

Localized Fock space cages in kinetically constrained models

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We investigate a mechanism for nonergodic behavior arising from destructive interference in Fock space, leading to *Fock space cages* (FSCs)—exact zero-energy eigenstates localized on $O(1)$ to $O(L^p)$ configurations within an exponentially large connected Krylov sector. Unlike many-body localization or Hilbert space fragmentation, FSCs emerge from graph-theoretic interference cancellations in disorder-free kinetically constrained systems with chiral symmetry. We develop systematic algorithms that enable explicit construction of these cages in four representative models, revealing this as a universal mechanism for kinetically constrained systems with chiral symmetry. Dynamically, FSCs produce persistent plateaus in the Loschmidt echo and magnetization: return probabilities scale as L^{-2} for $O(L)$ cages and remain $O(1)$ for ultralocal ones, while the magnetization saturates to $O(1)$ instead of decaying to zero. This may provide new routes for engineering long-lived nonthermal states in quantum simulation platforms.

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The question of whether and how closed quantum many-body systems thermalize, typically described by the Eigenstate Thermalization Hypothesis (ETH) [1–5], has obtained significant attention over the past years. While generic quantum systems typically reach thermal equilibrium for local observables, this scenario breaks down in certain cases. Notable examples include integrable systems [6,7] and many-body localized (MBL) phases [8–11]. In both cases, thermalization is circumvented due to an extensive set of conserved quantities [12–14]. These conservation laws lead in turn to nonergodic dynamics, even at high energy densities. A key question that has recently drawn significant attention concerns systems exhibiting intermediate behavior—neither fully localized nor completely ergodic. In this context, several novel classes of models displaying weak ergodicity breaking have reignited interest in quantum thermalization. These models are characterized by dynamics that depend strongly on initial conditions, distinguishing them from both fully thermal and strongly ergodicity-breaking systems. Quantum many-body scars [15–20], arising from a distinct subset of nonthermal eigenstates embedded within a thermal spectrum, are a prominent example that have been observed in Rydberg atom experiments [21,22]. Another example involves fractonic systems, such as one-dimensional (1D) models with conserved $U(1)$ charge and associated dipole moment [23–25], where the Hilbert space fragments into exponentially many disconnected subspaces—a phenomenon known as Hilbert space

fragmentation (HSF). Unlike HSF where dynamics is confined to disconnected sectors, the mechanism we investigate produces localized eigenstates within fully connected, exponentially large Krylov sectors—a fundamentally different route to nonergodicity.

In this work we investigate a mechanism that yields nonergodic behavior due to destructive many-body interference effects, which in turn lead to *exact eigenstates that are localized in Fock space*. This mechanism is akin to well-known phenomena in single-particle physics, such as localization in flat-band systems where destructive interference localizes particles. Notably, flat bands emerging from lattice geometries like the Kagome lattice [see Fig. 1(a)] or, more generally, from line graph constructions exhibit dispersionless energy bands, implying an extensive degeneracy of localized eigenstates [26–28]. In these systems, the kinetic energy is quenched, and interference effects dominate the dynamics. Another prominent example is Aharonov-Bohm cages, where an applied magnetic flux can lead to complete localization due to destructive interference on specific lattice structures [29–31]. We extend this concept to the many-body domain, demonstrating that many-body *Fock space cages* (FSCs) can emerge, leading to localized many-body eigenstates.

To this end, we develop a graph-theoretic framework for generic kinetically constrained models with chiral (particle-hole) symmetry. In Sec. I, we show how this symmetry endows the Fock space graph with a bipartite structure that lower-bounds the number of zero-energy modes via sublattice imbalance, and recast the search for these modes as a parity-check problem, which can be conducted either globally (at exponential cost) or locally (for nonexhaustive result). In Sec. II, we illustrate this framework in three types of kinetically constrained models, which differ in the number and locality of the cages they host. Certain states are fully

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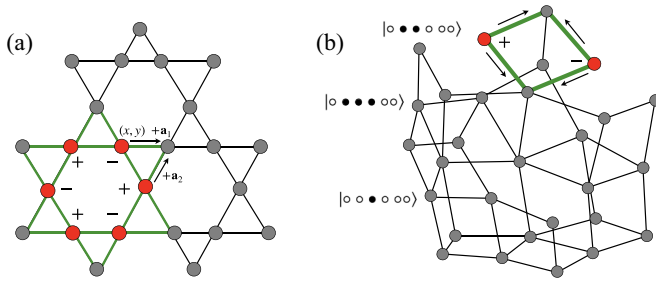


FIG. 1. Real-space flat-band caging $\leftrightarrow$ Fock space caging (a) Single particle on the kagome lattice: destructive interference of nearest-neighbor hoppings around a hexagon localize the single-particle wave function. Red sites are charged, gray ones are neutral. (b) Many-body Fock space: Here, nodes represent bitstring configurations, and edges represent nonzero matrix elements of the Hamiltonian. A similar interference cancellation can localize the many-body wave function here. We refer to such localized states as *Fock space cages* (FSC).

localized on $O(1)$ nodes of the exponentially large Fock space graph and others span $O(L^p)$ nodes (with $p > 0$). Although the Hamiltonians remain globally ergodic—with typical (thermal) eigenstates spreading over an exponentially large fraction of Fock space—each model supports a set of localized zero-energy cage states. We show that the existence of such states leads to nonergodic dynamics when the system is initialized in states with finite overlap with the FSCs. While our detailed case studies focus on these four models, the same graph-theoretic framework applies to a broader class of dynamically constrained models exhibiting an effective chiral or particle-hole symmetry, such as the East model [32–34], the East-West model [35], and the PXP model [19,36,37].

I. GRAPH-THEORETIC FRAMEWORK OF FSCS

Before exploring specific models, we outline a general approach to identify FSCs on bipartite graphs using graph theory. The Fock space naturally lends itself to a graph-based interpretation [see Fig. 1(b)]: bitstring states correspond to nodes s_n , while the nonzero matrix elements of the many-body Hamiltonian $H_{nm} = \langle s_n | H | s_m \rangle$ define the edges. Disregarding the amplitudes of H_{nm} , this gives an undirected graph, where each vertex has degree $O(L)$ for generic local Hamiltonians, and usually a smaller degree for kinetically constrained models. For systems with chiral or particle-hole symmetry, this graph is bipartite, splitting into even-parity (sublattice A) and odd-parity (sublattice B) states. This bipartition lets us bound the number of zero modes (via sublattice imbalance) and construct explicit ones via local interference cancellations.

A. Bipartite graphs

Hamiltonians with a chiral symmetry correspond to bipartite graphs, which in a suitable basis (organized by parity number) can be written as

$$H \rightarrow \begin{pmatrix} 0 & M \\ M^\dagger & 0 \end{pmatrix}, \quad (1)$$

where we write the vertex set as $A \cup B$ with $N_A = |A|$ and $N_B = |B|$, and define the biadjacency matrix

$$M \in \mathbb{C}^{N_B \times N_A}, \quad M_{ba} \equiv \langle b | H | a \rangle, \quad (2)$$

which contains the actual matrix elements that hop between sublattices $A \rightarrow B$ (this is a sub-block of H). Zero modes satisfy $M\psi_B = 0$ or $M^\dagger\psi_A = 0$, hence by rank-nullity

$$\dim \ker H = \dim \ker M + \dim \ker M^\dagger \geq |N_A - N_B|, \quad (3)$$

providing a minimal number of zero-energy states determined by the sublattice imbalance. The bipartite nature of the Fock space graph has another useful consequence: if $N_A \neq N_B$, any zero-energy eigenstate must lie entirely on the larger sublattice. We define the sublattice-parity operator $\Gamma = \text{diag}(\mathbb{I}_{N_A}, -\mathbb{I}_{N_B})$, which anticommutes with H , $\{\Gamma, H\} = \Gamma H + H\Gamma = 0$. Hence, for any $|\psi\rangle$ satisfying $H|\psi\rangle = 0$, $H(\Gamma|\psi\rangle) = -\Gamma H|\psi\rangle = 0$, Γ preserves the zero-mode subspace and diagonalizes it: its $+1(-1)$ eigenspace sits entirely on $A(B)$. We rely heavily on this property to find the FSC's algorithmically.

B. Zero mode manifold

Let us define the total number of zero modes in a system by μ . In systems with translation symmetry, we can resolve the imbalance in each sector, $\mu_k = \text{Tr}_{\mathcal{H}_k}(\Gamma) = \sum_{s \in \text{rep}} p(s_{\text{rep}})$, where $\Gamma = \prod_i Z_i = (-1)^{\sum_i Z_i}$ is the parity of representative state $|s_{\text{rep}}\rangle$ [38], and Z_i is a local Pauli Z operator. The chiral pairing argument guarantees at least $|\mu_k|$ zero-energy eigenstates in that sector. However, the bound in Eq. (3) from chiral and translation symmetry alone is actually very weak, it guarantees at most $O(1)$ or at best $O(L)$ zeros per momentum sector (this is because the sum over representative states gives a small imbalance between even and odd parity orbits).

The key to obtaining an exponentially large zero mode manifold is to further exploit inversion symmetry, $[H, I] = 0$, where $I : i \rightarrow L - i$. Since inversion commutes with translation only in the $k = \{0, \pi\}$ momentum sectors, we can simultaneously resolve both symmetries only in these blocks. However, this suffices to obtain an exponential bound. For example, Ref. [37] improved the $O(1)$ bound on zero modes in the PXP model [18,19] coming from translation and chiral symmetry alone, to an exponential one by resolving inversion symmetry. In Sec. II, we demonstrate the same mechanism for the East-West model, where the counting of inversion-symmetric computational basis states gives an exponential lower bound $\mu_{k=0,\pi} \gtrsim 2^{L/2-1}$ on the zero mode manifold. This strategy—combining chiral, translation, and inversion symmetries—provides a general framework for establishing exponential zero mode proliferation in kinetically constrained systems. Finally, note that the imbalance only sets a (lower) bound on the number of zeros, there can always be additional ones.

C. Fock space cages

A Fock space cage (FSC) is a zero-energy eigenstate, localized by destructive interference on the bipartite Fock space graph, that (i) lives entirely on one sublattice of a single connected Krylov sector of exponential dimension, and, within

this exponentially large space, (ii) has support on at most $O(L^p)$ nodes for some $p \geq 0$. Ultralocal cages with $O(1)$ support are a special case.

Condition (i) distinguishes FSCs from the disconnected sectors produced by Hilbert space fragmentation (HSF); in models with HSF, the search for cages should take place inside the largest sector. While we focus on zero-energy FSCs in this work, type cages can also occur at finite energies [39,40].

The symmetries discussed above (chirality, translation, and inversion) guarantee an exponentially degenerate zero-energy manifold, but not that any eigenstate therein is localized in Fock space. A generic basis for this degenerate subspace will consist of states whose support is spread over exponentially many configurations. Standard exact diagonalization cannot resolve this, since it returns an arbitrary basis for the degenerate subspace. Moreover, spectral graph theory—which relates graph structure to eigenvalue spectra via tools like graph automorphisms—primarily addresses spectral gaps and eigenvalue distributions rather than eigenstate structure. To systematically search for FSC candidates, we develop algorithms that exploit two key properties of zero modes on bipartite graphs: (i) they have support entirely on one sublattice of the chiral decomposition, and (ii) any nontrivial zero mode must form a closed interference loop in the Fock space graph. Condition (ii) of the FSC definition must then be verified separately—either by analytical construction or by confirming that support scales polynomially across system sizes. We employ two complementary algorithms: (1) a parity-check algorithm for global search, and (2) a charge-flow algorithm for local search.

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D. Parity check algorithm

Recall that chiral symmetry confines zero modes entirely to one sublattice [fact (a)]. This lets us recast the eigenvalue problem $M\psi_A = 0$ as a constraint-satisfaction search: we seek assignments $\psi_A \in \{-1, 0, +1\}^{|A|}$ such that each row of M sums to zero. For simplicity, we treat the biadjacency matrix M as a binary connectivity matrix: $M_{ba} = 1$ if nodes b and a are connected, and 0 otherwise. The algorithm fixes all nodes on one sublattice (say B) to zero and searches over ternary values $\{-1, 0, +1\}$ on the other sublattice (A). A valid zero mode satisfies $\sum_{a \in N(b)} \psi_a = 0$ for all $b \in B$, where $N(b)$ denotes the neighbors of b in the A sublattice. Enforcing this zero-sum constraint at every B -site automatically ensures any nontrivial solution forms closed interference loops on the bipartite graph [fact (b)]. To find the complete set of cages, we run the algorithm twice—once with B nodes fixed to zero

(finding cages on A), and once with A nodes fixed to zero (finding cages on B).

Remark. The parity-check search finds many condition (i) states, but only a subset are genuine cages as required by condition (ii). In Sec. II, we list only destructive-interference solutions that are localized according to (ii). Perhaps ironically, the states that do qualify as FSCs are precisely those one could have written down with pen and paper, at least with hindsight. It is their closed-form expressions, valid for all L , that make condition (ii) verifiable in the first place. Nevertheless, we decided to provide the algorithm so that the construction is systematic and reproducible.

E. Charge flow algorithm

When our goal is to identify primarily local cages rather than exhaustively enumerate every zero-energy state, we devise a local “charge-flow” algorithm that is far more efficient. We begin by fixing all B -sites to zero and selecting a random seed B -site (this choice can be refined by analyzing the connectivity of the graph). Next, we pick two of its neighboring A -sites and assign them opposite charges, $+1$ and -1 (leaving all other A -sites at 0). This preserves neutrality at the seed but creates imbalances on its next nearest neighbor B -sites. We continue the construction by iteratively neutralizing the B nodes. For each charged B -site, we assign compensating ± 1 charges to one or more of its unassigned A neighbors until it is neutral. If a B -site cannot be neutralized because no suitable A neighbors remain, we either backtrack to the previous assignment or restart with a new seed. Note that by construction (each B -site is neutralized by assigning two of its neighboring A -sites), this approach only allows us to find cages with two-step cancellation.

II. FSCS IN KINETICALLY CONSTRAINED MODELS

We study four different models to highlight distinct aspects of FSCs. The first two employ “East-minus-West” type constraints $X_i(P_{i-r} - P_{i+r})$: a model supporting a single FSC across $O(L)$ Fock states (Sec. II A), and (Sec. II B) its long-range generalization that hosts cages of different support. The latter two use “East-plus-West” type constraints $X_i(P_{i-r} + P_{i+r})$: (Sec. II C) a model with extensively many FSCs of size $O(L)$, and (Sec. II D) a model with ultralocal FSCs of size $O(1)$.

A. Model with single FSC of size $O(L)$

To illustrate the concept of FSCs, we begin with an extended “East-West model” [41], which, as we will show, hosts a single Fock space cage and displays weak ergodicity breaking. By this we mean that the Hamiltonian is globally ergodic—its spectral statistics and eigenstate entanglement follow random-matrix predictions—but the presence of an exact FSC produces robust nonergodic dynamical signatures for initial states with finite overlap with the cage. Ergodicity is therefore only weakly broken and depends sensitively on the choice of initial state. This makes it a minimal yet insightful setting for exploring the interplay between ergodicity and localization in dynamically constrained systems.

The model consists of a one-dimensional lattice with spin degrees of freedom, $|0\rangle = |\circ\rangle$ and $|1\rangle = |\bullet\rangle$ denoting empty and occupied sites, respectively. Spin flips X_i at site i are governed by constraints involving either the nearest or next-nearest neighbors to the left (right). The Hamiltonian reads

$$H = J \sum_i [X_i(P_{i-1} - P_{i+1}) + X_i(P_{i-2} - P_{i+2})], \quad (4)$$

where $P_i = \frac{1-Z_i}{2}$ is a projector onto the $|\bullet\rangle$ state. All terms in the Hamiltonian anticommute with $\Gamma = \prod_i Z_i$, and thus the model exhibits a chiral symmetry. The choice of boundary conditions plays an important role: to access the single-cage regime, translation invariance must be broken, which we achieve by imposing open boundaries. The model transitions from a multicage to a single-cage structure at zero energy either through such symmetry breaking or by introducing disorder, as discussed in Sec. II E.

Importantly, despite the strong kinetic constraints of the model, the system does not exhibit Hilbert space fragmentation [23–25]. In fact, apart from the trivial empty state $|\circ \circ \dots \circ\rangle$, all remaining $2^L - 1$ configurations lie in a single connected Krylov sector of H . We therefore restrict to the $(2^L - 1)$ -dimensional Krylov subspace generated from the seed state $|s_0\rangle = |\bullet \circ \dots \circ\rangle$.

Within the Krylov space, the Hamiltonian is chaotic and exhibits level-spacing statistics consistent with random-matrix theory. The average adjacent-gap ratio is defined as $r_n = \frac{\min(\Delta E_n, \Delta E_{n+1})}{\max(\Delta E_n, \Delta E_{n+1})}$, where $\Delta E_n = E_n - E_{n-1}$ [42,43], and yields $\langle r \rangle \approx 0.53$, as shown in Fig. 2(a). This is indicative of the Gaussian orthogonal ensemble (GOE); chiral symmetry suggests a slight shift toward chGOE, but numerically $\langle r \rangle$ remains near 0.53.

To further characterize the eigenstates in this Krylov sector, we consider their entanglement structure. For each eigenstate ψ_n at energy E_n , we compute the von Neumann entropy of the reduced density matrix on a half chain. As seen in Fig. 2(b), the entanglement entropy follows the Page curve [44] for midspectrum states, with two clear outliers: the empty state ($S = 0$), which lies in its own Krylov sector, and the nonthermal FSC.

While the Krylov space scales exponentially with L , the zero-energy FSC is localized on L nodes:

$$|FSC\rangle = \frac{1}{\sqrt{L}} \sum_{i=0}^{L-1} |2^i\rangle = \frac{1}{\sqrt{L}} (|\bullet \circ \dots \circ\rangle + \dots + |\circ \circ \dots \bullet\rangle). \quad (5)$$

Here we introduce a binary representation of the bit strings such that $|2^i\rangle$ corresponds to bitstrings with a single 1 at position i . The FSC behaves as a simple $k = 0$ single-particle state—despite the Hamiltonian not conserving particle number. Its exact eigenstate nature arises from perfect destructive interference, which dynamically confines the system to this localized cage state of L nodes. Figures 2(c) and 2(d) illustrate this localization for $L = 6$ and $L = 10$, respectively. Despite residing in the middle of the spectrum, the FSC exhibits area-law entanglement $S = \log 2$ well below the Page value [see Fig. 2(b)]. The FSC state thus represents an exact scar state.

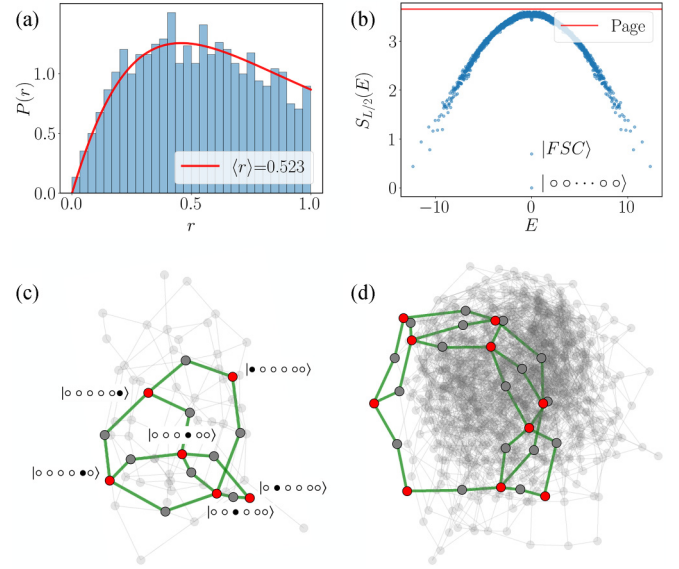


FIG. 2. (a) Distribution of the adjacent-gap ratio and (b) half-system entanglement entropy for the single-cage model Eq. (4) on $L = 12$ with open boundary conditions. The trivial state $|\circ \circ \dots \circ\rangle$ (which spans its own Krylov subspace) and the FSC appear as outliers at zero energy. Despite the model's global ergodicity, the cage shows sub-volume-law entanglement and remains localized in Fock space. (c) and (d) Visualization of the Fock space cage Eq. (4) on the bipartite state-space graph for system sizes with open boundary conditions $L = 6$ (c) and $L = 10$ (d). The single-FSC lives on the red A nodes, the blue B nodes are neutral.

Next we study the effect of FSC on the real-time dynamics. We begin by examining the return probability

$$\mathcal{L}(t) = |\langle \psi(0) | \psi(t) \rangle|^2, \quad (6)$$

starting from a simple bit-string configuration $|\psi(0)\rangle = |\circ \circ \dots \circ \bullet \circ \dots \circ\rangle$. This initial state has an overlap of $1/\sqrt{L}$ with the $|FSC\rangle$, leading us to expect that $|\langle \psi(0) | \psi(t \rightarrow \infty) \rangle|^2 \propto \frac{1}{L^2}$, in contrast to the exponentially small values characteristic of ergodic systems. This expectation is confirmed by the numerical results presented in Fig. 3(a). Next, we analyze the time evolution of the total magnetization,

$$Z_{\text{total}} = \sum_{i=1}^L \langle \psi(t) | Z_i | \psi(t) \rangle, \quad (7)$$

where Z_i denotes the Pauli Z operator at site i . As before, we use the same bit-string configuration as the initial state. Figure 3(b) displays the behavior of $Z_{\text{total}}(t)$, which saturates to a value of order one rather than decaying to zero, as would be expected in an ergodic system without conservation of Z_{total} . This behavior can once again be attributed to the non-negligible overlap between the initial and the FSC state.

B. Model with multiple FSCs of size $O(L^p)$

Let us generalize Eq. (4) to arbitrary distances r , and enforce periodic boundary conditions,

$$H = J \sum_i X_i(P_{i-r} - P_{i+r}). \quad (8)$$

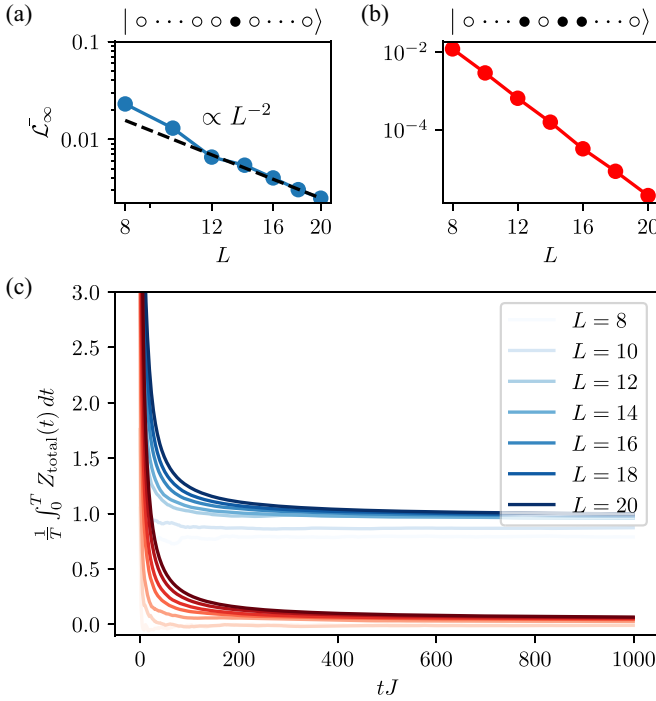


FIG. 3. Quench dynamics of the single cage model Hamiltonian Eq. (4) with open boundary conditions, starting from two different initial states for various system sizes L . (a) Saturation value of the return probability for a bit string that has order $\frac{1}{\sqrt{L}}$ overlap with the FSC state (blue) and (b) for a bit string that has no overlap with it (red). The dashed line in (a) shows the expected slope of $\propto L^{-2}$ as a guide to the eye. (c) Time-averaged expectation value of the magnetization is shown for the two states.

In this case, we have a plethora of Fock space cages, namely, the fully permutation-symmetric weight k states defined as

$$|S_k^L\rangle = \binom{L}{k}^{-1/2} \sum_{b \in \{0,1\}^L, |b|=k} |b\rangle, \quad (9)$$

where the sum is restricted over bitstring states with Hamming weight k . It is straightforward to see why: Fix any computational basis state $|a\rangle$. Its amplitude in $H|S_k^L\rangle$ is, up to normalization, $\sum_i (\delta_{y_{i-r},1} - \delta_{y_{i+r},1})$ if flipping site i takes $|\alpha\rangle$ to weight k , and 0 otherwise. That is, if $|a| = k+1$, the condition is $a_i = 1$, and the sum reduces to $\sum_{i:a_i=1} (\delta_{y_{i-r},1} - \delta_{y_{i+r},1}) = \sum_i a_i a_{i-r} - \sum_i a_i a_{i+r} = 0$. The cancellation comes from the relabeling that we can perform due to having periodic boundaries. The case $|a| = k-1$ is completely analogous: only sites with $a_i = 0$ contribute, and rewriting in terms of $1 - a_i$ gives the same cancellation. Intuitively, on a ring, for any configuration, the number of East-enabled flips equals the number of West-enabled flips, and the uniform superposition makes them cancel exactly. The normalization tells us exactly what the support of each such state is, and in general scales as $\Theta(L^k)$. This shows that long-range kinetically constrained interactions can host cages too.

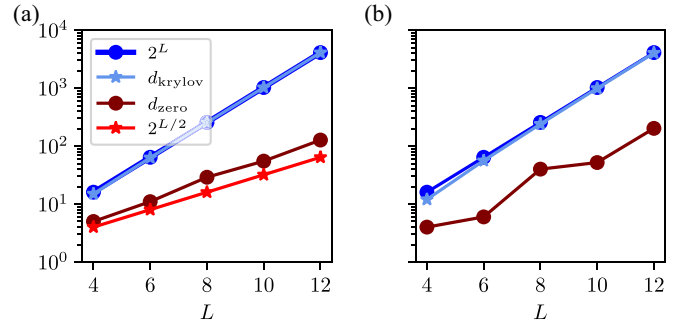


FIG. 4. Scaling of the Krylov subspace d_{krylov} dimension and the number of zero energy eigenstates d_{zero} for Hamiltonians (a) Eq. (10) and (b) Eq. (13) with periodic boundary conditions.

C. Model with multiple FSCs of size $O(L)$

We now discuss a model that exhibits an extensive number of $O(L)$ FSC states, described by the Hamiltonian

$$H = J \sum_i X_i (P_{i-1} + P_{i+1}). \quad (10)$$

As in the previously considered model, all terms in the Hamiltonian anticommute with $\mathcal{P} = \prod_i Z_i$, and thus the model exhibits chiral symmetry. Additionally, this model has translation and inversion symmetry. The combined effect of translation and chirality gives us a (weak) lower bound on the number of zero modes through the lattice imbalance of the states in each momentum sectors. The inversion symmetry tightens this bound in the $k = 0, \pi$ sectors (the two commute only in those subspaces). Since our model has no HSF beyond the all -0 state, which forms its own one-dimensional sector, we only need to count the number of inversion-invariant strings. For even $L = 2\ell$ (pbc), inversion-fixed bitstrings must have an even Hamming weight: the number with $2n$ ones is $\Delta_{2\ell,2n} = \binom{\ell}{n}$ (and $\Delta_{2\ell,2n+1} = 0$), summing to $\sum_n \Delta_{2\ell,2n} = 2^\ell$. These states split approximately evenly between $k = 0$ and $k = \pi$ (up to $O(1)$ corrections), giving the sectorwise bound $N_0(k = 0, \pi) \geq 2^{L/2-1}$.

The scaling of the number of zero states is shown in Fig. 4, showing good agreement with the analytic prediction and confirming an exponential growth with system size. We have verified that the level statistics within momentum and inversion sectors follow the GOE. The dynamics (not shown) behaves similar to the single cage model in that states which have an overlap with any of the FSC states show nonergodic behavior while other states appear to thermalize.

We can use the algorithm outlined in Sec. IC and explicitly construct the FSCs. Below, we list some closed-form expressions for FSC's of the model using the binary representation of bitstrings:

$$|\text{FSC}_1\rangle = \frac{1}{\sqrt{L}} \sum_{i=0}^{L-1} (-1)^i |2^i\rangle \quad (11)$$

$$|\text{FSC}_2\rangle = \frac{1}{\sqrt{L}} \sum_{i=0}^{L-1} (-1)^i |2^i + 2^{(i+1) \bmod L}\rangle. \quad (12)$$

All the states identified have a support of $O(L)$ nodes and are shown in Fig. 5.

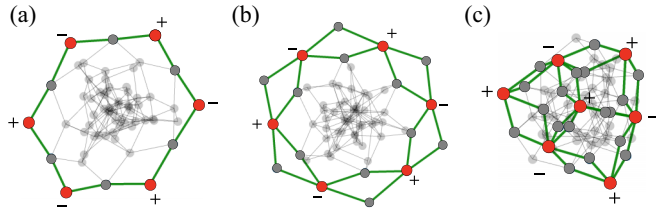


FIG. 5. FSC of size $O(L)$ for the multicage model Eq. (10) for $L = 6$ spins on the $2^L - 1$ dimensional Fock space graph: (a) $|FSC_1\rangle$, (b) $|FSC_2\rangle$, and (c) $|FSC_3\rangle$. Each panel displays a distinct cage on the bipartite graph. These cages grow in size with L and are present for all system sizes. Nodes with zero amplitude (i.e., outside the cage) are shown in gray.

D. Model with multiple FSCs of size $O(1)$

We now discuss a model that exhibits an extensive number of $O(1)$ FSC states, described by the Hamiltonian

$$H = J \sum_{i \text{ even}} X_i(P_{i-1} + P_{i+1}) + J \sum_i X_i(P_{i-1}P_{i-2}P_{i-3} + P_{i+1}P_{i+2}P_{i+3}). \quad (13)$$

Note that the first term is the same as in the previous model except that it is summed only over even sites i . Like the models considered previously, this model exhibits chiral, translation, and inversion symmetries, which protect an extensive number of zero modes, as demonstrated in Fig. 4(b). Apart from the trivial empty state, there exist additional $2^{L/2} - 1$ (frozen) states that are not connected to the largest Krylov sector, but they still constitute a set of measure zero compared to the Hilbert space dimension of 2^L . These states correspond to product states with $\circ$ on all odd sites and an arbitrary number of $\bullet$ on the even sites.

One can now identify $O(1)$ FSCs, which are of the form

$$|FSC\rangle = \frac{1}{\sqrt{2}}(|\circ \dots \circ \bullet \bullet \circ \circ \dots \circ\rangle - |\circ \dots \circ \circ \bullet \bullet \circ \dots \circ\rangle). \quad (14)$$

See also Fig. 6 for a visualization of the FSC. In addition to this pattern, there can be an arbitrary number of isolated $\bullet$ on the even sublattice or other spatially separated cages, leading to an exponential number of such cage states in the

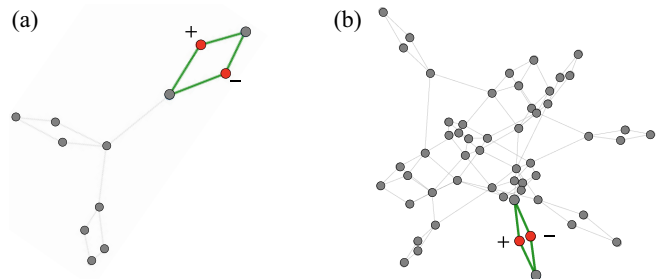


FIG. 6. FSC of size $O(1)$ for Hamiltonian Eq. (13) in the largest Krylov sector for system sizes (a) $L = 4$ and (b) $L = 6$. Nodes with zero amplitude (i.e., outside the cage) are shown in gray.

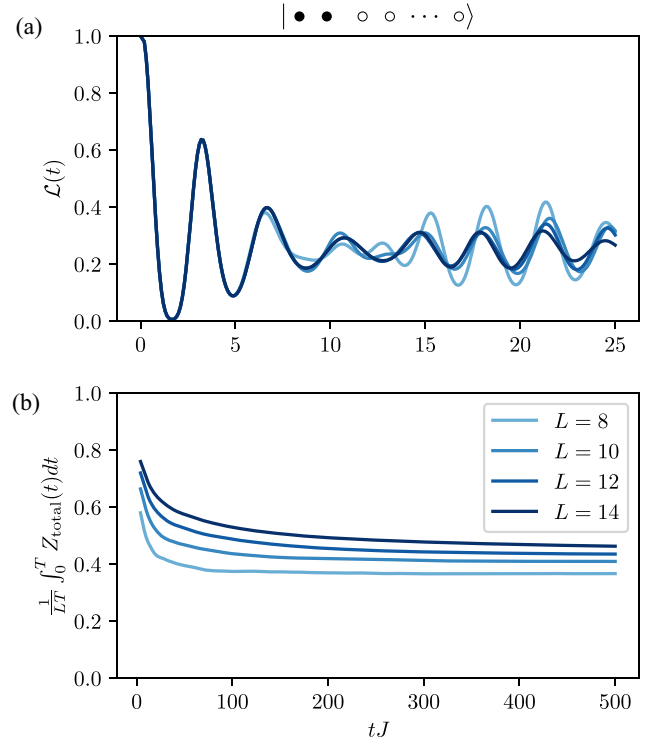


FIG. 7. Quench dynamics of the $O(1)$ cage model starting from an initial state that has order one overlap with the $O(1)$ FSC for Hamiltonian Eq. (13) for different system sizes L with periodic boundary conditions. (a) Return probability and (b) time-averaged expectation value of the magnetization Z_{total}/L .

spectrum. Moreover, there exist many larger FSCs that we do not list here. As in the previous model, the level spacings (not shown) are ergodic within momentum and parity sectors. Finally, we explore the effect of the $O(1)$ cages on the dynamics. Figure 7(a) shows the return probability of the simple bit-string configuration $|\circ \dots \circ \bullet \bullet \circ \circ \dots \circ\rangle$. Since it has an overlap of $1/\sqrt{2}$ with $|FSC\rangle$, we obtain $|\langle\psi(0)|\psi(t \rightarrow \infty)\rangle|^2 \approx \frac{1}{4}$, in contrast to the exponentially small values characteristic of ergodic systems. Figure 3(b) shows the evolution of the magnetization $Z_{\text{total}}(t)/L$, which saturates to a value of order one rather than decaying to zero. Both results are a direct consequence of the strong $[O(1)]$ localization of the FSC.

E. Stability to perturbations

We examine the robustness of FSCs under two classes of perturbations,

$$\delta H_d = \sum_i d_i X_i(P_{i-1} + P_{i+1}), \quad \delta H_x = \sum_i X_i, \quad (15)$$

with periodic boundary conditions.

Let us discuss disorder first. While the uniform model ($d_i = d$ for all i) hosts exponentially many zero modes protected by translation symmetry, introducing disorder ($d_i \neq d_j$) reduces the zero-mode manifold to a single nontrivial zero. Importantly, the corresponding eigenstate remains a cage, and is identical to the clean alternating sign pattern in (11). To see this, write a single-excitation state $|\psi\rangle = \sum_j c_j |j\rangle$. Using

$P_j = |1\rangle\langle 1|$, $\delta H_d|\psi\rangle = \sum_i d_i(c_i + c_{i+1})|i, i+1\rangle$. The interference condition for cancellation is $c_i + c_{i+1} = 0$ for every bond i , which is independent of d_i . Thus, the clean alternating sign FSC remains an exact zero mode even in the presence of disorder.

In contrast, the perturbation δH_x destroys the cage: unconstrained spin flips add $O(L)$ extra edges per node in the Fock space graph, breaking the bond-by-bond destructive-interference mechanism and thereby delocalizing the state.

III. CONCLUSION

In this work, we have investigated a mechanism that leads to nonergodic behavior due to destructive many-body interference effects. The setting is the connected many-body state space of dynamically constrained systems, which hosts localized, exact zero energy eigenstates. We refer to such states as (Fock space) cages. In systems with chiral and translation symmetry, we develop a graph theoretic framework for bounding the dimension of the zero energy manifold and an algorithm to find cage representations. The parity check algorithm does an extensive state space search and can find cages coming from higher order cancellations. In contrast, the charge flow algorithm searches for local two-step cancellations.

We apply these methods to four representative models: (i) a single-cage model with one $O(L)$ FSC, and three models with exponentially degenerate zero manifolds—(ii) a long-range variant hosting multiple $O(L^p)$ FSCs, (iii) a model where we construct several $O(L)$ -sized FSCs, and (iv) a model supporting numerous ultralocal $O(1)$ FSCs.

In every case, these cages imprint clear nonergodic signatures in the dynamics—most notably in the long-time return probability and steady-state magnetization—even though the Hamiltonian as a whole remains ergodic.

Our results reveal a universal interference-driven localization mechanism in Fock space, closely analogous to flat-band and Aharonov-Bohm cages in single-particle physics, yet acting in the exponentially large many-body Hilbert space. Unlike Hilbert space fragmentation, this mechanism produces localized states within fully connected sectors. Although we focused on localized FSCs at zero energy in the paper, they also occur at finite energies.

Looking forward, a number of open questions remain: a systematic spectral-graph analysis of cage subgraphs, a complete classification of all FSCs in broader model classes, and the inverse problem of engineering Hamiltonians with prescribed cages. Additionally, understanding the stability of the exponential zero-mode degeneracy—whose spectral properties are captured by chiral random matrix theory with its characteristic bathtub-shaped DOS and hard-edge scaling near zero energy—to experimental perturbations will be crucial for realizing these phenomena. The distinctive spectral signatures, including the scaling of the gap with sublattice imbalance and system size, may provide experimental diagnostics for identifying FSCs in quantum simulators. A promising setup for realizing FSCs experimentally is the Rydberg platform in which constrained dynamics can be implemented naturally.

Note added. Recently, we became aware of three related preprints [39,40,45] that independently explore interference-driven localization in Fock space. [40] formulate dynamics on Fock space graphs and, and focus on the same mechanism of interference cancellation as giving rise to “interference-caged quantum many-body scars” (what we call cages). They review analytically constructed many-body scars from earlier works [46,47], reinterpreting them in the interference-caging framework, and also construct new cages in 2-D $U(1)$ quantum link and quantum dimer models. Their work provides valuable insight into the topological stability of certain types of these cages: any real-space local perturbation that leaves the relevant interference pattern intact does not change the eigenvalue equation, and the cage remains an eigenstate. For systematic searches, they propose a graph-automorphism-guided search protocol for scars, which efficiently identifies cages with two-step cancellation patterns. In contrast, our parity-check algorithm captures cages with higher-order interference processes, enabling a more complete characterization of the FSC landscape.

[39] establish connections between sparse random graphs and physical lattice models, identifying flat bands at nonzero energies through tree-grafting arguments, and probe them using a signature (Edwards-Anderson parameter) rather than relying on constructing them explicitly. The tree-grafting nature of their cages resembles our setting with extensive $O(1)$ -support cages in Sec. II D. Similar to our work, they show Loschmidt-echo signatures including many-body Rabi oscillations when an initial state overlaps with multiple bands.

[45] study kinetically constrained East-West models with chiral and $U(1)$ symmetries, deriving exponential lower bounds on zero modes at fixed filling and constructing “bound” states that decouple in real space at boundaries. In contrast, our FSCs are localized in Fock space—confined to $O(L^p)$ nodes of the exponentially large Fock space graph. We thus focus on models with different symmetry classes and explore a different notion of locality.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [48], embargo periods may apply.

APPENDIX: PARITY CHECK ALGORITHM

We discuss some details of the parity-check backtracking algorithm from the main text (Sec. ID).

1. Sketch

We now give a sketch of the algorithm: Let S_b be the running row sum of node b (i.e., it sums the values of the assigned neighbors of b) and U_b the number of unassigned neighbors of b . An optional step is to order the A -sites by descending degree.

(1) Initialize: $\psi_A \leftarrow$ (all unassigned), $S_b \leftarrow 0$, $U_b \leftarrow \deg(b)$ for all b .

(2) Recursion at step k : pick the next A -site a_k and try $a_k \in \{-1, 0, +1\}$.

(3) Update neighbors: for each $b \in N(a_k)$ set $S_b \leftarrow S_b + a_k$ and $U_b \leftarrow U_b - 1$.

(4) Forward check: prune immediately if for any touched b either $S_b + U_b < 0$ or $S_b - U_b > 0$, i.e., $0 \notin [S_b - U_b, S_b + U_b]$; also prune if $U_b = 0$ and $S_b \neq 0$. If $U_b = 1$, force the last neighbor to $-S_b$ if it lies in $\{-1, 0, +1\}$; otherwise prune.

(5) On prune: backtrack—we undo this step's updates (and any forced assignments) and try the next value for a_k ; if all values fail, we return to $k - 1$.

(6) On success: when all A -sites are assigned and all $S_b = 0$, record ψ_A ; backtrack to continue (or stop after the first solution).

This is a standard constraint-satisfaction search (akin to parity-check decoding, but with ternary node values) [49,50]. While the sketch above assumes unit weights and ternary values, the same search extends to weighted constraints and arbitrary finite alphabets.

2. Algorithmic complexity

We briefly comment on the computational complexity of this algorithm (memory cost and runtime). Let E denote the number of edges in the bipartite Fock space graph and d_A (d_B) is the average degree of A (B) nodes ($O(L)$ for local models). Memory use is linear in the graph size: we store the edge list (one entry per edge, $\sim E$) plus a few counters per site (the row sums S_b , remaining counts U_b , and the current partial

assignment/undo list, $\sim N_A + N_B$). Since $N_B = O(N_A)$ here, the total scales as $O(E + N_A)$. Turning to runtime, we count how many search-tree nodes the algorithm visits on average: each node (one A assignment and its neighbor updates) costs $O(d_A)$, so the total time is $O(d_A \times \text{\#nodes})$. In the worst case (no pruning), each variable has three choices, so the search explores $O(3^{N_A})$ nodes and the time is exponential in N_A , $O(d_A 3^{N_A})$. For a mean-field estimate under *saturation-only* pruning (no interval forward check), we model the k neighbors of a just-saturated B row as i.i.d. uniform picks in $\{-1, 0, +1\}$ and assume random order of neighbor assignments, so a touched B row of degree d_B saturates with probability $1/d_B$.

The number of n strings with entries $\in \{-1, 0, +1\}$ and zero sum is given by the central trinomial coefficient T_n , which scales as $T_n \sim \frac{\sqrt{3}}{2\sqrt{\pi}} \frac{3^n}{\sqrt{n}}$ so the probability that a degree- n row sums to zero is $p_n := \frac{T_n}{3^n} \sim \frac{\sqrt{3}}{2\sqrt{\pi n}}$. Now consider a specific step in the algorithm: we assign one A node, which affects d_A B nodes. The probability that this results in fully assigned B rows is $1/d_B$, so the number of newly saturated B rows is $q := d_A/d_B$, which for our models is a constant. The chance that the attempted assignment survives all the newly triggered checks is then $(p_n)^q$, and the expected number of surviving branches per parent branch is

$$b_{\text{eff}} \approx 3(p_n)^q \sim 3\left(\frac{\sqrt{3}}{2\sqrt{\pi n}}\right)^q = \Theta(n^{-q/2}).$$

Here n denotes the typical degree of a checked B row at saturation; in our models $n \sim L$, and with $q \approx 1$ this gives $b_{\text{eff}} \sim \text{const}/\sqrt{L}$ (strictly smaller than the worst case $b = 3$).

With $b_{\text{eff}} < 1$ the expected number of visited nodes up to depth D is the geometric sum $1 + b_{\text{eff}} + b_{\text{eff}}^2 + \dots + b_{\text{eff}}^D = \frac{1 - b_{\text{eff}}^{D+1}}{1 - b_{\text{eff}}} \leq \frac{1}{1 - b_{\text{eff}}}$. The expected size of the explored tree is therefore set by $1/(1 - b_{\text{eff}})$, which remains $O(1)$ for moderate L when $b_{\text{eff}} \sim \text{const}/\sqrt{L}$, and the runtime scales as $O(d_A)$ —i.e., $O(L)$ in our models where $d_A = \Theta(L)$. (If degrees remain $O(1)$ so that $n = \Theta(1)$, this mean-field shrinkage does not apply and b_{eff} becomes a model-dependent constant.) Finally, the full interval forward check only strengthens pruning relative to this saturation-only estimate, so these bounds are conservative.

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- Correction:* The source information in Ref. [48] was incorrect and has been fixed.