

Hund-projected Kanamori model: An effective description of Hund's metals near the Mott insulating regime

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Hund's coupling plays a decisive role in shaping electron correlations of multi-orbital systems, giving rise to a class of materials—Hund's metals—that combine local-moment physics with metallic transport. Here, we derive an effective low-energy description of such a system near the Mott insulating regime, starting from the multi-orbital Hubbard-Kanamori Hamiltonian and projecting onto the high-spin manifold favored by Hund's first rule. The resulting Hund-projected Kanamori model captures the interplay between carrier motion and magnetic correlations in the presence of strong Hund's coupling. In the undoped limit, the model reduces to a spin- $N/2$ Heisenberg system with suppressed quantum fluctuations, approaching the classical limit for realistic five-band configurations. Upon doping, carrier motion couples strongly to the spin background and drives ferromagnetic correlations through a Hund-enhanced kinetic mechanism analogous to, but much stronger than, Nagaoka ferromagnetism. Owing to its reduced sign problem, the model can be addressed with advanced path-integral methods to determine quasiparticle structure and effective interactions between carriers—quantities that are challenging to obtain with other methods. This framework establishes a microscopic bridge between the Kanamori model and the emergent magnetic and transport phenomena characteristic of Hund's metals.

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I. INTRODUCTION

Strongly correlated electronic systems occupy a central position in condensed matter physics owing to their propensity to host emergent quantum phases such as unconventional superconductivity, spin and charge order, orbital-selective states, and non-Fermi-liquid behavior. For decades, the study of strong correlations was almost synonymous with high-temperature superconductivity and microscopic descriptions based on doped Mott insulators [1–3]. More recently, correlation-driven phenomena have also been realized in moiré heterostructures, where a small twist angle or layers with an incommensurate interatomic spacing produces superlattices with anomalously narrow bands and enhanced electronic interactions [4–9].

A central theme uniting these systems is the competition between magnetic order and carrier delocalization [10–13]. This interplay is believed to underlie a variety of emergent states, including high-temperature superconductivity [1,14,15], quantum spin liquids [16], and kinetic magnetism [17–20], where magnetic fluctuations can mediate effective pairing interactions leading to phase separation and condensation [21,22].

In conventional strongly correlated lattice descriptions such as the Hubbard or the t - J models, as well as multi-orbital

generalizations thereof, strong quantum fluctuations suppress magnetic order even in the absence of charge carriers, particularly in low-spin systems [23]. In contrast, multi-orbital compounds with strong Hund's coupling—such as the iron pnictides, chalcogenides, and ruthenates—exhibit a qualitatively different form of correlated behavior [24]. These so-called Hund's metals display large local magnetic moments owing to Hund's first rule that persist from Mott insulating states well into the metallic regime. The impact of Hund's coupling on charge fluctuations is somewhat contradictory: At half filling it increases the effective contact repulsion, driving Mott insulating behavior. Away from half filling, it enhances charge fluctuations, promoting metallicity instead—a dual behavior referred to as the Janus effect [25]. The tendency to form large local spin moments should be expected to dramatically reduce quantum spin fluctuations. At the same time, compliance with Hund's rule should clearly make delocalization especially sensitive to spin correlations. The emerging picture is thus of competition between almost classical magnetism, and a very potent form of kinetic ferromagnetism. This clearly sets Hund's metals apart from other strongly correlated systems.

Materials with strong Hund's coupling host an array of unconventional phenomena, including multiband superconductivity, orbital-selective Mott phases, and broken time-reversal symmetry [26–28]. Recent experiments even indicate the existence of a new fermion quadrupling state in the iron pnictide $\text{Ba}_{1-x}\text{K}_x\text{Fe}_2\text{As}_2$ [29]. The diversity of materials exhibiting these behaviors highlights the importance of a general theoretical framework capable of connecting Hund's coupling, magnetism, and charge motion. Because many of these systems lie close to the Mott insulating regime in terms of filling

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[30–32], it is natural to begin by characterizing the correlated magnetic parent state from which the metallic phase emerges and subsequently add carriers.

In this work, we derive such a framework, starting from the multi-orbital Hubbard-Kanamori Hamiltonian and projecting onto the high-spin manifold favored by Hund’s rule. The resulting Hund-projected Kanamori model (HPKM) constitutes an effective low-energy theory for doped Hund’s systems near the Mott regime derived with two assumptions: that charge fluctuations are short lived and can be integrated out, and that fluctuations to lower-spin states—violating Hund’s first rule—are small. Despite the complexity of the original Hamiltonian, the projected model takes the form of a doped spin system with strongly suppressed quantum fluctuations and a tractable structure well suited for path-integral and diagrammatic methods. It thereby provides a microscopic foundation for studying the magnetic dynamics and carrier-induced phenomena characteristic of Hund’s metals beyond mean-field theory.

The paper is organized as follows: In Sec. II, we introduce the multi-orbital Hubbard-Kanamori model and specify the hierarchy of energy scales relevant to Hund’s metals near the Mott insulating regime. In Sec. III, we derive the Hund-projected Kanamori model by explicitly projecting onto the high-spin manifold favored by Hund’s first rule and systematically integrating out short-lived charge and low-spin excitations; this section contains the main analytical development of the work. In Sec. IV, we discuss the physical properties of the resulting effective model, including the suppression of quantum spin fluctuations at half filling and the emergence of strong carrier-induced ferromagnetic correlations upon doping. In Sec. V, we delineate the regime of validity of the Hund-projected description and discuss its applicability to realistic Hund-coupled materials, as well as implications for numerical treatments enabled by the reduced sign problem. Finally, Sec. VI presents a summary and outlook.

II. THE KANAMORI MODEL

The paradigmatic model of Hund’s metals is the Hubbard-Kanamori Hamiltonian [24], with an on-site interaction of the form

$$H_K = U \sum_{\alpha} n_{\alpha\uparrow} n_{\alpha\downarrow} + U' \sum_{\alpha \neq \beta} n_{\alpha\uparrow} n_{\beta\downarrow} + (U' - J) \sum_{\alpha > \beta} n_{\alpha\sigma} n_{\beta\sigma} - J \sum_{\alpha \neq \beta} a_{\alpha\uparrow}^{\dagger} a_{\alpha\downarrow} a_{\beta\downarrow}^{\dagger} a_{\beta\uparrow} + P \sum_{\alpha \neq \beta} a_{\alpha\uparrow}^{\dagger} a_{\alpha\downarrow}^{\dagger} a_{\beta\downarrow} a_{\beta\uparrow}. \quad (1)$$

Here, α denotes the band index. The principal energy scales of this model are the Coulomb repulsion U , and the Hund’s coupling J . The interorbital repulsion between electrons with opposite spin is given by $U' = U - 2J$, while the interaction between electrons in different orbitals with the same spin is reduced further to $U' - J$. For the pair-conversion amplitude, we generally have $P = J$, though we will treat this term on a separate footing. The most general form of the hopping term is given by

$$H_0 = \sum_{ij\alpha\beta\sigma\sigma'} t_{ij\alpha\beta} a_{i\alpha\sigma}^{\dagger} a_{j\beta\sigma'}. \quad (2)$$

TABLE I. Typical estimates for model parameters relevant to iron-based superconductors [33,34]. Because of crystal field splitting, the bandwidths and chemical potentials of individual orbitals can vary significantly [35]. The parameter t refers specifically to nearest-neighbor hopping.

Term	Energy scale
U	2.2–4.2 eV
J	0.3–0.6 eV
t	0.2 – 0.3 eV

For simplicity, we restrict ourselves to nearest-neighbor processes that preserve both spin and orbital quantum numbers, though the generalization to other cases is straightforward.

The principal effect of Hund’s coupling is to maximize total spin, consistent with Hund’s first rule. With N bands and the constraint that the orbital occupancy is given by $\vec{o} = [o_1, o_2, \dots, o_N]$, the Kanamori Hamiltonian (1) predicts a ground state spanned by

$$|S_M^n, S_z\rangle, \quad \text{with } |S_z| \leq S_M^n, \quad \text{where} \quad (3)$$

$$S_M^n = \frac{1}{2}n, \quad n = \frac{1}{2} \sum_{i=1}^N [1 - (-1)^{o_i}]. \quad (4)$$

Here, n is the number of orbitals that contribute to total spin (by being singly occupied) and S_M^n is the corresponding maximal spin. For $H_K[P = 0]$, and occupation number n , the first excited state is given by

$$|S_M^n - 1, S_z\rangle \quad \text{with} \quad E(S_M^n - 1) = E(S_M^n) + nJ, \quad (5)$$

which defines a “Hund’s gap” $\Delta_H = nJ$, associated with violating Hund’s first rule. For real materials, this energy scale can be quite large: Recent estimates for iron-based superconductors (see Table I) indicate that $J \sim 0.3\text{--}0.6$ eV, implying a gap of $\Delta_H \sim 1.5\text{--}3$ eV at half filling, which is much greater than the energy scale of any itinerant terms in the Hamiltonian.

An additional effect of Hund’s coupling is its impact on charge fluctuations. Given two sites where all orbitals are singly occupied, in a ground state of the type (3), the energy cost of creating a particle-hole pair is given by

$$U_{\text{eff}} = E(N + 1) + E(N - 1) - 2E(N) = U + J(N - 1). \quad (6)$$

At half filling, a key consequence of Hund’s coupling is thus a substantial increase of the effective on-site repulsion, which drives Mott insulating behavior. It should be noted, however, that away from half filling, Hund’s coupling instead promotes metallicity [24].

III. HUND-PROJECTED KANAMORI MODEL

Based on the discussion in Sec. II, we focus on the regime where both the effective charge gap U_{eff} and the Hund’s gap Δ_H are large compared to the kinetic energy scale. In this limit, charge fluctuations as well as local excitations that violate Hund’s first rule are short lived and can be treated

TABLE II. Contact interaction for some common excitations above ground state on a half filled background (i.e., one electron per orbital and maximal spin), obtained for $H_K[P = 0]$. The contact energy for the biorbitor is relative to four unbound carriers (the energy barrier against decay into two separate orbitons is $2J$).

Object	Creator	V_{contact}	Interaction
Orbitor	$a_\alpha^\dagger a_\beta$	$-U'$	Attractive
Bipolaron	$a_\alpha^\dagger a_\beta^\dagger$	$(U' - J)$	Repulsive
Biorbitor	$a_\alpha^\dagger a_\beta a_\gamma^\dagger a_\delta$	$-2U' - 2J$	Attractive

as high-energy virtual processes. It is therefore natural to formulate a low-energy description by projecting onto the high-spin local manifold and integrating out these degrees of freedom. In the following, we introduce the corresponding Hund's projector and use it to derive an effective Hamiltonian acting within the reduced Hilbert space,

$$P^H = \Pi_j \sum_{\bar{o}^j} \sum_{S_z^j = -S_M^j}^{S_M^j} |S_M^j, S_z^j\rangle \langle S_M^j, S_z^j|, \quad (7)$$

where $\bar{o}^j$ is the orbital occupation of the site j and $n_j = n_j[\bar{o}^j]$ is given by Eq. (4). The operator (7) thus projects onto states satisfying the condition that the total spin on any given site is maximal for its given orbital occupation, consistent with the Kanamori Hamiltonian (1) and Hund's first rule. Eventually, we will constrain our Hilbert space to the subspace on which P^H projects.

A. Charge-sector excitations above the Mott insulating phase

In this subsection we classify local charge-sector excitations of the Kanamori model that comply with Hund's-rule; excitations that violate Hund's first rule are discussed separately in Appendix A 4.

Table II provides contact interaction for some elementary excitations above the ground state, derived for $H_K[P = 0]$. For realistic parameters, orbitons are strongly bound, while bipolarons see a slightly weaker repulsion. The biorbitor contact energy is relative to four unbound carriers. The contact interaction of these objects can be reproduced from a simple two-body term of the form

$$V_{pp} = V_{hh} = U' - J, \quad V_{ph} = -U', \quad (8)$$

where the subscript denotes particle-particle, hole-hole, and particle-hole interaction, respectively.

Finally, we consider the impact of the pair-conversion term in (1), which “inverts” orbitons. If we account for the contribution from isolated charge carriers, the energy of an orbitor is

$$E_{o,\alpha\beta} = U_{\text{eff}} - U' + \delta\mu_{\alpha\beta} = J(N + 1) + \delta\mu_{\alpha\beta}, \quad (9)$$

where $\delta\mu_{\alpha\beta} = \mu_\beta - \mu_\alpha$ is the difference in chemical potential between the particle types. For this object to be thermodynamically stable, we require that $\delta\mu_{\alpha\beta} \sim -J(N + 1)$. Applying pair conversion, we instead obtain an object with energy

$$E_{o,\beta\alpha} \sim E_{o,\alpha\beta} + 2J(N + 1). \quad (10)$$

This represents a high-energy state, with a correspondingly short lifetime. The primary decay path is via another pair conversion. For physically relevant model parameters, we have $P = J$, implying a slight hybridization of the two orbitons given by

$$|o_{GS}\rangle \sim \sqrt{1 - \frac{1}{(2N + 1)^2}} |o_{\alpha\beta}\rangle + \frac{1}{2(N + 1)} |o_{\beta\alpha}\rangle. \quad (11)$$

The reduction of the first amplitude is small (<0.01 for $N = 5$), and we conclude that the pair conversion term can safely be neglected. It should be stressed, however, that this holds true for a doped Mott insulator, and not for a general metallic system.

B. Gutzwiller projection

We next integrate out short-lived charge fluctuations to obtain projected hopping and superexchange terms, consistent with the assumption $U_{\text{eff}} \gg t$ [36]. In multi-orbital systems, we have the prospect of bands being both hole doped or particle doped, meaning that we either project out doublons or vacant sites in a given orbital. To simplify the representation, we therefore conduct a spin-preserving particle-hole transformation of the bands that are particle doped, given by

$$a_\sigma^\dagger \Rightarrow a_{-\sigma}, \quad a_\sigma \Rightarrow a_{-\sigma}^\dagger. \quad (12)$$

In the new representation, all carriers are hole-like. The delocalization part of the resulting model is described by projected hopping,

$$H_t = \sum_{\langle ij \rangle \alpha \sigma} t_\alpha \tilde{a}_{i\alpha\sigma}^\dagger \tilde{a}_{j\alpha\sigma}, \quad (13)$$

where $\tilde{a}_{i\alpha\sigma} = a_{i\alpha\sigma}(1 - n_{i\alpha,-\sigma})$. It should be noted that $t_\alpha \rightarrow -t_\alpha$ under the mapping (12), though this only matters on nonbipartite lattices.

From the charge fluctuations, we obtain superexchange processes of the form

$$H_S = \sum_{\langle ij \rangle \alpha} I_\alpha \left\{ \frac{\hat{S}_{\alpha-}^i \hat{S}_{\alpha+}^j + \hat{S}_{\alpha+}^i \hat{S}_{\alpha-}^j}{2} + \hat{S}_{\alpha z}^i \hat{S}_{\alpha z}^j - \frac{1}{4} n_{i\alpha} n_{j\alpha} \right\},$$

$$I_\alpha = \frac{4t_\alpha^2}{U_{\text{eff}}}, \quad (14)$$

where $\hat{S}_{\alpha\pm}^i$ raises/lowers the spin of the α orbital on site i [36]. Using Eq. (6) and the estimates from the Table I, we find that $I_\alpha = 0.025 - 0.1$ eV.

Interactions between carriers take the form

$$H_V = \sum_{i\alpha\beta} V_{\alpha\beta} n_{i\alpha} n_{i\beta}, \quad (15)$$

where $V_{\alpha\beta}$ is given by Eq. (8). Note that this interaction will depend on whether the carriers were particle or hole-like in the original description.

To take into account the Hund's coupling, we follow a strategy similar to Gutzwiller projection, and work in the subspace dictated by the projector P^H , [defined in Eq. (7)], while treating excitations across the Hund's gap as emergent processes.

C. Hund-projected magnetism

Operating with H_S on a ground state of the type (3), we obtain two principal contributions,

$$H_S |S_M^n, S_z\rangle = P^H H_S |S_M^n, S_z\rangle + (1 - P^H) H_S |S_M^n, S_z\rangle.$$

The latter term violates Hund's first rule to an energy cost of at least Δ_H . In Matsubara theory, this excited state will decay with an imaginary lifetime of $\sim 1/\Delta_H$, leading to emergent processes with an effective energy scale

$$\epsilon \sim \frac{I^2}{4\Delta_H} \sim 0.002 - 0.017 I \quad (16)$$

near half filling. Virtual magnetic interactions of this type are thus subleading by two to three orders of magnitude, and it is therefore an excellent approximation to discard them at temperatures $T \ll \Delta_H$. In what follows, we will thus project out high-energy contributions from the magnetic interactions by taking

$$H_S \rightarrow P^H H_S. \quad (17)$$

For the spin raising/lowering operator of the electron α , we note that $\hat{S}_{\alpha\pm}$ must necessarily alter S_z by unity. Therefore, we have

$$\begin{aligned} P^H \hat{S}_{\alpha\pm} |S_M^n, S_z\rangle &= |S_M^n, S_z \pm 1\rangle \langle S_M^n, S_z \pm 1 | \hat{S}_{\alpha\pm} |S_M^n, S_z\rangle \\ &\equiv |S_M^n, S_z \pm 1\rangle \gamma_{S_M^n, S_z}^{\pm}. \end{aligned} \quad (18)$$

Evaluation of the inner product (see Appendix A 1) gives

$$\begin{aligned} \gamma_{S_M^n, S_z}^- &= \frac{\sqrt{(S_M^n + S_z)(S_M^n - S_z + 1)}}{2S_M^n}, \\ \gamma_{S_M^n, S_z}^+ &= \frac{\sqrt{(S_M^n + S_z + 1)(S_M^n - S_z)}}{2S_M^n}. \end{aligned} \quad (19)$$

For the $\hat{S}_z$ terms in the model (14), we first note that these preserve S_z . Therefore, we must have

$$\begin{aligned} P^H \hat{S}_{\alpha z} |S_M^n, S_z\rangle &= P^H \left\{ u |S_M^n, S_z\rangle + v |S \neq S_M^n, S_z\rangle \right\} \\ &= |S_M^n, S_z\rangle \langle S_M^n, S_z | \hat{S}_{\alpha z} |S_M^n, S_z\rangle = \frac{S_z}{n} |S_M^n, S_z\rangle. \end{aligned} \quad (20)$$

Combining the results (18) and (20), we can write the projected Hamiltonian (17) as

$$\begin{aligned} P^H H_S &= I_{ij}^T \sum_{\langle ij \rangle} \left\{ \frac{\hat{S}_i^+ \hat{S}_j^+ + \hat{S}_i^- \hat{S}_j^-}{2n_i n_j} + \frac{\hat{S}_i^z \hat{S}_j^z}{n_i n_j} \right\} \\ &\quad - \frac{1}{4} \sum_{\alpha} I_{\alpha} n_{i\alpha} n_{j\alpha}, \quad I_{ij}^T = \sum_{\alpha} I_{\alpha} n_{i\alpha} n_{j\alpha}. \end{aligned} \quad (21)$$

At half-filling, the spin sector is thus governed by an emergent spin- $N/2$ Heisenberg model.

D. Charge carriers

To establish the role of mobile dopants, we construct the analog of the delocalization part of the Hamiltonian [Eq. (13)] within the Hund-projected subspace. We begin by defining

the normalization coefficients of the spin-ladder operators as follows:

$$\xi_{S, S_z}^{\pm} \hat{S}_{\pm} |S, S_z\rangle \equiv |S, S_z \pm 1\rangle, \quad |S_z \pm 1| \leq S, \quad (22)$$

$$\chi_{S, l}^{\pm} \hat{S}_{\pm}^l |S, \mp S\rangle \equiv |S, \pm(l - S)\rangle, \quad |S - l| \leq S. \quad (23)$$

Consider a site with n particles in a state that maximizes total spin. If the orbital α is vacant, then it can be filled by applying the corresponding creation operator. Assuming $S_z = -S_M^n + l$ in the initial state, we find

$$\begin{aligned} a_{\uparrow\alpha}^{\dagger} |S_M^n, -S_M^n + l\rangle &= a_{\uparrow\alpha}^{\dagger} \hat{S}_+^l |S_M^n, -S_M^n\rangle \chi_{S_M^n, l}^+ \\ &= \hat{S}_+^l \hat{S}_{\alpha+} |S_M^{n+1}, -S_M^{n+1}\rangle \chi_{S_M^n, l}^+, \end{aligned} \quad (24)$$

where we have used $[\hat{S}_+^l, c_{\uparrow\alpha}^{\dagger}] = 0$ and $a_{\uparrow\alpha}^{\dagger} \rightarrow \hat{S}_{\alpha+} \hat{a}_{\uparrow\alpha}^{\dagger}$. The operator $\hat{S}_{\alpha+}$ splits the initial state into two contributions,

$$\hat{S}_{\alpha+} |S_M^{n+1}, -S_M^{n+1}\rangle = \frac{1}{\sqrt{n+1}} |S_M^{n+1}, 1 - S_M^{n+1}\rangle + |R_{\alpha}\rangle, \quad (25)$$

where only the former lies in the Hund subspace. Under the subsequent spin-raising operations, the first part transforms as follows (see Appendix A 3):

$$\hat{S}_+^l |S_M^{n+1}, 1 - S_M^{n+1}\rangle \frac{\chi_{S_M^n, l}^+}{\sqrt{n+1}} = |S_M^{n+1}, l + 1 - S_M^{n+1}\rangle \sqrt{\frac{l+1}{n+1}}. \quad (26)$$

The second contribution takes the form

$$|R_{\alpha}\rangle = |S_M^{n+1} - 1, 1 - S_M^{n+1}\rangle_{\alpha}, \quad (27)$$

which is an eigenstate of H_K with energy Δ_H (even after applying $\hat{S}_+^l$). Since $\Delta_H \gg t, j$, this state must have a short lifetime, giving rise to emergent processes with energy scales

$$\frac{tI_{\beta}}{2\Delta_H\sqrt{N}} \sim 0.002 - 0.015t, \quad (28)$$

$$\frac{t^2}{\Delta_H} \sim 0.07 - 0.2t, \quad (29)$$

depending on whether it decays by magnetic or kinetic mechanisms. The former is subleading by two to three orders of magnitude, and may safely be neglected. The kinetic decay is an order of magnitude smaller than the hopping integral, but has the ability to nucleate a weak pair-propagation mechanism under certain circumstances; see discussion in Appendix A 4.

In the next stage, we consider the creation of a hole on a state with a filled α orbital. Assuming $S_z = S_M^n - m$, we find

$$\begin{aligned} a_{\uparrow\alpha} |S_M^n, S_M^n - m\rangle &= a_{\uparrow\alpha} \hat{S}_-^m |S_M^n, S_M^n\rangle \chi_{S_M^n, m}^- \\ &= \hat{S}_-^m |S_M^{n-1}, S_M^{n-1}\rangle \chi_{S_M^n, m}^- \\ &= |S_M^{n-1}, S_M^{n-1} - m\rangle \frac{\chi_{S_M^n, m}^-}{\chi_{S_M^{n-1}, m}^-}, \end{aligned} \quad (30)$$

where we have used $[c_{\uparrow, \alpha}, \hat{S}_-] = 0$. Next, we note that

$$\frac{\chi_{S_M^n, m}^-}{\chi_{S_M^{n-1}, m}^-} = \sqrt{\frac{n-m}{n}} \equiv \sqrt{\frac{l}{n}}, \quad (31)$$

since m counts the number of spin flips from $S_z = S_M^n$, while l counts the number of flips from $S_z = -S_M^n$. Combining (26)

and (31), we finally conclude

$$\begin{aligned} P^H a_{\uparrow\alpha}^\dagger |S_M^n, l - S_M^n\rangle &= |S_M^{n+1}, l + 1 - S_M^{n+1}\rangle \sqrt{\frac{l+1}{n+1}}, \\ a_{\uparrow\alpha} |S_M^n, l - S_M^n\rangle &= |S_M^{n-1}, l - 1 - S_M^{n-1}\rangle \sqrt{\frac{l}{n}}, \end{aligned} \quad (32)$$

where it is assumed that the α orbital of the initial state is vacant or occupied, respectively. In the Hund-projected subspace, a state is uniquely identified by the orbital occupation $\bar{o}$ and the spin S_z . Hence, we adapt the notation

$$|\bar{o}, l\rangle, \quad \text{with} \quad 0 \leq l \leq n[\bar{o}], \quad (33)$$

where n is obtained from Eq. (4). The spin is then mapped onto a bosonic field so that

$$\hat{l} = \hat{b}^\dagger \hat{b} \quad (34)$$

counts the number of spin-up electrons. The orbital occupation is encoded into N fields of spinless fermions in such a way that the charge-carrying holes are mapped onto particles rather than holes. This gives

$$\hat{n} = N - \sum_{\alpha} c_{\alpha}^\dagger c_{\alpha}, \quad (35)$$

where $c_{\alpha}^\dagger$, c_{α} are creation/annihilation operators of a spinless fermion. The original creation and annihilation operators can then be written as

$$\begin{aligned} P^H \hat{a}_{\uparrow\alpha}^\dagger P^H &= \hat{c}_{\alpha} \hat{b}^\dagger \frac{1}{\sqrt{\hat{n}+1}}, \quad \hat{a}_{\uparrow\alpha} = \hat{b} \hat{c}_{\alpha}^\dagger \frac{1}{\sqrt{\hat{n}}}, \\ P^H \hat{a}_{\downarrow\alpha}^\dagger P^H &= \hat{c}_{\alpha} \sqrt{\frac{\hat{n}+1-\hat{l}}{\hat{n}+1}}, \quad \hat{a}_{\downarrow\alpha} = \hat{c}_{\alpha}^\dagger \sqrt{\frac{\hat{n}-\hat{l}}{\hat{n}}}. \end{aligned} \quad (36)$$

Note that the bosonic operators $\hat{b}^\dagger$, $\hat{b}$ have matrix elements $\sim \sqrt{\hat{l}+1}$ and $\sqrt{\hat{l}}$, respectively, to reproduce Eq. (32). This gives

$$P^H H_t P^H = - \sum_{\langle ij \rangle \alpha} t_{\alpha} (c_{i\alpha}^\dagger c_{j\alpha} \hat{b}_i \hat{b}_j^\dagger \hat{\gamma}_{ij}^\uparrow + c_{i\alpha}^\dagger c_{j\alpha} \hat{\gamma}_{ij}^\downarrow), \quad (37)$$

where the sign results from commuting $cc^\dagger \rightarrow -c^\dagger c$, and

$$\hat{\gamma}_{ij}^\uparrow = \frac{1}{\sqrt{\hat{n}_i}} \frac{1}{\sqrt{\hat{n}_j+1}}, \quad \hat{\gamma}_{ij}^\downarrow = \sqrt{1 - \frac{\hat{b}_i^\dagger \hat{b}_i}{\hat{n}_i}} \sqrt{1 - \frac{\hat{b}_j^\dagger \hat{b}_j}{\hat{n}_j+1}}. \quad (38)$$

E. Hund-projected model

Combining the hopping integral (37), the intercarrier interaction (15) and the superexchange (21), we obtain the Hund-projected Kanamori model,

$$\begin{aligned} H &= \sum_{\langle ij \rangle \alpha} \{ \tilde{t}_{\alpha} c_{i\alpha}^\dagger c_{j\alpha} \hat{b}_i \hat{b}_j^\dagger \hat{\gamma}_{ij}^\uparrow + c_{i\alpha}^\dagger c_{j\alpha} \hat{\gamma}_{ij}^\downarrow \} + \sum_{i\alpha\beta} V_{\alpha\beta} n_{i\alpha} n_{i\beta} \\ &+ \sum_{\langle ij \rangle} I_{ij}^T \left\{ \frac{\hat{S}_-^i \hat{S}_+^j + \hat{S}_+^i \hat{S}_-^j}{2n_i n_j} + \frac{\hat{S}_z^i \hat{S}_z^j}{n_i n_j} \right\} - \frac{1}{4} \sum_{\langle ij \rangle \alpha} I_{\alpha} n_{i\alpha} n_{j\alpha}, \end{aligned} \quad (39)$$

where $\tilde{t}_{\alpha} = \pm t_{\alpha}$ and the sign depends on whether the orbital α was particle-like (+) or hole-like (−) in the original representation. The application of the map in Eqs. (34) and (35) to the spin operators constitutes a Holstein-Primakoff transformation [37], and gives

$$S_+ = \hat{b}^\dagger \sqrt{2S_M^n - \hat{l}}, \quad S_- = \sqrt{2S_M^n - \hat{l}} \hat{b}, \quad S_z = \hat{l} - S_M^n.$$

The spinless fermions in this model represent charge carriers, while the bosonic occupation number counts the number of spin-up electrons on a site. The latter is constrained to

$$\hat{l} = \hat{b}^\dagger \hat{b} \leq \hat{n} = N - \sum_{\alpha} c_{\alpha}^\dagger c_{\alpha}. \quad (40)$$

The interactions in the model (39) are given by

$$V_{\alpha\beta} = \begin{cases} U' - J, & \text{particle-particle or hole-hole channel,} \\ -U', & \text{particle-hole channel.} \end{cases} \quad (41)$$

The effective hopping integral is

$$\hat{\gamma}_{ij}^\uparrow = \frac{1}{\sqrt{\hat{n}_i}} \frac{1}{\sqrt{\hat{n}_j+1}}, \quad \hat{\gamma}_{ij}^\downarrow = \sqrt{1 - \frac{\hat{b}_i^\dagger \hat{b}_i}{\hat{n}_i}} \sqrt{1 - \frac{\hat{b}_j^\dagger \hat{b}_j}{\hat{n}_j+1}},$$

and the superexchange term takes the form

$$I_{ij}^T = \sum_{\alpha} I_{\alpha} c_{i\alpha} c_{i\alpha}^\dagger c_{j\alpha} c_{j\alpha}^\dagger. \quad (42)$$

For a low-carrier density, we can make the approximation

$$\hat{\gamma}^\uparrow \approx \frac{1}{N}, \quad \hat{\gamma}_{ij}^\downarrow \approx \sqrt{1 - \frac{\hat{b}_i^\dagger \hat{b}_i}{N}} \sqrt{1 - \frac{\hat{b}_j^\dagger \hat{b}_j}{N}}, \quad (43)$$

which becomes exact for isolated dopants. In this scenario, we can also use

$$I_{ij}^T \approx \sum_{\alpha} I_{\alpha}, \quad (44)$$

which becomes exact in the absence of carriers.

IV. PROPERTIES OF THE HUND-PROJECTED MODEL

At half-filling, the low-energy model reduces to a spin- $N/2$ Heisenberg model, implying that quantum spin fluctuations are dramatically smaller than in the single-band models, or indeed multi-orbital systems without Hund's coupling. This gives an antiferromagnetic ground state with large magnetic moments. The size of the quantum fluctuations around this state can be estimated via Holstein-Primakoff mean-field theory; see Appendix A 2: For a five-band model on a 2D square lattice, we find that the probability of a spin deviating from its classical value is approximately 4%. On the cubic lattice this figure is $\sim 1.6\%$. This places us close to the classical limit with a coupling strength

$$J_{\text{classical}} = \frac{1}{4} \sum_{\alpha} I_{\alpha} \sim 0.03 - 0.125 \text{ eV}. \quad (45)$$

On a cubic lattice, this would give an ordering temperature of $T_c \sim 500\text{--}2100$ K, though it should be stressed that many Hund's metals feature a weaker interlayer interaction

and that the presence of carriers tends to reduce the ordering temperature.

Once carriers are present, their motion couples strongly to spin correlations, as seen from Eq. (38). On a polarized background, carrier motion is unaffected by Hund's coupling, whilst paramagnetic or anticorrelated spins lead to a drastic decrease of the effective bandwidth (especially the latter). The immediate consequence of this is that carrier motion drives ferromagnetic correlations. In the strong coupling limit, we find

$$t/U \rightarrow 0 \Rightarrow t/J \rightarrow \infty, \quad (46)$$

implying that for sufficiently strong coupling and a finite carrier density, the system becomes ferromagnetic, providing an example of kinetic magnetism, i.e., the nucleation of a magnetic phase because of delocalization. This phenomenon, best known from the Nagaoka theorem [17], is generally understood to result from interference of carrier paths. In the current context, it instead emerges at the level of effective hopping, pointing to a mechanism far more potent than that of conventional single-band lattice models or multi-orbital systems devoid of Hund's coupling, where the coupling between magnetism and delocalization is typically weak [10].

Integrating or projecting out high-energy states has far reaching implications for computations. The original Kanamori model is notoriously challenging in this regard, because of a combination of multiple orbitals and complex interaction. Path-integral methods are not viable owing to the fermionic sign, and most existing theoretical results consequently rely on dynamical mean-field theory (DMFT). This method does frequently provide accurate results for band structure and local correlations, but cannot provide any insight into the nonlocal correlations that provide the structure of charge carriers/polarons for instance. Crucially, this makes it hard to identify pairing mechanism.

By contrast, the Hund-projected Kanamori model derived here offers a far simpler and more tractable framework for computations. By integrating out charge fluctuations, the sign problem becomes extensive only in the number of charge carriers, rather than in the system size. This permits the use of path-integral methods at relatively low temperatures to address problems of few carriers with extremely high precision. Notably, such calculation—based on the worm algorithm [38]—have been conducted for the t - J model to produce the polaron structure [39], as well as demonstrating attraction between carriers [40]. These observables are not available in DMFT, and our work thus complements existing protocols focusing on few carriers.

V. APPLICABILITY AND RELEVANCE OF THE HUND-PROJECTED KANAMORI MODEL

This work focuses on the regime of strong Hund's coupling, where charge fluctuations are short lived and may be consistently integrated out of the low-energy theory. This assumption is motivated by the growing body of evidence that iron-based superconductors are situated in close proximity to a Mott-insulating regime and are strongly influenced by this nearby insulating state.

Experimental support for this picture is substantial. X-ray spectroscopy [31] and angle-resolved photoemission spectroscopy (ARPES) [30] indicate that both electron- and hole-doped BaFe_2As_2 exhibit electronic properties consistent with strong influence from a Mott insulating state that would be realized at half-filling of the Fe $3d$ manifold. Motivated by these observations, several closely related iron-based compounds have been synthesized and indeed found to be Mott insulating, including $\text{A}_2\text{O}_3\text{Fe}_2\text{Se}_2$ and $\text{La}_2\text{O}_3\text{Fe}_2\text{S}_2$.

It should be emphasized, however, that iron-based superconductors themselves typically do not reach a Mott-insulating state under stoichiometric doping owing to chemical and structural constraints.

A particularly clean example of a continuous evolution between a Hund-coupled Mott insulator and a Hund metal is provided by the ruthenate series $\text{Ca}_{2-x}\text{Sr}_x\text{RuO}_4$. The end member Ca_2RuO_4 is an antiferromagnetic Mott insulator in which Hund's coupling plays a central role, with an estimated value $J \approx 0.4 \text{ eV}$ [41], comparable to that of iron-based superconductors. In contrast, Sr_2RuO_4 is a strongly correlated Hund's metal with a similar Hund's coupling strength, exhibiting pronounced mass renormalization and supporting spin-triplet superconductivity [42]. The $\text{Ca}_{2-x}\text{Sr}_x\text{RuO}_4$ series thus demonstrates that Hund-coupled metallic states may be continuously connected to Mott-insulating phases by tuning bandwidth, without a change in the underlying local interaction physics [43].

From the perspective of low-energy theory, the proximity to a Hund-coupled Mott insulator implies a hierarchy of energy scales in which charge excitations out of the high-spin local manifold are energetically costly and short lived, even when the system remains metallic. In this regime, charge fluctuations can be treated as virtual processes and systematically integrated out, leaving an effective low-energy description formulated within a reduced Hilbert space. This provides a natural justification for employing a Hund-projected Gutzwiller approach to describe the low-energy spin and carrier dynamics relevant to iron-based superconductors.

VI. SUMMARY

In conclusion, we have derived an effective low-energy model of a correlated Hund's metal near the Mott insulating regime. The undoped system exhibits strong antiferromagnetism with weak quantum fluctuations, placing it close to the regime of classical magnetism. Carrier motion, in contrast, strongly promotes ferromagnetism, resulting in competition. The low-energy model features a sign problem that is extensive only in the number of charge carriers, opening a route to using path-integral methods to characterize polarons and other problems of few carriers.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [44].

APPENDIX: TECHNICAL DETAILS OF THE PROJECTION, AND SUPPLEMENTARY CALCULATIONS

1. Matrix elements of the spin-lowering/raising operator

For the spin-lowering operator of the electron α , we note that S_{α}^{-} must necessarily reduce S_z by unity. Therefore, we have

$$P^H \hat{S}_{\alpha}^{-} |S_M, S_z\rangle = |S_M, S_z - 1\rangle \langle S_M, S_z - 1 | \hat{S}_{\alpha}^{-} |S_M, S_z\rangle. \quad (\text{A1})$$

The inner product can be rewritten as

$$\langle S_M, S_M | (\hat{S}^+)^{l+1} \hat{S}_{\alpha}^{-} (\hat{S}^-)^l |S_M, S_M\rangle \Pi_{a=0}^l \xi_a \Pi_{b=0}^{l-1} \xi_b, \quad (\text{A2})$$

where

$$\xi_a^{-1} = \sqrt{(S_M + [S_M - a])(S_M - [S_M - a] + 1)} \quad (\text{A3})$$

are the coefficients for the spin-lowering operators with $S_z = S_M - a$. Since $[\hat{S}_{\alpha}^{-}, \hat{S}^-] = 0$, we can write

$$\begin{aligned} \langle S_M, S_M | (\hat{S}^+)^{l+1} (\hat{S}^-)^l \hat{S}_{\alpha}^{-} |S_M, S_M\rangle \Pi_{a=0}^l \xi_a \Pi_{b=0}^{l-1} \xi_b \\ = \langle S_M, S_M | (\hat{S}^+)^{l+1} (\hat{S}^-)^l |S_M, S_M - 1\rangle \\ \times \langle S_M, S_M - 1 | \hat{S}_{\alpha}^{-} |S_M, S_M\rangle \Pi_{a=0}^l \xi_a \Pi_{b=0}^{l-1} \xi_b \\ = \langle S_M, S_M | (\hat{S}^+)^{l+1} (\hat{S}^-)^{l+1} |S_M, S_M\rangle \\ \times \frac{\xi_0}{\sqrt{N}} \Pi_{a=0}^l \xi_a \Pi_{b=0}^{l-1} \xi_b = \frac{\xi_0 \xi_l^{-1}}{\sqrt{N}} \equiv \gamma_{S_M S_z}^{-}. \end{aligned}$$

Using (A3) and $\xi_0 = N^{-1/2} = (2S_M)^{-1/2}$, we find the inner product

$$\begin{aligned} \gamma_{S_M S_z}^{-} &= \frac{\sqrt{(S_M + S_z)(S_M - S_z + 1)}}{2S_M}, \\ \gamma_{S_M S_z}^{+} &= \frac{\sqrt{(S_M + S_z + 1)(S_M - S_z)}}{2S_M}, \end{aligned} \quad (\text{A4})$$

where the latter follows by symmetry. Combining (A1) and (A4), we conclude

$$P^H \hat{S}_{\alpha}^{\pm} |S_M, S_z\rangle = \gamma_{S_M S_z}^{\pm} \hat{S}_N^{\pm} |S_M, S_z\rangle, \quad (\text{A5})$$

where $\hat{S}_N^{\pm} |S, S_z\rangle = |S, S_z \pm 1\rangle$ for $|S_z \pm 1| \leq S$ defines the normalized spin-raising/lowering operator.

2. Holstein-Primakoff mean-field theory

As a starting point, we consider the Hund's projected spin Hamiltonian at half filling

$$P^H H_S = I_T \sum_{\langle ij \rangle} \left\{ \frac{S_i^{-} S_j^{+} + S_j^{-} S_i^{+}}{2N^2} + \frac{S_i^z S_j^z}{N} \right\}$$

In the next stage, we apply a Holstein-Primakoff type map [37] of the form

$$\begin{aligned} S_i^z &= S - a_i^{\dagger} a_i, \quad S_i^{+} = a_i \sqrt{N}, \quad S_i^{-} = a_i^{\dagger} \sqrt{N} \quad \text{A-lattice} \\ S_j^z &= b_j^{\dagger} b_j - S, \quad S_j^{+} = b_j^{\dagger} \sqrt{N}, \quad S_j^{-} = b_j \sqrt{N} \quad \text{B-lattice} \end{aligned} \quad (\text{A6})$$

Where $S^{\dagger} = a_i \sqrt{N}$ etc result from the approximation $\langle a_i^{\dagger} a_i \rangle$ and $\langle b_i^{\dagger} b_i \rangle \ll N$. We find

$$P^H H_S = I_T \sum_{\langle ij \rangle} \left\{ \frac{a_i^{\dagger} b_j^{\dagger} + a_i b_j}{2N} + \frac{-S^2 - a_i^{\dagger} a_i b_j^{\dagger} b_j + S(a_i^{\dagger} a_i + b_j^{\dagger} b_j)}{N^2} \right\}. \quad (\text{A7})$$

The quartic term is the smallest, and scales as $\sim A_s^2/N$, where A_s denotes the spin-wave amplitude, so we proceed to drop it.

Noting that $S = N/2$, we find

$$P^H H_S \approx I_T \sum_{\langle ij \rangle} \left\{ \frac{a_i^{\dagger} b_j^{\dagger} + a_i b_j + a_i^{\dagger} a_i + b_j^{\dagger} b_j}{2N} - \frac{1}{4} \right\},$$

where the last term is the classical energy. Fourier transforming, the quantum part becomes

$$\begin{aligned} H_Q &= \sum_{\mathbf{k}} A_{\mathbf{k}} (a_{\mathbf{k}}^{\dagger} a_{\mathbf{k}} + b_{\mathbf{k}}^{\dagger} b_{\mathbf{k}}) + B_{\mathbf{k}} (a_{\mathbf{k}} b_{-\mathbf{k}} + a_{\mathbf{k}}^{\dagger} b_{-\mathbf{k}}^{\dagger}) \\ A_{\mathbf{k}} &= \frac{z I_T}{2N}, \quad B_{\mathbf{k}} = \frac{z I_T}{2N} \gamma_{\mathbf{k}}, \quad \gamma_{\mathbf{k}} = \frac{1}{z} \sum_{\delta} e^{i\mathbf{k} \cdot \delta}. \end{aligned} \quad (\text{A8})$$

Adopt a spinor notation $\Psi_{\mathbf{k}}^{\dagger} = [a_{\mathbf{k}}^{\dagger}, b_{-\mathbf{k}}]$, $\Psi_{\mathbf{k}} = [a_{\mathbf{k}}, b_{-\mathbf{k}}^{\dagger}]$ to obtain

$$H_Q = \sum_{\mathbf{k}} \Psi_{\mathbf{k}}^{\dagger} h_{\mathbf{k}} \Psi_{\mathbf{k}}, \quad h_{\mathbf{k}} = (A_{\mathbf{k}} \sigma_0 + B_{\mathbf{k}} \sigma_x). \quad (\text{A9})$$

The time evolution of an operator $\Psi_{\mathbf{k}}$ is given by

$$i\partial_t \Psi_{\mathbf{k}} = [\Psi_{\mathbf{k}}, H_Q] = \mathcal{L}_{\mathbf{k}} \Psi_{\mathbf{k}}, \quad \mathcal{L}_{\mathbf{k}} = \sigma_3 h_{\mathbf{k}}. \quad (\text{A10})$$

The appropriate Bogolyubov transform is

$$T = \begin{bmatrix} \cosh r & -\sinh r \\ -\sinh r & \cosh r \end{bmatrix}, \quad r = \frac{1}{2} \arctan \frac{B}{A}.$$

This gives

$$T^{-1} \mathcal{L}_{\mathbf{k}} T = \omega_{\mathbf{k}} \sigma_3 = D_{\mathbf{k}}, \quad \omega_{\mathbf{k}} = \sqrt{A_{\mathbf{k}}^2 - B_{\mathbf{k}}^2}. \quad (\text{A11})$$

Introducing new operators

$$\Psi_{\mathbf{k}} = T \Phi_{\mathbf{k}}, \quad \Phi_{\mathbf{k}} = \begin{bmatrix} \alpha_{\mathbf{k}} \\ \beta_{-\mathbf{k}}^{\dagger} \end{bmatrix},$$

the time evolution is given by

$$i\partial_t \Phi_{\mathbf{k}} = D_{\mathbf{k}} \Phi_{\mathbf{k}} \quad (\text{A12})$$

implying a Hamiltonian matrix

$$\tilde{k}_{\mathbf{k}} = \sigma_3 D_{\mathbf{k}} = \omega_{\mathbf{k}} \sigma_0. \quad (\text{A13})$$

In the ground state, we find

$$\alpha_{\mathbf{k}} |0\rangle = \beta_{\mathbf{k}} |0\rangle = 0, \quad (\text{A14})$$

implying

$$\langle a_{\mathbf{k}}^\dagger a_{\mathbf{k}} \rangle = v_{\mathbf{k}}^2 = \frac{1}{2} \left(\frac{A_{\mathbf{k}}}{\sqrt{A_{\mathbf{k}}^2 - B_{\mathbf{k}}^2}} - 1 \right). \quad (\text{A15})$$

The total fluctuation is then given by

$$\langle a_i^\dagger a_i \rangle = \sum_{\mathbf{k}} \langle a_{\mathbf{k}}^\dagger a_{\mathbf{k}} \rangle = \sum_{\mathbf{k}} \frac{1}{2} \left(\frac{1}{\sqrt{1 - \gamma_{\mathbf{k}}^2}} - 1 \right). \quad (\text{A16})$$

On a square/cubic lattice, we find

$$\gamma_{\mathbf{k}} = \frac{1}{2D} \sum_{l=1}^D 2 \cos k_l, \quad (\text{A17})$$

where D is the dimension. On the square lattice, we obtain

$$\langle a_{\mathbf{k}}^\dagger a_{\mathbf{k}} \rangle \approx 0.197. \quad (\text{A18})$$

This result is independent of the total spin. For a five-band model, we thus find that fluctuations are approximately 4% in a given band.

3. Spin rotation of Hund's ground state

Under the subsequent spin-raising operations, the first part of (25) transforms as follows:

$$\begin{aligned} & \hat{S}_+^l |S_M^{n+1}, 1 - S_M^{n+1}\rangle \frac{\chi_{S_M^+, l}^+}{\sqrt{n+1}} \\ &= \hat{S}_+^l \hat{S}_+ |S_M^{n+1}, -S_M^{n+1}\rangle \frac{\chi_{n, l}^+}{\sqrt{n+1}} \xi_{S_M^{n+1}, -S_M^{n+1}}^+ \\ &= |S_M^{n+1}, l+1 - S_M^{n+1}\rangle \frac{\chi_{n, l}^+}{\chi_{S_M^{n+1}, l+1}^+ \sqrt{n+1}} \xi_{S_M^{n+1}, -S_M^{n+1}}^+ \\ &= |S_M^{n+1}, l+1 - S_M^{n+1}\rangle \sqrt{\frac{(l+1)}{n+1}}, \end{aligned} \quad (\text{A19})$$

where $l+1 = S_M^N + S_z + \frac{1}{2}$.

4. Excitations across the Hund's gap

This appendix elaborates on transient kinetic processes involving virtual excitations across the Hund's gap, which are subleading in the low-energy theory discussed in Sec. III. Such processes involve intermediate states in which the local spin configuration temporarily violates Hund's rule, incurring an energy cost on the order of Δ_H . Starting from Eqs. (24) and (25), the corresponding excited state can be expressed as

$$\chi_{S_M^+, l}^+ \hat{S}_+^l |R_\alpha\rangle \equiv \lambda_{n, l} \chi_{S_M^+, l}^+ \hat{S}_+^l \left| \frac{n}{2} - 1, 1 - \frac{n}{2} \right\rangle_\alpha,$$

where λ is the amplitude of the state. The subscript α here indicates that the excited state was generated by creating a particle in the α orbital. This is important since states that do not maximize total spin are typically degenerate, so that $|R_\alpha\rangle \neq |R_\beta\rangle$ in general.

TABLE III. Numerical values of κ obtained for $n = N = 5$, i.e., a state $|R_\alpha\rangle$ with five electrons in five orbitals.

Operator	$l = 0$	$l = 1$	$l = 2$	$l = 3$	$l = 4$
$a_{\alpha\uparrow}$	0.8	0.54	0.31	0.13	0
$a_{\alpha\downarrow}$	0.4	0.44	0.38	0.26	0
$a_{\beta \neq \alpha \uparrow}$	0.2	0.13	0.08	0.03	0
$a_{\beta \neq \alpha \downarrow}$	0.1	0.11	0.095	0.063	0

Applying an annihilation operator and projecting on the Hund's ground state, we find

$$P^H a_{\beta\sigma} \lambda_{n, l} \chi_{S_M^+, l}^+ \hat{S}_+^l \left| \frac{n}{2} - 1, 1 - \frac{n}{2} \right\rangle_\alpha \quad (\text{A20})$$

$$\equiv \kappa_{\alpha\beta\sigma l} \left| \frac{n-1}{2}, l+1 - \frac{n+\sigma}{2} \right\rangle_{\alpha'}, \quad (\text{A21})$$

where α' denotes the orbital occupation after annihilating a particle in the band β , and κ is the amplitude of the resulting state.

To derive the emergent process associated with kinetic excitation and decay, consider two subsequent tunneling events,

$$t^2 a_{i\alpha\sigma}^\dagger(\tau) a_{j\alpha\sigma}(\tau) a_{j\beta\sigma'}^\dagger(\tau') a_{k\beta\sigma'}(\tau'), \quad \tau > \tau'. \quad (\text{A22})$$

Here, we are interested in the contribution when $a_{j\beta\sigma'}^\dagger$ excites the site j across the Hund's gap. The lifetime of this state is $\delta\tau = 1/\Delta_H$ and with $\sigma' = \uparrow$, we obtain an emergent pair propagation term

$$H_h \sim \frac{-t^2}{\Delta_H} c_{j\alpha}^\dagger c_{j\beta} \left(\delta_{\sigma, \uparrow} + \delta_{\sigma, \downarrow} \frac{\hat{b}_j^\dagger}{\sqrt{\hat{l}_j + 1}} \right) \kappa_{\alpha\beta\sigma l_j} P^H a_{i\alpha\sigma}^\dagger a_{k\beta\uparrow}, \quad (\text{A23})$$

where $a^\dagger, a$ take the form (36). Some values of κ are given in Table III. These indicate that pair propagation (A23) is strongly dependent on the spin background, and notably vanishes for a polarized system.

The ideal circumstances for pair propagation is on an antiferromagnetic background. Assuming that $S_z = S_M^N$ on i, k , $S_z = -S_M^N$ on j , and $\sigma = \sigma' = \uparrow$, we find

$$H_h \sim -t_p c_{j\alpha}^\dagger c_{j\alpha} \hat{c}_{i\alpha} \frac{\hat{b}_i^\dagger}{\sqrt{\hat{n}_i + 1}} c_{k\alpha}^\dagger \frac{\hat{b}_k}{\sqrt{\hat{n}_k}} \quad (\text{A24})$$

$$\text{with } t_p = \frac{t^2}{\Delta_H} \kappa_{AF}, \quad \kappa_{AF} = 1 - \frac{1}{n}. \quad (\text{A25})$$

Here, we obtain two principal contributions corresponding to $i = k$ and $i \neq k$. Assuming that the sites are not occupied by other carriers, we can use (43). For $i = k$, we find

$$H_{NN} = -t_p \sum_{(ij)\alpha} c_{j\alpha}^\dagger c_{j\alpha} \hat{c}_{i\alpha} c_{i\alpha}^\dagger, \quad (\text{A26})$$

which describes a nearest-neighbor repulsion with magnitude t_p . For $i \neq k$, we find

$$H_p = -t_p \sum_{(ij)(jk)\alpha} c_{i\alpha}^\dagger c_{j\alpha} c_{j\alpha}^\dagger \hat{c}_{k\alpha} \frac{\hat{b}_k^\dagger \hat{b}_i}{N}, \quad (\text{A27})$$

which describes pair propagation. For opposite spins (i.e., $S_z = -S_M^n$ on i, k etc) we analogously find

$$H_p = -t_p \sum_{(i,j)\langle jk\rangle\alpha} c_{i\alpha}^\dagger c_{j\alpha} c_{j\alpha}^\dagger c_{k\alpha} \quad (\text{A28})$$

by propagating spin-down electrons. We thus obtain two principal contributions from transient excitations across the

Hund's gap in an antiferromagnet: Nearest-neighbor repulsion and a form of pair propagation that preserves the magnetic order. The potency of this mechanism diminishes as antiferromagnetic correlations weaken, and vanishes completely for a ferromagnet.

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