

Superdiffusion and anomalous fluctuations in chiral integrable dynamics

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Symmetries strongly influence transport properties of quantum many-body systems and can lead to deviations from the generic case of diffusion. In this work, we study the impact of time-reversal symmetry breaking on the transport and its universal aspects in integrable chiral spin ladders. We observe that the infinite-temperature spin transport is superdiffusive with a dynamical critical exponent $z = 3/2$ matching the one of the Kardar-Parisi-Zhang (KPZ) universality class, which also lacks the time reversal symmetry. However, we find that fluctuations of the net magnetization transfer deviate from the KPZ predictions. Moreover, the full probability distribution of the associated spin current obeys fluctuation symmetry despite broken time-reversal and space-reflection symmetries. To further investigate the role of conserved quantities, we introduce an integrable quantum circuit that shares the essential symmetries with the chiral ladder and which exhibits analogous dynamical behavior in the absence of energy conservation. Our work shows that time-reversal symmetry breaking is compatible with superdiffusion but insufficient to stabilize the KPZ universality in integrable systems. This suggests that additional fundamental features are missing in order to identify the emergence of such dynamics in quantum matter.

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I. INTRODUCTION

Understanding charge transport is one of the archetypal problems in the physics of strongly correlated quantum matter. It has been addressed in various experimentally relevant contexts, ranging from quark dynamics in heavy-ion collisions [1–4] to correlated electrons and synthetic quantum matter [5–13]. Recently, there has been growing interest in understanding exceptions to the generic case of diffusive charge dynamics [14–30]. A notable one arises in integrable spin chains, where extensive theoretical [14–16] and experimental [31–34] evidence suggests the existence of superdiffusive magnetization transport at infinite temperature [35–39]. In particular, spin superdiffusion has been established in systems exhibiting both integrability and continuous non-Abelian symmetries [38], as reported, e.g., in the case of the isotropic

Heisenberg model [14,40] or its integrable semiclassical (large-spin) limit—the Landau-Lifshitz model [41,42].

However, the impact of other fundamental symmetries, such as the time-reversal and the space-reflection ones, on superdiffusive transport has largely remained obscure. This lack of understanding is particularly impactful, since one of the most discussed emergent transport universality classes is the Kardar-Parisi-Zhang one, which—contrary to all spin models studied so far—posits the *absence* of time reversal invariance [43]. There is, so far, no evidence that superdiffusion can be robust to time-reversal symmetry breaking, and if so, what changes in the universal behavior can this breaking induce.

A related question that has not been adequately addressed is how the space-time symmetries affect the fluctuations of charge and its associated current. While the time-reversal symmetry implies that the ballistic-scale current fluctuations in maximum-entropy states have no preferred direction [44], the effect of broken time reversal symmetry on the fluctuations remains less clear. In particular, there exist dynamical systems in which the fluctuation symmetry seems to be absent not due to the lack of time-reversal invariance but rather due to the particular way in which the space-reflection symmetry is broken [45]. What is the precise relation between the space-time symmetries and the symmetries of charge and current fluctuations is a particularly timely question, especially in light of recent theoretical and experimental studies, which have reported discrepancies with the KPZ predictions concerning the higher-order fluctuations of magnetization transfer in the Heisenberg spin chain [34,46].

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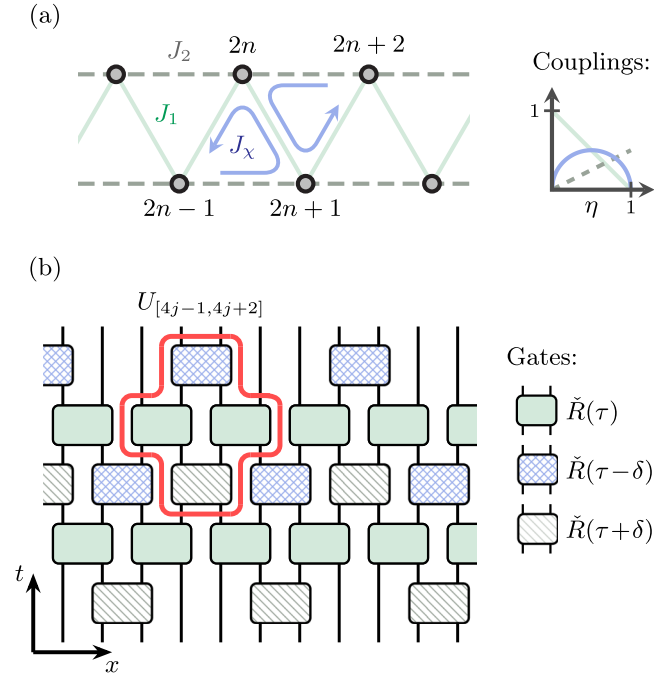


FIG. 1. SU(2)-invariant spin-1/2 systems considered in this work: (a) Triangular ladder parametrized by $\eta \in [0, 1]$, with nearest-neighbor exchange $J_1 = 1 - \eta$, next-nearest-neighbor exchange $J_2 = \eta/2$, and uniform three-spin chiral interaction $J_\chi = \sqrt{\eta(1-\eta)}$. (b) Brickwork circuit constructed from the composition of four elementary gates $\tilde{R}(\tau) = (\mathbb{1} + i\tau P)/(1 + i\tau)$, some of which are deformed by the parameter $\delta = -\sqrt{\eta/(1-\eta)}$. Here, P is the permutation of two neighboring spins 1/2. The composite four-site unitary gate $U_{[4j-1, 4j+2]}$, acting in sites $4j-1, 4j, 4j+1, 4j+2$, is encircled in red. Up to $O(\tau^3)$ corrections, it is equivalent to the exponential of the four-site Hamiltonian density of the chiral spin ladder shown in (a). The continuous- and discrete-time dynamics are equivalent in the limit $\tau \rightarrow 0$.

Here, we report the observation of superdiffusive spin transport at infinite temperature in a class of SU(2)-invariant integrable spin models where time-reversal symmetry breaking can be controlled. Specifically, we study both a continuous-time dynamics in an integrable chiral spin ladder and an integrable (discrete-time) quantum circuit that shares the same key symmetries, allowing us to also explore the effects of broken energy conservation. In both cases we show that, even in the absence of time-reversal symmetry, the spin transport is superdiffusive with a dynamical exponent $z = 3/2$, but with magnetization-transfer statistics that is incompatible with the KPZ universality class. Moreover, we observe that the associated spin current probability distribution obeys a fluctuation symmetry. Surprisingly, the latter is not a consequence of time-reversal and space-reflection symmetries, since they are both broken. Instead, the fluctuation symmetry follows from another spatial C_2 symmetry (invariance under π rotations of the ladder).

The platform for our study is a chiral spin ladder [47–49], depicted in Fig. 1(a), whose continuous-time dynamics and symmetries are described in Sec. II. To explore features of the charge transport, we employ two complementary approaches. In Sec. III, we first determine the transport regime by

considering the dynamical structure factor. This approach involves the use of the thermodynamic Bethe ansatz (TBA) combined with hydrodynamics [50–52]. In Sec. IV, we next identify the transport universality class by considering the full counting statistics of the magnetization transfer. To access it, we employ the recently developed quantum generating function approach, which is based on tensor-network methods [46]. Finally, in Sec. V we relax the assumption of energy conservation and study transport in an integrable quantum circuit that has the same essential symmetries as the chiral spin ladder and which fits the contemporary digital quantum simulation platforms. The brickwork-type circuit is depicted in Fig. 1(b). The key symmetries of the chiral spin ladder are retained by choosing the unitary gates which, up to the second order in the time step, match the exponential of the spin ladder’s Hamiltonian density. We summarize our findings in Sec. VI.

II. CONTINUOUS-TIME CHIRAL DYNAMICS

We consider a triangular two-leg spin ladder, with sites labeled by indices $\{1, \dots, L\}$ and arranged in a zigzag geometry, as shown in Fig. 1(a). The system dynamics is governed by the Hamiltonian [49]

$$\begin{aligned}
 H &= H_0 + H_\chi, \\
 H_0 &= \sum_{n=1}^L J_1 \mathbf{S}_n \cdot \mathbf{S}_{n+1} + J_2 \mathbf{S}_n \cdot \mathbf{S}_{n+2}, \\
 H_\chi &= J_\chi \sum_{n=1}^L (-1)^n \mathbf{S}_n \cdot (\mathbf{S}_{n+1} \times \mathbf{S}_{n+2}),
 \end{aligned} \tag{1}$$

where $\mathbf{S}_n$ is the vector of spin-1/2 operators on site n , with components $S_n^\alpha = \frac{1}{2}\sigma_n^\alpha$ for $\alpha \in \{x, y, z\}$. The first term, H_0 , describes a frustrated Heisenberg chain with nearest- and next-nearest-neighbor couplings J_1 and J_2 , respectively. The second term, H_χ , introduces a chiral interaction of strength J_χ [53]. The staggered factor $(-1)^n$ accounts for the alternating site orientations in the plaquettes, and ensures a uniform spin-circulation sense across the ladder.

The ladder preserves a global SU(2) spin symmetry for all parameter values. However, the chiral coupling J_χ reduces the translational symmetry from one-site to two-site periodicity. It also breaks time-reversal symmetry, $\mathcal{T}$, and the site parity, $\mathcal{P}_s$, while preserving their combined action $\mathcal{P}_s \mathcal{T}$. The ladder also retains the link parity $\mathcal{P}_\ell$, corresponding to a C_2 rotation about the central link. We note, in passing, that there is a significant interest in the low-energy properties of chiral spin systems [54], as they provide a valuable platform for investigating the stability of chiral spin-liquid phases [55–60], and that effective models with chiral three-spin interactions arise in the strong-coupling expansions of Mott insulators with a magnetic flux through triangular plaquettes [61,62] or can appear in effective Hamiltonians governing the high-frequency expansion of Floquet systems driven with circularly polarized light [63,64].

In the following, we focus on the integrable line

$$J_1 = 1 - \eta, \quad J_2 = \frac{\eta}{2}, \quad J_\chi = \sqrt{\eta(1-\eta)}, \tag{2}$$

with $\eta \in [0, 1]$, where the model admits an exact Bethe-ansatz solution [47–49]. As further illustrated in Fig. 1(a), by varying η continuously, one interpolates between a single Heisenberg model (at $\eta = 0$) and two decoupled chains (at $\eta = 1$). The chiral coupling J_χ vanishes at both ending points, and reaches its maximum at $\eta = 1/2$.

III. DYNAMICAL STRUCTURE FACTOR

To investigate charge transport, we will first consider the moments $m_n(t) := \int dx x^n S(x, t)$ of the dynamical structure factor

$$S(x, t) = \langle q(x, t)q(0, 0) \rangle - \langle q(x, t) \rangle \langle q(0, 0) \rangle. \quad (3)$$

Here, $q(x, t)$ is the charge density at coarse grained coordinates x and t , and $\langle \bullet \rangle$ denotes the expectation value in the thermodynamic state of interest. Transport regime is characterized by the dynamical exponent z , identified from the scaling of the second moment:

$$m_2(t) \sim t^{2/z}. \quad (4)$$

Depending on the value of z , we distinguish ballistic ($z = 1$), superdiffusive ($1 < z < 2$), diffusive ($z = 2$), and subdiffusive ($z > 2$) transport.

In integrable spin models, such as the chiral ladder that we consider herein, the structure factor can be decomposed in terms of ballistically spreading quasiparticles—magnons and their bound states—with rapidity λ and total spin s [50,51]. On a ballistic scale (large x and t , with the ratio x/t fixed), such a decomposition reads

$$S(x, t) \simeq \sum_{s=1}^{\infty} \int d\lambda \delta(x - v_s t) \rho_s^t n_s (1 - n_s) (q_s^{\text{dr}})^2, \quad (5)$$

where we have suppressed the dependence on λ . The total state density $\rho_s^t(\lambda)$ and the occupancy ratio $n_s(\lambda)$ determine the thermodynamic state $\langle \bullet \rangle$. Charge $q_s^{\text{dr}}(\lambda)$ is dressed due to scattering between the quasiparticles which move ballistically with effective velocities $v_s(\lambda)$. All the above ingredients are properties of the thermodynamic state and are obtained from the TBA. In particular, we will consider spin transport at infinite temperature and chemical potential μ , i.e., in the grand-canonical state

$$\langle \bullet \rangle = \text{Tr}[\langle \bullet \rangle \mathcal{Q}_\mu], \quad \mathcal{Q}_\mu = \frac{e^{-\mu \sum_j S_j^z}}{\text{Tr}(e^{-\mu \sum_j S_j^z})}, \quad (6)$$

in which TBA and hydrodynamics can be most easily applied.

The Hamiltonian of the integrable chiral spin ladder turns out to be a sum of the first two conserved quantities in the integrable Trotterization of the Heisenberg model [65] (see Appendix A 1 for details). The ingredients for the hydrodynamic decomposition of the spin structure factor in (5) therefore closely resemble those derived in Refs. [45,66], in the context of integrable circuits (i.e., integrable Floquet systems). At infinite temperature the occupancy ratio is [67]

$$n_s = \frac{1}{\chi_s^2}, \quad \chi_s := \frac{\sinh[(s+1)\frac{\mu}{2}]}{\sinh(\frac{\mu}{2})}. \quad (7)$$

The relation between the Hamiltonian in (1) and the integrable Trotterization of the Heisenberg model moreover allows us to

express the total state density as

$$\rho_s^t(\lambda) = \tilde{\rho}_s^t(\lambda) + \tilde{\rho}_s^t(\lambda - \delta), \quad \delta = -\sqrt{\frac{\eta}{1-\eta}},$$

$$\tilde{\rho}_s^t(\lambda) = \frac{\chi_s}{2\pi\chi_1} \left(\frac{\tilde{p}'_s(\lambda)}{\chi_{s-1}} - \frac{\tilde{p}'_{s+2}(\lambda)}{\chi_{s+1}} \right). \quad (8)$$

Here, $(\bullet)'$ is the rapidity derivative, while $\tilde{\rho}_s^t(\lambda)$ and $\tilde{p}_s(\lambda) = \frac{i}{2} \ln[(\lambda + i\frac{s}{2})/(\lambda - i\frac{s}{2})]$ are the total state density and the bare quasiparticle momentum in the Heisenberg model, respectively. Finally, the effective quasiparticle velocity and the dressed charge (i.e., magnetization, dressed due to the inter-particle scattering processes) are obtained as

$$v_s(\lambda) = -\frac{1}{2} \partial_\lambda \ln \rho_s^t(\lambda), \quad (9)$$

and

$$q_s^{\text{dr}} = \partial_\mu \ln(n_s^{-1} - 1), \quad (10)$$

respectively. We relegate the derivations of the above expressions and technical details to Appendix A 2.

A. Drude weights and ballistic transport

Due to the relation to the Heisenberg circuit [see, e.g., (8)], we expect the chiral spin ladder to support ballistic spin transport away from half filling, i.e., for $\mu > 0$. In a system with ballistic charge transport ($z = 1$), the second moment of the structure factor scales as $m_2(t) \simeq Dt^2$ at long times. Here, the finite coefficient $D > 0$ is the charge Drude weight, which can be expressed as

$$D = \lim_{t \rightarrow \infty} \int dx \left(\frac{x}{t} \right)^2 S(x, t)$$

$$= \sum_s \int d\lambda \rho_s^t v_s^2 n_s (1 - n_s) (q_s^{\text{dr}})^2, \quad (11)$$

where the hydrodynamic resolution in (5) has been applied. Another quantity of interest in ballistic systems is the first absolute moment of $S(x, t)$, which defines the so-called Drude self-weight through the scaling $\int dx |x| S(x, t) \simeq D_{\text{sw}} t$. The hydrodynamic resolution of the Drude self-weight is then

$$D_{\text{sw}} = \sum_s \int d\lambda \rho_s^t |v_s| n_s (1 - n_s) (q_s^{\text{dr}})^2. \quad (12)$$

Since the first absolute moment of $S(x, t)$ can also be obtained from the quantum generating function (QGF) method, described in Sec. IV, it will serve as a useful benchmark.

For the spin transport, $Q = S_{\text{total}}^z = \sum_{j=1}^L S_j^z$, we find both the Drude weight and the Drude self-weight to be finite away from half-filling ($\mu \neq 0$), consistently with ballistic spin transport. They are computed by truncating the sum in (12) at some s_{max} and numerically integrating over λ . We obtain, for example, $D_{\text{sw}}(\eta = 0, \mu = 0.5) \approx 0.0482(1)$ and $D_{\text{sw}}(\eta = 0.5, \mu = 0.5) \approx 0.0412(4)$. These results are benchmarked against the QGF method, described in Sec. IV.

B. Superdiffusion from ballistically propagating quasiparticles

Having established ballistic transport for $\mu \neq 0$, we now argue the presence of superdiffusion at half filling ($\mu = 0$),

following the arguments in Refs. [38,39]. For small μ , the dressed charge in (10) reads

$$q_s^{\text{dr}} = \frac{1}{6}(s+1)^2\mu + O(\mu^3). \quad (13)$$

Since it vanishes as $\mu \rightarrow 0$, so does the Drude weight in (11), and the spin transport at half filling is sub-ballistic. To determine the dynamical exponent z , we revisit the variance of the dynamical structure factor, assuming the hydrodynamic form

$$m_2(t) \simeq t^2 \sum_s \int d\lambda \rho_s^\dagger v_s^2 n_s(1-n_s)(q_s^{\text{dr}})^2, \quad (14)$$

and consider the dressed charge as a fluctuating quantity. Specifically, charge is carried by quasiparticles moving ballistically through a background that instead evolves sub-ballistically. Quasiparticles experience fluctuations in the magnetic field $h \sim \mu$, which directly translate into corresponding fluctuations of the dressed charge, according to (13). For sufficiently fast quasiparticles, the background can be treated as frozen. When such quasiparticles traverse the distance $|v_s t|$, they experience magnetic-field fluctuations that scale as $\langle h^2 \rangle \sim |v_s t|^{-1}$ [68]. The square of the dressed charge in (13) then scales as $(q_s^{\text{dr}})^2 \sim (s+1)^4/|v_s t|$. When $\mu \rightarrow 0$, the effective velocity in (9), for the fastest quasiparticles with spin s , scales as $\max_\lambda |v_s(\lambda)| \sim s^{-1}$. For the occupancy ratio in (7), we find $n_s \sim (s+1)^{-2}$, except for heavy (slow) quasiparticles with $s > s_* \sim h^{-1}$, where s_* serves as a cutoff. For these heavy quasiparticles, n_s is instead exponentially suppressed in s .

Using these scaling forms in (14), we obtain

$$m_2(t) \sim t \sum_{s=1}^{s_*} s(s+2)I_s, \quad (15)$$

where we have denoted $I_s = \int d\lambda \rho_s^\dagger |v_s|$. This integral scales as $I_s \sim s^{-2}$, as long as $s < s_* \sim h^{-1}$. We thus arrive at $m_2(t) \sim t s_*$, however, the cutoff $s_* \sim h^{-1}$ is itself a function of time. Indeed, the effective field that arises from the background fluctuations, and which is felt by the fastest among the quasiparticles with $s \sim s_*$, scales as $h \sim |v_{s_*} t|^{-1/2} \sim |t/s_*|^{-1/2}$. From here, we find $s_*(t) \sim t^{1/3}$ and $m_2(t) \sim t^{4/3}$ [69]. According to (4), the transport is thus superdiffusive, with dynamical exponent $z = 3/2$.

IV. FULL-COUNTING STATISTICS OF THE CHARGE TRANSFER

A complementary way to characterize transport is through the full counting statistics of charge transfer, encoded in the moment generating function

$$G(\lambda, t) = \langle e^{i\lambda \Delta Q_S(t)} \rangle, \quad \Delta Q_S(t) = Q_S(t) - Q_S(0), \quad (16)$$

where $Q_S(t) \equiv \int_S dx q(x, t)$ denotes the total charge in region S at time t . The moments of the net charge transfer, defined as $\mu_n(t) := [\Delta Q_S(t)]^n$, are obtained as derivatives of the generating function. Derivatives of $\ln G(\lambda, t)$ instead yield the corresponding cumulants $\kappa_n(t)$, which isolate the connected components of the charge-transfer fluctuations. The moments $\mu_n(t)$ encode transport properties and can be used to identify the transport universality class. By virtue of the

continuity equation, they are related to the moments of the time-integrated currents flowing out of the region S .

Similarly to the second moment of the structure factor, $m_2(t)$, the dynamical exponent z also governs the growth of the second moment of the transferred charge, $\mu_2(t) \sim t^{1/z}$. In fact, choosing the region S as the left-half of the system, $\mu_2(t)$ coincides with the first absolute moment of the structure factor $S(x, t)$. In ballistic systems, we therefore have $\mu_2(t) \simeq D_{\text{sw}} t$.

A. Quantum generating function

Following Ref. [46], we obtain the moments of the transferred charge using the QGF method. We consider observables of the form $\mathcal{O}_S = \sum_{j \in S} \mathcal{O}_j$ that correspond to the total charge in region S . We start within the tensor-network framework by constructing the matrix-product operator (MPO) representation of

$$R(\lambda, 0) = e^{i\lambda \mathcal{O}_S(0)}, \quad (17)$$

with a complex generating parameter, $\lambda \in \mathbb{C}$. For single-site observables, such as magnetization, $\mathcal{O}_j = S_j^z$, the MPO representation of $R(\lambda, 0) = \bigotimes_{j \in S} e^{i\lambda \mathcal{O}_j}$ has bond dimension $\chi = 1$. Energy density, however, has larger support, and it cannot generally be written as a tensor product of single-site operators. The time evolution is

$$R(\lambda, t) = U^\dagger(t) R(\lambda, 0) U(t), \quad (18)$$

and is implemented using a time-evolving block decimation (TEBD) scheme. The algorithm efficiency is largely controlled by the size of λ : for instance, for small enough $|\lambda|$, the operator $R(\lambda, t) = e^{i\lambda \mathcal{O}_S(t)}$ may remain sufficiently close to the identity at all times. An important and subtle bottleneck is the machine precision, as the extraction of higher-order moments is directly tied to high powers of $|\lambda| = \lambda_0$. Unless stated otherwise, we use $\lambda_0 = 0.03$ and a TEBD time step $dt = 0.1$ in the numerical simulations.

For a grand-canonical state ϱ_μ , (6), the moment-generating function is obtained as

$$G_{\text{QGF}}(\lambda, t) = \text{Tr}[R(\lambda, t) R^\dagger(\lambda^*, 0) \varrho_\mu]. \quad (19)$$

The charge-transfer moments are extracted by taking discrete derivatives of $G_{\text{QGF}}(\lambda, t)$ [70]. To this end, we consider $\lambda = \lambda_0 e^{i\phi}$ and run several simulations for different phases of the complex generating parameter, $\phi \in \{0, \pi/4, -\pi/4, \pi/2\}$, combining them as follows [46]:

$$\begin{aligned} \mu_2(t) + O(\lambda_0^8) &= -\frac{2!}{4\lambda_0^2} [\text{Im} G_{\frac{\pi}{4}}(\lambda_0, t) - \text{Im} G_{-\frac{\pi}{4}}(\lambda_0, t) \\ &\quad - \text{Re} G_{\frac{\pi}{2}}(\lambda_0, t) + \text{Re} G_0(\lambda_0, t)], \\ \mu_4(t) + O(\lambda_0^8) &= -\frac{4!}{4\lambda_0^4} [\text{Re} G_{\frac{\pi}{4}}(\lambda_0, t) + \text{Re} G_{-\frac{\pi}{4}}(\lambda_0, t) \\ &\quad - \text{Re} G_{\frac{\pi}{2}}(\lambda_0, t) - \text{Re} G_0(\lambda_0, t)]. \end{aligned} \quad (20)$$

Here, we adopted the shorthand notation $G_\phi(\lambda_0, t)$ to highlight the phase dependence. Finally, we mention that in computing the generating function $G_{\text{QGF}}(\lambda, t)$, one performs the direct contraction between $R(\lambda, t)$ and $R^\dagger(\lambda^*, 0)$, which differs from the operator expression $e^{i\lambda[\mathcal{O}_S(t) - \mathcal{O}_S(0)]}$ since $\mathcal{O}_S(t)$ and $\mathcal{O}_S(0)$ do not generally commute. The resulting

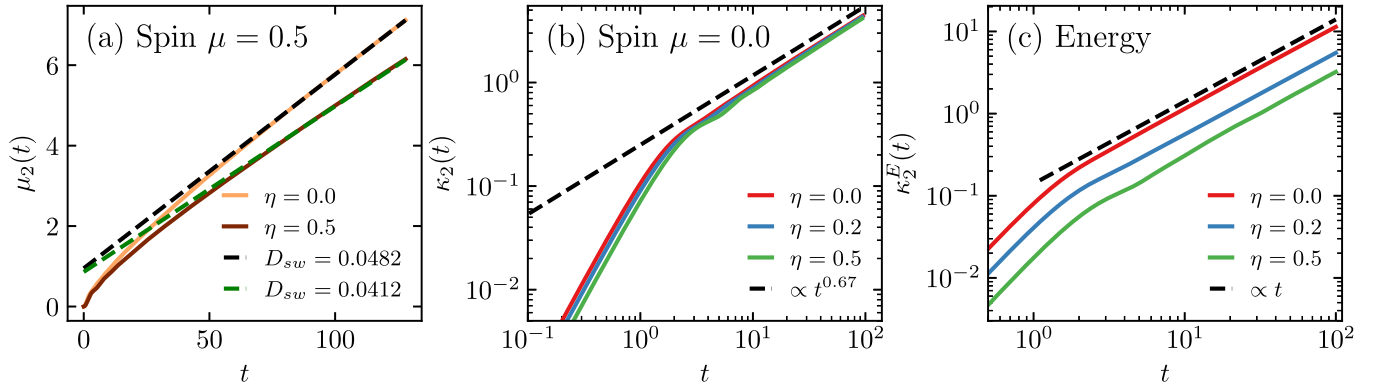


FIG. 2. Second moments and cumulants of the magnetization and energy transfer. (a) Second moment of the net magnetization transfer through the middle of the system, $\mu_2(t) = \langle [S_{\text{left}}^z(t) - S_{\text{left}}^z(0)]^2 \rangle$. The orange and dark red solid lines correspond to $\mu_2(t)$ computed at finite chemical potential $\mu = 0.5$ for $\eta = 0$ and $\eta = 0.5$, respectively. The black and green dashed lines correspond to $\sim D_{sw}t$, with the hydrodynamic predictions for the Drude self-weights D_{sw} reported in Sec. III A. (b) Second cumulant of the net magnetization transfer through the middle of the system, $\kappa_2(t)$, for different values of the model's parameter η , at zero chemical potential ($\mu = 0$). We observe the asymptotic scaling $\kappa_2(t) \sim t^{2/3}$, implying the dynamical exponent $z = 3/2$. (c) Second cumulant $\kappa_2^E(t)$ of the net energy transfer through the middle of the system at infinite temperature and for different values of η . Irrespectively of the model's parameter η , we find $\kappa_2^E(t) \sim t$, indicating ballistic energy transport.

discrepancy, defined as $\varepsilon(t) = |G_{\text{QGF}}(\lambda, t) - G(\lambda, t)|$, can be expanded in powers of λ . The leading nonvanishing contributions are of order $O(\lambda^4)$ and do not influence the computation of the second (or third) moments. They may, however, affect the estimation of the fourth moment, $\mu_4(t)$. Nevertheless, in the simulations presented herein, the difference $\varepsilon(t)$ introduces only subleading time-dependent corrections, which are washed out in the long-time estimates of the excess kurtosis, defined as $\gamma_4(t) = \mu_4(t)/\mu_2^2(t) - 3$.

B. Fluctuations of magnetization and energy

To validate our approach, we first perform numerical simulations at a nonzero chemical potential. Spin transport is probed through magnetization in the left-half of the system, $\mathcal{O}_S = S_{\text{left}}^z = \sum_{j=1}^{L/2} S_j^z$. We simulate a finite chain of length $L = 256$ at a chemical potential $\mu = 0.5$. The maximum bond dimension during the time evolution is set to $\chi = 512$. The time dependence of the second spin-transfer moment $\mu_2(t)$ is shown in Fig. 2(a) for two coupling values, $\eta = 0$ and $\eta = 0.5$. Numerical results are compared with the long-time hydrodynamic predictions reported in Sec. III A (dashed lines).

We next investigate the transport at half filling ($\mu = 0$), where superdiffusion is expected to emerge according to the predictions in Sec. III B. We again consider the second cumulant, $\kappa_2(t)$. The latter coincides with the second moment, since the first moment vanishes identically. Numerical simulations are performed for parameters $\eta = 0$, $\eta = 0.2$, and $\eta = 0.5$ for a system of size $L = 256$. We set the maximum bond dimension to $\chi = 512$. The extracted second moments are presented in Fig. 2(b). We find that spin transport is consistent with superdiffusion with dynamical exponent $z = 3/2$, irrespective of the value of η , i.e., irrespective of how strongly J_χ enters the dynamics. Since J_χ parametrizes the strength of the time-reversal breaking term, this suggests that the violation of time-reversal does not alter the universal properties of the superdiffusive regime, at least as far as the second-order correlations are concerned.

Before considering higher-order fluctuations of the transferred magnetization, we first report on the second cumulant of the energy transfer, $\kappa_2^E(t)$. Here, we set $\mathcal{O}_S = H_{\text{left}}$, where H_{left} is the truncation of the Hamiltonian (1) to the left half of the ladder. In Fig. 2(c), we show $\kappa_2^E(t)$ for system size $L = 128$ and various values of the model's parameter η . Irrespectively of the latter, we observe linear growth of the second cumulant $\kappa_2^E(t) \propto t^{1/z_E}$, the corresponding dynamical exponent being $z_E = 1$, consistently with ballistic energy transport. Finally, we confirm that non-Abelian symmetries alone are not enough to support superdiffusion away from the integrable line, (2). Indeed, we observe (not shown) diffusion of spin and energy when considering the dynamics of H_χ alone.

We next focus on how the breaking of time-reversal symmetry affects the higher moments, restricting the discussion to magnetization transfer. First, we note that, due to the C_2 symmetry, i.e., the link parity $\mathcal{P}_\ell$ rotating the ladder around the central link, the odd moments corresponding to the magnetization transfer vanish in the grand-canonical state, (6). Specifically, we have

$$\mu_n(t) = \langle [S_{\text{left}}^z(t) - S_{\text{left}}^z(0)]^n \rangle = 0 \quad (21)$$

for odd n . This follows from the fact that, on the one hand, we have $S_{\text{left}}^z(t) - S_{\text{left}}^z(0) = -S_{\text{right}}^z(t) + S_{\text{right}}^z(0)$ due to the conservation of the total spin $S_{\text{total}}^z = S_{\text{left}}^z + S_{\text{right}}^z$, and, on the other hand, $\mathcal{P}_\ell[S_{\text{left}}^z(t) - S_{\text{left}}^z(0)] = S_{\text{right}}^z(t) - S_{\text{right}}^z(0)$, as well as $\langle \mathcal{P}_\ell[\bullet] \rangle = \langle \bullet \rangle$ due to the C_2 invariance of the state. Expressing odd cumulants of the transferred magnetization in terms of moments, one finds that each term contains an odd moment, and the odd cumulants therefore vanish as well. Since, by the continuity equation, the net magnetization transferred from the left- to the right-half of the system is equal to the time-integrated current flowing through the middle of the system, we conclude that the full counting statistics of the spin current obeys fluctuation symmetry.

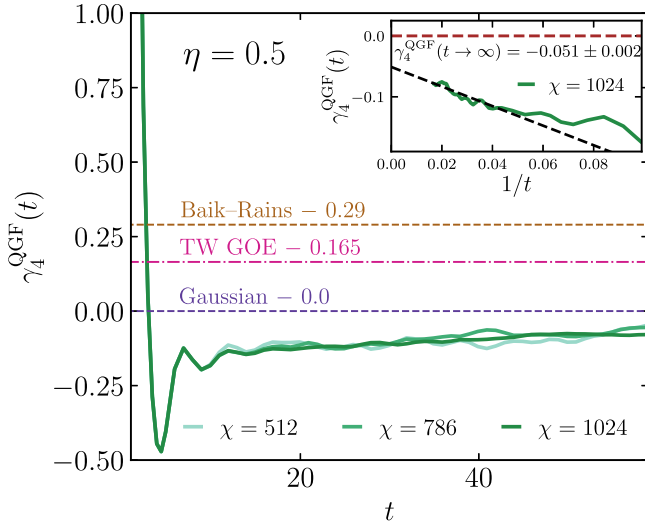


FIG. 3. Higher-order fluctuations corresponding to the magnetization transfer. Plotted is the excess kurtosis $\gamma_4^{\text{QGF}}(t)$ of the magnetization transfer for the chiral ladder with $L = 128$ sites, model parameter $\eta = 1/2$, and for various bond dimensions χ . The dashed lines correspond to the predictions within the KPZ universality class [either Baik-Rains or Tracy-Widom (TW-GOE)] and to Gaussian fluctuations. The inset shows $\gamma_4^{\text{QGF}}(t)$ as a function of $1/t$ for $\chi = 1024$. While the $t \rightarrow \infty$ extrapolation from the available data cannot conclusively establish non-Gaussianity, the results are consistent with non-KPZ behavior.

We now analyze the long-time behavior of the excess kurtosis, $\gamma_4(t)$, which quantifies deviations from Gaussian fluctuations, for which $\gamma_4(t) = 0$. In the KPZ universality class, the excess kurtosis is expected to be positive, with typical values approaching $\gamma_4 \approx 0.29$ and $\gamma_4 \approx 0.165$ [71], respectively, corresponding to the Baik-Rains [72] and Tracy-Widom Gaussian orthogonal ensemble distributions [73], depending on the initial conditions on the height function. These reference values are indicated by the dashed horizontal lines in Fig. 3. To compute $\gamma_4^{\text{QGF}}(t)$, we simulate a system of length $L = 128$ at coupling $\eta = 0.5$, using several maximum bond dimensions $\chi = 512$, $\chi = 786$, and $\chi = 1024$. The resulting data are shown in Fig. 3. Despite the second moment exhibiting a KPZ-type dynamical exponent, the excess kurtosis remains inconsistent with the KPZ predictions: $\gamma_4^{\text{QGF}}(t)$ remains negative for all accessible times (however, we cannot exclude the possibility that it approaches zero at long times). The discrepancies visible in Fig. 3 are amplified by error propagation in the evaluation of $\gamma_4^{\text{QGF}}(t)$, which involves dividing $\mu_4(t)$ by $\mu_2^2(t)$. Both $\mu_2(t)$ and $\mu_4(t)$ are, however, well converged across different bond dimensions, as shown in the Appendix B. The inset of Fig. 3 displays a simple $1/t$ extrapolation toward $t \rightarrow \infty$, yielding a small negative asymptotic value, $\gamma_4^{\text{QGF}}(t \rightarrow \infty) \approx -0.051 \pm 0.002$. The precise numerical value, as well as the reported error bar (which only includes the error propagation from the linear fit), should be taken *cum grano salis* and possibly sharpened by future large scale simulations. Nevertheless, our results strongly indicate that the observed fluctuations deviate from the KPZ behavior, similarly with recent findings in the XXX model [34,46].

V. CHIRAL UNITARY CIRCUIT

We now introduce an integrable brickwork-type unitary circuit designed to preserve the key symmetries of the continuous-time evolution generated by the Hamiltonian in (1). The key question that we want to address here is whether giving up energy conservation can dramatically affect the overall picture obtained in the case of Hamiltonian dynamics, i.e., whether a non-KPZ kurtosis and $z > 1$ still appear in the absence of time-reversal symmetry. Furthermore, this circuit formulation of the dynamics is naturally suited for implementation in digital simulators, as demonstrated in Refs. [74,75], where chiral three-body terms are engineered.

The Hamiltonian in (1) can be rewritten as $H = \sum_{j=1}^{L/2} h_{[2j-1, 2j+2]}$, where $[a, b]$ denotes the (closed) interval of integers between a and b , and

$$\begin{aligned} h_{[1,4]} = & J_1 \left(\frac{1}{2} \mathbf{S}_1 \cdot \mathbf{S}_2 + \mathbf{S}_2 \cdot \mathbf{S}_3 + \frac{1}{2} \mathbf{S}_3 \cdot \mathbf{S}_4 \right) \\ & + J_2 (\mathbf{S}_1 \cdot \mathbf{S}_3 + \mathbf{S}_2 \cdot \mathbf{S}_4) \\ & - J_\chi \mathbf{S}_1 \cdot (\mathbf{S}_2 \times \mathbf{S}_3) + J_\chi \mathbf{S}_2 \cdot (\mathbf{S}_3 \times \mathbf{S}_4) \end{aligned} \quad (22)$$

is the four-site local Hamiltonian density, parametrized by $J_{1,2}$ and J_χ given in (2). Time discretization is based on the Trotter-Suzuki formula

$$e^{-itH} = \lim_{n \rightarrow \infty} \left(e^{-i\frac{t}{n}H_A} e^{-i\frac{t}{n}H_B} \right)^n, \quad (23)$$

with $H_A = \sum_{j=1}^{L/4} h_{[4j-1, 4j+2]}$ and $H_B = \sum_{j=1}^{L/4} h_{[4j-3, 4j]}$. The right-hand side of (23) factorizes into a sequence of four-site unitary gates, each one corresponding to the exponential of the local density in (22). To preserve integrability, we would moreover like the unitary gate to satisfy the Yang-Baxter equation [65,66]. For the chiral spin ladder, this is only possible by choosing the unitary gate $U_{[1,4]}$, which matches $\exp\{-i(t/n)h_{[1,4]}\}$ up to an $O([t/n]^3)$ correction. In particular, we have to set

$$U_{[1,4]} = \check{R}_{2,3}(\tau - \delta) \check{R}_{1,2}(\tau) \check{R}_{3,4}(\tau) \check{R}_{2,3}(\tau + \delta), \quad (24)$$

where $\check{R}_{1,2}(\lambda) = (\mathbb{1} + i\lambda P_{1,2}) / (1 + i\lambda)$ is the Yang-Baxter gate in the integrable XXX circuit [65], and $P_{1,2} = \frac{1}{2}\mathbb{1} + 2\mathbf{S}_1 \cdot \mathbf{S}_2$ denotes the permutation of two spins $1/2$, and $\delta = -\sqrt{\eta/(1-\eta)}$. For the unitary gate in (24), we find $U_{[1,4]} - \exp\{i\tau[4h_{[1,4]} + (\eta - 2)\mathbb{1}]\} = O(\tau^3)$ (see Appendix A3). Setting $\tau = -t/n$, we can then reproduce the continuous-time dynamics of the chiral ladder in the Trotter limit $e^{-itH} = \lim_{n \rightarrow \infty} \mathbb{U}^n$, where

$$\mathbb{U} = \prod_{j=1}^{\frac{L}{4}} U_{[4j-1, 4j+2]} \prod_{j=1}^{\frac{L}{4}} U_{[4j-3, 4j]} \quad (25)$$

is the one-step propagator. Its circuit diagram is depicted in Fig. 1(b).

To characterize spin transport in the circuit, we apply the QGF method implementing the one-step propagator $\mathbb{U}$ in (25) with a TEBD scheme [76]. Throughout, we set $\tau = 1$ and examine two values of the coupling parameter, $\delta = 1$ and $\delta = 2$. The simulations are performed for a system of size $L = 256$ and a maximum bond dimension $\chi = 2048$. The second cumulant $\kappa_2(t)$ is shown in Fig. 4(a): it exhibits a power-law growth consistent with the superdiffusive exponent

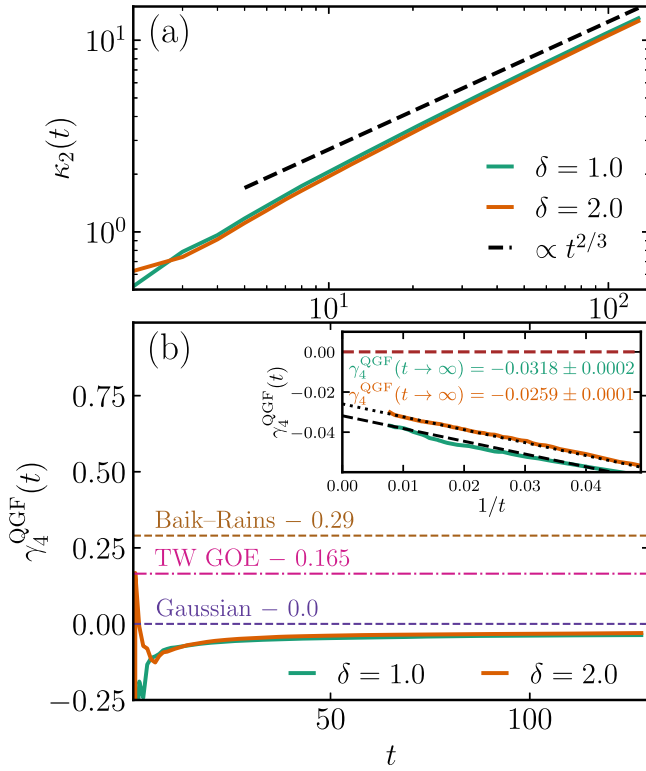


FIG. 4. Spin transport and higher-order fluctuations in the chiral circuit. (a) Second moment $\mu_2(t)$ for $\delta = 1$ and 2. The dashed line indicating the power-law scaling $\propto t^{2/3}$ serves as a guide to the eye. (b) Excess kurtosis $\gamma_4^{QGF}(t)$ for the same values of δ . Dashed lines show theoretical predictions for the standard KPZ (TW and BR) distributions, as well as for the Gaussian fluctuations. The inset highlights the $1/t$ infinite-time extrapolation. While not excluding the possibility of Gaussian fluctuations, it again suggests non-KPZ behavior. Results were computed using the QGF method with maximum bond dimension $\chi = 2048$, for a system of size $L = 256$, and circuit step $\tau = 1$.

$z = 3/2$. However, the corresponding excess kurtosis $\gamma_4^{QGF}(t)$, displayed in Fig. 4(b), remains negative at all accessible late times. The inset, plotted versus $1/t$, indicates only a slow drift toward larger values, with a naive infinite-time extrapolation yielding $\gamma_4^{QGF}(t \rightarrow \infty, \delta = 1) \approx -0.0318 \pm 0.0002$ and $\gamma_4^{QGF}(t \rightarrow \infty, \delta = 2) \approx -0.0259 \pm 0.0001$. Here, the error again comes from the linear extrapolation. The circuit results are thus compatible with the ones obtained in the continuous time evolution: on the accessible time scales the excess kurtosis has slightly negative values, again reinforcing the non-KPZ nature of the fluctuations.

VI. CONCLUSIONS

In this work, we have considered several aspects of transport in an integrable $SU(2)$ -invariant spin ladder with tunable chiral terms breaking the time-reversal symmetry. We have found that, even in the absence of time reversal, the spin transport at infinite temperature and zero chemical potential is superdiffusive, with dynamical critical exponent $z = 3/2$, but with fluctuations of the net magnetization transfer that are not consistent with the Kardar-Parisi-Zhang universality class.

Surprisingly, we also found that the statistics of the associated net spin current obeys fluctuation symmetry despite the lack of time reversal. Time reversal symmetry is therefore sufficient [44] but not a necessary condition for the fluctuation symmetry to appear. In the case of the chiral spin ladder, the latter instead appears due to a particular geometrical symmetry of the ladder.

First, our findings establish that the anomalous superdiffusion can be robust to the time-reversal symmetry breaking. Furthermore, they suggest that the mere absence of time-reversal symmetry is insufficient for the stabilization of KPZ-type superdiffusion, even though the KPZ universality class itself also lacks that symmetry [43]. Finally, we have introduced an integrable chiral quantum circuit exhibiting the same symmetries as the chiral ladder, apart from energy conservation. Here, the results regarding the spin transport were consistent with the ones in the continuous-time model. The chiral spin circuit introduced herein provides a natural platform for experimental investigation of the effects of broken space-time symmetries on the transport phenomena [74,75].

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: INTEGRABILITY, THERMODYNAMICS, AND INTEGRABLE CHIRAL CIRCUIT

Here, we discuss integrability of the chiral spin ladder Hamiltonian (1). In Appendix A 1, we introduce the transfer matrix used in the algebraic Bethe ansatz to diagonalize the Hamiltonian [48]. We also relate the Hamiltonian to the conserved charges of the (integrable) Trotterized Heisenberg model [65]. This correspondence allows us to use the

existing Bethe ansatz results for integrable circuits (see, e.g., Refs. [45,66]) in the description of the chiral spin ladder's thermodynamics. The latter is summarized in Appendix A 2: it is the essential ingredient in the hydrodynamic description of the transport. Lastly, in Appendix A 3 we describe how to obtain the unitary gate that can be used in the integrable Trotterization of the chiral spin ladder.

1. Transfer matrix of the chiral spin ladder

Integrability of the chiral spin ladder Hamiltonian (1) can be traced back to the SU(2) symmetric R matrix of the XXX model:

$$R_{1,2}(\lambda) = \frac{\lambda \mathbb{1} - i P_{1,2}}{\lambda - i}. \quad (\text{A1})$$

Here, $P_{1,2} = \frac{1}{2}\mathbb{1} + 2\mathbf{S}_1 \cdot \mathbf{S}_2$ is a permutation acting on two spins $1/2$, and $\lambda \in \mathbb{C}$ is a free spectral parameter. For any two $\lambda, \vartheta \in \mathbb{C}$, the R matrix satisfies the Yang-Baxter equation

$$R_{1,2}(\lambda - \vartheta) R_{1,3}(\lambda) R_{2,3}(\vartheta) = R_{2,3}(\vartheta) R_{1,3}(\lambda) R_{1,2}(\lambda - \vartheta). \quad (\text{A2})$$

We use the R matrix to define a family of transfer matrices,

$$T_\delta(\lambda) = \text{Tr}_a \left[\prod_{1 \leq x \leq L/2}^{\rightarrow} R_{a,2x-1}(\lambda) R_{a,2x}(\lambda - \delta) \right], \quad (\text{A3})$$

where $\text{Tr}_a(\bullet)$ is a partial trace over the auxiliary spin $1/2$. The arrow above the product indicates the ordering of the physical spins labeled by $1, 2, \dots, L$. The Yang-Baxter equation (A2) implies

$$[T_\delta(\lambda), T_\delta(\vartheta)] = 0 \quad (\text{A4})$$

for a fixed $\delta \in \mathbb{R}$, and for any pair of spectral parameters $\lambda, \vartheta \in \mathbb{C}$. The transfer matrices can be simultaneously diagonalized by means of the algebraic Bethe ansatz [81].

The integrable chiral spin ladder Hamiltonian (1), whose parameters ($J_1(\eta)$, $J_2(\eta)$, $J_x(\eta)$) lie on a curve specified in (2), is obtained as [48]

$$H = \frac{1}{4i} \frac{d}{d\lambda} \ln [T_\delta(\lambda + \delta) T_\delta(\lambda)] \Big|_{\lambda=0}, \quad (\text{A5})$$

provided that we set $\delta = -\sqrt{\eta/(1-\eta)}$. Besides the Hamiltonian, which generates translations in time, the two-site lattice translations can likewise be expressed using the transfer matrix (A3) as

$$T_\delta(\delta) T_\delta(0) = (P_{1,2} P_{1,3} \cdots P_{1,L})^2 \equiv e^{-i2\mathcal{P}}. \quad (\text{A6})$$

We use this equation as a definition of the momentum operator, $\mathcal{P}$, which generates two-site translations in space (we normalize $\mathcal{P}$ with a prefactor 2, reflecting the two-site translation). Bethe ansatz gives access to the eigenvalues of $T_\delta(\lambda)$, and thus to the energies and (quasi) momenta.

We note that the staggered transfer matrix (A3) also plays the central role in the construction of integrable Trotterization of the XXX model [65]. The latter is an integrable quantum unitary circuit that, in the so-called Trotter-Suzuki limit, reproduces the continuous-time evolution with the isotropic

spin-1/2 Heisenberg Hamiltonian. In fact, the chiral spin ladder Hamiltonian (A5) can be expressed as

$$H = \frac{1}{4i} (Q_1^+ + Q_1^-), \quad (\text{A7})$$

where

$$Q_1^\pm \equiv \frac{d}{d\lambda} \ln T \left(\lambda + \frac{\delta}{2}; \delta \right) \Big|_{\lambda=\pm\frac{\delta}{2}} \quad (\text{A8})$$

are precisely the first two local conserved charges of the integrable XXX circuit—cf. Eqs. (9)–(11) in Ref. [65]. It follows that the chiral spin ladder Hamiltonian and the integrable Trotterization of the Heisenberg spin chain share the same (quasi)local conserved charges. This means that, following a quantum quench, the chiral spin ladder Hamiltonian and the integrable XXX circuit evolve the initial state towards a steady state whose local properties are captured by the same generalized Gibbs ensemble.

2. Quasiparticle content of a thermodynamic state

The eigenvalues of local charges, such as the Hamiltonian in (A5), are extensive. They are sums of contributions from quasiparticles which correspond to bound states of s magnons ($s \in \mathbb{N}$), each magnon carrying spin 1 (a magnon is associated with a spin flip). Extensivity allows us to express the density of a local charge Q in the thermodynamic state as

$$\lim_{\substack{N, L \rightarrow \infty \\ N/L \text{ finite}}} \frac{\langle Q \rangle}{L} = \sum_{s=1}^{\infty} \int d\lambda \rho_s(\lambda) q_s(\lambda). \quad (\text{A9})$$

Here, $\rho_s(\lambda)$ is the density of quasiparticles, and $q_s(\lambda)$ is the bare charge carried by the type- s quasiparticle with rapidity λ . For example, denoting the rapidity derivative by $(\bullet)'$, the bare energy reads

$$\varepsilon_s(\lambda) = -\frac{1}{2} p'_s(\lambda). \quad (\text{A10})$$

Here, due to the staggered structure of the transfer matrix (A3), the bare momentum $p_s(\lambda)$ can be expressed as a sum of bare momenta $\tilde{p}_s(\lambda)$ and $\tilde{p}_s(\lambda - \delta)$ in the XXX model. Specifically, we have

$$p_s(\lambda) = \tilde{p}_s(\lambda) + \tilde{p}_s(\lambda - \delta), \quad (\text{A11})$$

with

$$\tilde{p}_s(\lambda) \equiv \frac{i}{2} \ln \left(\frac{\lambda + i\frac{\delta}{2}}{\lambda - i\frac{\delta}{2}} \right). \quad (\text{A12})$$

The same decomposition of the momentum occurs in the integrable Trotterization of the XXX model—see Ref. [66].

a. Quasiparticle density

The quasiparticle density entering (A9) is customarily factorized as $\rho_s(\lambda) = n_s(\lambda) \rho_s^t(\lambda)$, where $n_s(\lambda)$ is the occupancy ratio and $\rho_s^t(\lambda)$ is the total state density, i.e., the density of particles and holes. The equations that determine $n_s(\lambda)$ and $\rho_s^t(\lambda)$ are the TBA equations. They appear in the thermodynamic

limit of the equations which characterize the eigenvalues of the transfer matrix (A3).

In this work, we are mostly interested in the thermodynamic state at an infinite temperature T and a finite chemical potential μ , (6), for which the TBA equations read [66,67]

$$\ln(n_s^{-1} - 1) = \mu s - \sum_{\ell=0}^{\infty} \int d\vartheta K_{s,\ell}(\lambda - \vartheta) \ln(1 - n_\ell), \quad (\text{A13})$$

$$\rho_s^t(\lambda) + \sum_{\ell=0}^{\infty} \int d\vartheta K_{s,\ell}(\lambda - \vartheta) n_\ell \rho_\ell^t(\vartheta) = \frac{p'_s(\lambda)}{2\pi}. \quad (\text{A14})$$

Here, $K_{s,\ell}(\lambda - \vartheta)$ is the scattering kernel with an explicit form [67]

$$K_{s,m}(\lambda) = \frac{1}{\pi} \sum_{j=\frac{|s-m|}{2}}^{\frac{s+m}{2}-1} \left(\frac{j}{\lambda^2 + j^2} + \frac{j+1}{\lambda^2 + (j+1)^2} \right). \quad (\text{A15})$$

From (A13), one obtains a rapidity-independent occupancy ratio n_s , reported in (7)—see, e.g., Ref. [67]. Since n_s does not depend on the rapidity, (A14) is a linear equation for the total state density $\rho_s^t(\lambda)$. As a consequence, since the momentum (A11) decomposes into two parts, so does $\rho_s^t(\lambda)$. Similarly as in integrable circuits [66], we then find

$$\rho_s^t(\lambda) = \tilde{\rho}_s^t(\lambda) + \tilde{\rho}_s^t(\lambda - \delta), \quad (\text{A16})$$

where $\tilde{\rho}_s^t(\lambda)$, reported in (8), satisfies the TBA equation (A14) in the XXX model [37].

b. Dressed charges and effective velocities

Equation (A14) is a particular example of the dressing equation [51],

$$q_s^{\text{dr}}(\lambda) + \sum_{\ell=0}^{\infty} \int d\vartheta K_{s,\ell}(\lambda - \vartheta) n_\ell q_\ell^{\text{dr}}(\vartheta) = q_s(\lambda), \quad (\text{A17})$$

which determines how the bare charge carried by the quasi-particles changes in the presence of interactions: $q_s \mapsto q_s^{\text{dr}}$. In particular, according to (A14), we have $(p'_s)^{\text{dr}}(\lambda) = 2\pi \rho_s^t(\lambda)$. Below, we summarize also the derivation of the dressed energy derivative $(\varepsilon'_s)^{\text{dr}}$, which enters the effective velocity $v_s = (\varepsilon'_s)^{\text{dr}} / (p'_s)^{\text{dr}}$ alongside $(p'_s)^{\text{dr}}$, and the dressed charge—magnetization. We will stick with notation q_s^{dr} for the latter.

First, we obtain the dressing equation for the magnetization, which encodes the mapping $s \mapsto q_s^{\text{dr}}$, by applying ∂_μ to (A13). Comparing the result to (A17), we then identify the dressed magnetization reported in (10).

Second, applying ∂_λ to (A14), and noting that $\lim_{\lambda \rightarrow \pm\infty} K_{s,m}(\lambda) = 0$, we obtain

$$\partial_\lambda \rho_s^t(\lambda) + \sum_m \int d\vartheta K_{s,m}(\lambda - \vartheta) n_m \partial_\vartheta \rho_s^t(\vartheta) = \frac{p''_s(\lambda)}{2\pi}. \quad (\text{A18})$$

Recalling (A10), we now recognize

$$(\varepsilon'_s)^{\text{dr}}(\lambda) = -\pi \partial_\lambda \rho_s^t(\lambda). \quad (\text{A19})$$

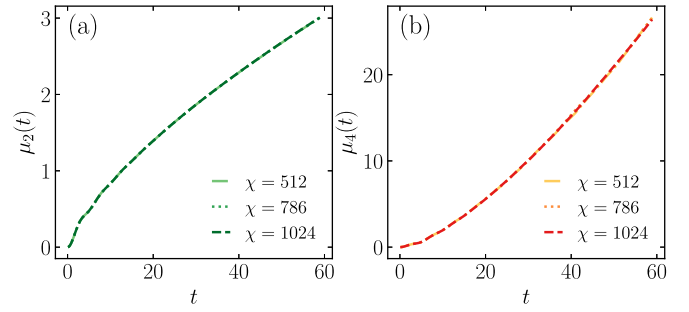


FIG. 5. Hamiltonian dynamics: Convergence test for the second and fourth moments of the magnetization transfer for different bond dimensions. The other parameters are $L = 128$, $dt = 0.1$, $\eta = 0.5$.

The dressed momentum and energy derivatives enter the hydrodynamic mode velocity

$$v_s(\lambda) = \frac{(\varepsilon'_s)^{\text{dr}}(\lambda)}{(p'_s)^{\text{dr}}(\lambda)}, \quad (\text{A20})$$

whose final form is reported in (9).

3. Integrable circuit for the chiral spin ladder

In the context of the integrable XXX circuit [65], the local Hamiltonian density $h_{[1,4]}$ from (22) enters the boost operator B , which generates the hierarchy of conserved charges. Specifically,

$$B = \sum_j j h_{[2j-1, 2j+2]}, \quad (\text{A21})$$

which is defined in a thermodynamically large system, translates within the hierarchy of charges of the integrable circuit according to the boost relation $[B, Q_n^\pm] = Q_{n+1}^\pm$. Here, the first two charges among $Q_n^\pm$ are $Q_1^\pm$ from (A8).

In Ref. [65], it was shown that one can obtain the local density $h_{[1,4]}$, entering the boost operator B , as

$$h_{[1,4]} = \frac{1}{4i} \partial_\lambda \mathbb{R}_{[1,4]}(\lambda; \delta) \Big|_{\lambda=0} + \frac{2-\eta}{4} \mathbb{1}, \quad (\text{A22})$$

where $\delta = -\sqrt{\eta/(1-\eta)}$ and

$$\mathbb{R}_{[1,4]}(\lambda; \delta) = \check{R}_{2,3}(\lambda - \delta) \check{R}_{1,2}(\lambda) \check{R}_{3,4}(\lambda) \check{R}_{2,3}(\lambda + \delta). \quad (\text{A23})$$

Here, $\check{R}_{1,2}(\lambda) = P_{1,2} R_{1,2}(\lambda)$ are unitary matrices, with $R_{1,2}(\lambda)$ defined in (A1). Expanding in λ , we find

$$\mathbb{R}_{[1,4]}(\lambda; \delta) = \mathbb{1} + i\lambda \mathcal{H}_{[1,4]} + \frac{(i\lambda)^2}{2!} \mathcal{H}_{[1,4]}^2 + O(\lambda^3), \quad (\text{A24})$$

where $\mathcal{H}_{[1,4]} = 4h_{[1,4]} + (\eta - 2)\mathbb{1}$. The third order in λ cannot be expressed in terms of powers of only. Therefore, up to the third order in λ , the matrix $\mathbb{R}_{[1,4]}(\lambda; \delta)$ is equal to the exponential of the local Hamiltonian. As such, it can be chosen as the unitary gate in the integrable brickwork circuit that, in the Trotter-Suzuki limit, reproduces the continuous-time dynamics of the chiral spin ladder. In particular, denoting

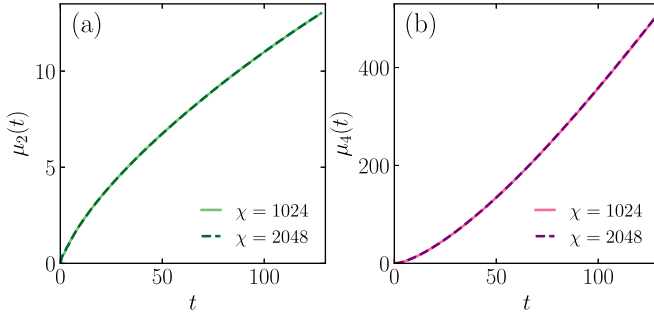


FIG. 6. Circuit dynamics: Convergence test for the second and fourth moments of the magnetization transfer for different bond dimensions. The other parameters are $L = 256$, $\tau = 1$, $\delta = 1$.

the gate as $U_{[1,4]} = \mathbb{R}_{[1,4]}(\tau; \delta)$, with $\delta = -\sqrt{\eta/(1-\eta)}$, the one-step propagator of the Trotterized chiral spin ladder becomes $\mathbb{U}$, reported in (25). The Trotter-Suzuki limit is then $e^{-itH} = \lim_{n \rightarrow \infty} \mathbb{U}^n$.

The transfer matrix from which the circuit and its conserved quantities are generated can be identified following Ref. [66], which deals with construction of inhomogeneous integrable-circuit architectures. Specifically, the transfer

matrix reads

$$\mathbb{T}_{\tau,\delta}(\lambda) = \text{Tr}_a \left[\prod_{1 \leq x \leq L/4}^{\rightarrow} R_{4j-3,a}(\lambda + \frac{\tau-\delta}{2}) R_{4j-2,a}(\lambda + \frac{\tau+\delta}{2}) \times R_{4j-1,a}(\lambda - \frac{\tau+\delta}{2}) R_{4j,a}(\lambda - \frac{\tau-\delta}{2}) \right], \quad (\text{A25})$$

where the R matrix is given in (A1).

APPENDIX B: CONVERGENCE OF THE TEBD SIMULATIONS

Here, we present additional numerical simulation results to check the convergence of the data for both the Hamiltonian and the circuit dynamics. We first report the results for the chiral ladder for $\eta = 0.5$. In particular, in Fig. 5 we plot the dynamics of the second, $\mu_2(t)$, and the fourth moment, $\mu_4(t)$, for system size $L = 128$ and for different bond dimensions $\chi = 512$, $\chi = 786$, $\chi = 1024$, with time step $dt = 0.1$. A similar analysis for the circuit dynamics is plotted in Fig. 6 for $L = 256$ and for two different bond dimensions $\chi = 1024$ and $\chi = 2048$, with $\tau = 1$ and $\delta = 1$. Both the second and the fourth moments are well converged in the bond dimension in the continuous- and in the discrete-time evolution.

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