

Magnetization induced by nonlinear response to temperature gradient in d' , g' , and i' altermagnets

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It is a highly nontrivial question whether magnetization is induced by a nonlinear response to a temperature gradient in systems where the linear response is forbidden by inversion symmetry. In this work, we address this issue and provide explicit examples demonstrating that such a response can indeed arise. The spin-split electron band structures induced by d -wave, g -wave, and i -wave altermagnets are characterized by $k^{N_X} \sin N_X \phi$ around the band bottom, where $N_X = 2, 4$, and 6 , respectively. In contrast, those of the corresponding d' -wave, g' -wave, and i' -wave altermagnets are described by $k^{N_X} \cos N_X \phi$. We show that a finite magnetization is induced in the d' -wave, g' -wave, and i' -wave altermagnets under a second-order nonlinear response to a temperature gradient $(\partial_x T)^2$ when the temperature gradient is linear and along either the x or y direction, whereas no such response occurs in the d -wave, g -wave, and i -wave altermagnets. We also study the case where the direction of the temperature gradient is arbitrary. In this case, a finite magnetization is induced also in d -wave altermagnets but not in g -wave and i -wave altermagnets. We discuss the difference in physics between the X -wave and X' -wave altermagnets with $X = d, g, i$ based on the tight-binding model. Furthermore, we show the emergence of magnetization as a response when the temperature profile is not linear, $\partial_x^2 T \neq 0$.

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I. INTRODUCTION

Nonlinear responses have attracted considerable attention. The most studied one is the nonlinear electric conductivity induced by a nonlinear response to an electric field [1–15]. The nonlinear spin conductivity has also been studied [16–21]. A nonlinear response to a temperature gradient $\nabla_x T$ can generate currents including the charge current [22–31] and the spin current [32]. It is notable that both the electric field and temperature gradient are polar vectors, where they change sign under inversion symmetry $\mathbf{E} \rightarrow -\mathbf{E}$ and $\nabla T \rightarrow -\nabla T$, while remaining invariant under time-reversal symmetry. Especially, the nonlinear Seebeck effect [24–30,32] and nonlinear Nernst effect [22–24,28,29,31,32] are such phenomena in which longitudinal and transverse currents are induced by a nonlinear response to a temperature gradient, respectively. The nonlinear Edelstein effect [33–36] is a phenomenon where magnetization is induced by a nonlinear response to an electric field. In contrast, magnetization induced by a nonlinear response to a temperature gradient is yet to be explored.

Altermagnets [37,38] have emerged as one of the most active fields in condensed matter physics. Because they possess zero net magnetization, they are promising candidates for future ultrafast and ultradense magnetic memories. We investigate the two-dimensional electron system coupled to an altermagnet. The prominent phenomenon is the emergence of a distinctive spin-split electron band structure which preserves

inversion symmetry but breaks time-reversal symmetry. In the presence of inversion symmetry, a linear magnetization response to a temperature gradient is forbidden, whereas a second-order nonlinear response is allowed.

Odd-parity magnets [39–43] share similarities with altermagnets in that they also exhibit characteristic spin-split electron band structures in the coupled electron system, where time-reversal symmetry is preserved while inversion symmetry is broken in the coupled electronic band structure. We note that time-reversal symmetry is preserved while inversion symmetry is broken in the coupled electronic band structure. On the other hand, time-reversal symmetry in odd-parity magnets themselves is broken as a feature common to all magnetic systems.

Altermagnets and odd-parity magnets together form a broader class known as X -wave magnets, which include d -wave, g -wave, i -wave altermagnets and p -wave, f -wave odd-parity magnets.

Spin-split band structures induced by d -wave, g -wave, and i -wave altermagnets are described by $k^{N_X} \sin N_X \phi$ around the band bottom, where $N_X = 2, 4$, and 6 , respectively. On the other hand, there are d' -wave, g' -wave, and i' -wave altermagnets [43] as well, which are described by $k^{N_X} \cos N_X \phi$ around the band bottom. d -wave altermagnets are also known as d_{xy} -wave, while d' -wave altermagnets are also known as $d_{x^2-y^2}$ -wave altermagnets.

In this paper, we first derive a general formula for the magnetization induced by a temperature gradient in the electron system, valid up to arbitrary orders in nonlinear response, when the temperature profile is linear. We apply this formula to the tight-binding model of the X -wave magnet. Among them, d' -wave, g' -wave, and i' -wave altermagnets exhibit magnetization induced by the second-order nonlinear

response to a temperature gradient. Using a high-temperature expansion, we obtain an analytic expression for the induced magnetization. In contrast, all odd-parity magnets do not exhibit magnetization induced by temperature gradient owing to the presence of time-reversal symmetry. We then present the results when the temperature profile is parabolic satisfying $\partial_x^2 T \neq 0$, which is realized when the diffusion constant κ has a temperature dependence. In particular, when $\kappa \propto 1/T$ as observed [44] around room temperature in an altermagnet candidate RuO₂, the results for the parabolic temperature profile are simply modified by factor 2.

II. X-WAVE MAGNETS

We investigate the two-dimensional electron system associated with a metallic *X*-wave magnet or a metallic thin film attached to an insulating *X*-wave magnet. The prominent phenomenon is the emergence of a distinctive spin-split electron band structure. The simplest model is the two-band Hamiltonian [20,32,43,45,46] given by

$$H = H_{\text{kin}}(\mathbf{k}) + J f_X(\mathbf{k}) \sigma_z, \quad (1)$$

where the first term represents the kinetic term of electrons, while the second term represents the band splitting described by the function $f_X(\mathbf{k})$ with the coupling constant J and the Pauli matrix σ_z . Around the band bottom, the spin-split function $f_X(\mathbf{k})$ is explicitly given by

$$f_p(\mathbf{k}) = ak_x = ak \cos \phi, \quad (2)$$

$$f_d(\mathbf{k}) = 2a^2 k_x k_y = a^2 k^2 \sin 2\phi, \quad (3)$$

$$f_f(\mathbf{k}) = a^3 k_x (k_x^2 - 3k_y^2) = a^3 k^3 \cos 3\phi, \quad (4)$$

$$f_g(\mathbf{k}) = 4a^4 k_x k_y (k_x^2 - k_y^2) = a^4 k^4 \sin 4\phi, \quad (5)$$

$$f_i(\mathbf{k}) = 2a^6 k_x k_y (3k_x^2 - k_y^2)(k_x^2 - 3k_y^2) = a^6 k^6 \sin 6\phi \quad (6)$$

for the *X*-wave magnet, and

$$f_{p'}(\mathbf{k}) = ak_y = ak \sin \phi, \quad (7)$$

$$f_{d'}(\mathbf{k}) = a^2 (k_x^2 - k_y^2) = a^2 k^2 \cos 2\phi, \quad (8)$$

$$f_{f'}(\mathbf{k}) = a^3 k_y (3k_x^2 - k_y^2) = a^3 k^3 \sin 3\phi, \quad (9)$$

$$\begin{aligned} f_{g'}(\mathbf{k}) &= a^4 (k_x^2 - k_y^2 - 2k_x k_y)(k_x^2 - k_y^2 + 2k_x k_y) \\ &= a^4 k^4 \cos 4\phi, \end{aligned} \quad (10)$$

$$\begin{aligned} f_{i'}(\mathbf{k}) &= 2a^6 (k_x^2 - k_y^2)(k_x^4 + k_y^4 - 14k_x^2 k_y^2) \\ &= a^6 k^6 \cos 6\phi \end{aligned} \quad (11)$$

for the *X'*-wave magnet, where $k_x = k \cos \phi$, $k_y = k \sin \phi$, and a is the lattice constant. We note that the *d*-wave altermagnet described by the function $f_d(\mathbf{k})$ is commonly called the d_{xy} -wave altermagnet and $f_{d'}(\mathbf{k})$ is commonly called the $d_{x^2-y^2}$ -wave altermagnet. The *X*-wave magnet has N_X nodes in the band structure, where $N_X = 1, 2, 3, 4$, and 6 for $X = p, d, f, g$, and i , respectively.

We make a symmetry analysis of Hamiltonian (1). Time-reversal symmetry is defined by

$$TH(\mathbf{k})T^{-1} = H(-\mathbf{k}) \quad (12)$$

with the time-reversal symmetry operator $T = i\sigma_y K$, where K is the complex conjugation operator. Inversion symmetry is defined by

$$PH(\mathbf{k})P^{-1} = H(-\mathbf{k}). \quad (13)$$

In the *d*-wave, *g*-wave, and *i*-wave altermagnets, time-reversal symmetry is broken but inversion symmetry is preserved

$$T f_X(\mathbf{k}) \sigma_z T^{-1} = -f_X(-\mathbf{k}), \quad (14)$$

$$P f_X(\mathbf{k}) \sigma_z P^{-1} = f_X(-\mathbf{k}) \sigma_z. \quad (15)$$

Hence, time-reversal symmetry is broken and inversion symmetry is preserved in Hamiltonian $H(\mathbf{k})$ for altermagnets. On the other hand, in the *p*-wave and the *f*-wave odd-parity magnets, inversion symmetry is broken but time-reversal symmetry is preserved

$$T f_X(\mathbf{k}) \sigma_z T^{-1} = f_X(-\mathbf{k}) \sigma_z, \quad (16)$$

$$P f_X(\mathbf{k}) \sigma_z P^{-1} = -f_X(-\mathbf{k}) \sigma_z. \quad (17)$$

Hence, time-reversal symmetry is preserved and inversion symmetry is broken in Hamiltonian $H(\mathbf{k})$ for odd-parity magnets.

Here, we make an important remark. We have introduced the notion of the *X*-wave magnet on the basis of the continuum model, where Hamiltonians (1) for the *X*-wave magnet and the *X'* magnet are related by the change of the angle $\phi \rightarrow \phi + \pi/(2N_X)$, as follows from Eqs. (2)–(6) and (7)–(11). However, the continuum model is valid only in low-energy physics. It is necessary to use the tight-binding model to investigate physics at high temperature, where such a symmetry is lost. In general, the *X*-wave magnet and the *X'*-magnet are independent objects.

III. TEMPERATURE GRADIENT INDUCED MAGNETIZATION

We study a two-dimensional electron system described by Hamiltonian (1). The linear response to magnetization induced by a temperature gradient is determined by

$$M_\mu = \chi_{\mu\nu} \nabla_\nu T, \quad (18)$$

where $\chi_{\mu\nu}$ is the susceptibility. Electric-field induced magnetization is prohibited in the inversion symmetric system because the magnetization is an axial vector, where it does not flip its sign under inversion symmetry operation but flips its sign under time-reversal symmetry operation. Next, we consider a second-order nonlinear response associated with electric-field induced magnetization

$$M_\rho = \chi_{\rho\mu\nu} (\nabla_\mu T) \nabla_\nu T, \quad (19)$$

where $\chi_{\rho\mu\nu}$ is the nonlinear susceptibility. Both the left- and right-hand sides are invariant under inversion symmetry. Hence, nonzero magnetization is not prohibited for inversion symmetric systems. On the other hand, the system must break time-reversal symmetry because the left-hand side is time-reversal symmetry odd but the right-hand side is time-reversal symmetry even.

The expectation value of the magnetization is given by

$$\mathbf{M} = \int \frac{d^2k}{(2\pi)^2} \mathbf{S}(\mathbf{k}) f(\mathbf{k}), \quad (20)$$

where $\mathbf{S}(\mathbf{k}) \equiv g\mu_B \langle \psi | \boldsymbol{\sigma} | \psi \rangle / 2$ is the expectation value of the spin, μ_B is the Bohr magneton, and g is the g factor.

We first assume a spatially uniform temperature gradient along the x axis such that $\partial_x^2 T = 0$. We discuss the case for $\partial_x^2 T \neq 0$ in Sec. VII.

By using the nonequilibrium Fermi distribution function f determined by the Boltzmann equation, magnetization induced by the ℓ th order nonlinear response to a temperature gradient is calculated from the formula

$$\mathbf{M}^{(\ell)} = \left(-\frac{\tau}{\hbar} \partial_x T\right)^\ell \int \frac{d^2k}{(2\pi)^2} \mathbf{S}(\mathbf{k}) \left(\frac{\partial \varepsilon}{\partial k_x}\right)^\ell \frac{\partial^\ell f^{(0)}}{\partial T^\ell}, \quad (21)$$

where τ is the relaxation time, ε is the energy, and

$$f^{(0)}(\mathbf{k}) = \frac{1}{\exp\left[\frac{\varepsilon(\mathbf{k}) - \mu}{k_B T(\mathbf{r})}\right] + 1} \quad (22)$$

is the Fermi distribution function at equilibrium. At high temperature, the magnetization is approximated as

$$\mathbf{M}^{(\ell)} \simeq -\left(\frac{\tau}{\hbar} \partial_x T\right)^\ell \frac{\ell!}{4k_B T^{\ell+1}} \int \frac{d^2k}{(2\pi)^2} \mathbf{S}(\mathbf{k}) \left(\frac{\partial \varepsilon}{\partial k_x}\right)^\ell (\varepsilon - \mu), \quad (23)$$

where we have used

$$\frac{\partial^\ell f^{(0)}}{\partial T^\ell} \simeq (-1)^{\ell+1} \frac{\ell!}{4k_B T^{\ell+1}} (\varepsilon - \mu). \quad (24)$$

See Appendix for derivation of Eq. (23).

In this system, the spin is diagonal $s = \pm 1$. Hence, the magnetization formula (23) is simplified as

$$\begin{aligned} M_z^{(\ell)} &\simeq \frac{g\mu_B}{2} \left(-\frac{\tau}{\hbar} \partial_x T\right)^\ell \frac{\ell!}{4k_B T^{\ell+1}} \\ &\times \sum_{s=\pm 1} s \frac{d^2k}{(2\pi)^2} \left(\frac{\partial \varepsilon_s}{\partial k_x}\right)^\ell (\varepsilon_s - \mu), \end{aligned} \quad (25)$$

where

$$\varepsilon_s = H_{\text{kine}}(\mathbf{k}) + sJf_X(\mathbf{k}) \quad (26)$$

is the energy for spin $s = \pm 1$.

For the second-order nonlinear response, it is explicitly given by

$$\begin{aligned} M_z^{(x^2)} &\simeq -\left(\frac{\tau}{\hbar} \partial_x T\right)^2 \frac{g\mu_B J}{2k_B T^3} \frac{d^2k}{(2\pi)^2} [2(H_{\text{kine}} - \mu) \partial_{k_x} f_X \partial_{k_x} H_{\text{kine}} \\ &+ f_X (\partial_{k_x} H_{\text{kine}})^2 + J^2 f_X (\partial_{k_x} f_X)^2]. \end{aligned} \quad (27)$$

If f_X is odd for k_x , we find $M_z^{(x^2)} = 0$ because H_{kine} is even for k_x . Hence, there are no second-order nonlinear responses in d -wave, g -wave, and i -wave altermagnets. On the other hand, it is nontrivial for d' -wave, g' -wave, and i' -wave altermagnets. We will analyze these altermagnets explicitly in the following sections.

We also study magnetization induced by the temperature gradient with an arbitrary angle φ , where the new coordinate

is defined by

$$X \equiv x \cos \varphi - y \sin \varphi, \quad Y \equiv x \sin \varphi + y \cos \varphi. \quad (28)$$

The magnetization is calculated by changing ∂_x to ∂_X with

$$\partial_x = \frac{\partial x}{\partial X} \frac{\partial}{\partial x} + \frac{\partial y}{\partial X} \frac{\partial}{\partial y} = \cos \varphi \frac{\partial}{\partial x} - \sin \varphi \frac{\partial}{\partial y} \quad (29)$$

in Eq. (23) together with

$$\partial_{k_x} = \frac{\partial k_x}{\partial k_X} \frac{\partial}{\partial k_x} + \frac{\partial k_y}{\partial k_X} \frac{\partial}{\partial k_y} = \cos \varphi \frac{\partial}{\partial k_x} - \sin \varphi \frac{\partial}{\partial k_y}. \quad (30)$$

There is a relation

$$M_z^{(x^2)} = M_z^{(x^2)} \cos^2 \varphi + M_z^{(y^2)} \sin^2 \varphi - M_z^{(xy)} \sin 2\varphi. \quad (31)$$

IV. d' -WAVE ALTERMAGNET

The tight-binding model (1) for the d' -wave altermagnet is given by [13,37,38,47–51]

$$H = H_{\text{kine,sq}} + J f_{d'} \sigma_z \quad (32)$$

with

$$H_{\text{kine,sq}} = \frac{\hbar^2}{ma^2} (2 - \cos ak_x - \cos ak_y), \quad (33)$$

and

$$f_{d'} = 2(\cos ak_y - \cos ak_x), \quad (34)$$

where a is the lattice constant. In the following, we set $\bar{m} \equiv ma^2/\hbar^2$. The Fermi surface is shown in Fig. 1(a).

The magnetization is analytically obtained from Eq. (23) as

$$\frac{M_z^{(x^2)}}{\frac{g\mu_B}{2(2\pi)^2} \left(\frac{\tau}{\hbar} \partial_x T\right)^2} \simeq -\frac{8\pi^2 J(\bar{m}\mu - 2)}{\bar{m}^2 k_B T^3}. \quad (35)$$

The magnetization is shown as a function of μ in Fig. 1(b), as a function of β in Fig. 1(c), and as a function of J in Fig. 1(d). The analytical results (cyan curves) obtained by using high-temperature expansion well agree with the numerical results (red curves) without using the expansion.

The angle dependence is analytically obtained as

$$\frac{M_z^{(y^2)}}{\frac{g\mu_B}{2(2\pi)^2} \left(\frac{\tau}{\hbar} \partial_y T\right)^2} = -\frac{M_z^{(x^2)}}{\frac{g\mu_B}{2(2\pi)^2} \left(\frac{\tau}{\hbar} \partial_x T\right)^2}, \quad (36)$$

$$\frac{M_z^{(xy)}}{\frac{g\mu_B}{2(2\pi)^2} \left(\frac{\tau}{\hbar}\right)^2 \partial_x T \partial_y T} = 0, \quad (37)$$

and

$$\frac{M_z^{(x^2)}}{\frac{g\mu_B}{2(2\pi)^2} \left(\frac{\tau}{\hbar} \partial_X T\right)^2} \simeq \frac{8\pi^2 J(\bar{m}\mu - 2)}{\bar{m}^2 k_B T^3} \cos 2\varphi. \quad (38)$$

The magnetization is shown as a function of μ in Fig. 1(e).

We also study the d -wave altermagnet, where

$$f_d = \sin ak_x \sin ak_y. \quad (39)$$

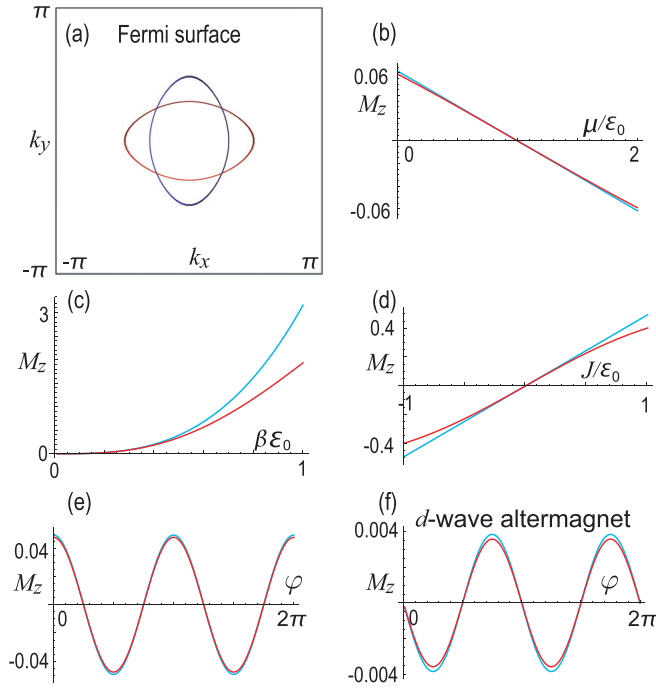


FIG. 1. d' -wave altermagnet. Magnetization induced by temperature gradient in units of $\frac{g\mu_B}{2(2\pi)^2} \left(-\frac{\tau}{\hbar} \partial_x T\right)^2$. (a) Fermi surfaces. The red oval represents up spin, while the blue oval represents down spin. (b) μ dependence. We have set $J/\varepsilon_0 = 0.1$ and $\beta\varepsilon_0 = 1/4$. (c) β dependence. We have set $J/\varepsilon_0 = 0.1$ and $\mu/\varepsilon_0 = 0.2$. (d) J dependence. Magnetization as a function of φ in units of $\frac{g\mu_B}{2(2\pi)^2} \left(-\frac{\tau}{\hbar} \partial_x T\right)^2$ of (e) d' -wave altermagnet and (f) d -wave altermagnet. We have set $\beta\varepsilon_0 = 1/4$ and $\mu/\varepsilon_0 = 0.2$. Red curves represent numerical results, while cyan curves represent analytic results based on high-temperature expansion. We have set $\varepsilon_0 = 4\hbar^2/(ma^2)$.

The angle dependence of the magnetization is analytically obtained as

$$\frac{M_z^{(x^2)}}{\frac{g\mu_B}{2(2\pi)^2} \left(\frac{\tau}{\hbar} \partial_x T\right)^2} = -\frac{M_z^{(y^2)}}{\frac{g\mu_B}{2(2\pi)^2} \left(\frac{\tau}{\hbar} \partial_y T\right)^2} = 0, \quad (40)$$

$$\frac{M_z^{(xy)}}{\frac{g\mu_B}{2(2\pi)^2} \left(\frac{\tau}{\hbar}\right)^2 \partial_x T \partial_y T} \simeq -\frac{\pi^2 J}{m^2 k_B T^3}. \quad (41)$$

The angle dependence is obtained as

$$\frac{M_z^{(x^2)}}{\frac{g\mu_B}{2(2\pi)^2} \left(\frac{\tau}{\hbar} \partial_x T\right)^2} \simeq -\frac{\pi^2 J}{m^2 k_B T^3} \sin 2\varphi \quad (42)$$

up to any order in J . The magnetization is shown as a function of μ in Fig. 1(b), as a function of β in Fig. 1(c), and as a function of J in Fig. 1(d). The analytical result obtained by using high-temperature expansion well agrees with the numerical result without using the expansion. We find

V. g' -WAVE ALTERMAGNETS

The tight-binding model (1) for the g' -wave altermagnet is

$$H = H_{\text{kin},\text{sq}} + J f_g \sigma_z \quad (43)$$

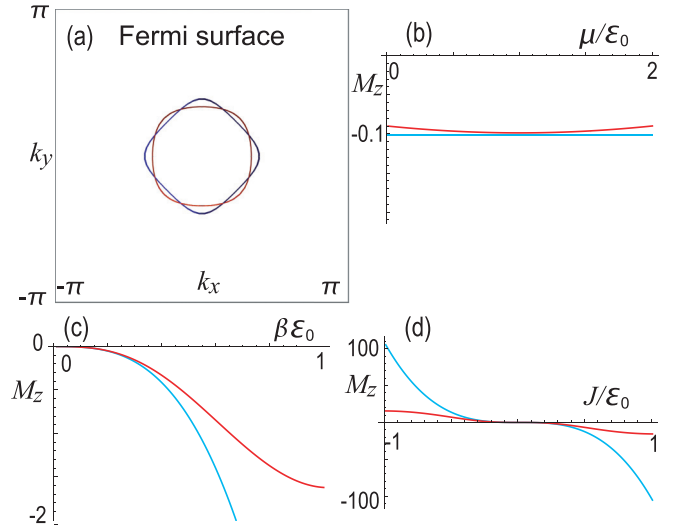


FIG. 2. g' -wave altermagnet. Magnetization induced by temperature gradient in units of $\frac{g\mu_B}{2(2\pi)^2} \left(-\frac{\tau}{\hbar} \partial_x T\right)^2$. (a) Fermi surface. (b) μ dependence. We have set $J/\varepsilon_0 = 0.1$ and $\beta\varepsilon_0 = 1/4$. (c) β dependence. We have set $J/\varepsilon_0 = 0.1$ and $\mu/\varepsilon_0 = 0.2$. (d) J dependence. We have set $\beta\varepsilon_0 = 1/4$ and $\mu/\varepsilon_0 = 0.2$. Red curves represent numerical results, while cyan curves represent analytic results based on high-temperature expansion. We have set $\varepsilon_0 = 4\hbar^2/(ma^2)$.

with Eq. (33) and

$$f_{g'} = 4(\cos ak_y - \cos ak_x - \sin ak_x \sin ak_y) \times (\cos ak_y - \cos ak_x + \sin ak_x \sin ak_y). \quad (44)$$

The Fermi surface is shown in Fig. 2(a).

The magnetization is analytically obtained as

$$\frac{M_z^{(x^2)}}{\frac{g\mu_B}{2(2\pi)^2} \left(\frac{\tau}{\hbar} \partial_x T\right)^2} \simeq -\frac{\pi^2 J(1 - 684\bar{m}^2 J^2)}{\bar{m}^2 k_B T^3} \quad (45)$$

up to any order in J . The magnetization is shown as a function of μ in Fig. 2(b), as a function of β in Fig. 2(c), and as a function of J in Fig. 2(d). The analytical result obtained by using high-temperature expansion well agrees with the numerical result without using the expansion. We find

$$M_z^{(y^2)} = M_z^{(x^2)}, \quad M_z^{(xy)} = 0 \quad (46)$$

Hence, there is no angle dependence for g' -wave altermagnets.

We also study the g -wave altermagnet with

$$f_g = 8 \sin ak_x \sin ak_y (\cos ak_y - \cos ak_x). \quad (47)$$

We find

$$M_z^{(x^2)} = M_z^{(y^2)} = M_z^{(xy)} = 0, \quad (48)$$

because the system is odd under mirror operations. Hence, the magnetization is zero for the g -wave altermagnet for an arbitrary angle φ .

VI. i' -WAVE ALTERMAGNETS

The tight-binding model (1) for the i' -wave altermagnet is

$$H = H_{\text{kin},\text{tri}} + J f_{i'} \sigma_z \quad (49)$$

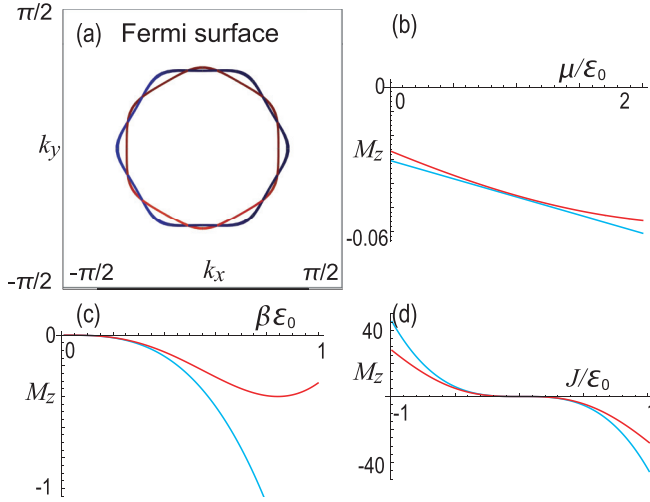


FIG. 3. i' -wave altermagnet. Magnetization induced by a temperature gradient in units of $\frac{g\mu_B}{2}(-\frac{\tau}{\hbar}\partial_x T)^2$. (a) Fermi surface. (b) μ dependence. We have set $J/\varepsilon_0 = 0.1$ and $\beta\varepsilon_0 = 1/4$. (c) β dependence. We have set $J/\varepsilon_0 = 0.1$ and $\mu/\varepsilon_0 = 0.2$. (d) J dependence. We have set $\beta\varepsilon_0 = 1/4$ and $\mu/\varepsilon_0 = 0.2$. Red curves represent numerical results, while cyan curves represent analytic results based on high-temperature expansion. We have set $\varepsilon_0 = 4\hbar^2/(ma^2)$.

with the kinetic term

$$H_{\text{kin},\text{tri}} = \frac{-2\hbar^2}{3ma^2} \left(\sum_{j=0}^2 \cos(\mathbf{a}_j^A \cdot \mathbf{k}) - 3 \right), \quad (50)$$

and

$$f_i = 16 \left[\left(\prod_{j=0}^2 \sin(\mathbf{a}_j^A \cdot \mathbf{k}) \right)^2 - \left(\frac{1}{3\sqrt{3}} \prod_{j=0}^2 \sin(\sqrt{3}\mathbf{a}_j^B \cdot \mathbf{k}) \right)^2 \right], \quad (51)$$

where we have defined

$$\begin{aligned} \mathbf{n}_j^A &= \left(\cos \frac{2\pi j}{3}, \sin \frac{2\pi j}{3} \right), \\ \mathbf{n}_j^B &= \left(\sin \frac{2\pi j}{3}, \cos \frac{2\pi j}{3} \right) \end{aligned} \quad (52)$$

with $j = 0, 1$, and 2 . The Fermi surface is shown in Fig. 3(a).

The magnetization is analytically obtained as

$$\frac{M_z^{(x^2)}}{\frac{g\mu_B}{2(2\pi)^2} \left(\frac{\tau}{\hbar} \partial_x T \right)^2} \simeq -\frac{4\pi^2 J}{3^3 \sqrt{3} m^2 k_B T^3} - \frac{372058\pi^2 J^3}{3^7 \sqrt{3} k_B T^3} \quad (53)$$

up to any order in J . We find

$$M_z^{(y^2)} = M_z^{(x^2)}, \quad M_z^{(xy)} = 0. \quad (54)$$

Hence, there is no angle dependence for i' -wave altermagnets. In addition, the magnetization is zero for i' -wave altermagnet for an arbitrary angle φ .

The magnetization is shown as a function of μ in Fig. 3(b), as a function of β in Fig. 3(c) and as a function of J

in Fig. 3(d). The analytical result obtained by using high-temperature expansion well agrees with the numerical result without using the expansion.

We also study the i -wave altermagnet, where

$$f_i = 16J\sigma_z \sin ak_z \prod_{j=0}^2 \sin(\mathbf{a}_j^A \cdot \mathbf{k}) \prod_{j=0}^2 \sin(\sqrt{3}\mathbf{a}_j^B \cdot \mathbf{k}). \quad (55)$$

We find

$$M_z^{(x^2)} = M_z^{(y^2)} = M_z^{(xy)} = 0, \quad (56)$$

because the system is odd for the mirror operations. Hence, the magnetization is zero for i -wave altermagnet for an arbitrary angle φ .

VII. PARABOLIC TEMPERATURE PROFILE

We have so far assumed that $\partial_x^2 T = 0$, which is not valid when the diffusion coefficient κ has a temperature dependence. The heat equation under steady-state conditions reads

$$\nabla(\kappa \nabla T) = 0, \quad (57)$$

where κ is the diffusion coefficient. We have $\nabla^2 T = 0$ only when κ is a constant. Otherwise, we have

$$\frac{\partial^2 T}{\partial x^2} = -\frac{1}{\kappa} \frac{\partial \kappa}{\partial T} \left(\frac{\partial T}{\partial x} \right)^2. \quad (58)$$

Consequently, $\partial_x^2 T \neq 0$ if $\partial_x T \neq 0$ when κ is not a constant. Indeed, it is pointed out [44] that

$$\kappa = c/T \quad (59)$$

with a constant c in RuO_2 , which is a candidate material for the d -wave altermagnet.

When $\partial_x^2 T \neq 0$ in general, the formula for the second-order response of the magnetization is modified as

$$\begin{aligned} M_z^{(x^2)} &= -g\mu_B \left(\frac{\tau}{\hbar} \right)^2 \int d^3 k \mathbf{S}(\mathbf{k}) \left(\frac{\partial \varepsilon}{\partial k_x} \right)^2 \frac{\partial^2 f^{(0)}}{\partial x^2} \\ &= -g\mu_B \left(\frac{\tau}{\hbar} \right)^2 \int d^3 k \mathbf{S}(\mathbf{k}) \left(\frac{\partial \varepsilon}{\partial k_x} \right)^2 \\ &\quad \times \left[\frac{\partial^2 f^{(0)}}{\partial T^2} (\partial_x T)^2 + \frac{\partial f^{(0)}}{\partial T} \partial_x^2 T \right], \end{aligned} \quad (60)$$

where we have used the relation

$$\begin{aligned} \frac{\partial^2 f^{(0)}}{\partial x^2} &= \frac{\partial}{\partial x} \left(\frac{\partial T}{\partial x} \frac{\partial f^{(0)}}{\partial T} \right) = \frac{\partial^2 T}{\partial x^2} \frac{\partial f^{(0)}}{\partial T} + \frac{\partial T}{\partial x} \frac{\partial T}{\partial x} \frac{\partial}{\partial T} \frac{\partial f^{(0)}}{\partial T} \\ &= \frac{\partial f^{(0)}}{\partial T} \partial_x^2 T + \frac{\partial^2 f^{(0)}}{\partial T^2} (\partial_x T)^2, \end{aligned} \quad (61)$$

and Eq. (A17) in Appendix. By using Eq. (24) with $\ell = 2$, we obtain

$$\begin{aligned} \mathbf{M}^{(x^\ell)} &\simeq \frac{g\mu_B}{2} \left(\frac{\tau}{\hbar} \right)^2 \left[(\partial_x T)^2 \frac{1}{2k_B T^3} - \frac{1}{4k_B T^2} \partial_x^2 T \right] \\ &\quad \times \int d^3 k \mathbf{S}(\mathbf{k}) \left(\frac{\partial \varepsilon}{\partial k_x} \right)^\ell (\varepsilon - \mu). \end{aligned} \quad (62)$$

Hence, the second-order response of the magnetization for $\partial_x^2 T \neq 0$ is given by the following replacement:

$$\left(\frac{\tau}{\hbar} \partial_x T\right)^2 \frac{1}{2k_B T^3} \mapsto \frac{1}{2k_B T^3} \left(\frac{\tau}{\hbar}\right)^2 (\partial_x T)^2 - \left(\frac{\tau}{\hbar}\right)^2 \frac{1}{4k_B T^2} \partial_x^2 T. \quad (63)$$

Namely, Eqs. (35), (45), and (53) are replaced from

$$\frac{M_z^{(x^2)}}{\frac{g\mu_B}{2(2\pi)^2} \left(\frac{\tau}{\hbar} \partial_x T\right)^2} = F \quad (64)$$

to

$$\frac{M_z^{(x^2)}}{\frac{g\mu_B}{2(2\pi)^2} \left(\frac{\tau}{\hbar}\right)^2 \left[\frac{1}{2k_B T^3} (\partial_x T)^2 - \frac{1}{4k_B T^2} \partial_x^2 T \right]} = F. \quad (65)$$

With the use of Eq. (58), we have

$$\begin{aligned} & \frac{1}{2k_B T^3} (\partial_x T)^2 - \frac{1}{4k_B T^2} \partial_x^2 T \\ &= \frac{1}{2k_B T^3} (\partial_x T)^2 + \frac{1}{4k_B T^2} \frac{1}{\kappa} \frac{\partial \kappa}{\partial T} \left(\frac{\partial T}{\partial x} \right)^2 \\ &= \frac{1}{2k_B T^3} (\partial_x T)^2 \left[1 + \frac{T}{2} \frac{1}{\kappa} \frac{\partial \kappa}{\partial T} \right]. \end{aligned} \quad (66)$$

For instance, in the case of Eq. (59), we obtain

$$\begin{aligned} & \frac{1}{2k_B T^3} (\partial_x T)^2 \left[1 + \frac{T}{2} \frac{1}{\kappa} \frac{\partial \kappa}{\partial T} \right] \\ &= \frac{1}{2k_B T^3} (\partial_x T)^2 \left[1 + \frac{T}{2} \frac{1}{c} \left(-\frac{c}{T^2} \right) \right] \\ &= \frac{1}{4k_B T^3} (\partial_x T)^2. \end{aligned} \quad (67)$$

Consequently, the results are modified only by a factor of 2.

VIII. DISCUSSION

We have investigated the emergence of the magnetization induced by the second-order response to a temperature gradient in the X -wave and X' -wave magnets with $X = d$, g , and i . It is noteworthy that there are nontrivial responses in d' -wave, g' -wave, and i' -wave altermagnets when the temperature gradient is along the x or y direction. In addition, there are nontrivial responses in d -wave altermagnets when the temperature gradient is along an arbitrary direction. On the other hand, no such responses occur in g -wave and i -wave altermagnets. It is understood in terms of the mirror symmetry with respect to the x and y axes. We also note that g' -wave and i' -wave altermagnets have been scarcely studied, despite being predicted on the basis of symmetry analysis [43].

Magnetization induced by temperature gradient is experimentally observed in Au [52]. We estimate the magnetization induced by the second-order nonlinear response to the temperature gradient. The magnetization per volume is estimated as

$$\frac{g\mu_B}{2a^3} \left(\tau v_F \frac{\partial_x T}{T} \right)^2 \frac{\varepsilon - \mu}{k_B T} \sim 0.15 \text{ A/m}, \quad (68)$$

where we have used a typical relaxation time $\tau = 3 \times 10^{-12}$ s, the Fermi velocity $v_F = 10^6$ m/s, $\partial_x T = 1$ K/mm, $T = 300$ K, $g\mu_B = 2 \times 10^{-23}$ A m², $a = 3$ Å, $k_B T = 25$ meV, and $\varepsilon - \mu = 10$ meV. Assuming a cubic sample with a side length of 1 mm, the magnetization is estimated as

$$0.15 \text{ A/m} \times (10^{-3} \text{ m})^3 = 1.5 \times 10^{-14} \text{ A m}^2. \quad (69)$$

It is smaller than a typical order of the magnetization by the SQUID of the order of 10^{-11} A m² [53], but is experimentally detectable because the minimum measurable magnetization by the SQUID is of the order of 10^{-14} A m² [54,55]. The spin splitting in the band structure for the d' -wave altermagnet is 1.4 eV in RuO₂ [56]. On the other hand, g -wave and i -wave altermagnets are theoretically proposed to be realized in van der Waals ferromagnets [57], where the spin splitting is estimated to be around 10 meV. Then, the magnetization may range from 10^{-15} to 10^{-13} A m².

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX: TEMPERATURE GRADIENT INDUCED MAGNETIZATION

We derive the formula for magnetization induced by temperature gradient ∇T based on the Boltzmann equation. By assuming the spatial dependence of temperature $T(\mathbf{r})$, the Fermi distribution function is given by

$$f(\mathbf{r}, \mathbf{k}) = \frac{1}{\exp\left[\frac{\varepsilon(\mathbf{k}) - \mu}{k_B T(\mathbf{r})}\right] + 1}. \quad (A1)$$

The Boltzmann equation is given by

$$\frac{df(\mathbf{r}, \mathbf{k})}{dt} = -\frac{f(\mathbf{r}, \mathbf{k}) - f^{(0)}}{\tau}, \quad (A2)$$

where τ is the relaxation time. The total derivative is expanded as

$$\begin{aligned} \frac{df}{dt} &= \frac{\partial f}{\partial t} + \frac{\partial \mathbf{k}}{\partial t} \cdot \frac{\partial f}{\partial \mathbf{k}} + \frac{\partial \mathbf{r}}{\partial t} \cdot \frac{\partial f}{\partial \mathbf{r}} \\ &= \frac{\partial \mathbf{k}}{\partial t} \cdot \left(\frac{\partial \varepsilon(\mathbf{k})}{\partial \mathbf{k}} \frac{\partial f}{\partial \varepsilon} \right) + \frac{\partial \mathbf{r}}{\partial t} \cdot \left(\frac{\partial T(\mathbf{r})}{\partial \mathbf{r}} \frac{\partial f}{\partial T} \right). \end{aligned} \quad (A3)$$

We use the kinetic equations

$$\frac{\partial \mathbf{r}}{\partial t} = \frac{1}{\hbar} \frac{\partial \varepsilon(\mathbf{k})}{\partial \mathbf{k}}, \quad \frac{\partial \mathbf{k}}{\partial t} = 0 \quad (A4)$$

to obtain

$$\frac{1}{\hbar} \left(\frac{\partial \varepsilon(\mathbf{k})}{\partial \mathbf{k}} \cdot \frac{\partial T(\mathbf{r})}{\partial \mathbf{r}} \right) \frac{\partial f}{\partial T} = -\frac{f - f^{(0)}}{\tau}. \quad (A5)$$

Assuming that the temperature gradient is along the x direction, we measure the induced current along the b direction. First, we consider the case $\partial_x^2 T = 0$. Next, we consider the case $\partial_x^2 T \neq 0$.

First, when $\partial_x^2 T = 0$, we expand the Fermi distribution in powers of $\partial_x T$, $f = f^{(0)} + f^{(1)} + \dots$, and we obtain

$$\frac{1}{\hbar} \left(\frac{\partial \varepsilon(\mathbf{k})}{\partial \mathbf{k}} \cdot \frac{\partial T(\mathbf{r})}{\partial \mathbf{r}} \right) \frac{\partial f^{(\ell-1)}}{\partial T} = -\frac{f^{(\ell)}}{\tau}. \quad (\text{A6})$$

It is straightforward to solve the Boltzmann equation recursively and derive the formula

$$f^{(\ell)} = \left(-\frac{\tau}{\hbar} \partial_x T \right)^\ell \left(\frac{\partial \varepsilon}{\partial k_b} \right)^\ell \frac{\partial^\ell f^{(0)}}{\partial T^\ell}, \quad (\text{A7})$$

where we have assumed that temperature gradient is along the x -axis.

At high temperature T , the derivative of the Fermi-distribution function at equilibrium is approximated as

$$\frac{\partial^\ell f^{(0)}}{\partial T^\ell} = (-1)^{\ell+1} \frac{\ell!}{4k_B T^{\ell+1}} (\varepsilon - \mu). \quad (\text{A8})$$

By inserting it into the Fermi distribution function at nonequilibrium, we obtain

$$f^{(\ell)} = -\left(\frac{\tau}{\hbar} \partial_x T \right)^\ell \left(\frac{\partial \varepsilon}{\partial k_b} \right)^\ell \frac{\ell!}{4k_B T^{\ell+1}} (\varepsilon - \mu). \quad (\text{A9})$$

The expectation value of the magnetization is given by

$$\mathbf{M} = \int d^3 k \mathbf{S}(\mathbf{k}) f, \quad (\text{A10})$$

where

$$\mathbf{S}(\mathbf{k}) \equiv \frac{g\mu_B}{2} \langle \psi | \boldsymbol{\sigma} | \psi \rangle \quad (\text{A11})$$

is the expectation value of the spin and μ_B is the Bohr magneton. By using the Fermi distribution function, magnetization induced by the ℓ -th order nonlinear response to a temperature

gradient is calculated from the formula

$$\begin{aligned} \mathbf{M}^{(x^\ell; b)} &= -g\mu_B \left(\frac{\tau}{\hbar} \partial_x T \right)^\ell \int d^3 k \mathbf{S}(\mathbf{k}) \left(\frac{\partial \varepsilon}{\partial k_b} \right)^\ell \frac{\partial^\ell f^{(0)}}{\partial T^\ell} \\ &\simeq -\frac{g\mu_B}{2} \left(\frac{\tau}{\hbar} \partial_x T \right)^\ell \frac{\ell!}{4k_B T^{\ell+1}} \\ &\quad \times \int d^3 k \mathbf{S}(\mathbf{k}) \left(\frac{\partial \varepsilon}{\partial k_b} \right)^\ell (\varepsilon - \mu). \end{aligned} \quad (\text{A12})$$

In the following, we consider the two-band Hamiltonian for a metallic X -wave magnet,

$$H = H_{\text{kin}}(\mathbf{k}) + J f_X(\mathbf{k}) \sigma_z, \quad (\text{A13})$$

where $H_{\text{kin}}(\mathbf{k})$ is the kinetic term and $f_X(\mathbf{k})$ is the spin-split function characteristic to the X -wave magnet. In this system, the spin is diagonal $\mathbf{S}(\mathbf{k}) = \pm 1$. Hence, the magnetization formula is simplified as

$$\begin{aligned} M_z^{(x^\ell; b)} &= -\frac{g\mu_B}{2} \left(\frac{\tau}{\hbar} \partial_x T \right)^\ell \frac{\ell!}{4k_B T^{\ell+1}} \\ &\quad \times \sum_{s=\pm 1} s \int d^3 k \left(\frac{\partial \varepsilon_s}{\partial k_b} \right)^\ell (\varepsilon_s - \mu), \end{aligned} \quad (\text{A14})$$

where

$$\varepsilon_s = H_{\text{kin}}(\mathbf{k}) + s J f_X(\mathbf{k}) \quad (\text{A15})$$

is the energy for spin $s = \pm 1$. Eq. (A14) is Eq. (23) in the main text.

Next, we consider the case $\partial_x^2 T \neq 0$. We expand the Fermi distribution, $f = f^{(0)} + f^{(1)} + \dots$, and we obtain

$$\frac{1}{\hbar} \frac{\partial \varepsilon(\mathbf{k})}{\partial \mathbf{k}} \cdot \frac{\partial f^{(\ell-1)}}{\partial \mathbf{r}} = -\frac{f^{(\ell)}}{\tau}. \quad (\text{A16})$$

It is straightforward to solve the Boltzmann equation recursively and derive the formula

$$f^{(\ell)} = \left(-\frac{\tau}{\hbar} \frac{\partial f^{(0)}}{\partial x} \right)^\ell \left(\frac{\partial \varepsilon}{\partial k_b} \right)^\ell, \quad (\text{A17})$$

where we have assumed that temperature gradient is along the x axis. We use it to derive Eq. (61) in the main text.

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