

Emergent chiral Higgs mode in π -flux frustrated lattices

M. Lanaro^{1,2}, L. Maffi^{1,2}, and M. Di Liberto^{1,2,3,*}

¹*Dipartimento di Fisica e Astronomia “G. Galilei,” Università degli Studi di Padova, I-35131 Padova, Italy*

²*INFN Istituto Nazionale di Fisica Nucleare, Sezione di Padova, I-35131 Padova, Italy*

³*Padua Quantum Technologies Research Center, via Marzolo 8, I-35131 Padova, Italy*



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Neutral-atom quantum simulators provide a powerful platform for realizing strongly correlated phases, enabling access to dynamical signatures of quasiparticles and symmetry breaking processes. Motivated by recent observations of quantum phases in flux-frustrated ladders with nonvanishing ground-state currents, we investigate interacting bosons on the dimerized Benalcazar-Bernevig-Hughes (BBH) lattice in two dimensions—originally introduced in the context of higher-order topology. After mapping out the phase diagram, which includes vortex superfluid (V-SF), vortex Mott insulator (V-MI), and featureless Mott insulator (MI) phases, we focus on the integer filling case. There, the MI/V-SF transition breaks the $\mathbb{Z}_2^T$ and U(1) symmetries, where $\mathbb{Z}_2^T$ corresponds to time-reversal symmetry (TRS). Using a slave boson description, we resolve the excitation spectrum across the transition and uncover a chiral Higgs mode whose mass softens at criticality, providing a dynamical hallmark of emergent chirality that manifests via the oscillation of the loop currents in quench dynamics. Our results establish an experimentally realistic setting for probing unconventional TRS-broken phases and quasiparticles with intrinsic chirality in strongly interacting quantum matter.

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I. INTRODUCTION

Neutral-atom quantum simulators offer a versatile playground to study strongly correlated phenomena [1,2]. Beyond identifying equilibrium properties, monitoring real-time dynamics has given access to their collective excitations and quantum correlations [3–7]. A prominent role in symmetry-broken phases is played by amplitude (Higgs) excitations of the order parameter [8], observed, for example, in interacting bosonic [9,10] and fermionic systems [11,12], long-range cavity systems [13], Rydberg spin systems [14], and predicted for dipolar supersolids [15]. Identifying unconventional quasiparticles in strongly correlated quantum systems, including those appearing in lattice gauge theories [16,17], is currently a major goal of quantum simulation platforms.

Synthetic gauge fields [18–20] provide a key ingredient to simulate correlated phases that break time-reversal symmetry (TRS) as for fractional quantum Hall states [21–23]. These states can host quasiparticle excitations that include Abelian and non-Abelian anyons [24,25] as well as chiral graviton modes [26,27], thus stimulating efforts for their identification in synthetic matter systems [28,29]. Recent progress in this direction has been demonstrated through the realization of Laughlin and Pfaffian states with few atoms [30–32]. To date, many-body topologically ordered phases however remain still a challenge for analog quantum simulators, as

compared to digital realizations [33]. However, TRS-broken phases without topological order can also display strongly correlated properties manifesting via unconventional quasiparticles, which is the focus of this work.

Recent experimental advances have demonstrated that many-body quantum phases that break TRS via specific ground-state current patterns and correlations are within reach. For example, quasi-one-dimensional (1D) optical lattice geometries with fermions in synthetic dimensions [34] have shown the signature of a Hall response [35]. Bosonic ladder experiments [36,37] have demonstrated the preparation of a Meissner phase [38], a strongly interacting gapped phase with broken TRS. Such a phase displays leg currents that can be viewed as a precursor of fractional quantum Hall (FQH) state edge modes in the sense of the wire construction [39,40]. In this context, correlated bosonic quantum phases with ground-state currents or vortices offer an intriguing quantum simulation scenario. The interplay of interactions with magnetic and geometrical frustration can indeed give rise to a variety of quantum phases with broken TRS whose excitation properties remain largely unexplored. These include the Meissner phase mentioned above, or phases characterized by loop currents forming staggered, commensurate, or incommensurate vortex arrays [41–56], which can display distinct Hall signatures [57]. The recent experimental developments [37] suggest some of them to be within reach. In 2D, this interplay can also give rise to topological bosonic insulating phases [58–62] characterized by quantized Hall responses and topological order [63], which remain challenging to realize, as we discussed above.

In this work, we examine ground-state and low-lying excitations of interacting bosons on a flux-frustrated dimerized model in two dimensions [Fig. 1(a)], relevant to

*Contact author: marco.diliberto@unipd.it

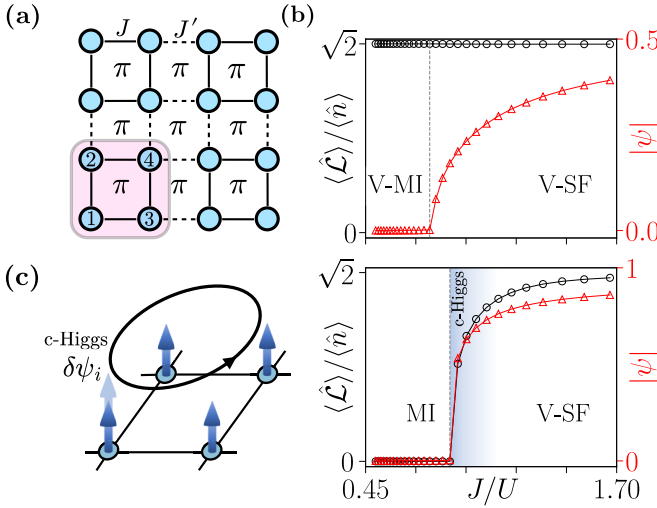


FIG. 1. Model and main results. (a) Schematics of the BBH lattice with dimerized hopping J, J' and uniform π flux for interacting bosons. The unit cell is highlighted. (b) Ground-state loop current $\langle \hat{\mathcal{L}} \rangle$ (black circles) and superfluid order parameter $|\psi|$ (red triangles) within the cluster Gutzwiller approximation marking phase transitions between vortex superfluid (V-SF) and vortex Mott-insulating (V-MI) phases for filling $\nu = 1/4$ (top) and between V-SF and Mott-insulating (MI) phases for $\nu = 1$ (bottom) for $J'/J = 0.03$. The region displaying a chiral Higgs excitation is highlighted. (c) Pictorial representation of chiral Higgs mode dynamics within a plaquette, where $\delta\psi_i$ indicates the amplitude excitation and the mode exhibits a finite loop current.

higher-order topological insulator models [64,65] and recent optical lattice experiments [37]. We identify vortex superfluid (V-SF), vortex Mott insulator (V-MI), and featureless Mott insulator (MI) phases via density matrix renormalization group (DMRG) numerical simulations and a cluster Gutzwiller approach [Fig. 1(b)]. The MI/V-SF transition at integer filling spontaneously breaks the $\mathbb{Z}_2^T$ and $U(1)$ symmetries, where $\mathbb{Z}_2^T$ indicates TRS. A similar mechanism has been suggested to occur in the context of flux-frustrated 2D Josephson junction arrays [41] and identified, for example, in p -band models [66] and twisted superfluids [67,68]. In our 2D model, the key features of this phase transition are reflected in the excitation spectrum, where we identify a chiral Higgs mode carrying nonvanishing vorticity [Fig. 1(c)]. This mode softens across the transition and can be revealed by quench dynamics manifesting in observable oscillations of the currents. Our results, based on a minimal and experimentally accessible construction, provide a pathway for investigating unexplored properties of strongly correlated quantum matter, as for the emergence of unconventional quasiparticles with intrinsic chirality.

The paper is organized as follows. In Sec. II, we introduce the model, the cluster mean-field approximation, and discuss the corresponding phase diagram in 2D; in Sec. III, we perform a DMRG analysis of the model in a ladder geometry; in Sec. IV, we employ a slave boson method to compute the excitations in the superfluid phase near the critical point; in Sec. V, we discuss the results and draw the conclusions.

II. MODEL AND QUANTUM PHASES

We consider soft-core bosons on a dimerized lattice depicted in Fig. 1(a) with uniform π flux per plaquette, and thus corresponding to the Benalcazar-Bernevig-Hughes (BBH) geometry [64,65]. In this work, we study the strongly dimerized limit $J \gg J'$, where J and J' describe the intra- and the inter-plaquette coupling amplitudes, respectively. The free-particle Hamiltonian is thus

$$\hat{\mathcal{H}}_0 = \hat{\mathcal{H}}_J + \hat{\mathcal{H}}_{J'}. \quad (1)$$

On-site interactions are described by

$$\hat{\mathcal{H}}_{\text{on-site}} = \frac{U}{2} \sum_p \sum_{i=1}^4 \hat{n}_{i,p} (\hat{n}_{i,p} - 1) - \mu \sum_p \sum_{i=1}^4 \hat{n}_{i,p}, \quad (2)$$

where the number operator $\hat{n}_{i,p} = \hat{b}_{i,p}^\dagger \hat{b}_{i,p}$ acts on the sublattice site i of the plaquette p according to the convention in Fig. 1(a), $U > 0$ is the interaction strength, and μ the chemical potential. The model $\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_{\text{on-site}}$ describes a BBH Bose-Hubbard model and can be readily realized in optical superlattices via synthetic magnetic fields [37].

In the strongly dimerized limit, each plaquette plays the role of a supersite. Here, we perform our analysis using a cluster Gutzwiller ansatz (cGWA) [69–73], generalizing the more conventional single-site Gutzwiller ansatz [74,75], allowing us to capture quantum phases at fractional filling as well as to retain quantum correlations within a plaquette [76]. Specifically, we perform a decoupling by considering a plaquette product state of the form

$$|\Phi\rangle_{\text{cl}} \equiv \bigotimes_p \left(\sum_{n_1 n_2 n_3 n_4} c_{n_1 n_2 n_3 n_4}^{(p)} |n_1 n_2 n_3 n_4\rangle_p \right), \quad (3)$$

where n_i denotes the Fock occupation number on the sublattice site i . Within this cluster mean-field decoupling, the Hamiltonian reads

$$\hat{\mathcal{H}} \approx \hat{\mathcal{H}}_{\text{cl}} - \sum_{\substack{(p,q) \\ (i,j)}} \{ J'_{ij} [\hat{b}_{i,p}^\dagger \psi^{j,q} - (\psi^{j,q})^* \psi^{i,p}] + \text{H.c.} \}, \quad (4)$$

where $\psi^{i,p} \equiv \langle \hat{b}_{i,p} \rangle$ is the superfluid order parameter and $\hat{\mathcal{H}}_{\text{cl}}$ is the cluster Hamiltonian obtained by considering terms in $\hat{\mathcal{H}}$ that contain only operators acting on a single plaquette (see Supplemental Material [77]). This allows us to write $\hat{\mathcal{H}} \approx \sum_p \hat{\mathcal{H}}_{\text{MF}}^{(p)}$, where $\hat{\mathcal{H}}_{\text{MF}}^{(p)}$ contains only operators acting on a single plaquette p . We solve for the ground state by assuming (plaquette) translational symmetry for a two-dimensional (2D) lattice, namely $\psi^{i,p} = \psi^i$ and by diagonalizing $\hat{\mathcal{H}}_{\text{MF}}^{(p)}$ to find the cluster wave-function parameters $c_{n_1 n_2 n_3 n_4}^{(p)}$ via a self-consistent algorithm (truncating $0 \leq n_i < n_{\text{max}}$ corresponding to a plaquette Hilbert space of dimension $d = n_{\text{max}}^4$). The resulting phase diagram is shown in Fig. 2(a).

In the strongly dimerized case, Ref. [53] showed that at low energies each plaquette p encodes two degenerate orbitals, $|\pm\rangle_p$, that are eigenstates of an emergent (plaquette) angular momentum operator $\hat{L}_z^{(p)}$ commuting with $\hat{\mathcal{H}}$, namely $\hat{L}_z^{(p)} |\pm\rangle_p = \pm |\pm\rangle_p$, and carry (opposite) nonvanishing loop currents. At the plaquette level, interactions take the approximate form $\hat{\mathcal{H}}_{\text{on-site}} \approx -(U/16) \sum_p [\hat{L}_z^{(p)}]^2 + \dots$ and favor the

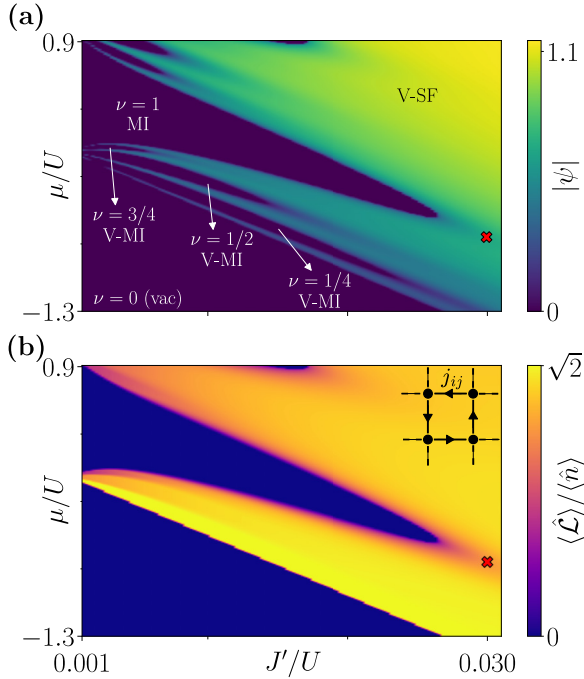


FIG. 2. Phase diagram for $J'/J = 0.03$ of the 2D BBH model via the cluster Gutzwiller approximation ($n_{\max} = 5$). (a) Color map of the superfluid order parameter $|\psi|$ showing the Mott phase (MI), vortex Mott phases (V-MI) at different fillings, and the vortex superfluid phase (V-SF). (b) Corresponding loop current per particle across the phase diagram. Notice that only the Mott lobe at $\nu = 1$ has zero loop current, while fractional fillings exhibit $\langle \hat{L} \rangle \neq 0$, indicating a V-MI phase. The inset shows the current pattern in V-SF and V-MI phases.

formation of ground-state currents,

$$\hat{j}_{ij} = i(J_{ij} \hat{b}_i^\dagger \hat{b}_j - \text{H.c.}), \quad (5)$$

maximizing the angular momentum and thus leading to spontaneous breaking of TRS. Indeed, the loop current operator within a plaquette, $\hat{L}^{(p)} = J^{-1} \sum_{\square} \hat{j}_{ij} \approx \sqrt{2} \hat{L}_z^{(p)}$ takes a nonzero expectation value, where $\sum_{\square}$ indicates the oriented sum on the plaquette p . In the weakly interacting limit, this corresponds to a macroscopic occupation of either $|+\rangle$ or $|-\rangle$, similarly to what happens in higher-band systems [78–82]. However, this feature persists in the strongly correlated superfluid phase. The value of $\langle \hat{L} \rangle$ across the phase diagram is shown in Fig. 2(b) and the corresponding phase is a V-SF that breaks $\mathbb{Z}_2^T \times \text{U}(1)$. Strong interactions renormalize the loop current, such that its value per particle is no longer exactly quantized in units of $\sqrt{2}$ as in the low-energy theory.

When interactions dominate, $U \gg J'$, different quantum phase transitions occur depending on the filling ν (bosons per site). At filling $\nu = 1/2$, we confirm the transition identified via DMRG in Ref. [53] where the system enters a vortex Mott-insulating phase (V-MI), namely an incompressible translationally invariant phase lacking phase coherence and with loop currents [37,48] localized at the plaquettes [see the inset of Fig. 2(b)]. At filling $\nu = 1/4$, namely one particle per plaquette, we also find a V-MI phase. This is in contrast with what happens in p -band models, where the insulating phase corresponds to a staggered paramagnet with no orbital

order [66,78,81]. To understand why the quantum fluctuations in our model still favor a V-MI phase we perform second-order perturbation theory at $J'/U \ll 1$ for filling $\nu = 1/4$. For this, we consider the chiral basis introduced above, $|\pm\rangle$, which represents the two chiral single-particle modes of a π -flux plaquette and can be mapped onto an effective spin-1/2 degree of freedom. Perturbative calculations yield the quantum XXZ 2D Hamiltonian

$$\hat{H}_{\text{eff}} = -\frac{5}{2} \frac{J'^2}{U} - \frac{J'^2}{4U} \sum_{\langle p,q \rangle} [3\hat{\sigma}_p^z \hat{\sigma}_q^z + \hat{\sigma}_p^x \hat{\sigma}_q^x + \hat{\sigma}_p^y \hat{\sigma}_q^y]. \quad (6)$$

Differently from p -band systems, where $\hat{H}_{\text{eff}}$ is classical at leading order, we obtain a quantum effective model. Furthermore, the parameters in $\hat{H}_{\text{eff}}$ set the system in the easy-axis regime [83] and put the ground state in the ferromagnetic phase [84,85], which corresponds to a V-MI phase. Finally, we also identified a transition to an insulating phase at $\nu = 3/4$ in a small regime of phase space.

At integer filling, $\nu = 1$, a completely different scenario takes place, which is the core of this work. Correlations within a plaquette will be suppressed when $U \gg J, J'$ and the system displays a transition onto a Mott-insulating (MI) phase with a unique ground state, asymptotically connected to the product state $|1111\dots\rangle$. Unlike the previous cases, a distinct symmetry-breaking pattern thus takes place. In addition to $\text{U}(1)$ [equivalently $\text{O}(2)$], the superfluid ground state also displays different time-reversal properties as compared to the nearby Mott phase, which therefore correspond to the breaking of $\mathbb{Z}_2^T \times \text{U}(1)$ (see Fig. 1). This fact will have dramatic consequences in the excitation spectrum, as we will discuss down below.

III. DMRG ANALYSIS ON A LADDER

We complement our findings by performing DMRG simulations via the itensor library [86] on a ladder of dimensions $L = 2 \times L_x$ shown in Fig. 3(a) for $J' = 0.1J$. We identify the transitions by computing the fidelity susceptibility,

$$\chi_{\mathcal{F}} = -\frac{\langle \psi(U + \delta U) | \psi(U) \rangle - 2 + \langle \psi(U - \delta U) | \psi(U) \rangle}{\delta U^2 L}, \quad (7)$$

expected to diverge at the transition point [87]. We find two distinct peaks displayed in Figs. 3(b) and 3(c): One occurs at $U/J \sim 0.65$ for $\nu = 1/4$ and another one at $U/J \sim 1.13$ for $\nu = 1$, whereas the transition at $\nu = 1/2$ was identified at $U/J \sim 1.2$ in Ref. [53]. The plaquette loop current $\langle \hat{L} \rangle$ manifests distinct behavior for the two types of transitions [see Figs. 3(d) and 3(e)]. While for $\nu = 1/4$ the loop current remains finite and smooth across the critical point, for $\nu = 1$ we observe a clear drop signaling that the ground state restores time-reversal symmetry in the insulating phase [88]. In the insets, we estimate the loop current correlations between plaquettes, $C_p = \langle \hat{L}_0 \hat{L}_p \rangle$, showing that the $\nu = 1/4$ insulating phase displays ferromagnetic long-range order as predicted by the perturbative analysis discussed above; instead, for $\nu = 1$, C_p decays exponentially as expected in a featureless MI phase. The identification of the transition at filling $\nu = 3/4$ with

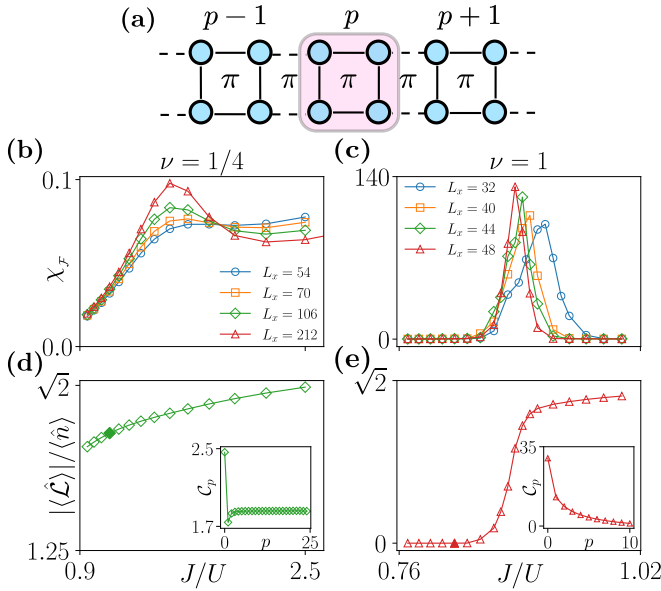


FIG. 3. DMRG numerical results for a ladder with $J'/J = 0.1$. (a) Ladder geometry for DMRG simulations. (b), (c) Fidelity susceptibilities χ_F for $\nu = 1/4$ and $\nu = 1$. The peaks growing with system size are indication of the phase transition. (d), (e) Loop current per particle $|\langle \hat{L} \rangle|/\langle \hat{n} \rangle$ for $L_x = 106$ ($\nu = 1/4$) and for $L_x = 48$ ($\nu = 1$). The $\nu = 1$ curve and the drop of loop current show the V-SF/MI phase transition. Insets show the C_p correlator corresponding to the solid symbols.

DMRG has instead proven to be computationally challenging, as already anticipated by the narrow parameters range predicted by the cGWA phase diagram.

IV. EXCITATIONS NEAR THE CRITICAL POINT

Quantum phase transitions with different symmetry-breaking mechanisms are expected to display low-energy excitations with distinct signatures [89]. The fractional filling transitions only break $U(1)$ symmetry and thus display analogous properties as the conventional Mott-superfluid transitions [8,89,90]. In this case, besides a low-energy gapless Goldstone mode, an amplitude (Higgs) excitation softens, as it was experimentally confirmed for the standard Bose-Hubbard model [9,10]. At integer filling, the ground state additionally breaks TRS across the critical point and one expects novel modes and excitation channels to appear, which may signal possible unconventional critical behavior.

To describe the low-energy excitation spectra, we apply the slave boson formalism described in Ref. [91] to the cluster framework used in this work. This and similar approaches [71,92–96] have proven to accurately capture the excitation gaps observed experimentally [10] and can include quantum fluctuations in a controlled manner [91,97]. Within this framework, for each plaquette we introduce slave boson operators $\hat{\beta}_{p,n}$, spanning the plaquette Hilbert space $|n_1 n_2 n_3 n_4\rangle_p \equiv |\mathbf{n}\rangle_p = \hat{\beta}_{p,n}^\dagger |0_{\text{SB}}\rangle$, where $|0_{\text{SB}}\rangle$ is a fictitious slave boson vacuum and $\sum_n \hat{\beta}_{p,n}^\dagger \hat{\beta}_{p,n} = 1$. The diagonalization of the cluster mean-field Hamiltonian $\hat{\mathcal{H}}_{\text{MF}}^{(p)}$ provides a set of plaquette normal modes described by the unitary

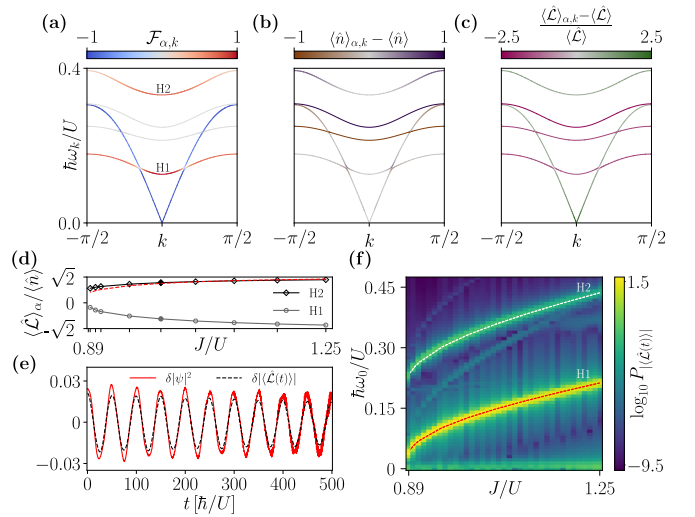


FIG. 4. Low-energy excitations in the V-SF phase for filling $\nu = 1$. (a)–(c) Quasiparticle excitations for $k_x = k_y = k$, $J'/U = 0.03$, $J/U = 1.0$, and $\mu/U = -0.68$, corresponding to the cross symbols in Fig. 2. Spectra are colored based on the values of (a) flatness $\mathcal{F}_{\alpha,k}$, (b) plaquette density, and (c) loop current $\langle \hat{L} \rangle_{\alpha,k}$. The index $\alpha = 1, \dots, 5$ labels the modes for increasing value of energy. (d) Loop current per particle, $\langle \hat{L} \rangle_\alpha / \langle \hat{n} \rangle$, for H1, H2, and superfluid ground state (red dashed line). Solid symbols correspond to spectra shown in the upper panel. (e) cGWA time evolution of loop current (black dashed line) and local order parameter (red solid line) oscillations normalized to their ground-state values following a quench of a staggered flux $\pm \delta\theta$ per plaquette at unit filling. Parameters as in (a)–(c) and $\delta\theta = 0.01(J - \bar{J})/\bar{J}$ with $\bar{J} = 0.88U$. (f) Power spectrum of the loop current time evolution for different values of J/U . Dashed lines highlight the frequencies corresponding to H1 (red) and H2 (white) modes.

transformation $U_{m,n}^{(p)}$ that we use to rotate the slave boson operators via the linear transformation $\hat{\gamma}_{p,m}^\dagger = \sum_n U_{m,n}^{(p)} \hat{\beta}_{p,n}^\dagger$, where m labels each plaquette normal mode with increasing energy. The bosonic creation operator can then be expressed as

$$\hat{b}_{i,p}^\dagger = \sum_{\mathbf{n}\mathbf{n}'} \langle \mathbf{n} | \hat{b}_{i,p}^\dagger | \mathbf{n}' \rangle \hat{\beta}_{p,\mathbf{n}}^\dagger \hat{\beta}_{p,\mathbf{n}'} = \sum_{n,m} \hat{\gamma}_{p,n}^\dagger \tilde{F}_{nm}^{i,p} \hat{\gamma}_{p,m}. \quad (8)$$

Within the slave boson approach, we approximate $\hat{\mathcal{H}}_{\text{MF}}^{(p)}$ by condensing $\hat{\gamma}_{p,m=0}^{(\dagger)} \approx 1$ and by treating $\hat{\gamma}_{p,m \neq 0}^{(\dagger)}$ as weak quantum fluctuations up to quadratic order. In momentum space, we obtain the quadratic Hamiltonian (see Supplemental Material [77])

$$\hat{\mathcal{H}} \approx \hat{\mathcal{H}}^{(2)} = \frac{1}{2} \sum_{\mathbf{k}} (\hat{\gamma}_{\mathbf{k}}^\dagger, \hat{\gamma}_{-\mathbf{k}}) H_{\mathbf{k}} (\hat{\gamma}_{\mathbf{k}}, \hat{\gamma}_{-\mathbf{k}}^T)^T, \quad (9)$$

where $H_{\mathbf{k}}$ is a $(2d-2) \times (2d-2)$ Bogoliubov–de Gennes (BdG) Hamiltonian matrix, $\hat{\gamma}_{\mathbf{p}} = 1/\sqrt{N} \sum_{\mathbf{k}} e^{i\mathbf{k} \cdot \mathbf{r}_{\mathbf{p}}} \hat{\gamma}_{\mathbf{k}}$ and $\hat{\gamma}_{\mathbf{p}} = (\hat{\gamma}_{p,1}, \dots, \hat{\gamma}_{p,n}, \dots, \hat{\gamma}_{p,d})^T$. As shown in Figs. 4(a)–4(c) near the critical point of the transition at $\nu = 1$, the excitation spectra obtained from $H_{\mathbf{k}}$ display several low-energy branches in the superfluid phase. The gapless branch corresponds to the Goldstone (phase) mode. However, other low-energy gapped modes appear. The analysis of the phase and amplitude character of the order parameter, obtained via the flatness

$\mathcal{F}_{\alpha,\mathbf{k}}$ [10,77,95], reveals that two of the gapped excitations (H1 and H2) are amplitude (Higgs) modes with $\mathcal{F}_{\alpha,\mathbf{k}} \approx 1$ corresponding to oscillations of the superfluid density and only one softening at the transition [see Fig. 4(d)], whereas only the lowest mode is a phase (Goldstone) mode ($\mathcal{F}_{\alpha,\mathbf{k}} \approx -1$). This result is in contrast to the more conventional behavior of Mott-superfluid transitions with only a single Higgs excitation. Two other dispersive modes appear with neither amplitude nor phase character. As shown in Fig. 4(b), these modes are particle or hole excitations as indicated by computing the density variation carried by each of them and their particle-hole character (see Ref. [95]). The chiral nature of the excitations can be revealed by estimating the loop current for each branch $\langle \hat{\mathcal{L}} \rangle_\alpha$ as compared to the ground-state value $\langle \hat{\mathcal{L}} \rangle$, as shown in Fig. 4(c). The two Higgs modes both carry a finite loop current. Sufficiently within the superfluid phase its value for $\mathbf{k} = \mathbf{0}$ tends to the effective theory expectation $\langle \hat{\mathcal{L}} \rangle_{\text{H1,H2}} \rightarrow \pm\sqrt{2}$ (per particle) [see Fig. 4(d)]. Crucially, the lowest-energy Higgs mode (H1) exhibits a loop current of opposite sign as compared to the ground-state one, thus bringing a finite *large* differential value $|\Delta\mathcal{L}_{\text{H1}}| \equiv |\langle \hat{\mathcal{L}} \rangle_{\text{H1}} - \langle \hat{\mathcal{L}} \rangle| \approx 2\sqrt{2}$ per particle, and manifesting a chiral nature. This hierarchy is motivated by the fact that the nearby insulating phase does not break TRS, and the lowest Higgs H1 mode is there to carry away the ground-state loop current across the transition. Notice that this multi-Higgs structure only appears near the $\nu = 1$ critical point, whereas at fractional fillings a more conventional picture with a single nonchiral Higgs mode ($\Delta\mathcal{L}_{\text{H}} \approx 0$) occurs (see Supplemental Material [77]).

In order to probe the lowest Higgs chiral mode H1, we perform a quench by adding a weak staggered flux perturbation $\pm\delta\theta$ [19], which preserves U(1) and explicitly breaks TRS. As shown in Fig. 4(e), this selectively excites coherent in-phase oscillations of loop current and order parameter. A Fourier spectral analysis [Fig. 4(f)] reveals a dominant peak corresponding to the chiral Higgs mode H1, negligible excitation of H2, and no particle (hole) weight, consistent with the fact that the quench preserves the global U(1) symmetry [10,98]. Similar results are also obtained by the more traditional quench of the hopping amplitude J [10]. The visibility of the Higgs mode via current oscillations makes its detection relevant in light of the recently developed protocol for (local/site-resolved) current measurement in optical lattices [99].

V. DISCUSSION AND CONCLUSIONS

In this work we have shown that interacting bosons on a dimerized flux-frustrated lattice display distinct quantum phases that spontaneously break TRS manifesting in vortex Mott and superfluid phases. The transition at integer filling breaks TRS and U(1) symmetries, and the excitation spectrum in the chiral superfluid phase consists of two Higgs quasiparticles, including a chiral one. Our results provide

an example of a strongly correlated system with nontrivial quasiparticles that is especially relevant in the context of very recent experimental developments employing dimerized lattices with synthetic gauge fields [37] and the corresponding direct measurement of the current patterns. Our work opens several questions concerning, for example, the decay and lifetime of the quasiparticles here identified [98], the protocol to selectively excite the second Higgs mode, the effective quantum field-theory description of the multi-Higgs structure in the presence of coexisting orders, and the critical properties of the $\nu = 1$ transition [89] to be investigated with other methods, as for example large-scale quantum Monte Carlo techniques [100]. Further work will clarify the possibility that the two symmetries [TRS and U(1)] are broken separately at integer filling through an intermediate phase in a narrow parameter region, as identified via refined numerical DMRG analyses in related nondimerized models [44–46]. Further investigations are needed to elucidate the interplay between topology and interactions in the BBH model at different fillings [101,102], as for the Su-Schrieffer-Heeger (SSH) case [103], including the possibility to stabilize topological phases via interactions [62]. In addition, it would be interesting to explore effects beyond strong dimerization that are not captured by our analysis and the role played by gauge symmetries [104] and chiral bound pairs [105] in transport phenomena. Our work paves the way to the identification of other unconventional critical points in chiral systems [46], including those connected to deconfined critical points [41,106,107].

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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