [25] and references therein.

In the hot-electrons limit (negligible electron-phonon coupling), the SL results have been extended in several aspects. The influences of Joule heating and heat injection are considered in Ref. [26]. The case of a Dirac fluid, for which the ratio of thermal to electric conductivity (Lorenz ratio) is much larger than predicted by the Wiedemann-Franz (WF) law, as observed in Ref. [27] near the charge-neutrality point, requires a hydrodynamic description. A hydrodynamic treatment, away from charge neutrality, was undertaken in Ref. [28]; in this case Ohm's law is nonlocal and the Lorenz ratio can be smaller than predicted by the WF law. Talanov *et al.* [29] indeed observed smaller than WF Lorenz ratio, and also negative thermal magnetoresistance, arising from viscous rather than local Joule heating, in a Corbino disk geometry; the analysis presented in Ref. [30] explains this anomalous behavior of the magnetoresistance.

The nature of "nonlocality" considered in the works mentioned above, which is generated by electronic heat flow within the conductor, differs essentially from that considered in the current study, in which the temperature at every point in the conductors will be imposed by tight thermal contact with heat baths along most of their lengths. Moreover, whereas the aforementioned works relate to systems in which electron interactions with the lattice are marginal, such interactions are here a key mechanism [31]. To the best of our knowledge, setups like those in Sec. II A or Sec. II B have not been studied within the SL framework.

III. SIMULATIONS

Since the experimental evidence is scarce, we would like to reinforce it by means of model simulations. For our purposes, a "resistor" is an object that obeys Ohm's law. If it obeys Eqs. (1) and (2) for uniform temperature, but does not obey Eq. (6) for nonuniform temperature, we can regard these results as evidence supporting the nonlocal scenario.

A. Choice of the model

We would like to mimic the typical features of a conducting wire: charge carriers are free to move but undergo collisions against defects and phonons that lead to their thermalization; these carriers repel each other, thus becoming spread with a nearly uniform density; the repulsion force enables signal propagation at a speed that is faster than the mean carrier speed; the medium in which the carriers move screens long-range interactions. In addition to these features, we would like the model to be as tractable as possible.

We model a resistor as a one-dimensional system of N classical particles, each with charge q and mass m , in a wire in the region $0 \leq x \leq L$. We define a sort of screening length ℓ such that the particles repel each other for distances r smaller than ℓ and do not interact for $r > \ell$. For $r < \ell$, the size of the interparticle force is taken as $k_c(\ell - r)$, where k_c is a positive constant. Each particle may also feel a force due to an applied voltage.

We define a free flight time τ such that for times that are integer multiples of τ every particle "collides" and the velocity of each particle acquires a random value, with zero average, variance $k_B T/m$, and normal distribution. Between collisions, the motion of the particles is dictated by the forces that act on them.

For this modified Drude model [32] (the distribution of times between consecutive collisions is sharp), the dc resistance is

$$R_D = 2mL^2/q^2N\tau. \quad (8)$$

For $k_c\ell^2 \ll k_B T$ or $N\ell \ll L$, we expect that the system will behave qualitatively as a gas (collisions act as a heat bath, the interaction between particles has little influence, and particles can pass through each other); in the opposite case, we expect that the system will behave as an incompressible fluid and there will be fast energy transfer among the particles. Here we will focus on the case $k_c\ell^2 \gg k_B T$.

In addition to the bulk properties of the model, we should stipulate the behavior at the ends of the wire. This will be done below for each particular situation.

B. Definitions and statistical properties

An inherent difficulty in the description of electrical variables in a circuit by means of numerical study of the particles' motion is that the number of particles will be far from macroscopic, and therefore unable to reproduce the underlying assumption that current is uniform. It is therefore necessary to specify what we mean by current, voltage, and so on.

1. Current

We define the instantaneous current in a resistor as its spatial average along the resistor. This gives

$$\tilde{I} := (q/L) \sum v_i, \quad (9)$$

where v_i is the velocity of particle i and the sum is over all the particles in the resistor. The measured current I is obtained by averaging $\tilde{I}$ over a lapse of time Δt . We note that (imagining the resistor as straight) the current equals the total momentum of the particles multiplied by q/mL .

Let us now consider a closed system of particles, as in the case that the resistor is a closed loop. Since there are no external forces, the current remains constant between collisions. Since by model assumptions $\langle v_i^2 \rangle = k_B T / m$ and the velocities of the particles are uncorrelated, for $\Delta t = \tau$ we obtain $\langle I^2 \rangle = N q^2 k_B T / m L^2$ and, using (8), $\langle I^2 \rangle = 2 k_B T / R_D \tau$. Finally, for measurement durations Δt that contain several collision cycles, $\langle I^2 \rangle = 2 k_B T / R_D \Delta t$, in agreement with (2).

2. Voltage

If the wire were perfectly homogeneous (density of particles N/L , independent of x), the electrical potential at $x = 0$ relative to that at $x = L$ would be $(L/qN) \int_0^L f(x) dx$, where $f(x) dx$ is the force exerted on all the particles in a segment of length dx . For a limited number of particles, in order to push the current, the electric field at a position without particles is irrelevant and only the actual forces on the particles count, so the integral expression should be replaced with $(L/qN) \sum F_i$, where F_i is the force exerted on particle i and the sum applies to all the particles. Since the internal forces cancel out, only the external forces F^{ext} have to be taken into account. Finally, we note that the influence of a force on the time-averaged current is proportional to the time left until the next collision. In summary, we write

$$V = (1/\Delta t) \int_0^{\Delta t} \tilde{V}(t) dt, \quad (10)$$

with

$$\tilde{V}(t) := (L/qN)(2 - 2t'/\tau) \sum F^{\text{ext}}, \quad (11)$$

where $t' = t - \lfloor t/\tau \rfloor \tau$ is the time elapsed since the latest collision.

We note that the present definition of voltage deviates from the usual definition as the difference of electrical potentials. In particular, the voltage drop along two segments in series is guaranteed to equal the sum of voltage drops only in the limit that both segments have the same number of particles per length. Although the deviation from the electrical potential may appear disturbing, we should bear in mind that the quantity that voltmeters measure is not the electrical but rather the electrochemical potential, which specifies the capability to drive current.

3. Units

Lengths, times, voltages, currents, temperatures, and k_c will, respectively, be taken in units of ℓ , τ , $m\ell^2/q\tau^2$, q/τ , $m\ell^2/k_B\tau^2$ and m/τ^2 ; in cases where writing these units explicitly would be cumbersome, they will be omitted.

4. Segments

For any segment PQ , we will denote by V_{PQ} the voltage at P relative to Q , by L_{PQ} the length of the segment and, in general, will use the subscript PQ to indicate that a quantity refers to the segment from P to Q . Neither the voltage nor the current can be sharply fixed in a segment, and we will study the emf $\varepsilon_{PQ} = V_{PQ} + I_{PQ} R_{PQ}$.

Since particles can enter or exit the segment, the number of particles N_{PQ} , and therefore the resistance R_D are not

fixed. When current is driven, fluctuations in the resistance are known to lead to flicker (1/f) noise [33,34], but in our case, no current will be driven.

In the absence of a theory, we call upon a dimensional argument. From Eqs. (8), (9), and (11), we note that the variances of $IR_D^{1/2}$ and VR_D^{-1} are independent of N . Motivated by this feature, we define the normalized emf

$$\hat{\varepsilon} := (1/\Delta t) \int_0^{\Delta t} [\tilde{V}(t) R^{-1}(t) R^{1/2}(\langle N \rangle) + \tilde{I}(t) R^{1/2}(t)] dt, \quad (12)$$

where all the quantities refer to the same segment and $R(t)$ is obtained by setting in (8) the number of particles in the segment at time t , whereas $R(\langle N \rangle)$ is obtained by setting the average number of particles in the segment. If N_{PQ} is constant, the Nyquist result is

$$\langle \hat{\varepsilon}_{PQ}^2 \rangle \Delta t = 2 k_B T. \quad (13)$$

In the case $\tilde{V} = 0$, this result also follows from the argument provided in Sec. III B 1. However, Eq. (13) is not valid in general. We will stay on the safe side by taking sufficiently small temperature and sufficiently long segments, so that the diffusion of particles into or out of the segment during the duration τ is much smaller than the number of particles that remain inside the segment.

C. Results

In our calculations, we focused on force coefficients in the range $25 \leq k_c \leq 50$ and followed the motion of the particles between collisions with a leapfrog algorithm with time steps of size 0.01τ , much shorter than $\sqrt{m/k_c}$.

1. Test of resistor behavior

We considered a system of particles in a loop and external forces were applied to them, as would be the case if a time-dependent magnetic flux pierced the loop.

Ohm's law. We followed the motion of $N = 20$ particles, took the values $k_c = 50m/\tau^2$ and $\Delta t = 10^5 \tau$, and the ranges $V \leq 1m\ell^2/q\tau^2$, $k_B T \leq 10^3 m\ell^2/\tau^2$, and $3\ell \leq L \leq 40\ell$. In most cases, the same external force was applied to every particle and remained constant over time, but we also considered the case in which the external force was applied to a single particle and the case in which the force was on for a time $\tau/2$ after each collision and off during the following half of the time.

In all the cases that we examined, we obtained $V - R_D I \approx 0$, where the deviation from zero was of the order expected from Eq. (1).

Electroneutrality. We tested electroneutrality on a loop of length $L = 10\ell$, conceptually divided into two halves. Denoting by N_- the number of particles in the region $0 < x \leq L/2$, the deviation from electroneutrality can be characterized by the variance $\langle (N_- - N/2)^2 \rangle$. We evaluated this variance for several values of N , T and k_c . In the limit $k_c \ell^2 / k_B T \rightarrow 0$, the system becomes a gas of non-interacting particles and $\langle (N_- - N/2)^2 \rangle = 2^{-N} N! \sum_{i=0}^N (i - N/2)^2 / i! (N - i)! = N/4$. In the opposite limit, $k_B T / k_c \ell^2 \rightarrow 0$, the system becomes a symmetric rigid structure (provided that $N\ell > L$), leading to

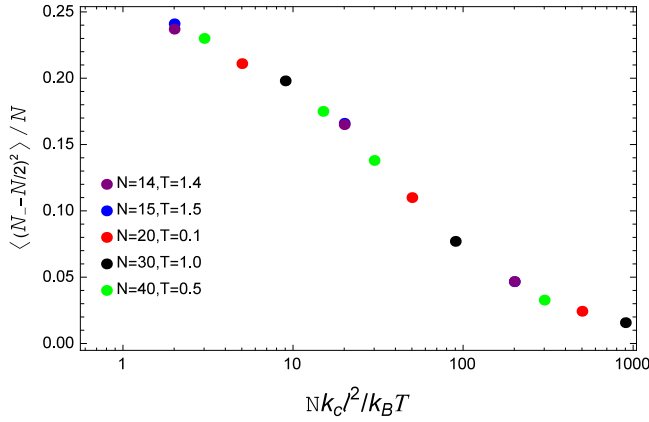


FIG. 3. Expected deviations from electroneutrality in a closed circuit, for several parameters. In all cases, $k_c = 50m/\tau^2$, $L = 10\ell$, and there was a constant emf applied by an external source, equal to $0.5m\ell^2/q\tau^2$.

$\langle (N_- - N/2)^2 \rangle = 0$ for even N and to $\langle (N_- - N/2)^2 \rangle = 1/4$ for odd N . Our results are shown in Fig. 3. Our results suggest that, for all the cases considered, $\langle (N_- - N/2)^2 \rangle / N$ is approximately given by the same function of $Nk_c\ell^2/k_BT$.

2. Noise at uniform temperature

We simulated “measurements” during periods of time $\Delta t = 50\tau$ in the absence of applied voltage. In each measurement, the current, the voltage, and the emf were evaluated as defined in Sec. III B. To evaluate the variance, measurements were repeated 10^5 times. Before the measurements, the system typically evolved for $2 \times 10^5\tau$ to ensure stabilization, and idle times of 10τ were left between consecutive measurements to avoid correlation. For the parameters we used, the influence of the factor $2 - 2t'/\tau$ in definition (11) and that of $R^{-1}(t)R^{1/2}(\langle N \rangle)$ rather than just $R^{-1/2}(t)$ in definition (12) was marginal.

Closed circuit. We studied a loop with $N = 50$ particles and length $L = 40\ell$ for the cases $k_c = 25$ and $k_c = 50$, and evaluated the variance of the normalized emf in the entire loop, and also in the segments described by the inset in Fig. 4. Figure 4 shows these variances as functions of the temperature.

We see that for $k_BT \lesssim 1.5m\ell^2/\tau^2$ our results are in good agreement with Eq. (3), but for the smaller value of k_c and $k_BT \gtrsim 1.5m\ell^2/\tau^2$, there is a systematic deviation, that increases as the length of the segment decreases. We attribute the deviation from Eq. (3) for small values of $Nk_c\ell^2/k_BT$ to the deviation from electroneutrality, that is a built-in assumption in the Nyquist scenario.

Open circuit. We open the circuit considered above by cutting the wire at the point O and creating two ends, O_1 and O_2 , as shown in the inset of Fig. 5.

Some force is required at the ends to counter the repulsion from the internal particles and thus prevent the escape of particles from the wire. With the purpose of having similar environments for those particles close to an end and those in the bulk, we impose that for every particle at position x there are mirror charges at $-x$ and $2L - x$, and these charges exert the same repulsion force as a real particle. If, despite this

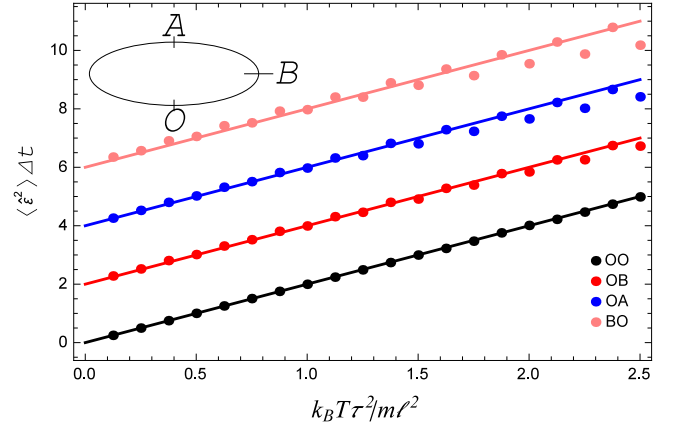


FIG. 4. Variances of the normalized emf [as defined in Eq. (12)] in a closed circuit, as functions of the temperature. As shown in the inset, OO (OB , OA , BO) has length L ($3L/4$, $L/2$, $L/4$), where $L = 40\ell$. The points for which the temperature is an odd (even) multiple of 0.125 were obtained setting $k_c = 50m/\tau^2$ ($k_c = 25m/\tau^2$); for other parameters, see text. For visibility, the results for different segments have been shifted upwards in steps of 2 units. The straight lines show the values predicted by Eq. (3). Since all the points in the circuit are equivalent, $\langle \varepsilon \rangle = 0$ for every segment.

force, a particle reaches an end, we assume that it bounces elastically.

Our results are shown in Fig. 5. Again, there is good agreement with Eq. (3) for $k_BT < 1.5m\ell^2/\tau^2$.

3. Circuits with resistors at different temperatures

In the case of an open circuit with a limited number of particles and $T = 0$, the particles assume equilibrium positions and form a lattice. For $0 < k_BT \ll k_c\ell^2$, there is still a lattice and the particles vibrate about their equilibrium positions, so that space is not homogeneous. As a consequence, the results are expected to depend on the particular places where the

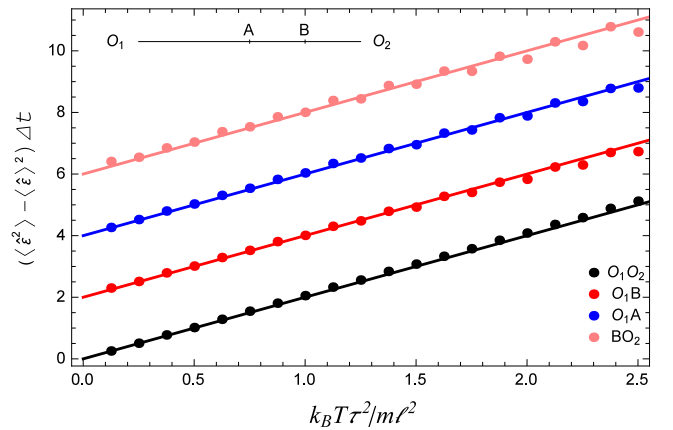


FIG. 5. Variances of the normalized emf [as defined in Eq. (12)] in an open circuit, as functions of the temperature. As shown in the inset, O_1O_2 (O_1B , O_1A , BO_2) has length L ($3L/4$, $L/2$, $L/4$), where $L = 40\ell$. All the parameters are the same as in Fig. 4. For visibility, the results for different segments have been shifted upwards in steps of 2 units. The straight lines show the values predicted by Eq. (3).

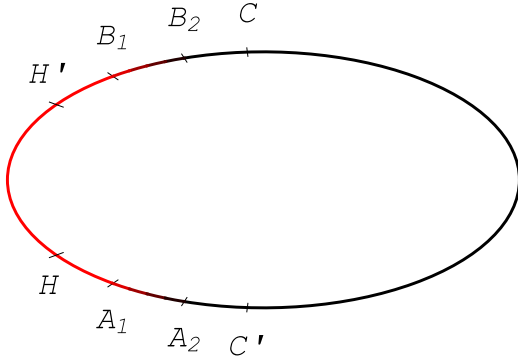


FIG. 6. Closed circuit with nonuniform temperature. Segment A_1B_1 (B_2A_2) has temperature T_1 (T_2); in A_2A_1 and B_1B_2 , the temperature changes gradually.

temperature changes and the particular locations of the ends of any considered segment. Inhomogeneity of space may become negligible for sufficiently long segments; this question will be postponed for a future study, and here we focus on closed circuits.

The circuit considered is sketched in Fig. 6. Region 1, from A_1 to B_1 , is held at temperature T_1 , and region 2, from B_2 to A_2 , is held at temperature T_2 . Along segments A_2A_1 and B_1B_2 , the temperature changes gradually; for numerical convenience, we take $T^{1/2}$ as a linear function of position. Segment HH' (CC') has temperature T_1 (T_2).

Our results are reported in Table I. Unless stated otherwise, we took $T_1 = 1$, $T_2 = 0.25$, $\Delta t = 50$, $N = 50$, $L_{A_2A_1} = L_{B_1B_2} = 2$, $L_{A_1H} = L_{H'B_1} = L_{B_2C} = L_{C'A_2} = 1$, $L_{HH'} = 10$, and $L_{CC'} = 22$, so that the sum of the segment lengths is $L = 40$. We emphasize that segments HH' and CC' have uniform temperatures. By symmetry, $\langle \varepsilon_{HH'} \rangle = \langle \varepsilon_{CC'} \rangle = \langle V_{HH'} \rangle = \langle V_{CC'} \rangle = 0$ and $\langle \varepsilon_{C'H}^2 \rangle - \langle \varepsilon_{C'H} \rangle^2 = \langle \varepsilon_{H'C}^2 \rangle - \langle \varepsilon_{H'C} \rangle^2$.

Nyquist scenario (NS). The variances of $\varepsilon_{HH'}$ and $\varepsilon_{CC'}$ were evaluated using Eq. (3), and those of $\varepsilon_{H'C}$ and $\varepsilon_{C'H}$, using Eq. (6). The variance of V_{PQ} is obtained noting that $V_{PQ} = \varepsilon_{PQ} - IR_{PQ} = (\varepsilon_{PQ} \sum' R_{P'Q'} - R_{PQ} \sum' \varepsilon_{P'Q'})/R$, where $\sum'$ denotes sum over all the segments except PQ and R is the

resistance of the entire circuit. From here, Eq. (3), and the assumption that the emf in different segments are uncorrelated,

$$\frac{S(V_{PQ})}{R_{PQ}} = \frac{S(\varepsilon_{PQ})(\sum' R_{P'Q'})^2/R_{PQ} + R_{PQ} \sum' S(\varepsilon_{P'Q'})}{R^2} \quad (14)$$

where $S(X)$ is the variance of X . For our evaluations in the NS we have assumed that the resistivity is uniform.

We note that Eq. (14) applies to a closed circuit; in an open circuit, $I \approx 0$ and $V_{PQ} \approx \varepsilon_{PQ}$. We also note that for a uniform wire in the limit $R_{C'H,H'C} \ll R_{HH',CC'}$, $S(V_{HH'})R_{CC'}/S(V_{CC'})R_{HH'} = L_{CC'}/L_{HH'}$, so that in this limit Eq. (14) coincides with Eq. (B5).

Limit $k_c \rightarrow \infty$. This is the case of a rigid structure analyzed in Appendix B. In order to have values comparable with those of our simulations, while complying with the assumptions of Appendix B, we took $L_{C'H} = L_{H'C} = 0$ and $N_1/L_{HH'} = N_2/L_{CC'} = \text{integer}$ (as required by the assumptions in Appendix B), and $L_{CC'}/L_{HH'} = 2.2$ (as the ratio in the most similar cases in the simulation). The variances of ε_{PQ} follow from Eq. (B4), and those of V_{PQ} , from Eq. (B3). Note that counterintuitively, in this extreme case the variance of the emf in the colder segment can be larger than that in the hot segment.

Large k_c . This and the following rows were obtained from our simulations. This case is intended to imitate the situation studied in Appendix B. We therefore require $L_{H'C} = L_{C'H} = 0$ (which implies a discontinuous passage between T_1 and T_2) and integer value of $N_1/L_{HH'} = N_2/L_{CC'}$. We took $N = N_1 + N_2 = 48$, $L_{HH'} = 7.5$ and $L_{CC'} = 16.5$, so that $N_1/L_{HH'} = N_2/L_{CC'} = 2$. As a large value of k_c , we took $k_c = 100$ and, accordingly, small evolution steps every $\tau/150$.

Cases $k_c = 50$ and $k_c = 25$. We see that the variances $S(\varepsilon_{HH'})$ and $S(\varepsilon_{CC'})$ strongly differ from the NS, and assume intermediate values between the NS and the rigid structure case. Remarkably, the normalized variance is larger in the cold region than in the hot region, as in the strong-interaction limit.

Cases $N = 100, 200, 400$. As seen above, the value $k_c = 25$ leads to a very rigid structure. We would like to study

TABLE I. Variances of the emf in some of the segments shown in Fig. 6. The first two rows are the predictions of two competing models and the following rows are simulation results. NS stands for the Nyquist scenario and the case $k_c \rightarrow \infty$ is studied in Appendix B. Since the number of particles in each segment is not fixed, the variances were evaluated using (12). For additional parameters and details, see Sec. III C 3.

Case	$\frac{\langle \varepsilon_{HH'}^2 \rangle \Delta t}{R_{HH'}}$	$\frac{\langle \varepsilon_{CC'}^2 \rangle \Delta t}{R_{CC'}}$	$\frac{(\langle \varepsilon_{C'H}^2 \rangle - \langle \varepsilon_{C'H} \rangle^2) \Delta t}{R_{C'H}}$	$\frac{\langle V_{HH'}^2 \rangle R_{CC'}}{\langle V_{CC'}^2 \rangle R_{HH'}}$	$\frac{N_{HH'} L_{CC'}}{N_{CC'} L_{HH'}}$
NS	2.00	0.50	1.21	2.46	
$k_c \rightarrow \infty, L_2/L_1 = 2.2$	0.71	1.79		2.20	1
Large $k_c, L_2/L_1 = 2.2$	1.00	1.82		2.23	0.99
$k_c = 50, N = 50$	1.25	1.52	1.32	2.66	0.99
$k_c = 25, N = 50$	1.12	1.46	1.24	2.50	0.96
$k_c = 25, N = 100$	1.13	1.22	1.29	2.40	0.96
$k_c = 25, N = 200$	1.42	0.87	1.30	2.39	0.95
$k_c = 25, N = 400$	1.70	0.67	1.24	2.35	0.95
$k_c = 10, N = 200$	1.64	0.66	1.00	2.19	0.84
$\rho_1 \neq \rho_2, k_c = 25$	1.11	1.53		2.47	0.96
Perfect, $k_c = 25$	0.98	1.55		2.38	0.96
Perfect, $k_c = 10$	1.64	0.66		2.02	0.85

softer structures, but taking k_c below 25 would confine the validity of our simulations to lower temperatures. Instead, we keep $k_c = 25$ and take longer circuits. $L_{H'C}$ and $L_{C'H}$ remain unchanged, but for $N = 100$ (200, 400) $L_{HH'} = 23$ (49, 101), $L_{CC'} = 49$ (103, 211), so that $L_{CC'}/L_{HH'} \approx 2.1$ and $L = 0.8N$. Indeed, as the length of the circuit increases, $S(\varepsilon_{HH'})$ and $S(\varepsilon_{CC'})$ become closer to the NS values. We consistently find that $S(\varepsilon_{HH'}) [S(\varepsilon_{CC'})]$ is smaller [larger] than predicted by the NS, and, for $N \gtrsim 100$, the deviations from the NS values are roughly inversely proportional to N . Our results suggest that the presence of a resistor at a different temperature may be felt at distances (from center to center) that exceed the screening length by three orders of magnitude.

For comparison, we sometimes repeated our simulations setting $T_2 = T_1 = 1$. For example, in the case $N = 200$, we obtained $S(\hat{\varepsilon}_{HH'})\Delta t = 1.91$ and $S(\hat{\varepsilon}_{CC'})\Delta t = 1.95$; both results differ from the NS prediction by less than 5%. We also evaluated the variance of $V_{\text{total}} = V_{HH'} + V_{H'C} + V_{CC'} + V_{C'H}$, which ideally should vanish; $S(V_{\text{total}})\Delta t$ came out to be smaller than $Rk_B T_{1,2}$ by two orders of magnitude.

Case $k_c = 10$. For longer circuits, and accordingly longer $L_{HH'}$ and $L_{CC'}$, we can take smaller values of k_c . The lengths of the segments in this case were the same as in the case “ $k_c = 25$, $N = 200$ ”. Comparison of the rows “ $k_c = 25$, $N = 200$,” “ $k_c = 25$, $N = 400$,” and “ $k_c = 10$, $N = 200$ ” indicates that increasing the length of the circuit has an effect similar to that of decreasing the value of k_c .

Resistors made of different materials ($\rho_1 \neq \rho_2$). At uniform temperature, the variances of the emf depend on the resistances but do not depend separately on the resistivity and geometry of each resistor. As a test for the extension of this property to nonuniform temperature, we considered a circuit with two parts. Part i had resistance ρ_i per unit length and the central segment in it had temperature T_i . We took $\rho_2 = 2\rho_1$, $k_c = 25$, and the number of particles and lengths were chosen to produce resistances similar to the case “ $k_c = 25$, $N = 50$.” The resistivity $2\rho_1$ was achieved by means of collisions every lapse of time $\tau/2$ rather than τ . The gradient of $T^{1/2}$ was taken proportional to the resistivity and the temperature changes between T_1 and T_2 took place over the distance 1.5ℓ . To keep the same average density as in the case “ $k_c = 25$, $N = 50$,” we took $L_{A_1H} = L_{H'B_1} = 1$, $L_{B_2C} = L_{C'A_2} = 0.5$, $L_{HH'} = 10$, $L_{CC'} = 11.2$ and $N = 1.25L = 34$. The results we obtained, as shown in Table I, are similar to those of case “ $k_c = 25$, $N = 50$ ”. The voltages and emf in segments $H'C$ and $C'H$, where the wire is not uniform, are not evaluated.

Resistors connected by perfect conductors (“perfect”). This situation is simulated by replacing the regions with temperature gradient considered above with regions where there are no collisions (and therefore the lattice temperature in these regions is irrelevant). We considered two cases: one almost identical to the case “ $k_c = 25$, $N = 50$ ” and the other almost identical to the case “ $k_c = 10$, $N = 200$ ”; the only difference from the above cases is that segments A_2A_1 and B_1B_2 became collisionless. We note that in the present cases there is a slight tendency (only for the shorter circuit) for a larger departure from the NS values than in the corresponding former cases, suggesting that the effective interaction between the hot and the cold regions is transmitted more freely along a perfect conductor than along a resistive wire.

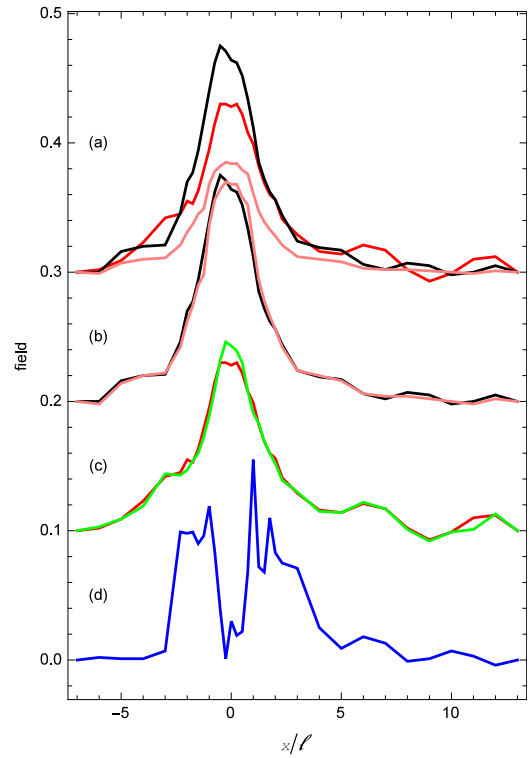


FIG. 7. Analogue of the electric field induced by the nonuniform temperature, in a half of a symmetric closed circuit. The origin $x = 0$ was taken at the middle of A_2A_1 , so that the temperature is uniform for $|x| > \ell$. For visibility, (a) [(b) and (c)] was raised by 0.3 [0.2 and 0.1] units. Red: $k_c = 50$, $T_2 = 0.25$; black: $k_c = 25$, $T_2 = 0.25$; pink: $k_c = 25$, $T_2 = 0.625$; green: $k_c = 25$, $T_2 = 0.25$, smooth temperature; blue: $k_c = 25$, $T_2 = 0.25$, resistors joined by perfect conductors. See text for additional parameters.

4. Influence of the temperature gradient

Heat flow. A temperature gradient causes heat flow, and Sukhorukov and Loss [22] found that heat flow can be a source of electric noise. Their result relates the noise at a given terminal to the thermal currents through all the terminals, but we may expect that the larger the local thermal current, the stronger its influence in its vicinity. As a check for this prediction, we evaluated the variances of $\varepsilon_{C'H}$ and $\varepsilon_{H'C}$, but the short length over which the temperature changes makes it hard to draw any conclusion.

Seebeck effect [35]. Since diffusivity increases with temperature, a force from lower to higher temperature is required to maintain a steady state. We define “field” at position x (in analogy to electric field) as the time-average of the force applied on the particles in the vicinity of x , divided by q . This field was evaluated with our simulations by letting them run during $1.4 \times 10^6 \tau$ for stabilization, and then taking the average during $5.6 \times 10^6 \tau$. We studied the circuits with $N = 50$ described in Sec. III C 3 and took $T_1 = 1$. In Fig. 7, we present the field as a function of x in half of the circuit, between the middles of HH' and CC' . The origin $x = 0$ was taken at the middle of A_2A_1 , leading to the range $-7 \leq x \leq 13$. By symmetry of the circuit, the field at $26 - x$ equals minus the field at x .

The case $k_c = 25$ ($k_c = 50$) and $T_2 = 0.25$ is depicted by the black (red) line in panel (a) of Fig. 7. The pink line corresponds to $k_c = 25$ and $T_2 = 5/8$, i.e., the value of $T_1 - T_2$ is half that of the other lines. We note that the Seebeck field is present not only in the region $|x| < \ell$ in which the temperature is not uniform, but extends several screening lengths beyond A_2A_1 . We also note that despite the larger interparticle force in case $k_c = 50$, it results in fields smaller than in case $k_c = 25$; the reason is that k_c imposes a more uniform density of particles.

From the Seebeck effect, we expect that the integral of the field be proportional to $T_1 - T_2$. Panel (b) of Fig. 7 shows the line for $k_c = 25$ and $T_2 = 2/8$ (black) together with the line for $k_c = 25$ and $T_2 = 5/8$ multiplied by a factor 2 (pink). These lines almost coalesce, indicating that the proportionality holds not only for the integrals, but locally for the fields.

The jaggedness of the lines is due to the discontinuity of the temperature gradient at $x = \pm\ell$. In panel (c) of Fig. 7, the red line stands for our usual case $k_c = 50$, $T_2 = 0.25$, whereas the green line corresponds to the same values, but in the regions A_2A_1 and B_1B_2 , $T^{1/2}$ does not depend linearly on position, but rather as a cubic polynomial such that dT/dx vanishes at $x = \pm\ell$. The remaining jaggedness of the green line is mainly due to the relatively large distance between consecutive evaluation points.

A smoother function $T(x)$ leads to a smoother dependence of the field on position, but does not suppress the oscillations that are conspicuous near the middle of the cold segment. We conjecture that these oscillations are due to a standing acoustic wave, that is excited by the hot region. Support for this conjecture is provided by the fact that the relative size of these oscillations decreases when $T_1 - T_2$ becomes smaller (more precisely, the size of the oscillations decreases by a larger factor than the spatial average of the field). An example of this behavior can be observed in panel (b) of Fig. 7.

Panel (d) of Fig. 7 takes again $k_c = 25$ and $T_2 = 0.25$, but this time A_2A_1 and B_1B_2 are perfect conductors. As might naively be expected, the field within a perfect conductor is small, and peaks at $|x| \geq \ell$.

5. Correlations

We denote the correlation between two random variables $X_{1,2}$ with zero average by $r(X_1, X_2) := \langle X_1 X_2 \rangle / \sqrt{S(X_1)S(X_2)}$; for consistency, X_1 and X_2 are understood as time averages of the corresponding physical quantities during common lapses in time of size Δt . We consider a closed circuit as in Fig. 6. In the following of this section, quantities that refer to segment HH' (CC'), which are kept at temperature T_1 (T_2), will be marked with the index 1 (2).

The usual circuit theory predicts several relations. Electroneutrality implies $r(I_1, I_2) = 1$. The NS predicts $r(\varepsilon_1, \varepsilon_2) = 0$ and, for $T_1 = T_2$, thermodynamics leads to $r(V_1, I_1) = r(V_2, I_2) = 0$. If segments $H'C$ and $C'H$ are perfect conductors or have vanishing length, then $r(V_1, V_2) = -1$.

We have investigated to what extent our simulations comply with the above predictions. Taking $N = 50$, $L_1 = 10$, $L_2 = 22$, $L_{C'H} = L_{H'C} = 4$, $L_{A_2A_1} = L_{B_1B_2} = 2$ with A_2A_1 and B_1B_2 perfect conductors, fixing the average temperature $(T_1 +$

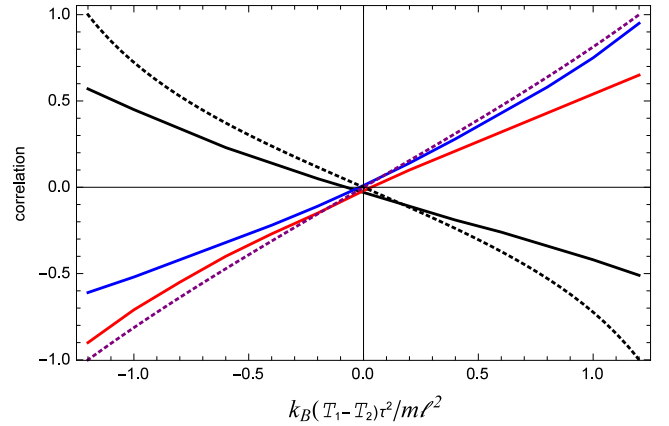


FIG. 8. Correlations between the emf's in two detached segments and between the voltage and the current in the same segment, as functions of the temperature difference. The dotted line corresponds to the strong-interaction limit and the solid line to $k_c = 50$. Black: $r(\varepsilon_1, \varepsilon_2)$; red: $-r(V_1, I_1)$; blue: $r(V_2, I_2)$; purple: Eq. (B7). The average temperature is $(T_1 + T_2)/2 = 0.6$. For additional parameters see Sec. III C 5. We attribute the deviations of $r(V_1, I_1)$ and $r(V_2, I_2)$ from 0 for $T_1 = T_2$ to inaccuracy of our algorithm.

$T_2)/2 = 0.6$, and varying the temperature difference $T_1 - T_2$, we obtained the results as follows. $r(I_1, I_2)$ varies between 0.93 at $T_1 - T_2 = -1.2$ and 0.84 at $T_1 - T_2 = 1.2$; the deviation from 1 is expected because electroneutrality is not perfect. $r(V_1, V_2)$ varies between -0.64 at $T_1 - T_2 = -1.2$ and -0.78 at $T_1 - T_2 = 1.2$; the deviation from -1 is expected because segments $H'C$ and $C'H$ have resistance. The differences between the cases $T_1 - T_2 = -1.2$ and $T_1 - T_2 = 1.2$ are expected because HH' and CC' have different lengths.

$r(\varepsilon_1, \varepsilon_2)$ and $r(V_i, I_i)$ are shown in Fig. 8. The solid lines correspond to the present case and the dotted lines to the strong-interaction limit (Appendix B). $r(\varepsilon_1, \varepsilon_2)$ is a nearly odd function of the temperature difference and behaves qualitatively as in the strong-interaction limit. The fact that $r(\varepsilon_1, \varepsilon_2) \neq 0$, even for detached segments, invalidates a central assumption on which Eq. (6) is based. Denoting by j the segment that is not i , $r(V_i, I_i)$ is positive (negative) if $T_i < T_j$ ($T_i > T_j$). This is expected because $\langle V_i I_i \rangle$ is the rate of work performed by segment j on the center of mass [36] of the particles in segment i , and the hotter segment has to deliver energy to the colder segment. $|r(V_i, I_i)| < |r(V_j, I_j)|$ for $T_i > T_j$ because for lower temperature the particles behave as a more rigid structure and a higher fraction of their kinetic energy can be ascribed to their center of mass; according to this argument, Eq. (B7) sets an upper bound on $|r(V_i, I_i)|$.

D. Nonstationary situations

Our model is insufficient for the study of situations in which temperatures vary in time, because not only the charges, but also the medium in which they are located, contribute to the internal energy and to heat transport.

In order to have a meaningful concept of temperature (even local temperature), the typical times for temperature changes have to be much longer than τ .

Since the time required by a circuit to approach thermal equilibrium does depend on the calorimetric properties of the medium, whereas the time required for propagation of an electric signal along the wire does not depend on these properties, a circuit can be engineered to allow the measurement proposed in Fig. 2. If the initial temperatures of the resistors in Fig. 2 are well defined and if the time required for charge equilibration is much shorter than the time required for heat equilibration, then immediately after charge equilibration, we should recover the results obtained for the stationary situation.

IV. DISCUSSION AND PERSPECTIVE

Since Johnson's measurement of thermal noise in a resistor and the immediate thermodynamic explanation by Nyquist, this subject has been regarded as closed. Even the case of nonuniform temperature, for which the thermodynamic argument does not apply, is sometimes taken into account assuming that the medium can be divided into pieces, each with a definite local temperature, thus extending the Nyquist formula to Eq. (6).

Unlike other cases of nonuniform temperature found in the literature (but like the experiments described in Sec. II B), in our case, nonuniformity does not arise from Joule heating or hot-electron injection but is already present in the heat baths. In this context, we emphasize that the temperatures $T(x)$ or $T_{1,2}$ considered above are local temperatures of the bath, and not necessarily the electronic temperature that builds up at a given position. We have considered resistors in the ohmic regime with charged particles that are frequently thermalized with the lattice, which is assumed to be in perfectly tight thermal contact with the local bath. Unlike Eq. (6), or Eq. (3.16) in Ref. [22], or Eq. (10) in Ref. [26], in our case, *the variance of the emf is not determined by the local temperatures inside the resistor* but depends also on the temperature beyond it. It might be conjectured that the dependence on the temperature beyond the resistor is taken into account by the heat flow, as described by, e.g., Eq. (27) in Ref. [26], although in our case heat current is not conserved but sinks everywhere in the resistor. It is worth noting that in our scenario energy transfer between resistors is not due only to the passage of particles, but also to the work that each particle performs on its neighbors.

Although the macroscopic behavior of the Johnson noise is well understood, we suggest that the microscopic mechanism for its emergence still requires investigation. Different microscopic mechanisms could be distinguished in circuits at nonuniform temperatures, in which the interparticle interaction could lead to spatial correlation and nonlocal temperature dependence.

The available experimental results hint at a nonlocal interplay between resistors in contact, which leads to deviations from Eq. (6), but these deviations are not substantial and do not manifest themselves unless the experimenter looks for them. We have suggested experiments in which these deviations would be enhanced.

Our simulations are intended to provide an additional source of motivation to look for the effects that we suggest. Besides the necessary requirements for providing an acceptable description of resistors, these simulations demonstrate several nontrivial phenomena. As the interaction strength k_c increases, we observe a gradual transition from Nyquist-like behavior to that of the strong-interaction limit, as obtained analytically in Appendix B. Under strong interactions (large k_c), there is a counterintuitive reversal between hot and cold regions in the size of the fluctuations of the emf. While in the Nyquist scenario the variance of the emf is proportional to the temperature, hence is higher in the high-temperature region, our simulations show that, for sufficiently strong interaction between the particles, the emf in the low-temperature region has higher variance than in the high-temperature region (see Table I). We also find correlation between the emf fluctuations in the two regions, the sign of which depends on their relative lengths: the correlation is positive when the high-temperature region is the longer one and negative when it is the shorter. The size of this correlation increases with the temperature difference (see Fig. 8). Explanations for these phenomena are provided by the analytical results in the strong-coupling limit (Appendix B).

The main limitation of our simulations is that they are strictly one-dimensional. In a 1D system of interacting fermions, a description by means of quasiparticles fails to provide an adequate qualitative picture, and a Luttinger liquid [37] with intrinsically nonlocal collective modes emerges instead. It is tempting to dismiss this objection because the particles in our model are classical objects, and due to the very wide scope of Nyquist's analysis this is not expected to be a limitation if $\Delta t \gg \hbar/k_B T$, or equivalently, the Luttinger liquid model describes the behavior of the system only when quantum effects dominate [38] over thermal fluctuations. Nevertheless, a mathematically 1D system of particles differs from a thin wire because [39] individual particles cannot move without pushing the other particles, so only collective excitations can exist. Our particles are not constrained by the Pauli exclusion principle, but since we took $k_c \ell^2 \gg k_B T$, they rarely bypass each other (in real rather than reciprocal space). Therefore even a qualitative extension of our results to higher dimensionality is not guaranteed; but the fact that at uniform temperature the 1D result was not anomalous (specifically in the limit $k_c \ell^2 \gg k_B T$), as well as the results reviewed in Sec. II, lead us to expect that this extension will hold. For $T_1 > T_2 = 0.25$, $\Delta t = 50\tau$, $k_c = 25$ and $N/L = 1.25$, the condition $\Delta t \gg \hbar/k_B T$ becomes $20/\tau \gg \hbar N^2/mL^2$ or, in a more transparent form, $\hbar\omega \gg 0.05E_F$, where $\omega = \sqrt{k_c/m}$ is the typical frequency of the dynamics of the particles and $E_F = \pi^2 \hbar^2 N^2/2mL^2$ is the Fermi energy (assuming spinless particles).

For larger dimensionality and Coulomb interaction, Pines and Bohm (PB) obtained for a gas of classical particles that collective modes will be present for wavelengths larger than the Debye length [40,41] (in the quantum case [42], the thermal energy is replaced by the plasmon energy). Since we assume $k_c \ell^2 \gg k_B T$ and $N\ell \sim L$, in an extension of our model to higher dimensionality we could expect an analog of the Debye length much shorter than ℓ , favoring collective modes; however, since the interaction we consider has no long range and each particle interacts with just

a few particles, and since there are frequent collisions with the lattice, the applicability of the PB result is unclear.

An additional limitation of our simulations is that, as any simulation, they are based on a limited number of particles, and therefore (1) the definitions of current and voltage, which are appropriate for a continuum of charge, require extension to the discrete case, and (2) charge variations in a segment, due to thermal inhomogeneity or fluctuations, can be a significant fraction of the charge in the segment.

Future projects could use the model introduced here to deal with systems such as (1) two-dimensional wires, in which the lateral walls could prevent escape of the particles by the same mechanisms assumed here at the ends of an open circuit, (2) wire meshes, that would enable the particles to bypass each other, (3) open circuits, in which the emf is more closely related to the voltage, which is directly measurable, (4) system with weak interparticle interaction, which could hopefully be treated analytically and provide a complementary limit to that obtained in Appendix B, or (5) wires in which the lattice has time-dependent temperature as a result of heat diffusion.

The results of Sec. III C 3 suggest that the presence of a resistor with different temperature could be felt at a distance along the circuit of the order of 10^3 times the screening length. For instance, if this result is applicable to strontium titanate [43,44] with dopant concentration of 10^{17}cm^{-3} , at temperatures in the range 25–100 K, this distance would be of the order of a tenth of a millimeter.

In summary, the available experimental and theoretical results that we have reviewed, as well as our simulation results, hint at a nonlocal interplay between resistors in electrical contact, altering the predictions of the Nyquist scenario. The conceptual significance of this result is that, unless measuring circuits offset nonlocal influence [14], corrections may be required in Johnson noise thermometry. The technological significance of the scenario conjectured here is the possibility of reducing thermal noise in a part of a conductor by lowering the temperature in another part of it.

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DATA AVAILABILITY

Fortran codes may be requested from the corresponding author.

APPENDIX A: JOHNSON NOISE IN THE TIME DOMAIN

For simplicity, let us consider a uniform resistor in the shape of a wire of length L and cross section A , shorted to enforce $V = 0$. Let the flowing charges be a gas of particles with charge q and volume density n , and let Δt be sufficiently larger than the average time between collisions, so that Brownian motion of the flowing charges can be described by the diffusion equation.

If a particle moves a short distance x along the wire, it will result on the average in the passage of a charge $\Delta Q_1 = qx/L$ across an arbitrary surface that cuts the wire. From the diffusion equation we know that $\langle x^2 \rangle = 2D\Delta t$, where D is the diffusion coefficient, and therefore the variance of the charge passed by one particle is

$$\langle (\Delta Q_1)^2 \rangle = 2q^2 D \Delta t / L^2. \quad (\text{A1})$$

There are nAL charged particles in the gas. Assuming that their motions are uncorrelated, the variance $\langle (\Delta Q)^2 \rangle$ of the total charge that passes the surface will be $nAL \langle (\Delta Q_1)^2 \rangle$, i.e.,

$$\langle (\Delta Q)^2 \rangle = 2nAq^2 D \Delta t / L. \quad (\text{A2})$$

Invoking the Einstein relation $D = k_B T \mu / q$ [1,45] where μ is the mobility, noting that $R = L / nq\mu A$, and dividing (A2) by $(\Delta t)^2$, we recover Eq. (2).

Equation (1) follows from (2) by noting that, if the current in the absence of voltage is $\eta_I \sqrt{2k_B T / R \Delta t}$, then the voltage required to cancel this current is $-\eta_I \sqrt{2k_B T R / \Delta t}$. More generally, the emf required to induce a current I is $RI - \eta_I \sqrt{2k_B T R / \Delta t}$, leading to Eq. (3).

On passing from (A1) to (A2) we assumed that the motions of the charged particles are uncorrelated. This condition may not be fulfilled, because charges cannot accumulate. Nevertheless, the validity of (A2) may be more general than the assumed requirement. Let us consider the opposite situation, in which the charged particles move in a stiff formation. This situation is actually equivalent to that of a single particle, in which both the charge and the mass are larger by a factor nAL than in the case of the gas. Since D is inversely proportional to the mass of the particle, (A2) remains unchanged.

For $\Delta t \gg \hbar / k_B T$, Eq. (1) follows also from the Nyquist expression in the frequency domain. In the notation of Ref. [45], the spectral density of the fluctuating voltage is $J_+(\omega) = 2k_B T R / \pi$ for any angular frequency $\omega \ll k_B T / \hbar$. Therefore the autocorrelation $\langle V(t)V(t+s) \rangle$ is

$$K(s) = (2k_B T R / \pi) \int_0^\Omega \cos \omega s d\omega = (2k_B T R / \pi s) \sin \Omega s, \quad (\text{A3})$$

where Ω is a cutoff.

The average voltage measured during a period of time Δt is $(1/\Delta t) \int_{-\Delta t/2}^{\Delta t/2} V(t) dt = (1/\Delta t) \int_{-\Delta t/2-t}^{\Delta t/2-t} V(t+s) ds$, so the ensemble average of its square is

$$\langle V^2 \rangle = \frac{1}{(\Delta t)^2} \left\langle \int_{-\Delta t/2}^{\Delta t/2} V(t) dt \int_{-\Delta t/2-t}^{\Delta t/2-t} V(t+s) ds \right\rangle \quad (\text{A4})$$

$$= \frac{1}{(\Delta t)^2} \int_{-\Delta t/2}^{\Delta t/2} dt \int_{-\Delta t/2-t}^{\Delta t/2-t} ds K(s). \quad (\text{A5})$$

Using (A3), evaluating the integrals, and taking the limit $\Omega \rightarrow \infty$, we recover Eq. (1).

APPENDIX B: STRONG-INTERACTION LIMIT

We study a loop of length $L = L_1 + L_2$, where the region with length L_i is kept at temperature T_i (with discontinuous temperature rather than regions of gradual variation). We assume that the particles are kept at fixed interparticle distances

L/N by rigid rods, and assume further that $N_i = NL_i/L$ are integer. This feature enables us to know exactly the number of particles in region i , namely, N_i . Every lapse of time τ , after absorbing the momentum from the particles, the lattice gives an impulse Σ_i to the particles in region i , with variance

$$\langle \Sigma_i^2 \rangle = N_i m k_B T_i. \quad (\text{B1})$$

The velocity acquired by the structure is $(1/Nm)(\Sigma_1 + \Sigma_2)$ and the current (the same in both regions) is $I = (q/Lm)(\Sigma_1 + \Sigma_2)$.

The momentum acquired by the particles in region 1 is $(N_1/N)(\Sigma_1 + \Sigma_2)$. Σ_1 was provided by the lattice in the region, and the remaining $(N_1/N)\Sigma_2 - (N_2/N)\Sigma_1$ is due to external forces. Using (11), integrating over the lapse of time τ , noting that the force is applied at the beginning of this lapse of time and that the density of particles is uniform, the average voltage in region 1 is $V_1 = (2L/qN^2\tau)(N_1\Sigma_2 - N_2\Sigma_1)$. We note that $V_2 = -V_1$ follows from Newton's third law.

With the expressions above for the current and the voltage, and with the resistance $R_1 = 2mL^2N_1/q^2N^2\tau$ obtained from (8), we can evaluate the average emf $\varepsilon_1 = V_1 + R_1I$ in region 1 during the lapse of time τ :

$$\varepsilon_1 = (2L/qN^2\tau)[(N_1 - N_2)\Sigma_1 + 2N_1\Sigma_2]. \quad (\text{B2})$$

Finally, using (B1), and for $\Delta t/\tau$ collision cycles, we obtain the variances

$$\langle V_1^2 \rangle = (2R_1N_2k_B/N^2\Delta t)(N_1T_2 + N_2T_1), \quad (\text{B3})$$

$$\langle \varepsilon_1^2 \rangle = (2R_1k_B/N^2\Delta t)[(N_1 - N_2)^2T_1 + 4N_1N_2T_2], \quad (\text{B4})$$

and the analogous relations hold for region 2. For $T_1 = T_2$, Eq. (B3) leads to (4). From Eq. (B3) we immediately obtain the ratio

$$\langle V_1^2 \rangle R_2 / \langle V_2^2 \rangle R_1 = N_2/N_1, \quad (\text{B5})$$

which is independent of the temperatures and reflects the fact that $V_2 = -V_1$. The most intriguing case in Eq. (B4) is that of $N_1 = N_2$, in which the Johnson noise does not depend on the local temperature, but rather on that of the neighboring region.

Noting that the correlation $r(\varepsilon_1, \varepsilon_2) = \langle \varepsilon_1 \varepsilon_2 \rangle / \sqrt{S(\varepsilon_1)S(\varepsilon_2)}$ does not depend on Δt , from (B2) and (B1), we also obtain

$$r(\varepsilon_1, \varepsilon_2) = \frac{\beta(T_1 - T_2)}{\sqrt{(\beta^2T_1 + T_2)(T_1 + \beta^2T_2)}}, \quad (\text{B6})$$

with $\beta = (N_1 - N_2)/2\sqrt{N_1N_2}$. This correlation may be either positive or negative, depending on whether the hotter segment is the longer or the shorter segment. For $T_1 = T_2$, we recover the NS.

Similarly, from the expressions for I and V_1 , we obtain

$$r(V_1, I) = \frac{T_2 - T_1}{\sqrt{(N_1T_1 + N_2T_2)(N_2T_1 + N_1T_2)/N_1N_2}}. \quad (\text{B7})$$

We note that V_1I is the power passed to region 1 by the forces that act from region 2; in the present limit, the heat transferred by the passage of particles vanishes. Note also that $r(\varepsilon_1, \varepsilon_2)$ and $r(V_1, I)$ are antisymmetric under the exchange of T_1 and T_2 , and their absolute values reach 1 when one of the temperatures vanishes.

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