

Extreme-value criticality and gain decomposition at the integer quantum Hall transitionWei-Han Li^{*} and Abbas Ali Saberi[†]*School of Science, Constructor University, Campus Ring 1, 28759 Bremen, Germany
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Extreme-value fluctuations at quantum critical points remain poorly understood in the presence of strong correlations and open-system amplification. We study the maximum wave-function amplitude in the open Chalker-Coddington network at the integer quantum Hall transition. Because the stationary scattering state is not normalized to unit total intensity, its largest amplitude contains both a sample-dependent global intensity scale and an intrinsic spatial extreme. We separate these contributions by writing the state as a total gain times a normalized spatial pattern. The raw maximum shows near-log-normal statistics over the studied sizes and an approximately parabolic extreme-moment exponent in a moderate moment window. We show that this behavior is strongly shaped by the fluctuating gain. After gain normalization, the maximum has a qualitatively different exponent function and does not collapse to a standard generalized extreme-value form under the same centering and scaling. These results identify extreme amplitudes as sensitive probes of open quantum criticality and separate collective amplification from intrinsic rare-event fluctuations.

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Introduction. Extreme fluctuations often expose physics that remains hidden at the level of typical observables. In disordered and random systems, extreme-value (EV) statistics have become an important tool for characterizing rare events, from low-energy excitations and trapping phenomena to fluctuating interfaces and random-matrix spectra [1–7]. This perspective is especially useful in strongly correlated systems, because the standard EV universality classes are derived for independent or weakly dependent variables, so systematic departures from them can directly encode correlation effects [8–10]. Yet, despite the broad development of EV methods in statistical physics [11,12], the EV statistics of quantum eigenstates in strongly correlated or critical settings remain less explored, with only a few studies in special ensembles and related models [13,14].

Extreme-value statistics of multifractal patterns have been explored in broader contexts such as logarithmically correlated fields and the associated “freezing” scenario, where departures from i.i.d. generalized extreme-value (GEV) universality can be intrinsic and convergence may be anomalously slow, with logarithmic corrections [15]. This motivates developing extreme-value observables for quantum-critical states, where multifractal correlations can strongly reshape the statistics of maxima and, in open or driven geometries, additional sample-dependent global amplification/normalization

factors may arise that must be separated from intrinsic extremal fluctuations.

A paradigmatic arena in which critical eigenstates are the central objects is the integer quantum Hall (IQH) plateau transition, which has also been approached from complementary spectral and network-based perspectives [16]. At criticality, electronic wave functions are delocalized and exhibit multifractal fluctuations, a defining feature of localization-delocalization criticality in two dimensions. Over the past decades, multifractal analysis has been developed extensively for the IQH problem, both numerically and analytically, and has revealed highly nontrivial scaling structure beyond simple single-parameter descriptions [17–23]. Most of this literature, however, focuses on bulk moment observables (inverse participation ratios and related quantities), which characterize amplitude statistics through ensemble moments and their scaling. A complementary question remains largely open for IQH critical wave functions: what is the scaling theory of the most extreme wave-function amplitudes? This question is physically natural, in principle experimentally relevant, and conceptually nontrivial because maxima are governed by the interplay of rare events, scale-invariant critical correlations, and finite-size effects.

In this Letter, we study extreme wave-function amplitudes of driven stationary states at the integer quantum Hall transition in the Chalker-Coddington (CC) network [19,24], focusing on the open, point-contact geometry [25]. The key difference from a closed eigenstate problem is that the stationary state in the open network is a driven scattering state, not a wave function normalized to unit total intensity. As a result, its total stationary intensity fluctuates across samples and sets the global scale for all local amplitudes. This motivates separating the raw amplitude field into a sample-dependent gain and a normalized spatial pattern.

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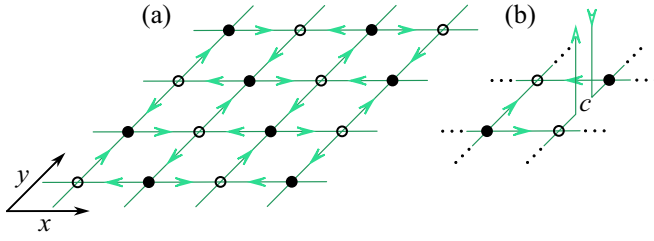


FIG. 1. Chalker-Coddington (CC) network on a 2D square lattice. (a) The wave function lives on directed links and propagates along arrows, forming closed loops with alternating circulation on neighboring plaquettes. At each node, a local 2×2 scattering matrix maps two incoming to two outgoing amplitudes; $\bullet$ and $\circ$ indicate the two node orientations. (b) Open CC network with an external point contact attached to link c .

We find that the maximum in the driven stationary state is not a purely intrinsic critical observable: it combines a sample-dependent gain factor and an intrinsic extremal component. More explicitly, for a stationary state with local amplitudes ψ_l , we define the gain factor

$$A^2 = \sum_{l \neq c} |\psi_l|^2, \quad (1)$$

which is the total intensity excluding the contact link c . The amplitudes and the maximum are then analyzed through $\tilde{\psi}_l = \psi_l/A$ and $|\tilde{\psi}|_{\max} = |\psi|_{\max}/A$. These relations follow from normalization, but their physical use is nontrivial: they separate collective amplification of the open stationary state from the intrinsic statistics of the normalized spatial pattern. This gain decomposition is the central new element of our work and complements known exact relations linking wave-function multifractality to the multifractality of Wigner delay times in lead-attached critical systems [26].

Through this gain decomposition, we formulate an extreme-value multifractality framework for the open CC model. We define extreme-moment scaling for the maximum and extract an exponent function $\tau_{\max}(q)$ and the associated large-deviation spectrum for maxima. Over the accessible moment window, the raw extrema exhibit an approximately parabolic $\tau_{\max}(q)$, and $\ln |\psi|_{\max}$ is close to Gaussian, or equivalently $|\psi|_{\max}$ is near-log-normal, in marked contrast to standard EV expectations for weakly dependent variables [8–10]. We show that these raw extreme statistics are strongly shaped by the gain factor A . Gain normalization reorganizes both the exponent function and the probability density, isolating an intrinsic, correlation-dominated extreme sector beyond collective amplification. Consequently, the raw maximum probes the amplified response encoded in the gain of the open critical system, whereas the gain-normalized maximum probes the spatial rare-event structure after the sample-specific global scale has been normalized out. This distinction is the central conclusion of our analysis.

CC networks. We study the Chalker-Coddington network model on a square lattice, shown in Fig. 1(a), which provides a standard lattice description of the IQH plateau transition [19,24]. The wave function lives on directed links $|\psi\rangle = \sum_l \psi_l |l\rangle$ with $\psi_l \in \mathbb{C}$, and evolves in discrete time under

a unitary operator $\hat{U}$ built from local scattering matrices at network nodes, see Supplemental Material [27]. At the critical point, the scattering probabilities are tuned to be isotropic and the network lies in the IQH universality class.

We focus on the open CC network [25], implemented by attaching an external contact link $|c\rangle$ [Fig. 1(b)]. The stationary state is obtained from

$$(1 - Q\hat{U})|\Psi\rangle = |c\rangle, \quad Q = 1 - |c\rangle\langle c|, \quad (2)$$

which defines a unique driven steady state in the open geometry see Supplemental Material [27].

Extreme-value multifractality. We analyze the stationary solution of Eq. (2) through the maximum link amplitude

$$|\psi|_{\max} := \max\{|\psi_l|_{l \neq c}\}, \quad (3)$$

excluding the contact link since $|\psi_c| = 1$ is fixed by the injection convention of the open dynamics.

The open setting yields two useful consequences, both observed in our critical-point simulations: (i) $|\psi|_{\max}$ is typically larger than unity and (ii) its typical scale increases with system size. Since the solution of Eq. (2) is a coherent sum over propagation paths launched from the contact and absorbed upon return, the local amplitudes ψ_l are not bounded by unity. Disorder realizations with long dwell times or resonant propagation paths can produce high local intensities through constructive interference, accounting for (i). Increasing L increases the number of available critical paths and rare high-intensity regions, accounting for (ii).

To characterize its finite-size scaling, we introduce the extreme moments

$$\mathbb{E}[(|\psi|_{\max})^{2q}] \sim L^{-d} \tau_{\max}(q), \quad (4)$$

where $\mathbb{E}[\dots]$ denotes disorder averaging and $d = 2$ is kept explicit as our scaling convention. By definition, the $q = 0$ moment equals unity; hence $\tau_{\max}(0) = 0$. The two consequences above imply that $\mathbb{E}[(|\psi|_{\max})^{2q}]$ increases with L for $q > 0$ and decreases with L for $q < 0$, implying $\tau_{\max}(q > 0) < 0$ and $\tau_{\max}(q < 0) > 0$. This sign structure already distinguishes the EV sector of the open network from conventional multifractal moments of normalized wave functions.

Parabolic extreme-moment exponent. Our first central result is that, in the accessible moment window $|q| \lesssim 1$, the extreme-moment exponent is accurately described by a parabolic form,

$$\tau_{\max}(q) \approx -\gamma q^2 + \nu q, \quad (5)$$

with $\nu = -0.430 \pm 0.002$ and $\gamma = 0.137 \pm 0.0007$, obtained from finite-size scaling, see Supplemental Material [27].

To expose the quadratic structure more transparently, we define the shifted extreme moments

$$\mathbb{E}[p^{2q}] \sim L^{-d} \Delta_q^{\max} \quad (6)$$

with $p := L^{vd/2} |\psi|_{\max}$, so that Eq. (5) implies

$$\Delta_q^{\max} \approx -\gamma q^2. \quad (7)$$

Figure 2 shows that $\Delta_q^{\max}$ follows the predicted parabola with excellent accuracy throughout the measured range. The agreement is quantified by adjusted $\bar{R}^2 > 0.999$ for the parabolic fits, see Supplemental Material [27]. At the same

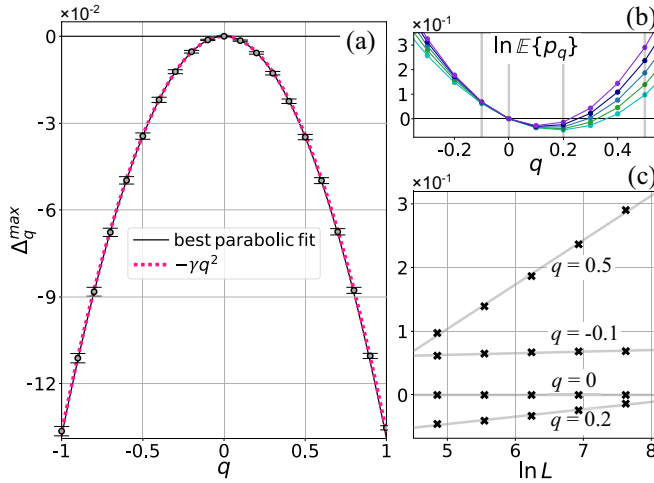


FIG. 2. Parabolic extreme-moment exponent. (a) Measured $\Delta_q^{\max}$ for $|q| \lesssim 1$. The dashed curve shows the prediction $\Delta_q^{\max} \approx -\gamma q^2$, while the solid curve is an unconstrained best parabolic fit. (b) $\ln \mathbb{E}[p^q]$ versus $\ln L$ (equivalently, the moments of the shifted variable entering the fit). Data from bottom to top correspond to $L = 64, 128, \dots, 2048$; ensemble sizes are of order $O(10^7)$ for $L < 1000$ and $O(10^6)$ for $L > 1000$. (c) Representative extractions of $\Delta_q^{\max}$ from the slopes of linear fits of $\ln \mathbb{E}[p^q]$ versus $\ln L$.

time, this parabolicity is established only over the finite moment interval studied here. It cannot be a global expression for all q : the observed growth of the raw maximum requires the sign of $\tau_{\max}(q)$ to remain opposite to the sign of q , whereas the parabola in Eq. (5) has a turning point at $q_t = \nu/(2\gamma)$ and exhibits a second zero at $2q_t \approx -3.14$, outside the measured window. We therefore interpret Eq. (5) as an accurate effective description of the moderate- q regime, not as an asymptotic spectrum valid for arbitrarily large $|q|$.

Importantly, the parabolic form in Eq. (5) is *distinct* from the conventional multifractal exponents of the full (all-link) wave-function statistics, see Supplemental Material [27], underscoring that the maximum probes a different rare-event sector. At the same time, this parabolicity pertains to the *raw* maximum in the open geometry and therefore incorporates the effect of a fluctuating global gain field. As we show below, gain normalization reorganizes the extreme-moment exponent and drives systematic deviations from a simple parabola, demonstrating that the raw parabolicity is not purely intrinsic to the extremal sector.

Parabolic extreme-value singularity spectrum. To complement the extreme-moment exponents, we characterize sample-to-sample fluctuations of the maximum by a large-deviation description on the scale $\ln L$. For each realization, we define α by $|\psi|_{\max}^2 := L^{-d\alpha}$; equivalently

$$\alpha = -\frac{\ln |\psi|_{\max}^2}{d \ln L}, \quad (8)$$

which need not be positive since the open-network gain can produce $|\psi|_{\max} > 1$. We introduce an extreme-value singularity spectrum $f_{\max}(\alpha)$ through

$$\text{PDF}_L(\alpha) \asymp L^{d f_{\max}(\alpha)} \quad (L \rightarrow \infty), \quad (9)$$

where $\asymp$ denotes equality at the level of the leading exponent in $\ln L$ [32,33]. Normalization implies $\max_{\alpha} f_{\max}(\alpha) = 0$.

The connection to $\tau_{\max}(q)$ follows from Laplace (saddle-point) evaluation of

$$\begin{aligned} \mathbb{E}[(|\psi|_{\max})^{2q}] &= \int d\alpha \text{PDF}_L(\alpha) L^{-dq\alpha} \\ &\asymp \int d\alpha \exp\{d \ln L [f_{\max}(\alpha) - q\alpha]\}. \end{aligned} \quad (10)$$

Comparing with $\mathbb{E}[(|\psi|_{\max})^{2q}] \sim L^{-d\tau_{\max}(q)}$ yields the usual Legendre relations

$$f_{\max}(\alpha) = q\alpha - \tau_{\max}(q), \quad \alpha = \partial_q \tau_{\max}(q), \quad (11)$$

with saddle condition $q = \partial_{\alpha} f_{\max}(\alpha)$. Here $f_{\max}(\alpha)$ describes large deviations of the *single* maximum per sample (an extreme-value spectrum), not the conventional multifractal spectrum that encodes the geometric distribution of intensities over all links.

Using the parabolic form in Eq. (5), we obtain

$$f_{\max}(\alpha) \approx -\frac{1}{4\gamma}(\alpha - \nu)^2 \quad (12)$$

(see Supplemental Material [27]). Thus, within the measured regime $|q| \lesssim 1$, the raw maxima are described by a local parabolic large-deviation spectrum. This should not be viewed as a complete spectrum over all α , because the parabolic form of $\tau_{\max}(q)$ holds only within the moderate- q window discussed above. A quadratic $f_{\max}(\alpha)$ implies that $\text{PDF}_L(\alpha)$ is approximately Gaussian near its typical value on the $\ln L$ scale. Therefore, $\ln |\psi|_{\max}$ has an approximately Gaussian bulk, or equivalently $|\psi|_{\max}$ is near-log-normal, over the same moment window. This is nontrivial: for weakly dependent variables, one expects convergence to the standard GEV universality classes rather than near-log-normal behavior [8–10]. In the present open geometry, the observed near-Gaussian bulk is further shaped by the fluctuating gain discussed below.

Figure 3 demonstrates this behavior directly. After centering and scaling, $\text{PDF}(\ln |\psi|_{\max})$ collapses toward a Gaussian curve. We quantify the approach to Gaussianity using the skewness, excess kurtosis, and $\sqrt{2\pi} - (\sigma h)^{-1}$ (with h the peak height of the PDF), which vanish for an exact Gaussian. The excess kurtosis and height-based diagnostic remain close to zero across sizes, with mild deviations at the largest L attributable to finite sampling. The skewness decays as aL^{-b} with $\bar{R}^2 > 0.999$, see Supplemental Material [27], providing strong evidence that the standardized bulk of $\text{PDF}(\ln |\psi|_{\max})$ approaches a Gaussian form as L increases (within the sampled window); tail asymptotics may involve additional finite-size or correlation-induced structure. In our largest systems ($L = 1024, 2048$), the ensemble size is of order $O(10^6)$, which sets the scale of the residual statistical noise.

Gain normalization and intrinsic extreme sector. The central ingredient of the open network is the sample-dependent global gain A defined in Eq. (1), whose square gives the total stationary intensity excluding the fixed contact amplitude. A close analog is the integrated intensity in lead-attached critical systems, tied to scattering observables such as Wigner delay times and their moments [26]; our “gain” factor isolates the

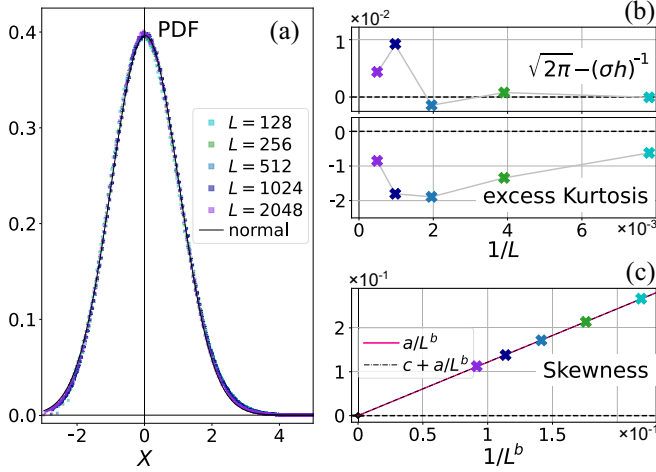


FIG. 3. Near-log-normal statistics of the raw maxima. (a) Probability densities of $\ln|\psi|_{\max}$ shown in terms of the standardized variable $X = (\ln|\psi|_{\max} - \mu)/\sigma$, where μ and σ are the mode and standard deviation of $\text{PDF}(\ln|\psi|_{\max})$, respectively. The curves approach a normal distribution. (b) Two diagnostics of Gaussianity, $\sqrt{2\pi} - (\sigma h)^{-1}$ and the excess kurtosis (with h the peak height of the PDF), remain close to zero; the larger scatter at the largest L is due to finite ensemble size. (c) Skewness versus L . The data are well fitted by aL^{-b} (solid line) with $b = 0.314 \pm 0.002$ and $a = 1.218 \pm 0.016$, while the more general form $aL^{-b} + c$ gives $c \approx 3 \times 10^{-4}$ (dash-dotted line), consistent with vanishing skewness as $L \rightarrow \infty$.

global amplification component entering the extremal statistics.

This global factor A contributes directly to the raw maximum and, as shown in Supplemental Material [27], strongly shapes the observed scaling and near-log-normality. In other words, the raw maximum contains both the global amplification and the intrinsic fluctuation; it therefore probes the amplified response of the open system. In contrast, the normalized maximum probes the spatial concentration of the stationary pattern once the global amplification has been factored out. To focus on these intrinsic fluctuations, we use the gain-normalized maximum

$$|\tilde{\psi}|_{\max} := \frac{|\psi|_{\max}}{A}. \quad (13)$$

The normalized extremes obey their own scaling law, $\mathbb{E}[|\tilde{\psi}|_{\max}^{2q}] \sim L^{-d\tilde{\tau}_{\max}(q)}$, with the resulting exponent function qualitatively different from the raw one. As shown in Fig. 4, $\tilde{\tau}_{\max}(q)$ is dominated by a linear term, well approximated by $-\nu q$, while the residual nonlinear part $\tilde{\Delta}_q^{\max} \equiv \tilde{\tau}_{\max}(q) + \nu q$ is small and asymmetric in q , with visible quadratic and cubic contributions. This should not be viewed as gain normalization destroying an underlying universal law. Rather, it shows that the simple parabolicity of the raw maximum is not an intrinsic property of the extremal fluctuations alone; it emerges from the interplay between critical extremes and the global gain field in the open geometry. Once the gain is normalized out, the maximum reveals an intrinsic extreme sector with its own exponent function.

The probability density of the normalized maxima corroborates this picture. In contrast to the raw variable,

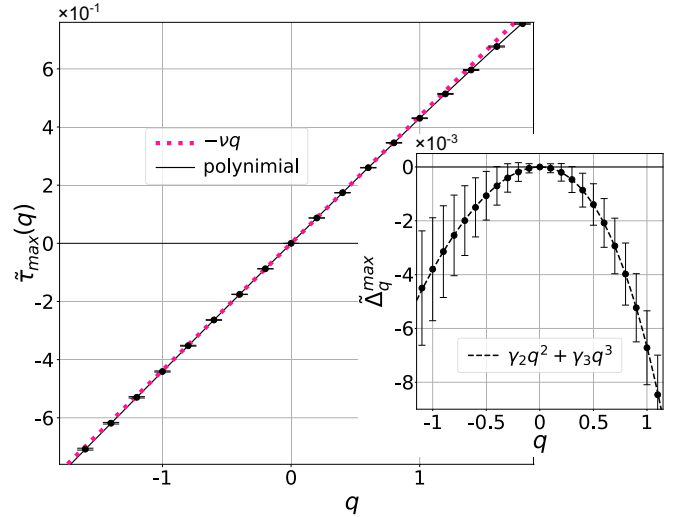


FIG. 4. Scaling exponents for gain-normalized maxima $|\tilde{\psi}|_{\max}$. The exponent $\tilde{\tau}_{\max}(q)$ (solid line) is dominated by a linear contribution $\approx -\nu q$ (dashed line). Inset: the nonlinear remainder $\tilde{\Delta}_q^{\max}$ shows a small but systematic parabolic term and a cubic correction, generating an asymmetry between q and $-q$. The fitted coefficients are $\gamma_2 = -0.005 \pm 2.54 \times 10^{-5}$ and $\gamma_3 = -0.0014 \pm 8.06 \times 10^{-6}$; higher-order terms are also present but subleading in the plotted range.

$\text{PDF}(\ln|\tilde{\psi}|_{\max})$ is no longer close to a single Gaussian form. Instead, it exhibits a compound structure with a Gaussian-like tail at large values together with systematic deviations from a single-parameter GEV description under the same centering/scaling protocol across sizes, see Supplemental Material [27]. This supports that the normalized maximum defines a distinct intrinsic extreme sector, consistent with correlation-dominated critical fluctuations.

Conclusions. We have introduced an extreme-value scaling framework for critical wave functions at the integer quantum Hall transition and applied it to the open Chalker-Coddington network. The raw maximum $|\psi|_{\max}$ exhibits an extreme-moment exponent $\tau_{\max}(q)$ that is approximately parabolic in the accessible moderate- q range, and an extreme singularity spectrum with the corresponding local quadratic form. Consistently, the standardized bulk of $\ln|\psi|_{\max}$ is well described by a Gaussian distribution over accessible sizes, i.e., the raw maxima are near-log-normal in this regime rather than governed by standard i.i.d. extreme-value theory.

The key physical result is that the open geometry endows the stationary state with a fluctuating global gain factor A , so the raw maximum combines collective amplification and intrinsic extremal fluctuations. After normalizing out this gain, the resulting maximum $|\tilde{\psi}|_{\max}$ displays the intrinsic extreme sector. The corresponding intrinsic exponent function and probability density show qualitative differences from those of the raw maximum. Thus the gain decomposition is not merely a change of variables: it separates the amplified response of the open driven system from the spatial rare-event structure of the normalized critical pattern. This distinction is absent in conventional analyses of closed, self-normalized eigenstates and is therefore specific to the open setting of the model.

The raw intensity profile is the quantity most directly connected to a driven or scattering measurement of an open network. The gain factor and the gain-normalized maximum can be constructed once the raw intensity profile is available. These two observables provide complementary probes of the same critical state: one sensitive to global amplification and the other to intrinsic spatial extremes.

Our results establish extreme observables as a sensitive probe of correlated quantum criticality in open systems and open a route to studying rare-event sectors at localization transitions beyond standard moment-based multifractality. Future work includes testing extreme-value criticality and gain decomposition in open CC networks with other symmetry

classes, as well as in broader open quantum models. Another important direction is to establish the corresponding framework away from the critical point, where the scale-invariant multifractal correlations studied here should no longer persist.

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Data availability. The data that support the findings of this Letter are available from the authors upon reasonable request.

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- [1] J.-P. Bouchaud and M. Mézard, Universality classes for extreme-value statistics, *J. Phys. A: Math. Gen.* **30**, 7997 (1997).
 - [2] P. Le Doussal and C. Monthus, Exact solutions for the statistics of extrema of some random 1D landscapes, application to the equilibrium and the dynamics of the toy model, *Physica A* **317**, 140 (2003).
 - [3] Y. V. Fyodorov and J.-P. Bouchaud, Freezing and extreme-value statistics in a random energy model with logarithmically correlated potential, *J. Phys. A: Math. Theor.* **41**, 372001 (2008).
 - [4] S. Raychaudhuri, M. Cranston, C. Przybyla, and Y. Shapir, Maximal height scaling of kinetically growing surfaces, *Phys. Rev. Lett.* **87**, 136101 (2001).
 - [5] G. Györgyi, P. C. W. Holdsworth, B. Portelli, and Z. Rácz, Statistics of extremal intensities for Gaussian interfaces, *Phys. Rev. E* **68**, 056116 (2003).
 - [6] J. Rambeau and G. Schehr, Extremal statistics of curved growing interfaces in 1+1 dimensions, *Europhys. Lett.* **91**, 60006 (2010).
 - [7] M. L. Mehta, *Random Matrices*, 3rd ed. (Elsevier (Academic Press), Amsterdam, 2004).
 - [8] R. A. Fisher and L. H. C. Tippett, Limiting forms of the frequency distribution of the largest or smallest member of a sample, *Math. Proc. Cambridge Philos. Soc.* **24**, 180 (1928).
 - [9] B. Gnedenko, Sur la distribution limite du terme maximum d’une série aléatoire, *Ann. Math.* **44**, 423 (1943).
 - [10] M. R. Leadbetter, G. Lindgren, and H. Rootzén, *Extremes and Related Properties of Random Sequences and Processes*, Springer Series in Statistics (Springer-Verlag, New York, 1983).
 - [11] F. Iglói and H. Rieger, Recent advances in the theory of disordered systems, *Eur. Phys. J. B* **93**, 90 (2020).
 - [12] E. Hernández-Lemus, Random fields in physics, biology and data science, *Front. Phys.* **9**, 641859 (2021).
 - [13] A. Lakshminarayan, S. Tomsovic, O. Bohigas, and S. N. Majumdar, Extreme statistics of complex random and quantum chaotic states, *Phys. Rev. Lett.* **100**, 044103 (2008).
 - [14] W.-H. Li and A. A. Saberi, Eigenstate plateau transition and equilibration in one-dimensional quantum lattice models, *Phys. Rev. B* **110**, 184104 (2024).
 - [15] Y. V. Fyodorov and O. Giraud, High values of disorder-generated multifractals and logarithmically correlated processes, *Chaos Solit. Fractals* **74**, 15 (2015).
 - [16] S. Kettemann and A. M. Tsvelik, Information about the integer quantum Hall transition extracted from the autocorrelation function of spectral determinants, *Phys. Rev. Lett.* **82**, 3689 (1999).
 - [17] F. Evers, A. Mildenberger, and A. D. Mirlin, Multifractality of wave functions at the quantum Hall transition revisited, *Phys. Rev. B* **64**, 241303(R) (2001).
 - [18] F. Evers and A. D. Mirlin, Anderson transitions, *Rev. Mod. Phys.* **80**, 1355 (2008).
 - [19] B. Kramer, T. Ohtsuki, and S. Kettemann, Random network models and quantum phase transitions in two dimensions, *Phys. Rep.* **417**, 211 (2005).
 - [20] H. Obuse, A. R. Subramaniam, A. Furusaki, I. A. Gruzberg, and A. W. W. Ludwig, Boundary multifractality at the integer quantum Hall plateau transition: Implications for the critical theory, *Phys. Rev. Lett.* **101**, 116802 (2008).
 - [21] H. Obuse, A. R. Subramaniam, A. Furusaki, I. A. Gruzberg, and A. W. W. Ludwig, Conformal invariance, multifractality, and finite-size scaling at Anderson localization transitions in two dimensions, *Phys. Rev. B* **82**, 035309 (2010).
 - [22] K. S. Huang, S. Raghu, and P. Kumar, Numerical study of a dual representation of the integer quantum Hall transition, *Phys. Rev. Lett.* **126**, 056802 (2021).
 - [23] S. S. Babkin, J. F. Karcher, I. S. Burmistrov, and A. D. Mirlin, Generalized surface multifractality in two-dimensional disordered systems, *Phys. Rev. B* **108**, 104205 (2023).
 - [24] J. T. Chalker and P. D. Coddington, Percolation, quantum tunnelling and the integer Hall effect, *J. Phys. C* **21**, 2665 (1988).
 - [25] R. Bondesan, D. Wieczorek, and M. R. Zirnbauer, Pure scaling operators at the integer quantum Hall plateau transition, *Phys. Rev. Lett.* **112**, 186803 (2014).
 - [26] A. D. Mirlin, Y. V. Fyodorov, A. Mildenberger, and F. Evers, Exact relations between multifractal exponents at the Anderson transition, *Phys. Rev. Lett.* **97**, 046803 (2006).
 - [27] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/wzs6-3s61> for the details of the time evolution and stationary states of open CC networks, the parabolic scaling of maxima $|\psi|_{\max}$, the adjusted R^2 values, the full-wave multifractal exponent, and the statistics of the global gain A and the normalized maxima $|\tilde{\psi}|_{\max}$, which includes Refs. [17,18,23,26,28–31].
 - [28] F. Evers, A. Mildenberger, and A. D. Mirlin, Multifractality at the quantum Hall transition: Beyond the parabolic paradigm, *Phys. Rev. Lett.* **101**, 116803 (2008).

- [29] M. R. Zirnbauer, The integer quantum Hall plateau transition is a current algebra after all, *Nucl. Phys. B* **941**, 458 (2019).
- [30] H. Obuse, I. A. Gruzberg, and F. Evers, Finite-size effects and irrelevant corrections to scaling near the integer quantum Hall transition, *Phys. Rev. Lett.* **109**, 206804 (2012).
- [31] R. Klesse and M. R. Zirnbauer, Point-contact conductances from density correlations, *Phys. Rev. Lett.* **86**, 2094 (2001).
- [32] T. C. Halsey, M. H. Jensen, L. P. Kadanoff, I. Procaccia, and B. I. Shraiman, Fractal measures and their singularities: The characterization of strange sets, *Phys. Rev. A* **33**, 1141 (1986).
- [33] H. Touchette, The large deviation approach to statistical mechanics, *Phys. Rep.* **478**, 1 (2009).
- [34] Advanced study group “Strongly correlated extreme fluctuations,” <https://www.pks.mpg.de/asg2024>.