

Spin subdiffusion in perturbed infinite- U Hubbard chainJ. Rękas¹,¹ M. Mierzejewski¹,¹ Z. Lenarčič²,² and P. Prelovšek²¹*Department of Theoretical Physics, Faculty of Fundamental Problems of Technology,
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The t model represents the Hubbard model in the limit $U \rightarrow \infty$ and is one of the basic models of strongly correlated electrons. On a one-dimensional chain, the model is integrable, and its charge dynamics is equivalent to that of free spinless fermions. However, within its dynamics the sequence of spins is frozen, implying Hilbert-space fragmentation and nontrivial spin dynamics. We focus primarily on perturbed versions of the model in which the perturbations break integrability while preserving fragmentation. Considered examples of such perturbations are spin-dependent hopping and additional spin interaction. We show that these models exhibit a variety of spin-transport regimes. In contrast to the integrable model, which displays either ballistic transport or dissipationless diffusion, the perturbed systems exhibit either normal diffusion or subdiffusion. Due to Hilbert-space fragmentation, spin transport is mediated by charge transport in all cases considered. This leads to a characteristic dependence on the magnetization, most notably resulting in subdiffusive spin transport in the grand-canonical ensemble of the perturbed model. The underlying mechanism is distinct from the subdiffusion observed in disordered systems or in models with dipole conservation.

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Introduction. The transport in integrable quantum lattice models has been in the focus of numerous theoretical studies with, both, analytical and numerical approaches. The vast majority dealt with (in general anisotropic) Heisenberg XXZ spin chain $1/2$ [1]. Another example is the one-dimensional (1D) Hubbard model [2,3], which is even closer to most challenging questions of materials with correlated electrons, such as cuprates [4] and fermionic cold-atom systems [5], but much less investigated theoretically with respect to transport [6–9]. The complexity of the 1D Hubbard model is strongly reduced going into the regime $U \gg t$ where it maps to the t - J model [10] and finally in the $U \rightarrow \infty$ limit into the integrable t model, which has been analytically considered in several studies [3,11,12]. The t model bears some similarity to the much more extensively studied easy-axis XXZ model [13–15], and in particular to the folded XXZ model in the limit $\Delta \rightarrow \infty$ [16–18]. It is well established that spin transport in the easy-axis XXZ model is ballistic at finite magnetization m and diffusive at $m = 0$. The latter regime is anomalous in several respects: it is dissipationless, depends explicitly on the order of the limits ($L \rightarrow \infty, \omega \rightarrow 0$) [1,7] and exhibits anomalous moments [19]. Similar anomalous diffusion is expected within the integrable t model [7,9,20–23].

Like the folded XXZ model [16,17], the t model is an example of the system with the Hilbert-space fragmentation [17,24,25]. Decoupled subspaces correspond to

(and can be labeled by) distinct spin sequences. The latter sequences are conserved for any magnetization because particles are impenetrable and there is no spin-exchange interaction. The fragmentation persists within certain generalized but nonintegrable t model. Here, we consider a system with spin-dependent hopping, $t_{\uparrow} \neq t_{\downarrow}$, as well as the $t - J_z$ model [24,26], where integrability is broken by the Ising-type spin-spin interaction. In spite of the integrability-breaking perturbation, the spin dynamics and transport are nontrivial because of the Hilbert-space fragmentation. We show that such perturbed models offer the manifestation of different transport scenarios for spin excitations.

At half-filling the integrable t model is trivial (charge and spin) insulator, while away from half-filling it reveals ballistic charge transport with the spin transport depending on magnetization m . In spite of Hilbert-space fragmentation, at $m \neq 0$ the spin transport is ballistic due to the overlap with the charge current. However, in the canonical $m = 0$ sector there is no spin current even in systems with periodic boundary conditions (PBCs). We show this both analytically and numerically by Thouless argument of sensitivity to magnetic flux (twisted boundary conditions) [27,28]. Similarly, we show that boundary Lindblad spin-flipping terms cannot induce a steady-state (dc) spin current. However, the thermodynamic grandcanonical (GC) averaging over all sectors reveals for the integrable t model a dissipationless spin diffusion [23,29], similar to that in the gapped regime of the XXZ model [7,9,20–22].

Like in the XXZ models [30,31], breaking of integrability in the perturbed model reduces the spin transport at finite magnetization from ballistic to diffusive. Even more interesting is the GC average of transport over all m sectors (corresponding to $\langle m \rangle_{\text{GC}} = 0$, where our results suggest a subdiffusive spin

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transport. A similar behavior has been previously theoretically anticipated in Ref. [30] for perturbed folded XXZ model. Besides more phenomenological approaches [32], there are only few other scenarios for the emergence of subdiffusion, e.g., due to dipole-moment conservation [33–36] or disorder [37,38]. The mechanism for subdiffusion in our study differs from the latter two and can be explained in terms of solution to the porous medium equation.

Perturbed t model. We study the generalized 1D t model, which can be considered as the $U \rightarrow \infty$ limit of the Hubbard model,

$$H_t = - \sum_{l,\sigma=\uparrow,\downarrow} t_\sigma \tilde{c}_{l+1,\sigma}^\dagger \tilde{c}_{l,\sigma} + \text{H.c.} \quad (1)$$

Double occupation of sites is not allowed and, consequently, we use projected fermionic operators $\tilde{c}_{l,\sigma} = P_l c_{l,\sigma} P_l$ with $P_l = 1 - n_{l\uparrow} n_{l\downarrow}$ and $n_{l\sigma} = c_{l,\sigma}^\dagger c_{l,\sigma}$. For $t_\uparrow = t_\downarrow$ the model is integrable, as is also the parent Hubbard model $H_H = -t \sum_{l,\sigma} (c_{l+1,\sigma}^\dagger c_{l,\sigma}^\dagger + \text{H.c.}) + U \sum_l n_{l\uparrow} n_{l\downarrow}$ [3]. However, unlike the parent H_H , it preserves the Hilbert-space fragmentation since it does not allow the change of spin sequences. As possible perturbation that break integrability but preserve the fragmentation, we will consider the $t - \Delta t$ model with $t_{\uparrow,\downarrow} = t \pm \Delta t/2$ and the $t - J_z$ model with spin interaction without the exchange of spins of the full $t - J$ model,

$$H_z = H_t + J_z \sum_i S_i^z S_{i+1}^z, \quad S_i^z = \frac{1}{2}(n_{i\uparrow} - n_{i\downarrow}). \quad (2)$$

The charge and spin currents are given, respectively, by

$$j_c = \sum_l j_{l,l+1,\uparrow} + j_{l,l+1,\downarrow}, \quad (3)$$

$$j_s = \frac{1}{2} \sum_l j_{l,l+1,\uparrow} - j_{l,l+1,\downarrow}, \quad (4)$$

where $j_{l,l',\sigma} = (it_\sigma \tilde{c}_{l',\sigma}^\dagger \tilde{c}_{l,\sigma} + \text{H.c.})$ are the local current operators.

Charge current. First, we discuss the integrable t model with $\Delta t = 0$, where the charge current is a conserved quantity (CQ), as observed long ago [39]. It is interesting to note that this CQ emerges from conserved energy current in the Hubbard model, [6,40,41],

$$Q_3 = t \sum_{l,\sigma} \left[j_{l-1,l+1,\sigma} - U \left(n_{l,-\sigma} - \frac{1}{2} \right) (j_{l,l+1,\sigma} + j_{l-1,l,\sigma}) \right]. \quad (5)$$

In the case of strong interaction, $U/t \gg 1$, the first terms can be neglected along with the contribution $n_{l,-\sigma} (j_{l,l+1,\sigma} + j_{l-1,l,\sigma})$ that requires a double occupancy of the site l . This yields $Q_3/U = j_c$, which implies the conservation of j_c for $U \rightarrow \infty$. Consequently, the charge transport within the t model is ballistic [3,11,12]. Perturbations $\Delta t \neq 0$ or $J_z \neq 0$ break integrability so that j_c is not conserved any more, leading to scattering and diffusive dc transport in the resulting fermionic models.

On the other hand, the spin dynamics in the t model is non-trivial. While the positions of spin-up and spin-down fermions evolve in time, the sequence of spins remains frozen (up to translations, for PBC), giving rise to Hilbert-space fragmentation [24]. The latter originates from the absence of doubly

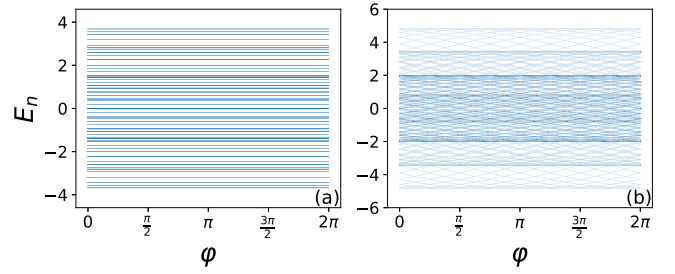


FIG. 1. Spectrum of the integrable t model vs flux φ for a system of $L = 8$ sites. (a) shows results for magnetization $m = 0$ with $N_\uparrow = N_\downarrow = 3$, and (b) for $m \neq 0$ with $N_\uparrow = 2$ and $N_\downarrow = 3$.

occupied sites and from the fact that hopping is restricted to neighboring sites. Consequently, fragmentation persists also for $\Delta t \neq 0$ and $J_z \neq 0$ which break the integrability along with the conservation of j_c .

Spin current. We study a chain of L sites with PBC containing $N = N_\uparrow + N_\downarrow$ particles with total spin $S_{\text{tot}}^z = (N_\uparrow - N_\downarrow)/2$. First, we show that for zero magnetization, $m = 2S_{\text{tot}}^z/L = 0$, all diagonal matrix elements of spin current vanish, i.e., $\langle n | j_s | n \rangle = 0$, where $H_t | n \rangle = E_n | n \rangle$. To this end, we discuss the sensitivity to spin-dependent flux [27,28]. That is, we replace $t_\uparrow \rightarrow t_\uparrow e^{i\varphi}$ and $t_\downarrow \rightarrow t_\downarrow e^{-i\varphi}$ and note that the spin current equals $j_s = -\frac{1}{2} dH_t/d\varphi$ and then $\langle n | j_s | n \rangle = -\frac{1}{2} dE_n/d\varphi$. Appropriate gauge transformation shifts the φ dependence of t_σ to an arbitrary bond. One may also choose different bonds for spin-up and spin-down particles without modifying the energy spectrum. In particular, one may choose that φ dependence is on the bond between site 1 and site L and the hopping term on this bond is specified as a boundary condition. For this reason, the derivative $dE_n/d\varphi$, can be viewed as the Thouless level sensitivity (LS) to the boundary conditions. The specific feature of the present LS is that the corresponding boundary conditions are spin-dependent, so the LS contains information on the spin transport instead of the charge transport. In the following, we distinguish between the results obtained within a single magnetization sector, m , from GC averaging over various magnetization sectors. For the latter, we explicitly indicate the average magnetization, $\langle m \rangle_{\text{GC}}$.

Figure 1 shows the energy levels, $E_n(\varphi)$, in a small system described by the t model. For $m = 0$, the LS vanishes as shown in Fig. 1(a). It is evident from Fig. 1(b) that $m \neq 0$ is required to obtain the nonvanishing flux dependence of $E_n(\varphi)$ and consequently also $\langle n | j_s | n \rangle \neq 0$. The same conclusions hold for other system sizes and particle densities, as well as for $\Delta t \neq 0$ and $J_z \neq 0$ (not shown). In the Supplemental Material (SM) [42], we present analytical arguments for the vanishing of LS at $m = 0$.

To gain physical insight into the LS results, one may consider a time-dependent flux with a sufficiently small slope, $d\varphi/dt = \text{const.}$, such that the system remains in the linear-response regime. This flux drives spin-up particles to move along the chain in one direction and spin-down particles in the opposite direction. At $m = 0$, the charge current must vanish due to symmetry, while the vanishing of the LS also implies the absence of the steady spin current. For nonzero m , the

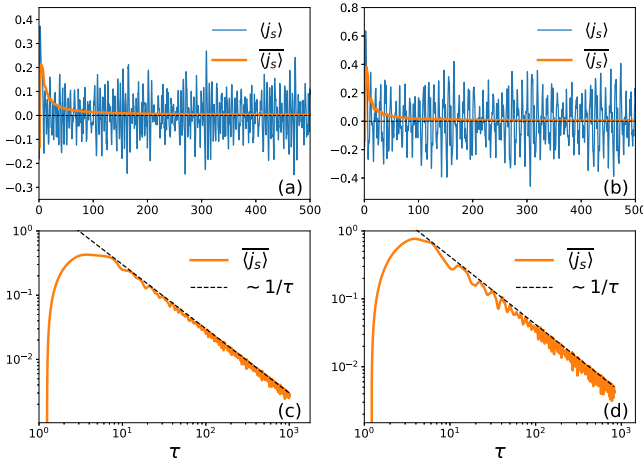


FIG. 2. (a), (b) Time dependence of the spin current, $\langle j_s \rangle(\tau)$, obtained from numerical solution of Eq. (6) for H_t , given by Eq. (1) with $\Delta t = 0$ on $L = 8$. We chose the initial state $|\downarrow 0 \uparrow 0 \downarrow 0 \uparrow 0\rangle$ in (a) and $|\downarrow 0 \uparrow 0 \uparrow 0 \uparrow 0\rangle$ in (b). (c) and (d) show cumulative averages of the spin currents shown in (a) and (b), respectively.

same flux induces a charge current flowing in the direction of the particles with the majority spin polarization and nonvanishing LS indicates a finite spin current. Consequently, the LS results shown in Fig. 1 strongly suggest that spin and charge currents in the t model are intrinsically coupled to one another. In other words, if a system does not allow for a steady charge current, stationary driving cannot generate a steady spin current. Below, we verify this conclusion by means of explicit calculations for an open system subject to edge dissipators.

Open systems. As an alternative to the time-dependent driving, we consider an open system with edge coupling to the baths

$$\frac{\partial \rho}{\partial \tau} = -i[H_{t(z)}, \rho] + \sum_{k=1}^2 L_k \rho L_k^\dagger - \frac{1}{2}(L_k^\dagger L_k \rho + \rho L_k^\dagger L_k). \quad (6)$$

We use edge dissipators $L_1 = S_1^+ = \tilde{c}_{1\uparrow}^\dagger \tilde{c}_{1\downarrow}$ and $L_2 = S_L^-$ that can induce spin current without causing any imbalance in the charge sector. There is a class of initial states $\rho_i = |\uparrow, \vec{s}, \downarrow, \alpha\rangle\langle\uparrow, \vec{s}, \downarrow, \alpha|$ that are annihilated by the dissipator and thus immune to current generation via coupling to baths at any time. These are projectors on eigenstates of the Hamiltonian H_t or H_z where the spin of the first particle is up spin and that of the last particle is down. In the above notation, $\vec{s} = (\sigma_2, \dots, \sigma_{N-1})$ labels the spin configuration of other $N - 2$ fermions in the system, while α enumerates eigenstates within a fragmented subspace. Since ρ_i are the steady states of Eq. (6) as well as of closed systems (described by H_t or H_z) with open boundary conditions, they do not support any steady current.

For other initial states, we determine the spin current from the numerical solution of Eq. (6), $\langle j_s \rangle(\tau) = \text{Tr}[\rho(\tau)j_s]$. The results in Figs. 2(a) and 2(b) show strong undamped oscillations of the spin current for two different initial states. In SM [42] we link these oscillations to a strongly structured spectrum of the Liouvillian (6) containing multiple nondecaying states (with zero real and finite imaginary part of eigenvalue)

with finite current expectation value. In Figs. 2(c) and 2(d) we plot the cumulative averages $\overline{\langle j_s \rangle}(\tau) = \frac{1}{\tau} \int_0^\tau d\tau' \langle j_s \rangle(\tau')$ that decay in time as τ^{-1} , indicating the absence of a stationary spin current. This is also consistent with spectrum analysis, which reveals that steady states with zero eigenvalue has zero spin current expectation value. It implies absence of the dc spin current independent of the initial state, expected also on increasingly large system sizes unless one includes Lindblad operators that allow for dc charge current, as shown in Ref. [42]. To summarize this part we note that that charge and spin currents are inherently coupled. Systems that do not allow for a dc charge current likewise do not support a dc spin current.

Spin diffusion in the integrable t model. From now on, we consider closed systems with $\varphi = 0$ and PBC, which allow for a dc charge current. The dynamical spin diffusion can be evaluated within the linear-response theory, $\mathcal{D}_s(\omega) = \int_0^\infty d\tau e^{i\omega\tau} \langle j_s(\tau)j_s \rangle / (L\tilde{\chi}_s)$, where $\tilde{\chi}_s = n/4$ is the renormalized spin susceptibility at $T \rightarrow \infty$. In the following we discuss results for $\mathcal{D}_s(\omega)$, obtained numerically via the microcanonical Lanczos method (MCLM) [43–45]. The method allows for system sizes up to $L = 20$ and the frequency resolution $\delta\omega \simeq 10^{-3}$ by employing $\sim 10^4$ Lanczos steps in the present study.

Low- ω behavior of $\mathcal{D}_s(\omega)$ distinguishes between: (i) ballistic transport when $\mathcal{D}_s(\omega) \sim D_s \delta(\omega)$ and D_s is the spin stiffness, (ii) diffusive transport when $D_s = 0$ and $\mathcal{D}_s^0 = \mathcal{D}_s(\omega \rightarrow 0) > 0$, (iii) subdiffusive transport when $D_s = 0$ and $\mathcal{D}_s^0 = 0$. To recognize these regimes, it is convenient to discuss the integrated conductivity $I_s(\omega) = \int_0^\omega d\omega' \mathcal{D}_s(\omega')$. Cases (i)–(iii) correspond to $I_s(\omega \rightarrow 0) = D_s$, $I_s(\omega \ll t) = \omega \mathcal{D}_s^0$ and $I_s(\omega \ll t) \propto \omega^a$ with $1 < a \leq 2$, respectively ($a > 2$ would correspond to insulating case).

The nontrivial spin transport in the t model emerges from the overlap with j_c . The relevant overlaps can be evaluated analytically at $T \rightarrow \infty$ and $L \rightarrow \infty$,

$$\langle j_c j_s \rangle = -2Lm(1-n)\bar{t}^2, \quad \langle j_c j_c \rangle = 2Ln(1-n)\bar{t}^2, \quad (7)$$

where $\langle \dots \rangle = \text{Tr}(\dots)/\text{Tr}(1)$, $\bar{t}^2 = (t_\uparrow^2 + t_\downarrow^2)/2 = t^2 + \Delta t^2/4$, and $n = N/L$ is the particle density. In the integrable model, j_c is conserved, leading to a finite spin stiffness D_s in each sector with $m \neq 0$ according to the Mazur-based inequality [6],

$$TD_s \sim \frac{1}{2L} \frac{\langle j_c j_s \rangle^2}{\langle j_c j_c \rangle} = \frac{m^2(1-n)t^2}{n}. \quad (8)$$

The same m^2 dependence of D_s occurs within the easy-axis XXZ model [9,14,22]. In the GC ensemble at $\langle m \rangle_{\text{GC}} = 0$ such m^2 dependence leads in the $L \rightarrow \infty$ limit to a finite $\mathcal{D}_s^0 > 0$, even though the diffusive transport remains anomalous since it is dissipationless [23,29]. One can also interpret the finite value of $\mathcal{D}_s^0 > 0$ as a consequence of boundary scattering, with an effective relaxation time $\tau_L \propto L$. Combined with the GC average $\langle m^2 \rangle_{\text{GC}} \propto 1/L$ this yields an L -independent dc diffusion constant, $\mathcal{D}_s^0 \propto \tau_L \langle \mathcal{D}_s \rangle_{\text{GC}}$.

As is now well established [1], the determination of diffusion in the integrable t model from finite systems calculations must be performed carefully. Result on finite L may suggest that $\mathcal{D}_s^0 = 0$ for $m = 0$, as presented on Fig. 3(a) showing $I_s(\omega)$ in the $m = 0$ sector at quarter filling, i.e., at particle density $n = 1/2$. However, it has been recognized [1,7]

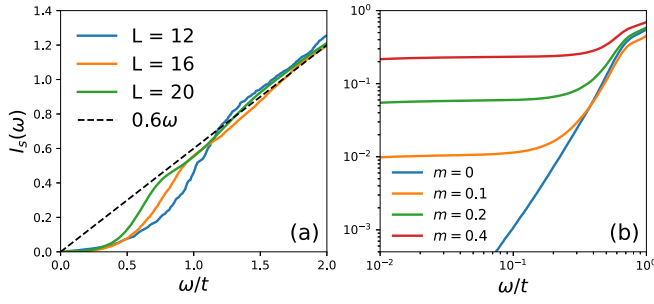


FIG. 3. (a) Integrated spin diffusion $I_s(\omega)$, as calculated within the integrable t model at $n = 1/2$ for magnetization $m = 0$ using MCLM for systems with PBC on $L = 12, 16, 20$ sites. Dotted line denotes the simplest $L \rightarrow \infty$ extrapolation. (b) $I_s(\omega)$ (in log-log scale) for $L = 20$ sites, but for different magnetizations $m = 0, 0.1, 0.2, 0.4$.

that the standard order of limits $\lim_{\omega \rightarrow 0} \lim_{L \rightarrow \infty} \mathcal{D}_s^0(\omega)$ converges to diffusive behavior $I_s(\omega) \sim \omega \mathcal{D}_s^0$, as evident also from Fig. 3(a). From such extrapolation we extract $\mathcal{D}_s^0 \sim 0.6t$, consistent with the analytical result $\mathcal{D}_{s,\text{ana}}^0 = 2t/\pi$ from Ref. [23]. On the other hand, results in Fig. 3(b) for $m \neq 0$ clearly reveal at fixed $n \sim 1/2$ a finite spin stiffness $D_s > 0$, visible as $I_s(\omega \rightarrow 0) \propto D_s \propto m^2$.

Transport in the perturbed t model. Away from half-filling, $n \neq 1$, the integrability breaking perturbations reduce the charge transport from ballistic to normal diffusive, depending on the strength of perturbation, i.e., either $\mathcal{D}_c^0 \propto 1/\Delta t^2$ or $\mathcal{D}_c^0 \propto 1/J_z^2$. This is expected to hold in each sector with $m \neq 0$. The restrictions on spin transport due to fragmentation remain the same, i.e., the spin current appears through the overlap with the charge current. The results in Fig. 4(b) for $m = 0.1$ at different $\Delta t/t = 0.2 - 0.8$ confirm that the perturbation broadens the spin-stiffness $\delta(\omega)$ -peak of the integrable t model to a relaxation peak with finite $\mathcal{D}_s^0 \propto D_s(m)/\Delta t^2$. Very similar conclusions follow from the results for $t - J_z$ model, presented in SM [42]. However, the canonical sector with $m = 0$ differs markedly from $m \neq 0$ sectors. A clear difference from the integrable case should also appear in the GC ensemble with $\langle m \rangle_{\text{GC}} = 0$. In the integrable model, in the thermodynamic limit $L \rightarrow \infty$ (and with PBC), the low-frequency behavior $I_s(\omega) \propto \omega$, shown in Fig. 3(a), is consistent with a finite diffusion constant $\mathcal{D}_s^0 > 0$. In contrast, for a finite

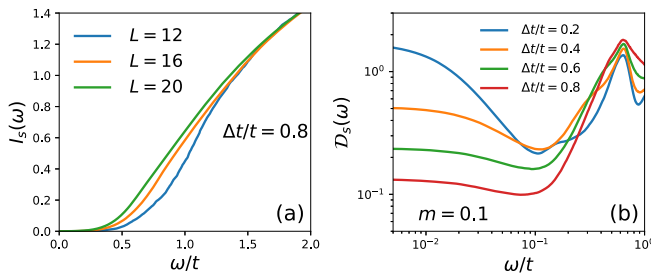


FIG. 4. (a) Integrated $I_s(\omega)$, as calculated within perturbed t model with $\Delta t/t = 0.8$ at $n = 1/2$ and $S_{\text{tot}}^z = 0$ on $L = 12, 16, 20$ sites. (b) Dynamical $\mathcal{D}_s(\omega)$ (in log-log scale) calculated for $L = 20$ sites at $n = 1/2$ and $m = 0.1$, for different $\Delta t/t = 0.2 - 0.8$.

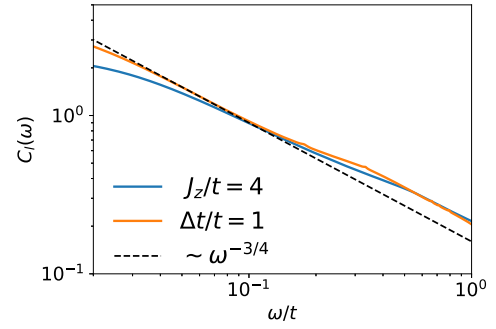


FIG. 5. Local spin correlations $C_l(\omega)$ (in the log-log scale), at $n = 1/2$ and $m = 0$ calculated on $L = 20$ sites within the $t - J_z$ model at $J_z/t = 4$, as well as within $t - \Delta t$ at $\Delta t/t = 1$. The dotted line represents the power-law dependence $\propto \omega^{-3/4}$.

perturbation $\Delta t \neq 0$, the scaling of $I_s(\omega)$ is qualitatively different, as illustrated in Fig. 4(a) for $\Delta t = 0.8t$, and implies a vanishing dc diffusion constant even in the thermodynamic limit, $\mathcal{D}_s^0 \rightarrow 0$.

Evidence for subdiffusion. In the thermodynamic limit ($L \rightarrow \infty$), the integrable t model shows the same type of spin transport as the easy axis XXZ chain [7,9,20–22], i.e., ballistic transport at $m \neq 0$ and dissipationless diffusion (within the GC ensemble) at $\langle m \rangle_{\text{GC}} = 0$. On the other hand, the perturbed t model should exhibit a different scenario. At fixed $m \neq 0$ the spin diffusion is normal and originates from the broadening of the stiffness- $\delta(\omega)$ -peak, as it is for charge transport. According to Eq. (8), the weight associated with the $\delta(\omega)$ -peak grows with magnetization as m^2 , while the relaxation time depends on the perturbation, $\propto 1/\Delta t^2$. Then, for each magnetization sector one expects the spin diffusion $\mathcal{D}_s^0 \propto m^2/\Delta t^2$. Consequently, the diffusion constant relevant for spreading of the coarse grained density of magnetic excitations in hydrodynamic region x , $u(x) = \langle S_i^z \rangle_{i \in x}$, is proportional to squared local magnetization, $u^2(x)$, and one obtains a nonlinear subdiffusion equation

$$\partial_\tau u(x, \tau) \propto \partial_x [u(x, \tau)^2 \partial_x u(x, \tau)] \propto \partial_{xx} u(x, \tau)^3. \quad (9)$$

Equation (9) is known as the *porous medium equation* and it has been intensively studied in the context of nonlinear transport, see Ref. [46] for review. In particular, the spread of the density profile $d(x, 0) \propto \delta(x)$ is described by the Barenblatt solution $u(x, \tau) = \tau^{-1/4} f(|x|\tau^{-1/4})$.

In finite systems, obtaining direct numerical evidence for subdiffusion is challenging. It requires sufficiently large L and long times for the relevant scattering processes to dominate over finite-size effects. In the following, we calculate the local spin correlation function, $C_l(\omega) = \int_0^\infty e^{i\omega\tau} \langle S_l^z(\tau) S_l^z \rangle d\tau$ at $m = 0$, in a system with PBC. Based on Eq. (9), one expects the relation $C_l(\omega) \propto \int_0^\infty e^{i\omega\tau} u(0, \tau) \propto \omega^{-\gamma}$ with the exponent $\gamma = 1 - 1/4 = 3/4$. In finite-size systems the scaling should be visible for intermediate times τ or, equivalently, for intermediate frequencies ω , provided that perturbation is large enough. In Fig. 5 we present $C_l(\omega)$ both for $t - J_z$ model (with $J_z/t = 4$) and for $t - \Delta t$ model ($\Delta t/t = 1$) which both are compatible with the exponent $\gamma \simeq 3/4$.

Reference [30] previously gave an argument for subdiffusion in the perturbed folded XXZ model. While the

subdiffusive transport in the $\Delta \rightarrow \infty$ limit of XXZ model should be mediated by diffusion of magnons through domains of varying but fixed size, the origin of subdiffusive transport in perturbed t model could be related to diffusive transport of holes through subregions of spins configurations pointing in the same direction, with lengths of such polarized regions varying randomly.

Allowing the spin exchange. It is evident that any perturbation which would break the Hilbert-space fragmentation by allowing change of spin sequence, would lead also to normal diffusion at $m = 0$. In the SM [42] we show this explicitly by including the spin exchange via $J_{\perp}$ term.

Conclusions. We have studied spin transport in the integrable and generalized t model, considering perturbations that break the integrability of the parent Hamiltonian while preserving the Hilbert-space fragmentation. We find that in the thermodynamic limit $L \rightarrow \infty$ (or within the GC ensemble), the integrable t model shows the diffusive spin transport while the perturbed but fragmented t models exhibit subdiffusive spin transport, a very unique phenomenon in translationally invariant models, recently claimed also for the folded XXZ model [30].

Hilbert space fragmentation manifests itself through a frozen spin sequence and leads to a crucial role of the coupling between charge and spin currents. We have demonstrated this numerically via the (Thouless-type) level sensitivity to spin-dependent flux in a system with PBC and through calculations of the spin current in a boundary-driven open system. Both approaches show that within a fixed magnetization sector $m = 0$ spin current strictly vanishes. This is the

case even at $m \neq 0$ in a open system, if boundary-driving is solely by spin exchange. Nevertheless, the inclusion of a boundary term permitting charge exchange enables a finite spin current.

The coupling between the spin and charge currents increases with magnetization as m^2 . Within the integrable t model, a diffusive yet in several respects anomalous spin current emerges in the GC ensemble [23,29] as a consequence of ballistic charge transport combined with the averaging over magnetization sectors, as is the case also within the easy-axis XXZ chain [7,9,13,19–22]. For the perturbed but fragmented t model, charge transport becomes diffusive, with the diffusion constant dependent on the magnetization as m^2 , yielding subdiffusive spin transport at $\langle m \rangle_{GC} = 0$ within the GC ensemble, confirmed also by explicit calculations of local correlation functions. We can conclude that spin transport in the t model exhibits many similarities to that of the folded XXZ model [16,17], while offering a more intuitive and physical picture due to its origin in the infinite- U Hubbard model [2,3].

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Data availability. The data that support the findings of this article are openly available at [47].

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