

Phase diagram and quench dynamics of a Floquet-engineered one-dimensional quantum droplet

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We theoretically investigate a one-dimensional quantum droplet with spin-orbit (SO) coupling and periodically modulated Raman coupling. Using a Floquet approach, we derive an effective time-independent model that reveals a controlled phase diagram. Crucially, the interplay of quantum fluctuations encoded in the Lee-Huang-Yang correction and SO coupling suppresses the plane-wave phase that is typical of mean-field SO-coupled condensates. The ground state is instead restricted to two distinct phases: a stripe phase, with periodic density modulations, and a zero-momentum phase, with a flat-top density profile. The plane-wave droplet is a metastable state of the system. We demonstrate that the boundary between these phases can be dynamically manipulated by tuning the driving parameters. Furthermore, we explore the system's response to quantum quenches, uncovering a wealth of nonequilibrium phenomena, including the dynamical formation of stable, solitonlike structures. Our work establishes periodic driving as a powerful tool for the dynamical control of self-bound quantum matter and elucidates the fundamental role of quantum fluctuations in shaping the phase diagram of SO-coupled droplets.

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I. INTRODUCTION

Self-bound liquid droplets, arising from a delicate balance of attractive and repulsive interactions, span scales from everyday fluids like water and oil to quantum systems such as atomic nuclei and liquid helium [1,2]. Recently, ultracold atomic gases have emerged as a promising platform for realizing stable, self-bound droplet states in the dilute quantum regime [3]. Such droplets have been observed in Bose-Einstein condensates (BECs) of dipolar gases [4–9] and in Bose-Bose mixtures [10–15]. These self-bound quantum droplets arise from the balance between mean-field interactions and beyond mean-field contributions due to quantum fluctuations, namely, the Lee-Huang-Yang (LHY) correction, which is attractive in one dimension but repulsive in two and three dimensions [16,17]. This kind of quantum matter has already revealed a range of unexpected phenomena, such as the pattern formation of quantum ferrofluid droplets [4–6], driven by long-range anisotropic dipolar interactions, and experimental demonstrations of supersolidity [18–20]. The phase transition from a single quantum-droplet state to a density-modulated supersolid phase [18,19,21,22] has been revealed in dipolar BEC systems, arising from a delicate in-

terplay between long-range anisotropic dipolar interactions, short-range isotropic interactions, and quantum fluctuations.

In BECs, internal atomic states can be engineered to form binary or multicomponent condensates, consisting of different isotopes or hyperfine states of the same or distinct atomic species, coherently coupled via external fields [23]. This versatility provides a universal platform for investigating macroscopic quantum many-body phenomena [24], while the introduction of artificial gauge potentials in ultracold atoms offers powerful tools for realizing and simulating complex quantum systems [25]. In particular, the realization of artificial spin-orbit (SO) coupling in both Bose and Fermi gases—arising from the interplay between a particle's momentum and spin—has established a versatile platform for exploring frontier topics in condensed matter physics, including the spin Hall effect [26], topological insulators [27], atomic electronics [28], topological superfluids [29], and supersolids [30–32]. This synthetic coupling has therefore attracted extensive theoretical and experimental attention, enabling the study of a wide range of exotic quantum phenomena.

Periodic driving serves as a fundamental tool for coherently controlling quantum systems, enabling the realization of novel phases and dynamical behaviors beyond equilibrium [33]. For ultracold quantum gases in optical lattices, periodic driving has been used to manipulate many-body states and their dynamics, such as controlling Mott-superfluid transitions [34] and engineering spinor phases through Floquet modulation of a quadratic Zeeman field [35–37]. Moreover, when combined with artificial gauge fields in BECs, periodic modulation of the s -wave scattering length induces rich nonlinear phenomena, including vortices, Faraday patterns, and parametric instabilities [38–40]. In the context of quantum

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droplets, applying periodic driving could give rise to intriguing dynamical behaviors.

The incorporation of SO coupling into binary quantum droplets not only enriches their ground-state properties but also gives rise to a range of novel dynamical phenomena, such as self-bound bright solitons dominated by quantum fluctuations [41,42], supersolid striped droplets stabilized by beyond mean-field corrections [43], and parameter-quench induced dynamical behaviors including breathing [44,45], fragmentation, and phase transitions [46,47]. In conventional SO-coupled BECs without the LHY correction, three phases are well established: the zero-momentum phase, the plane-wave phase, and the stripe phase [48]. In SO-coupled quantum droplets, flat-top and stripe droplets have been obtained [30,46,49,50]. However, important questions remain open, including the lack of a complete phase diagram and the unconfirmed existence and underlying mechanism of plane-wave quantum droplets. Furthermore, approaches for dynamically controlling the ground state and excitations of quantum droplets via periodic driving have yet to be systematically explored. To address these challenges, we investigate a one-dimensional SO-coupled quantum droplet system with periodically driven Raman coupling. Our study aims to uncover the physical mechanism responsible for the absence of plane-wave droplets using analytical methods and to demonstrate how Floquet engineering can be employed to manipulate both the structure and dynamics of quantum droplets, potentially enabling the realization of new droplet phases.

In this work, we investigate a one-dimensional SO-coupled quantum droplet subject to periodically driven Raman coupling. We address two central, open questions: first, we determine the complete ground-state phase diagram and elucidate the physical mechanism responsible for the absence of the plane-wave droplet phase. Second, we demonstrate how Floquet engineering can dynamically control both the equilibrium properties and the nonequilibrium dynamics of these self-bound states. We show analytically that the LHY correction energetically suppresses spin-polarized configurations, precluding the plane-wave phase and leaving only stripe and zero-momentum droplets. Furthermore, we reveal that the phase boundary is tunable via the driving parameters and that quenches can induce rich dynamics, including the formation of solitonlike structures.

The paper is organized as follows. In Sec. II, we establish a theoretical framework for one-dimensional SO-coupled quantum droplets subject to periodically driven Raman coupling. Employing the high-frequency approximation, we derive a time-independent effective Hamiltonian and the corresponding extended Gross-Pitaevskii (GP) equation that incorporates the LHY correction. In Sec. III, we use a superposition of two plane waves as the trial wave function and, through variational analysis, determine two ground-state phases (the zero-momentum droplet and stripe droplet), together with their phase boundaries. In Sec. IV, we present the system's phase diagrams across various parameter spaces, showing that the phase boundaries can be tuned by the periodic driving strength. In Sec. V, we examine the formation of self-bound solitonlike structures induced by quenching the periodic driving and SO coupling strengths. The paper is summarized in Sec. VI.

II. THEORETICAL MODEL

We consider a Raman-laser induced spin-1/2 BECs with equally weighted Rashba and Dresselhaus SO coupling. The system is confined to a quasi-one-dimensional geometry by a strong transverse potential ($\omega_\perp \gg \omega_x$), which suppresses dynamics in the transverse directions. While the GP equation, derived by minimizing the mean-field energy functional with respect to the condensate wave function, typically provides an accurate description of the trapped BECs, we focus here on a regime where the LHY correction dominates. This occurs for weak interspecies attraction and weak intraspecies repulsion.

Within the mean-field framework, the dynamics of the SO-coupled quantum droplets are described by an extended GP equation [41,42,46]:

$$i\partial_t \Phi = (H_0 + H_I + H_{\text{LHY}})\Phi, \quad (1)$$

where the single-particle and interatomic interaction Hamiltonians are given by

$$H_0 = \frac{1}{2}(\hat{p}_x - k_0 \hat{\sigma}_z)^2 + \frac{\Omega}{2} \hat{\sigma}_x + \frac{\Delta}{2} \hat{\sigma}_z \quad (2)$$

and

$$H_I = \text{diag}(g_{11}|\Phi_1|^2 + g_{12}|\Phi_2|^2, g_{22}|\Phi_2|^2 + g_{12}|\Phi_1|^2), \quad (3)$$

respectively. Here, $\Phi = [\Phi_1, \Phi_2]^T$ is the two-component spinor wave function, $\hat{\sigma}_{x,y,z}$ are the Pauli matrices, and $\hat{p}_x = -i\hbar\partial_x$ is the momentum operator along the x direction. The SO coupling strength k_0 is set by the wavelength and incident angle of the Raman laser, while Ω and Δ denote the Raman coupling strength and two-photon detuning, respectively. We set the detuning to $\Delta = 0$, achievable by appropriately tuning the frequency difference between the two Raman lasers. The LHY correction is

$$H_{\text{LHY}} = \frac{\delta g}{2}(|\Phi_1|^2 - |\Phi_2|^2)\hat{\sigma}_z - \frac{g_{\text{LHY}}^{3/2}}{\pi}\sqrt{|\Phi_1|^2 + |\Phi_2|^2}, \quad (4)$$

where $\delta g = g + g_{12}$ quantifies the imbalance between the inter- and intraspecies mean-field interactions. The system can form a self-bound quantum droplet state under quantum fluctuations when the attractive interspecies interactions ($g_{12} < 0$) and repulsive intraspecies interactions ($g > 0$) are balanced. The strength of the first-order quantum correction of LHY is $g_{\text{LHY}} = g$. This approximation is mainly valid near the mean-field collapse point and may overestimate quantum fluctuations for moderate or weak interspecies coupling.

We adopt dimensionless units to simplify calculations; scaling length, time, energy, and density are denoted by $a_\perp$, $\omega_\perp^{-1}$, $\hbar\omega_\perp$, and $a_\perp^{-1}$, respectively, where $a_\perp = \sqrt{\hbar/(m\omega_\perp)}$ is the transverse harmonic oscillator length. The spinor components, SO coupling strength, and Raman coupling strength are rescaled as $\Phi_{1,2}/\sqrt{a_\perp}$, $k_0/a_\perp$, and $\Omega/\omega_\perp$, respectively. The interaction constants are $g_{ij} = 2Na_{ij}/a_\perp$, which depend on the atom number N and the s -wave scattering lengths a_{ij} , which can be tuned via Feshbach resonances [51]. For simplicity, we assume spin-symmetric interactions, $g_{11} = g_{22} = g$, an approximation well-justified in current experiments [10,12,52].

Our parameters match recent realizations of quantum droplets in a ^{39}K binary mixture [10–12,52]. The pseudospin states $|\uparrow\rangle$ and $|\downarrow\rangle$ correspond to the internal hyperfine states

$|F, m_F\rangle = |1, -1\rangle$ and $|1, 0\rangle$, respectively, where F is the total angular momentum and m_F its projection. The droplet, typically comprising between 10^4 and 10^6 atoms, is confined in a harmonic trap with frequencies $\omega_x = 2\pi \times 50$ Hz and $\omega_\perp = 2\pi \times 800$ Hz, yielding a characteristic length $a_\perp = 0.569 \mu\text{m}$. The dimensionless number of atoms is $N = [1, 100]$. The s -wave scattering lengths are tuned via Feshbach resonance to $a_{11} = a_{22} = -a_{12} = 0.2686a_0/N$ (where a_0 is the Bohr radius), rendering the mean-field interaction nearly negligible. This gives a dimensionless intracomponent interaction strength of $g = g_{\text{LHY}} \approx 0.5$ and $\delta g = 0$. To study the effects of intercomponent imbalance, we introduce a finite δg by varying g_{12} , corresponding to the dimensionless constants $g_{12} = [-0.5, -0.05]$, while keeping $g = 0.5$ fixed.

In conventional setups, the SO coupling is fixed by the laser configuration [53]. To achieve dynamic control, we employ a time-periodic Raman coupling [54,55],

$$\Omega(t) = \Omega_0 + \Omega_M \cos(\omega t), \quad (5)$$

where Ω_0 is the static component, Ω_M is the modulation amplitude, and ω is the modulation frequency. The modulation thus introduces two characteristic energy scales, Ω_M and ω . We apply the high-frequency approximation, requiring ω to be much larger than all other energy scales in the system, a condition validated experimentally [54,55].

We transform the system into an oscillating frame via the unitary transformation:

$$U = \exp\left[-\frac{i\Omega_M \sin(\omega t)}{2\omega} \hat{\sigma}_x\right]. \quad (6)$$

The transformed Hamiltonian is

$$H' = U^\dagger(H_0 + H_I + H_{\text{LHY}})U - iU^\dagger \partial_t U. \quad (7)$$

Averaging over one modulation period $T = 2\pi/\omega$ yields an effective time-independent Hamiltonian:

$$\bar{H} = \frac{1}{T} \int_0^T H' dt = \bar{H}_0 + \bar{H}_1 + \bar{H}_{\text{LHY}}, \quad (8)$$

which is dimension independent. The LHY term in Eq. (4) is specific to one-dimensional (1D) fluctuations, scaling as $n^{3/2}$, with $n = |\psi_1|^2 + |\psi_2|^2$. In higher dimensions, the LHY correction adopts a different density dependence (e.g., $\propto n^{5/2}$ in three dimensions), whereas the Floquet-engineering approach described above remains applicable. The effective single-particle and interaction Hamiltonians are as follows:

$$\bar{H}_0 = \frac{1}{2}(\hat{p}_x^2 + k_0^2) - J_0(\chi)k_0\hat{p}_x\hat{\sigma}_z + \frac{\Omega_0}{2}\hat{\sigma}_x, \quad (9)$$

$$\bar{H}_1 = \begin{pmatrix} g_1|\psi_1|^2 + g_2|\psi_2|^2 & g_3(\psi_1\psi_2^* - \psi_1^*\psi_2) \\ g_3(\psi_1^*\psi_2 - \psi_1\psi_2^*) & g_2|\psi_1|^2 + g_1|\psi_2|^2 \end{pmatrix}, \quad (10)$$

where $\psi = U^\dagger \Phi$ is the transformed wave function. The effective interaction coefficients are

$$\begin{aligned} g_1 &= \{[3 + J_0(2\chi)]g + [1 - J_0(2\chi)]g_{12}\}/4, \\ g_2 &= \{[1 - J_0(2\chi)]g + [3 + J_0(2\chi)]g_{12}\}/4, \\ g_3 &= [1 - J_0(2\chi)](g - g_{12})/4, \end{aligned} \quad (11)$$

where $J_0(\chi)$ is the zero-order Bessel function and $\chi = \Omega_M/\omega$ is the periodic driving strength. The presence of Bessel

functions indicates that both the SO coupling strength and two-body interactions can be tuned dynamically.

Finally, the effective LHY Hamiltonian is

$$\begin{aligned} \bar{H}_{\text{LHY}} &= \frac{\delta g}{2} J_0(\chi)(|\psi_1|^2 - |\psi_2|^2)\hat{\sigma}_z \\ &\quad - \frac{g_{\text{LHY}}^{3/2}}{\pi} \sqrt{|\psi_1|^2 + |\psi_2|^2}. \end{aligned} \quad (12)$$

The unitary transformation defined in Eq. (6) corresponds to moving into a rotating spin frame that undergoes a time-dependent rotation about the x axis by an angle of $\theta(t) = \chi \sin(\omega t)$. In this rotating frame, $\hat{\sigma}_x$ is invariant, while $\hat{\sigma}_y$ and $\hat{\sigma}_z$ rotate as $U^\dagger \hat{\sigma}_y U = \cos \theta \hat{\sigma}_y + \sin \theta \hat{\sigma}_z$ and $U^\dagger \hat{\sigma}_z U = \cos \theta \hat{\sigma}_z - \sin \theta \hat{\sigma}_y$. The explicit time dependence of the rotating basis introduces an additional gauge term, $-iU^\dagger \partial_t U = -\frac{\Omega_M}{2} \cos(\omega t) \hat{\sigma}_x$, which cancels the corresponding periodic driving term. Expanding each term in Eq. (8) and averaging over one period using the identities $\frac{1}{T} \int_0^T \cos \theta dt = J_0(\chi)$ and $\frac{1}{T} \int_0^T \cos^2 \theta dt = \frac{1}{2}[1 + J_0(2\chi)]$ yields the effective Hamiltonians in Eqs. (9)–(12). The Bessel functions $J_0(\chi)$ and $J_0(2\chi)$ arise naturally from this averaging procedure, renormalizing both the SO coupling and the interaction coefficients. Notably, periodic driving renormalizes not only the SO coupling [via $J_0(\chi)$ in Eq. (9)] but also the interaction coefficients g_1 , g_2 , g_3 [via $J_0(2\chi)$ in Eq. (11)] and the spin-dependent LHY term [via $J_0(\chi)$ in Eq. (12)]. This simultaneous renormalization of both single-particle and interaction terms provides two independent control knobs, $J_0(\chi)$ and $J_0(2\chi)$, for manipulating the droplet phases.

We emphasize that the LHY correction used in this work is derived from an expansion around the mean-field collapse point and is rigorously valid only when the system is near this threshold [14,15]. Accordingly, we restrict our analysis to parameters where the mean-field energy is weakly negative or slightly positive—the regime where quantum droplets are observed experimentally [10–12,52]. Our conclusions about phase diagrams and dynamics are therefore specific to this droplet regime and should not be extrapolated to systems with moderate or weak interspecies interactions where mean-field effects dominate.

III. THE VARIATIONAL ANALYSIS

The ground-state properties of a quantum system form the basis for understanding its dynamics. In this work, we employ both an analytical variational method and numerical imaginary-time evolution to identify the ground state. The variational approach allows us to quantitatively reveal the competition between different energy contributions. Since the wave function uniquely characterizes a quantum system's microscopic state, from which all physical properties can be derived, we use the variational method to analyze the ground-state wave function and the associated phase transitions.

The mean-field energy functional for the effective Hamiltonian is given by

$$E = \int dx \left[(\psi_1^*, \psi_2^*) \left(\bar{H}_0 + \frac{1}{2} \bar{H}_1 \right) (\psi_1, \psi_2)^T + \varepsilon_{\text{LHY}} \right], \quad (13)$$

where the energy density from the LHY correction is

$$\varepsilon_{\text{LHY}} = \frac{\delta g}{4} J_0(\chi) (|\psi_1|^2 - |\psi_2|^2)^2 - \frac{2g_{\text{LHY}}^{3/2}}{3\pi} (|\psi_1|^2 + |\psi_2|^2)^{3/2}. \quad (14)$$

The ground state of the system is found by minimizing the energy functional in Eq. (13). A critical step in the variational method is the choice of an appropriate trial wave function. For $\Delta = 0$ and $\Omega_0 < 2J_0^2(\chi)k_0^2$, the lower branch of the single-particle dispersion relation exhibits two minima at

$$k_{\min} = \pm J_0(\chi)k_0 \sqrt{1 - \Omega_0^2/[2J_0^2(\chi)k_0^2]}. \quad (15)$$

For $\Delta = 0$ and $\Omega_0 > 2J_0^2(\chi)k_0^2$, there is only a single minimum at $k_{\min} = 0$. Accordingly, we select a two-component spinor trial wave function of the following form:

$$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \sqrt{\bar{n}} \left[C_1 \begin{pmatrix} \cos \theta \\ -\sin \theta \end{pmatrix} e^{ikx} + C_2 \begin{pmatrix} \sin \theta \\ -\cos \theta \end{pmatrix} e^{-ikx} \right], \quad (16)$$

where k is the condensate momentum, $\bar{n} = N/V$ is the average density of atoms, N is the total number of atoms, and V is the system volume. The normalization condition requires that the superposition coefficients satisfy $|C_1|^2 + |C_2|^2 = 1$. The four variational parameters (C_1 , C_2 , k , θ) are then determined by minimizing the energy functional. The parameter θ , which determines the wave function's spin structure, is obtained by minimizing the single-particle energy with respect to θ , yielding $\cos 2\theta = k/[J_0(\chi)k_0]$ for $0 \leq \theta \leq \pi/4$.

Substituting Eq. (16) into Eq. (13) gives the energy per particle:

$$\begin{aligned} \varepsilon &= E/N \\ &= \frac{k_0^2}{2} - \frac{\Omega_0}{2} \sqrt{1 - \left[\frac{k}{J_0(\chi)k_0} \right]^2} - \frac{H(\beta)k^2}{2J_0^2(\chi)k_0^2} + (1 + 2\beta)G_1 \\ &\quad + \frac{\beta}{2} [J_0(2\chi) - 1]G_2 - \frac{2g_{\text{LHY}}^{3/2}}{3\pi} \sqrt{\bar{n}}, \end{aligned} \quad (17)$$

where $\beta = |C_1|^2|C_2|^2$ with $0 \leq \beta \leq 1/4$. The function $H(\beta)$ is defined as

$$\begin{aligned} H(\beta) &= J_0^2(\chi)k_0^2 + [\beta - 1 + J_0(2\chi)(7\beta - 1)]G_2 + 4G_1\beta \\ &\quad + \frac{\delta g}{2} J_0(\chi)(4\beta - 1)\bar{n}, \end{aligned} \quad (18)$$

with $G_1 = (g + g_{12})\bar{n}/4$ and $G_2 = (g - g_{12})\bar{n}/4$.

Minimizing the energy with respect to k by taking its first derivative yields

$$\frac{\partial \varepsilon}{\partial k} = \frac{k}{2J_0^2(\chi)k_0^2} \left[\Omega_0 \left(1 - \frac{k^2}{J_0^2(\chi)k_0^2} \right)^{-1/2} - 2H(\beta) \right]. \quad (19)$$

Setting $\partial \varepsilon / \partial k = 0$ gives $k = 0$ or

$$k = k_0 J_0(\chi) \sqrt{1 - \left[\frac{\Omega_0}{2H(\beta)} \right]^2}. \quad (20)$$

We can see that, for $\Omega_0 > 2H(\beta)$, the energy minimum occurs at $k = 0$, which requires either C_1 or C_2 to vanish to ensure a

valid trial wave function. It indicates that the ground state is in the zero-momentum droplet phase.

For $\Omega_0 < 2H(\beta)$, substituting Eq. (20) into Eq. (17) gives

$$\begin{aligned} \varepsilon(\beta) &= \frac{k_0^2}{2} - \frac{\Omega_0^2 + 4H(\beta)^2}{8H(\beta)} + (1 + 2\beta)G_1 \\ &\quad + \frac{\beta}{2} [J_0(2\chi) - 1]G_2 - \frac{2g_{\text{LHY}}^{3/2}}{3\pi} \sqrt{\bar{n}} \end{aligned} \quad (21)$$

The ground state of the system is determined by minimizing $\varepsilon(\beta)$ in Eq. (21). By examining the first and second derivatives of ε with respect to β , the phase of the ground state can be identified. An energy minimum at $\beta = 1/4$ corresponds to a stripe droplet phase ($C_1 \neq 0$ and $C_2 \neq 0$), while a minimum at $\beta = 0$ corresponds to a plane-wave droplet phase or a zero-momentum droplet phase. However, we find that, in the quantum droplet regime, the energy at $\beta = 0$ is always higher than that at $\beta = 1/4$. Consequently, the ground state consists solely of stripe quantum droplets ($\beta = 1/4$, $k \neq 0$) and zero-momentum droplets ($\beta = 0$, $k = 0$), while plane-wave droplets ($\beta = 0$ and $k \neq 0$) do not appear. This contrasts with SO-coupled BECs without the LHY correction, which exhibit three ground-state phases: stripe, zero-momentum, and plane-wave [48,53]. The LHY correction thus suppresses the plane-wave droplet phase. Physically, these solutions correspond to distinct density profiles. The stripe droplet is formed by the interference of two plane waves, yielding a periodic density modulation with wave vector $2k$. In contrast, the zero-momentum droplet features a uniform, flat-top density profile. A plane-wave droplet would also be spatially uniform but carry finite momentum, yet it is not energetically favored in the ground state.

To illustrate the differences induced by the LHY correction and facilitate comparison with conventional SO-coupled BECs, we hypothetically consider a plane-wave quantum droplet (Phase III). The energy per particle for these phases can be expressed as follows:

$$\begin{aligned} \varepsilon_{\text{I}} &= \frac{k_0^2}{2} - \frac{\Omega_0^2 + 4H(1/4)^2}{8H(1/4)} + \frac{3}{2}G_1 - \frac{2g_{\text{LHY}}^{3/2}}{3\pi} \sqrt{\bar{n}} \\ &\quad + \frac{1}{8} [J_0(2\chi) - 1]G_2, \end{aligned} \quad (22a)$$

$$\varepsilon_{\text{II}} = \frac{k_0^2}{2} - \frac{\Omega_0}{2} + G_1 - \frac{2g_{\text{LHY}}^{3/2}}{3\pi} \sqrt{\bar{n}}, \quad (22b)$$

$$\varepsilon_{\text{III}} = \frac{k_0^2}{2} - \frac{\Omega_0^2 + 4H(0)^2}{8H(0)} + G_1 - \frac{2g_{\text{LHY}}^{3/2}}{3\pi} \sqrt{\bar{n}}. \quad (22c)$$

The critical Raman coupling strength for the phase transition between stripe (Phase I) and zero-momentum (Phase II) droplets is found by equating their energies:

$$\Omega_{0c} = 2H\left(\frac{1}{4}\right) - \sqrt{H\left(\frac{1}{4}\right)\{4G_1 + [J_0(2\chi) - 1]G_2\}}. \quad (23)$$

Equation (23) shows that the phase transition can be effectively tuned by the external driving parameter χ . In the absence of driving ($\chi = 0$) and LHY correction, the transition reduces to the static case of traditional SO-coupled BECs [48].

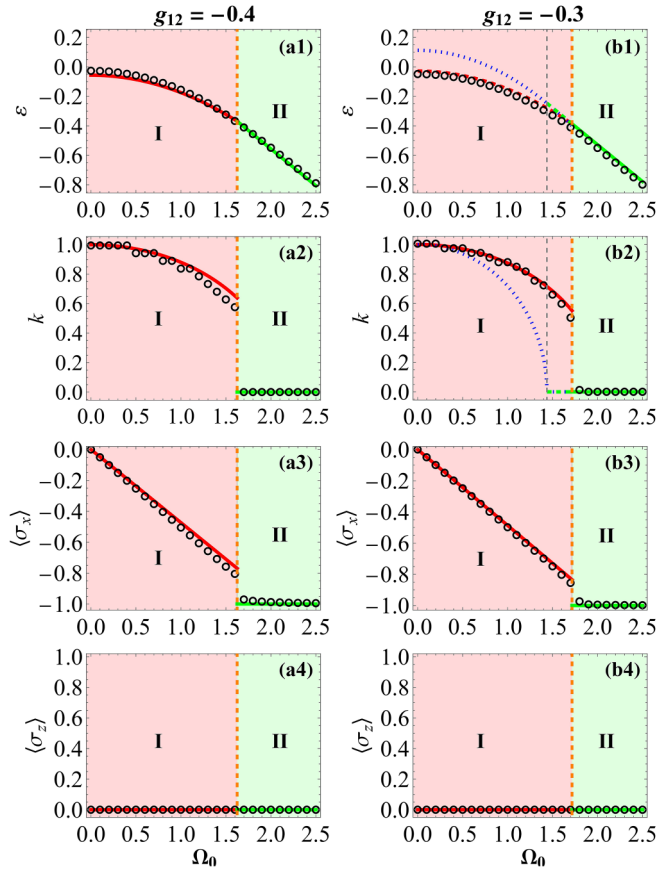


FIG. 1. Ground-state properties of the SO-coupled quantum droplet. The condensate energy per particle ε , the momentum k , and the average spin polarizations $\langle\sigma_x\rangle$ and $\langle\sigma_z\rangle$ are plotted as a function of Raman coupling strength Ω_0 . The left column [panels (a1)–(a4)] corresponds to $g_{12} = -0.4$, $\bar{n} = 2.252$, and the right column [panels (b1)–(b4)] corresponds to $g_{12} = -0.3$, $\bar{n} = 0.563$. Analytical results from the variational method (red and green solid lines) are compared with numerical simulations (black open circles). The vertical orange dotted line marks the phase boundary Ω_{0c} between the stripe droplet (Phase I, light pink background) and the zero-momentum droplet (Phase II, light green background). The gray dotted lines indicate $\Omega_{0\text{II-III}} = 2H(0)$, the critical point for the hypothetical plane-wave state. Analytical curves are as follows: green solid lines for the zero-momentum droplet ($k = 0$, $\beta = 0$); red solid lines for the stripe droplet ($k \neq 0$, $\beta = 1/4$); and blue dotted lines for the hypothetical plane-wave droplet ($k \neq 0$, $\beta = 0$). Other parameters are $g = 0.5$, $k_0 = 1$, and $N = 30$.

IV. THE SELF-BOUND GROUND-STATE PHASES

In this section, to validate the analytical predictions from the variational method, we now turn to numerical simulations of the extended GP equation. Our primary objective is to confirm the absence of a stable plane-wave droplet phase and to verify the phase boundary between the stripe and zero-momentum droplets.

Figure 1 presents a comprehensive comparison between the analytical and numerical results for the ground-state properties as a function of the Raman coupling strength Ω_0 , in the absence of periodic driving ($\chi = 0$). The key observables, the condensate momentum k , the energy per particle ε , and the

average spin polarizations $\langle\sigma_x\rangle$ and $\langle\sigma_z\rangle$, are shown for two different interspecies interaction strengths, $g_{12} = -0.4$ and $g_{12} = -0.3$.

The phase boundary between the stripe and zero-momentum droplet regimes is determined by comparing their respective energy curves. The solid red and green lines in Figs. 1(a1) and 1(b1) represent the energy per particle for the stripe droplet [Eq. (22a)] and the zero-momentum droplet [Eq. (22b)], respectively. The critical Raman coupling strength (phase boundary) Ω_{0c} is identified as the point where these two energy curves cross. For $\Omega_0 < \Omega_{0c}$, the energy of the stripe droplet (red line) is lower, defining Phase I. For $\Omega_0 > \Omega_{0c}$, the zero-momentum droplet energy (green line) becomes lower, defining Phase II. This critical coupling Ω_{0c} matches the value predicted by Eq. (23). The excellent agreement between these analytical results and the numerical data (open circles) obtained from solving the extended GP equation [Eq. (1)] validates our variational analysis. Furthermore, within the stripe droplet phase [pink regions in Figs. 1(a2) and 1(b2)], the momentum k is finite and follows the relation given by Eq. (20), as shown by the red solid lines. Upon crossing into the zero-momentum phase (green regions), k drops to zero (green solid lines). Similarly, the numerical and analytical results for the average spin polarizations [Figs. 1(a3) and 1(b4)] are in good agreement. The blue dotted line in Figs. 1(b2) represents the momentum for a hypothetical plane-wave state ($\beta = 0$, $k \neq 0$), which is consistently higher in energy than the stripe state, as confirmed by the energy comparison in Fig. 1(b1) (the blue dotted line). Under conditions of strong interspecies attraction [Figs. 1(a1)–1(a4)], the critical Raman coupling required for the energetic degeneracy of the plane-wave and zero-momentum states is mathematically negative, thereby ruling out any phase transition between them. The system undergoes a first-order phase transition as Ω_0 increases, characterized by a discontinuous change in the ground-state symmetry. Consistently, the order parameters k and $\langle\sigma_x\rangle$ show discontinuous jumps across the transition.

The consistent absence of the plane-wave phase, both analytically and numerically, can be understood from three complementary perspectives. First, our variational analysis demonstrates that, for $\Omega_0 < 2H(0)$, the energy of a hypothetical plane-wave configuration ($\beta = 0$) is always higher than that of the stripe configuration ($\beta = 1/4$). The system therefore relaxes to the energetically favored stripe droplet. Physically, a plane-wave state in a 1D SO-coupled BEC is nearly fully polarized in a single spin component. This polarization suppresses the intercomponent interaction balance that is crucial for counteracting the combined mean-field and LHY attractions. The stripe phase, in contrast, maintains a balanced population between components, allowing for a more efficient stabilization of the quantum droplet. Consequently, the plane-wave configuration incurs a higher total energy than the stripe droplet, preventing its formation as a stable ground state.

Second, in conventional SO-coupled BECs, the plane-wave phase disappears once the density exceeds the critical value $n_c = k_0^2/(2\gamma g)$ and thus exists only below it. However, the combination of weak intraspecies repulsion and attractive interspecies interaction shifts this tricritical point, n_c , to lower densities [48]. In the self-bound quantum droplet regime, the

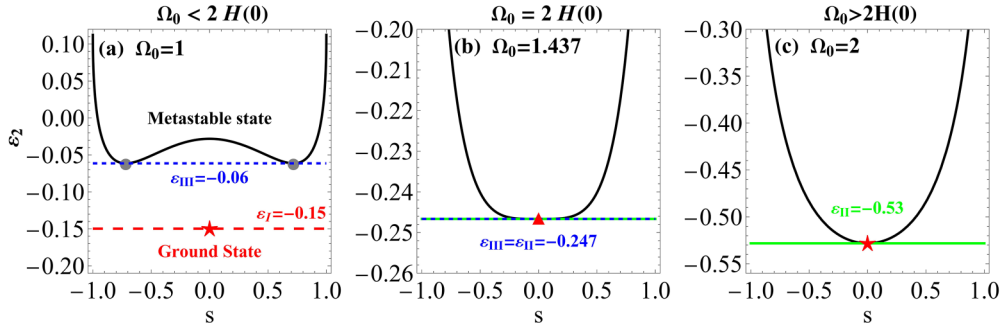


FIG. 2. The energy per particle ε_2 as a function of s for three values of Raman coupling: (a) $\Omega_0 = 1$, (b) $\Omega_0 = 1.437$, and (c) $\Omega_0 = 2$. The red dashed, green solid, and blue dotted lines correspond to ε_I , ε_{II} , and ε_{III} in Eq. (22), respectively. Gray dots indicate the metastable plane-wave droplet state, while red pentagrams mark the ground state—the stripe droplet in panel (a) and the zero-momentum droplet in panel (c). The red triangle in panel (b) identifies the phase transition point. Other parameters are $k_0 = 1$, $\chi = 0$, $g = 0.5$, $g_{12} = -0.3$, and $\bar{n} = 0.563$.

equilibrium density is intrinsically low, often falling below this shifted threshold. Consequently, the density window supporting a plane-wave phase is eliminated, and the stripe phase prevails.

Third, we introduce the phase-transition order parameter $s = \Delta\bar{n}/\bar{n} = \pm \cos 2\theta$ ($-1 \leq s \leq 1$) to characterize the spin-polarized and nonpolarized phases, where $\Delta\bar{n} = \bar{n}_1 - \bar{n}_2$ is the average particle number difference between components. This analysis is conducted within the $\beta = 0$ subspace, which encompasses both the zero-momentum and the hypothetical plane-wave droplets. The corresponding energy per particle, derived from Eq. (17), is

$$\varepsilon_2 = \frac{k_0^2}{2} - \frac{\Omega_0}{2} \sqrt{1-s^2} - \frac{1}{2} H(0) s^2 + G_1 - \frac{2s_{\text{LHY}}^{3/2}}{3\pi} \sqrt{\bar{n}}. \quad (24)$$

Minimizing ε_2 with respect to s yields the condition for equilibrium states:

$$\frac{d\varepsilon_2}{ds} = s \left[\frac{\Omega_0}{2\sqrt{1-s^2}} - H(0) \right] = 0. \quad (25)$$

Then the nonpolarized solution ($s = 0$) represents the zero-momentum droplet when $\Omega_0 > 2H(0)$, while the polarized solution ($s \neq 0$) exists only when $\Omega_0 < 2H(0)$ and represents the plane-wave droplet. To determine the stability of the plane-wave state, we examine the second derivative:

$$\left. \frac{d^2\varepsilon_2}{ds^2} \right|_{s \neq 0} = H(0) \left[\left(\frac{2H(0)}{\Omega_0} \right)^2 - 1 \right]. \quad (26)$$

This reveals a critical insight: For $\Omega_0 < 2H(0)$, the second derivative is positive, indicating that the plane-wave state ($s \neq 0$) is a local energy minimum. However, it is not the global ground state. As shown in Fig. 2(a), when $\Omega_0 < 2H(0)$, the energy minimum of the nonpolarized phase is $\varepsilon_{2\text{min}} = \varepsilon_{III}$. Crucially, the energy of the stripe phase is always lower ($\varepsilon_I < \varepsilon_{III}$). Therefore, while a plane-wave configuration can exist as a metastable excited state, the system's true ground state is the stripe droplet. At the critical point $\Omega_{0\text{II-III}} = 2H(0)$, the metastable plane-wave branch merges with the zero-momentum branch. For $\Omega_0 > 2H(0)$, the only energy minimum in the $\beta = 0$ subspace is the nonpolarized, zero-momentum droplet $\varepsilon_{2\text{min}} = \varepsilon_{II}$. This multifaceted analysis demonstrates that the absence of the plane-wave phase

is not merely a quantitative shift of a phase boundary, but a qualitative change in the phase diagram structure induced by quantum fluctuations.

The phase transition between the stripe and zero-momentum droplets is governed by a competition between different energy contributions. Figure 3 shows the evolution of the LHY correction E_{LHY} , the mean-field energy E_{mean} , the kinetic energy $\tilde{E}_k$, and the total interaction energy $E_T = E_{\text{LHY}} + E_{\text{mean}}$ with respect to the Raman coupling strength Ω_0 . Near the critical point (marked by the orange dotted lines), E_{LHY} exhibits a pronounced minimum, indicating a significant change in quantum fluctuations at the phase transition. In contrast, E_{mean} peaks at the same location, showing that stronger repulsive interactions are needed to counterbalance the enhanced LHY attraction and stabilize the self-bound droplet. The kinetic energy $\tilde{E}_k$ reveals the nature of the structural change. It decreases gradually with Ω_0 in the stripe droplet regime (Phase I), then drops sharply at the critical point to a stable value in the zero-momentum droplet regime (Phase

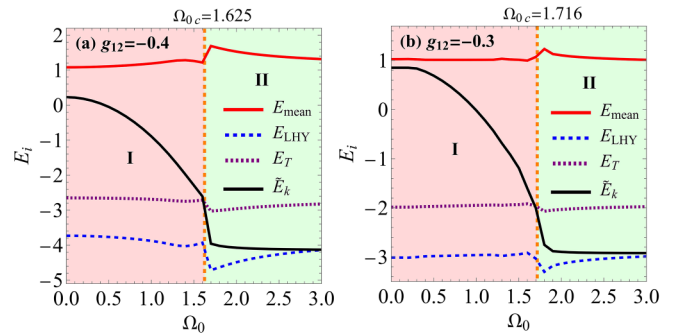


FIG. 3. The single-particle LHY energy E_{LHY} , the mean-field energy E_{mean} , the total interaction energy E_T , and the kinetic energy $\tilde{E}_k$ are shown as functions of Raman coupling strength Ω_0 for different interspecies interactions: (a) $g_{12} = -0.4$ and (b) $g_{12} = -0.3$. The kinetic energy is linearly mapped as $\tilde{E}_k = aE_k + b$ for comparison with other energies in the same plot, where E_k represents the original value obtained from numerical calculations, with (a) $a = 0.292$, $b = -4.193$ and (b) $a = 0.191$, $b = -2.937$. The vertical orange dotted line indicates the phase boundary between the zero-momentum droplet (Phase II, shaded light green) and the stripe droplet (Phase I, shaded light red). Other parameters are $g = 0.5$, $k_0 = 1$, and $N = 30$.

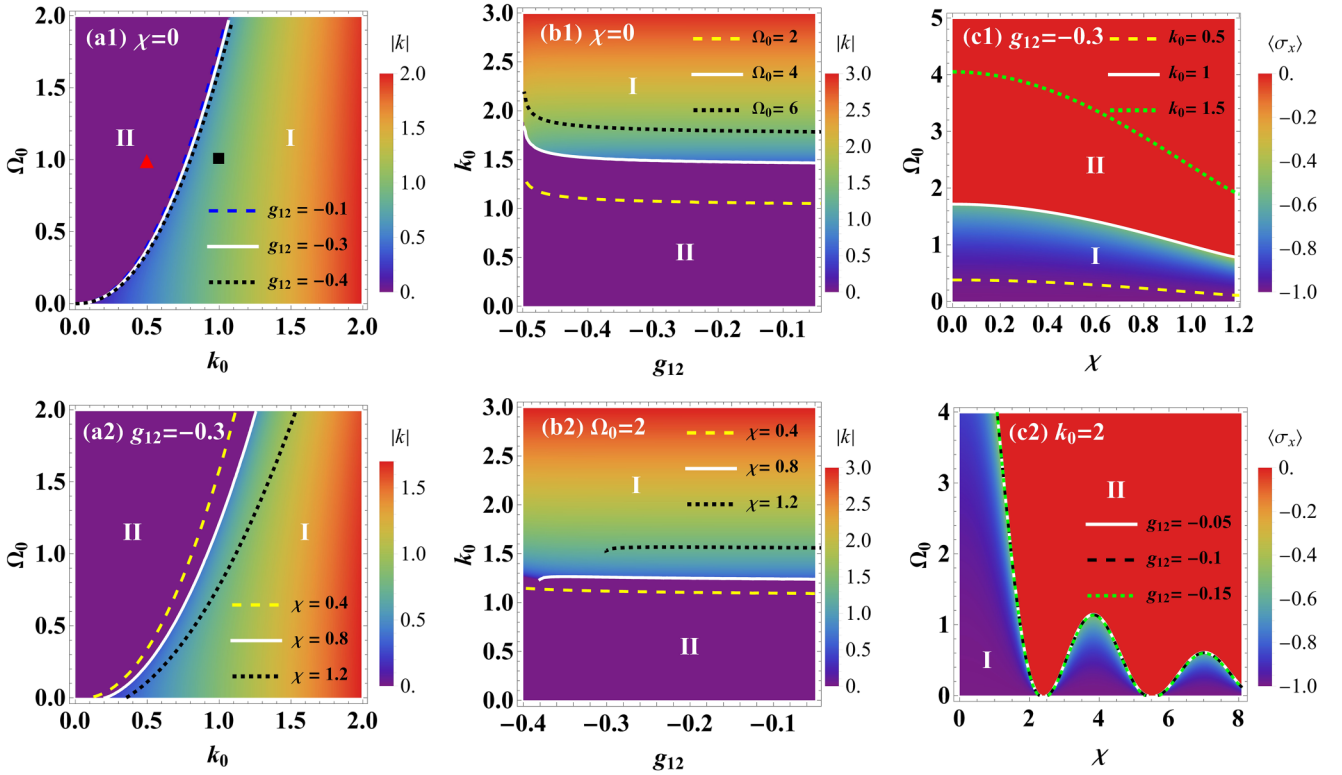


FIG. 4. Ground-state phase diagrams of 1D SO-coupled quantum droplets, showing zero-momentum (Phase II) and stripe (Phase I) droplets. (a1) and (a2): Phase diagrams in the (k_0, Ω_0) plane for different interspecies interactions g_{12} in (a1) and periodic driving strengths χ in panel (a2). In panel (a1), the red triangle and black square denote the zero-momentum droplet and the stripe droplet with momentum $k = 0.875$, respectively. (b1) and (b2): Phase diagrams in the (g_{12}, k_0) plane for different Raman coupling strengths Ω_0 in panel (b1) and χ in panel (b2). Contour plots show the condensate quasimomentum, corresponding to $g_{12} = -0.3$ in panel (a1), $\chi = 0.8$ in panel (a2), $\Omega_0 = 4$ in panel (b1), and $\chi = 0.8$ in panel (b2), respectively. (c1) and (c2): Phase diagrams in the (χ, Ω_0) plane for different SO coupling strengths k_0 in panel (c1) and g_{12} in panel (c2). The contour plots represent the average spin polarization $\langle \sigma_x \rangle$ for $k_0 = 1$ in panel (c1) and $g_{12} = -0.05$ in panel (c2).

II). This abrupt change reflects the different density profiles: the stripe phase, with its spatial modulations, involves significant density gradients and thus higher kinetic energy, whereas the homogeneous zero-momentum phase minimizes this cost. The sharp drop in $\tilde{E}_k$ signifies a first-order phase transition from a modulated state to a uniform state. Notably, the rapid increase of E_{mean} at the transition, facilitated by the drop in kinetic energy, results in denser, more compact droplets in Phase II compared to Phase I. This represents an internal redistribution of energy: the released kinetic energy enhances quantum fluctuations, promoting a smaller, denser self-bound state. Despite these obvious shifts in individual components, the total interaction energy E_T changes slightly across the transition, confirming that the self-bound nature of the droplet is preserved, with only its internal energy balance being re-configured.

Figure 4 presents the global phase diagrams of a 1D SO-coupled quantum droplet with distributions of momentum and average spin polarization, illustrating the interplay between Raman coupling Ω_0 , SO coupling strength k_0 , interspecies interaction g_{12} , and periodic driving strength χ . The system exhibits two ground-state phases: the stripe droplet (Phase I), characterized by periodic density modulations, and the zero-momentum droplet (Phase II), which features a flat-top density profile. As shown in Fig. 4(a), a larger k_0 enhances

density modulations, stabilizing the stripe phase, whereas a stronger Ω_0 suppresses gradients, favoring the uniform zero-momentum phase. This behavior mirrors the mechanism in conventional SO-coupled condensates, where the ground-state momentum is determined by the balance between SO and Raman couplings. The interspecies attraction g_{12} further modulates this balance by reducing the effective mean-field repulsion. As g_{12} becomes more negative [Figs. 4(a1) and 4(b1)], the system relies more heavily on repulsive contributions from kinetic and LHY terms, thereby widening the stability region of the zero-momentum droplet. An interesting special case occurs when the driving strength satisfies $J_0(\chi) = 0$, i.e., $\chi \approx 2.4048, 5.5201, \dots$. At these points, the effective SO coupling term in Eq. (9) vanishes, and the spin-dependent part of the LHY Hamiltonian [Eq. (12)] also disappears. The system consequently reduces to a binary BEC featuring only Raman coupling and renormalized interactions, while the SO coupling drops out. This realizes a complete crossover from a SO-coupled droplet to a non-SO-coupled one, controlled entirely by the driving parameter χ . Experimentally, these points are accessible by adjusting Ω_M/ω , offering a clean method to switch SO coupling on and off *in situ*. In this limit, the ground state is expected to be either a zero-momentum droplet or a phase-separated state, depending on the interaction parameters.

To verify the theoretical predictions, we computed the ground states using imaginary-time evolution and assessed their dynamical stability through real-time simulations with the spectral method. For instance, at a fixed Raman coupling strength ($\Omega_0 = 1$) but with different SO coupling ($k_0 = 0.5$ and $k_0 = 1$), the numerical results represented by the red triangle and black square in Fig. 4(a), respectively, fall into distinct phase regions, consistent with our theoretical predictions. In contrast, periodic driving χ renormalizes the SO coupling via the Floquet mechanism. Increasing χ suppresses spatial modulation, shrinking the stripe phase region while expanding the zero-momentum phase [Fig. 4(c1)]. At small Ω_0 , the periodic driving induces an oscillatory reshaping of the spin polarization, leading to reentrant phase behavior where stripe and zero-momentum droplets alternate with increasing χ [Fig. 4(c2)]. The rich structure in the (χ, Ω_0) plane, including the reentrant phase behavior at small Ω_0 [Fig. 4(c2)], arises from the independent dependence of the phase boundary on $J_0(\chi)$ and $J_0(2\chi)$ in Eq. (23), demonstrating that periodic driving provides two tunable parameters for precisely controlling the competition between quantum fluctuations and mean-field interactions beyond the capabilities of static SO coupling.

The zero-momentum droplet exhibits a distinct structural evolution with increasing particle number N [Figs. 5(a1) and 5(a2)]. At small N , the density profile approximates a Gaussian shape, indicating dominance by mean-field attraction. As N increases, quantum fluctuations grow in importance, and the LHY correction stabilizes a flat-top density distribution. In momentum space, this evolution corresponds to enhanced localization around $k_x = 0$, consistent with the uncertainty principle. In contrast, the stripe droplet follows a different stabilization mechanism. Its periodic density modulation results from interference between spin components induced by SO coupling, with the momentum distribution showing characteristic twin peaks at $|k_x| = 0.881$ [Figs. 5(b1) and 5(b2)]. This confirms the finite quasimomentum nature of the condensate. Compared to the zero-momentum droplet, the stripe phase occupies a larger spatial region. The average density $\bar{n}_0$ of the stripe droplet is smaller than the peak density of the zero-momentum droplet, indicating that a substantial portion of its energy is stored in kinetic gradients associated with the stripe order. This behavior underscores the delicate energy redistribution among kinetic, mean-field, and quantum fluctuation terms as the system optimizes its ground state. Real-time simulations confirm that both droplet phases maintain dynamical stability over extended evolution periods [Figs. 5(a3) and 5(b3)], demonstrating their robustness against quantum fluctuations and validating them as genuine self-bound ground states within the studied parameter regime.

The analytical framework consistently describes the equilibrium droplet density. The periodic driving induces an effective two-body interaction, which in turn modifies the mean-field energy density and thereby alters the flat-top density. To carry out the analysis, we examined the flat-top density for $\chi = 0$. Using the definition of the chemical potential

$$\mu = \frac{\delta g}{2} n - \frac{g_{\text{LHY}}^{3/2}}{\pi} \sqrt{n}, \quad (27)$$

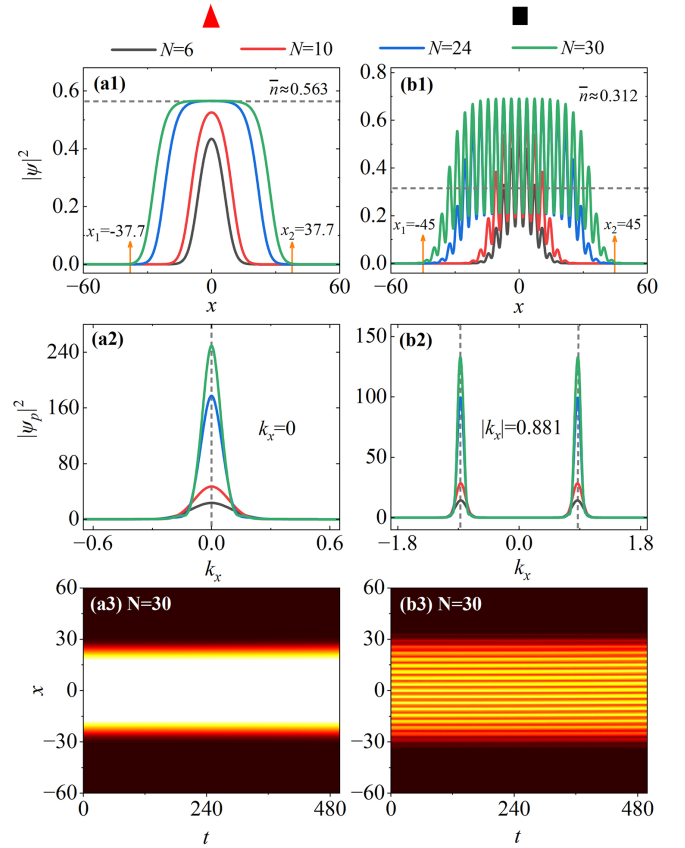


FIG. 5. Ground-state structures and dynamical stability of quantum droplets obtained through imaginary-time evolution. Panels (a1)–(a3) and (b1)–(b3) correspond to the zero-momentum droplet (red triangle, $k_0 = 0.5$) and the stripe droplet (black square, $k_0 = 1$), respectively, as identified in Fig. 4(a). The top and middle rows display density distributions for different particle numbers N in real space and momentum space, respectively. Gray dotted lines in panels (a1) and (b1) indicate the peak density and average density, respectively, while they indicate the condensate quasimomentum in panels (a2) and (b2). The bottom row shows the temporal evolution of the perturbed ground state for fixed particle numbers. Other parameters are $g = 0.5$, $g_{12} = -0.3$, $\chi = 0$, and $\Omega_0 = 1$.

the pressure of a self-bound droplet can be expressed as

$$P = \frac{\delta g}{4} n^2 - \frac{g_{\text{LHY}}^{3/2}}{3\pi} n^{3/2}. \quad (28)$$

By imposing the condition $P = 0$, the flat-top density of the droplet can be determined as

$$\bar{n}_{P=0} = \frac{16}{9\pi^2} \frac{g_{\text{LHY}}^3}{\delta g^2}. \quad (29)$$

For $g_{12} = -0.3$ and $\chi = 0$, the calculated flat-top density $\bar{n} \approx 0.563$ agrees quantitatively with numerical simulations, confirming the reliability of the analytical expression and its consistency with numerical results.

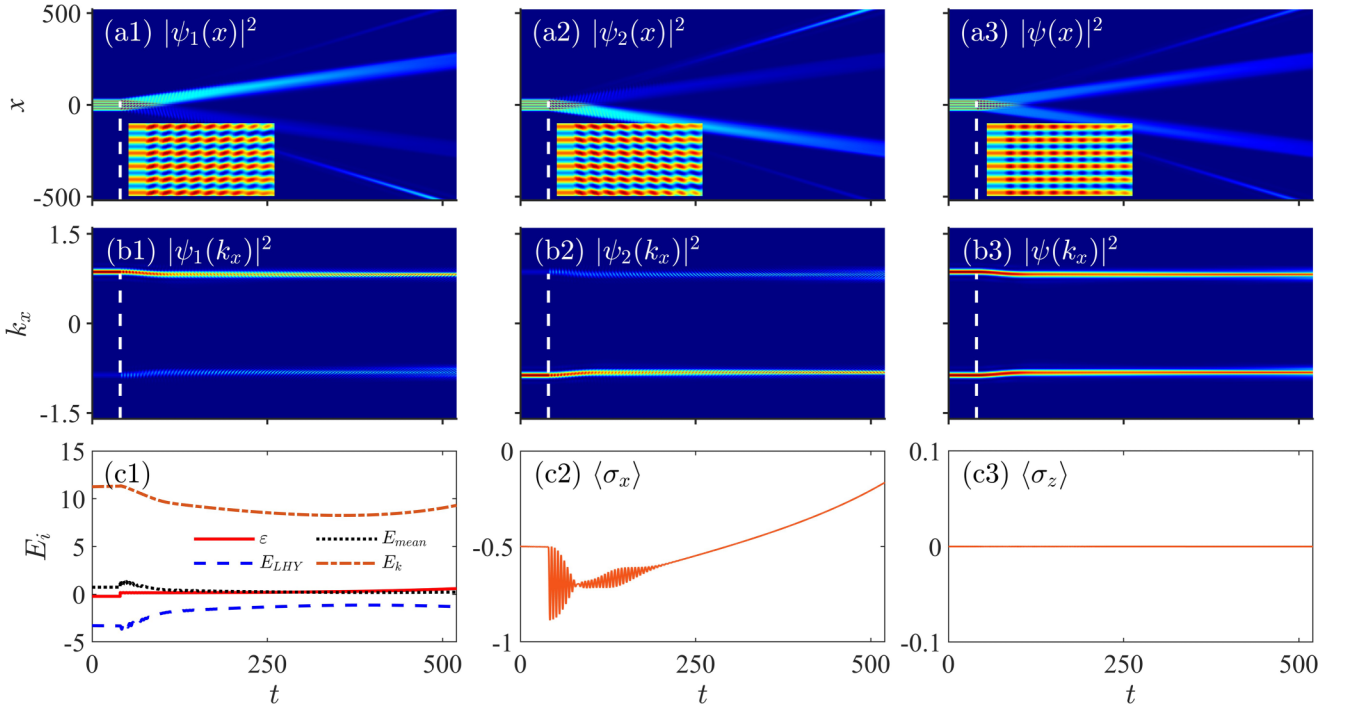


FIG. 6. Dynamical response to a sudden quench of the periodic driving strength from $\chi = 0$ to $\chi = 1.5$ at $t = 40$ (white dashed lines). Panels (a1)–(a3) show the spatiotemporal density evolution in real space, while panels (b1)–(b3) display the corresponding evolution in momentum space. Insets provide detailed views of the real-time evolution during $t = [38, 80]$ within the spatial range $x = [-10, 10]$. Panels (c1)–(c3) track the evolution of the single-particle energy ε , the LHY energy E_{LHY} , the mean-field energy E_{mean} , the kinetic energy E_k , and the average spin polarizations $\langle \sigma_x \rangle$ and $\langle \sigma_z \rangle$ during the real-time dynamics. Other parameters are $\Omega_0 = 1$, $g_{12} = -0.3$, and $k_0 = 1$.

V. THE QUENCH DYNAMICS

Having established that the ground state consists solely of zero-momentum and stripe droplet phases, we now investigate the nonequilibrium dynamics through quantum quenches—sudden or gradual parameter changes that drive the system out of equilibrium. These quenches provide powerful insight into many-body correlations and enable the realization of novel dynamical states, such as superfluid oscillator circuit [56], excitation and regulation of dark solitons [57,58], and persistent current [59]. Leveraging the exceptional tunability of ultracold atomic gases, we examine quenches of the periodic driving strength χ and SO coupling strength k_0 , both experimentally accessible via Raman-laser control.

We implement two quench protocols: sudden jumps and linear ramps. The linear quench follows the temporal profile:

$$Q(t) = \begin{cases} Q_1 + \frac{Q_2 - Q_1}{t_q} t, & t_1 \leq t \leq t_2, \\ Q_2, & t > t_2, \end{cases} \quad (30)$$

where $Q \in \{\chi, k_0\}$ denotes the quenched parameter, with initial and final values Q_1 and Q_2 at times $t_1 = 40$ and $t_2 = 60$, respectively, giving a quench duration of $t_q = 20$. We examine both directions of phase conversion: from stripe to zero-momentum droplets and the reverse process.

Figure 6 shows the dynamics following a sudden quench of the periodic driving strength from $\chi = 0$ to $\chi = 1.5$ at $t = 40$. The system initially maintains its stripe pattern with quasimomentum $|k_x| = 0.881$, but quickly develops breathing oscillations in each stripe [Fig. 6(a3) inset] and spatial separation between components [Figs. 6(a1) and 6(a2),

insets]. This redistribution stems from the modified effective SO coupling $J_0(\chi)k_0$, which alters the condensate's quasimomentum [Figs. 6(b1) and 6(b3)]. As evolution continues ($t > 150$), the striped patterns interact, splitting each component into self-bound solitonlike structures with finite lifetimes. These structures undergo further evolution, forming narrower, higher-amplitude solitons while maintaining momentum-space stability. Their emergence reflects a balance between dispersion and nonlinearity, mediated by competition between mean-field interactions and the LHY correction. Energy and spin dynamics reveal further insights [Figs. 6(c1)–6(c3)]: kinetic and mean-field energies initially decrease then increase, while the LHY energy shows the opposite trend. The spin polarization $\langle \sigma_x \rangle$ oscillates before gradually increasing, whereas $\langle \sigma_z \rangle$ remains zero, confirming balanced population between components throughout the dynamics.

Figure 7 examines a linear quench of χ between the same parameter values. Unlike the sudden quench, the linear ramp preserves the ground-state stripe pattern without breathing oscillations [Figs. 7(a1)–7(a3)] and eliminates momentum-space stripes [Figs. 7(b1)–7(b3)]. The spin polarization $\langle \sigma_x \rangle$ evolves smoothly without strong oscillations [Fig. 7(c2)]. The resulting solitonlike structures exhibit enhanced stability and longevity, with a prolonged kinetic energy decrease indicating their robust nature.

Finally, we consider the reverse process: quenching from zero-momentum to stripe droplet parameters. Figure 8 shows a linear quench of SO coupling strength from $k_0 = 0$ to $k_0 = 2$. The droplet maintains its flat-top profile during ($t <$

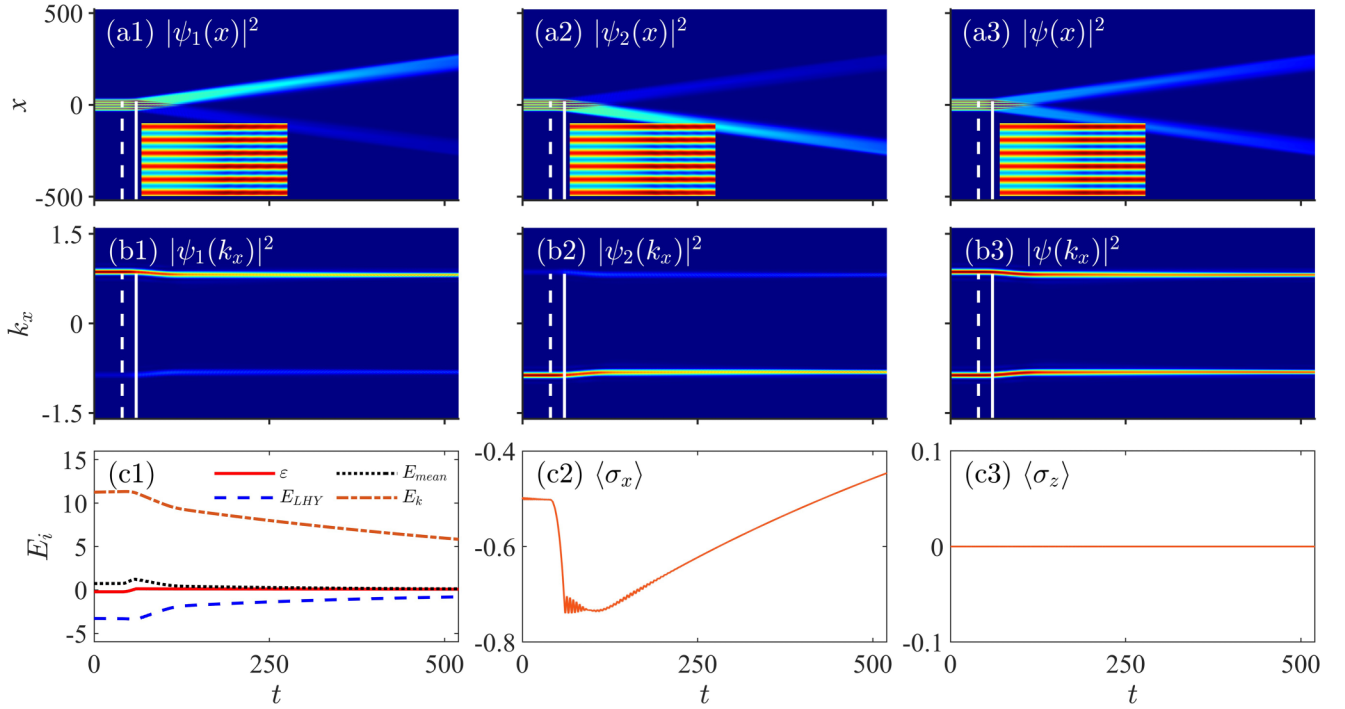


FIG. 7. Dynamical response to a linear quench of the periodic driving strength from $\chi = 0$ to $\chi = 1.5$ during the time interval between $t_1 = 40$ (white dashed lines) and $t_2 = 60$ (white solid lines). Panels (a1)–(a3) show the spatiotemporal density evolution in real space, while (b1)–(b3) display the corresponding evolution in momentum space. Insets provide detailed views of the dynamics during $t = [38, 80]$ within the spatial range $x = [-10, 10]$. Panels (c1)–(c3) track the evolution of the energy components (ϵ , E_{LHY} , E_{mean} , E_k) and spin polarizations ($\langle\sigma_x\rangle$, $\langle\sigma_z\rangle$) during the real-time dynamics. Other parameters are $\Omega_0 = 1$, $g_{12} = -0.3$, and $k_0 = 1$.

100) and after the quench [Figs. 8(a1)–8(a3)], demonstrating the zero-momentum phase's robustness. Subsequently, each component develops striped distributions [Fig. 8(c1)] that interleave to form an overall flat-top structure. When stripe contrast becomes pronounced, the total density exhibits modulation [Figs. 8(c2) and 8(c3)]. Notably, before complete fragmentation, the droplets emit solitonlike structures. This occurs because the quench imparts momentum to portions of each component while the majority remains at zero momentum. The system thus gradually splits into solitonlike structures, with the remaining zero-momentum component eventually acquiring momentum and breaking apart. We identify these emergent structures as solitonlike based on several features: self-bound, nondispersive propagation (insets of Figs. 6 and 7), localization in both real and momentum space (Figs. 6–8), and a time-invariant profile [Figs. 8(c1)–8(c3)].

The emergence of solitonlike structures from quench dynamics reveals fundamental aspects of the system's nonlinear behavior. First, it demonstrates a dynamical pathway for creating self-bound objects from a homogeneous droplet, showing that the balance between mean-field interactions and quantum fluctuations, which stabilizes the ground-state droplet, can also support excited-state solitonic configurations. Second, the quench triggers a redistribution among kinetic, mean-field, and LHY energies [Figs. 6(c1) and 7(c1)], illustrating how quantum fluctuations mediate the formation of nonlinear excitations in real time. Third, these results connect two previously separate phenomena: equilibrium quantum droplets and propagating solitons, suggesting a deeper unity between these forms of self-bound matter.

VI. CONCLUSIONS

In conclusion, we have presented a comprehensive theoretical framework for Floquet-engineered, SO-coupled quantum droplets in one dimension. Our analysis, combining a variational approach with numerical simulations of the extended GP equation, yields several key results with implications that extend beyond the immediate context of ultracold gases.

We have established a complete phase diagram revealing that the ground state consists exclusively of two phases: the stripe droplet, characterized by periodic density modulations, and the zero-momentum droplet, with a uniform flat-top profile. Crucially, the plane-wave phase, a robust ground state in conventional mean-field SO-coupled BECs, is completely absent. This is not merely a quantitative shift of a phase boundary, but a qualitative change in the phase diagram structure induced by quantum fluctuations. Through a multifaceted analysis (energetic comparison, phase-diagram arguments, and spin-balance minimization with respect to the order parameter s), we demonstrate that the LHY correction actively favors spin-balanced configurations, rendering the polarized plane-wave state energetically unfavorable and relegating it to a metastable excitation. This provides a clean, tractable example of a general principle: beyond-mean-field effects can qualitatively alter phase diagrams, not just shift boundaries, an insight relevant to nuclear physics, helium nanodroplets, and neutron star crusts where similar balances between attraction and repulsion occur.

Furthermore, we have shown that periodic driving of the Raman coupling provides a powerful tool for dynamical

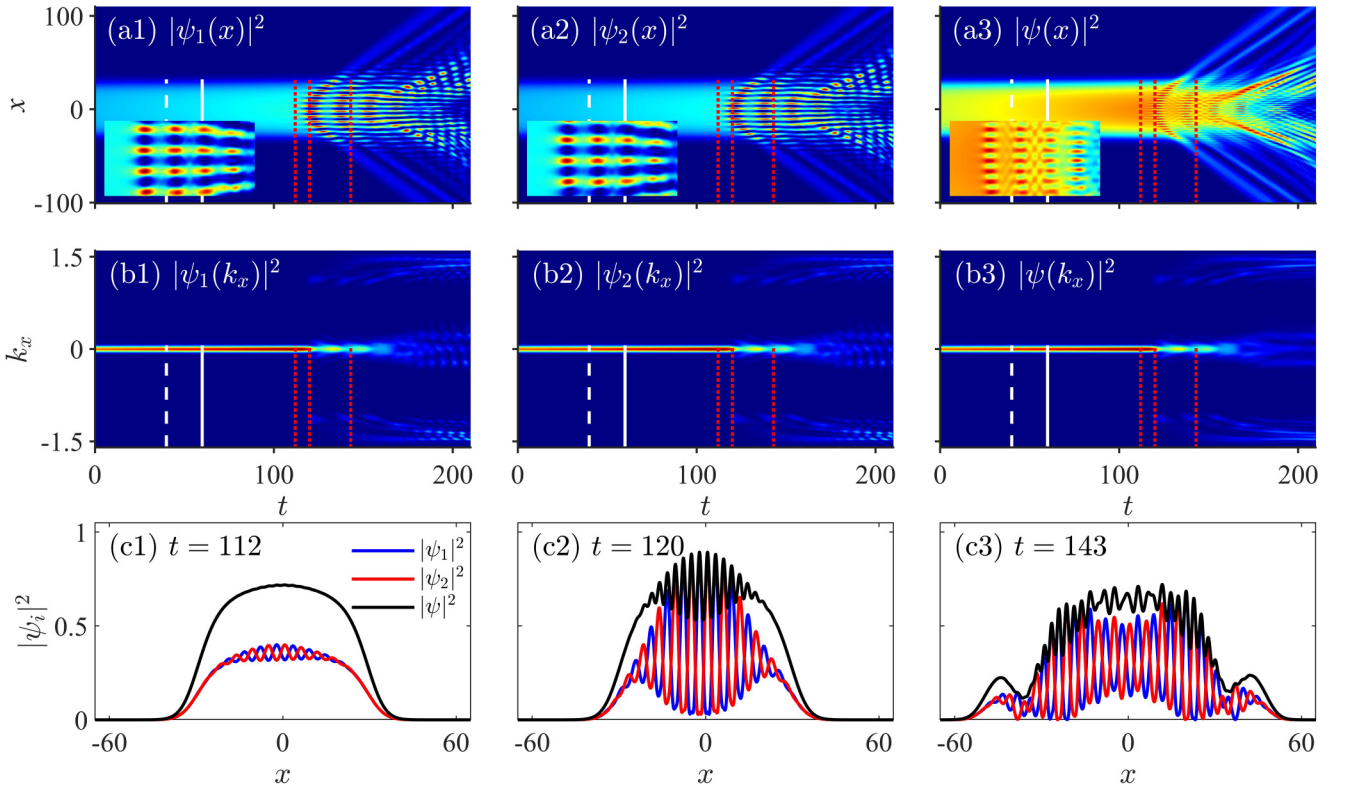


FIG. 8. Real-time evolution under a linear quench of the SO coupling strength from $k_0 = 0$ to $k_0 = 2$ between $t_1 = 40$ (white dashed lines) and $t_2 = 60$ (white solid lines). Panels (a1)–(a3) show the spatiotemporal density evolution in real space, while panels (b1)–(b3) display the corresponding evolution in momentum space. The inset provides a detailed view of the dynamics during $t = [100, 120]$ within the spatial range $x = [-10, 10]$. Panels (c1)–(c3) present density profiles at selected times (red dotted lines: $t = 112$, 120 , and 143). Parameters are $\Omega_0 = 1$, $g_{12} = -0.3$, and $\chi = 0$.

control. The effective Floquet Hamiltonian reveals that driving renormalizes not only the SO coupling [via $J_0(\chi)$] but also the interaction coefficients g_1 , g_2 , and g_3 [via $J_0(2\chi)$] and the spin-dependent LHY term [via $J_0(\chi)$]. This simultaneous renormalization of both single-particle and interaction terms provides two independent control knobs, $J_0(\chi)$ and $J_0(2\chi)$, for manipulating the phase boundary Ω_{0c} [Eq. (23)]. The resulting phase diagrams across the (k_0, Ω_0) , (g_{12}, k_0) , and (χ, Ω_0) planes exhibit rich structure, including reentrant behavior arising from the independent dependence on these two Bessel functions. Beyond spinor BECs, this Floquet engineering framework applies to any periodically driven nonlinear system, from exciton-polariton condensates to photonic metamaterials, demonstrating how periodic driving can simultaneously renormalize multiple couplings to control complex phase competitions.

Our exploration of nonequilibrium dynamics reveals that both sudden and linear quenches of the driving strength χ or the SO coupling k_0 can induce dynamical phase transitions and lead to the formation of robust, solitonlike structures. These emergent structures exhibit the hallmarks of solitonic behavior: self-bound, nondispersive propagation; localization in both real and momentum space; and shape preservation over time. Their formation reveals fundamental aspects of the system's nonlinear dynamics: it demonstrates a dynamical pathway for creating self-bound excitations from homogeneous droplets, illustrates the real-time competition

between kinetic, mean-field, and LHY energies mediated by quantum fluctuations, and connects two previously separate phenomena—equilibrium quantum droplets and propagating solitons, suggesting a deeper unity between these forms of self-bound matter. More broadly, this connects our system to the universal physics of nonlinear waves, which appear across hydrodynamics, nonlinear optics, plasma physics, and field theory. The sensitivity of these dynamics to the LHY correction also suggests potential applications as a probe of quantum fluctuations.

Beyond the specific context of spinor BECs, this work offers several wider insights. Our system provides a clean, tunable realization of SO-coupled matter, offering a platform for exploring condensed matter phenomena associated with SO coupling, including topological phases, the spin Hall effect, and spin transport. The complete phase diagrams serve as a benchmark for theoretical methods and a guide for experiments, while illustrating generic features of multiparameter phase competitions relevant across physics. The controlled generation of solitonlike structures via quench protocols suggests methods for creating ordered nonlinear excitations with potential applications in atomic transport, interferometry, and sensing. Floquet engineering offers a route to dynamically program Hamiltonian parameters for quantum simulation. These connections establish Floquet-driven quantum droplets as a versatile platform with implications reaching well beyond ultracold atomic gases, contributing to the broader under-

standing of periodically driven systems, quantum fluctuations, nonlinear dynamics, and SO-coupled matter.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available because they contain sensitive personal information. The data are available from the authors upon reasonable request.

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