

Superextensive charging speeds in a correlated quantum charger

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We define a quantum charger as an interacting quantum system that transfers energy between two drives. The key figure of merit characterizing a charger is its charging power. Remarkably, the presence of long-range interactions within the charger can induce a collective steady-state charging mode that depends superlinearly on the size of the charger, exceeding the performance of noninteracting, parallel units. Using the driven Lipkin-Meshkov-Glick model and power-law interacting spin chains, we show that this effect persists up to a critical system size set by the breakdown of the high-frequency regime. We discuss optimal work output as well as experimentally accessible initial states. The superlinear charging effect can be probed in trapped-ion experiments, and positions interacting Floquet systems as promising platforms for enhanced energy conversion.

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I. INTRODUCTION

A critical engineering objective for developing quantum technology is controlling the energy flow in quantum devices [1,2]. Given that quantum platforms are based on diverse physical implementations [3–8], the coherent transfer of energy is essential for interplatform crosstalk [9,10].

Floquet engineering provides a practical tool to bridge this communication gap [11,12]. Under strong periodic drives, quantum systems can act as pumps that transfer a quantized amount of energy per period (in units of the driving frequencies) [13–18]. In the example of a single qubit with two incommensurately oscillating fields, the work quantization can be understood via mapping to a topological insulator on a two-dimensional frequency lattice. Viewing one of the drives as an energy source, and the second drive as a battery (realized as an oscillator storing energy), the resulting system realizes a *quantum charger* that operates in a Floquet steady state. Recent theoretical work on energy conversion with periodic drives has focused on questions of nonadiabaticity [19–22], and proposed implementations in Weyl semimetals [23] as

well as rhombohedral multilayer graphene [24]. An experiment on superconducting qubits was reported in Ref. [25]. However, these studies were restricted to noninteracting systems.

Here, we generalize energy conversion with periodic drives to interacting many-body quantum systems. Our focus is on nonequilibrium steady states, so that we work within a Floquet description, which remains applicable in the interacting case. The system then realizes a *many-body quantum charger*, capable of transferring a large number of quanta (“photons”) between the drives during each period.

Our central objective is to investigate whether the presence of interactions can parametrically enhance the performance of the charger. We demonstrate that the transferred energy W can scale as

$$W \propto N^{1+\delta} \quad (1)$$

over a broad range of system sizes N , with $\delta > 0$, when interactions are present. This is schematically illustrated in Fig. 1, where we compare the pumping of independent, noninteracting chargers [Fig. 1(a)] to a collective charger in the presence of interactions [Fig. 1(b)].

Our notion of a quantum charger is related to quantum batteries, systems designed for energy storage and release [26–37]. Previous work has shown that many-body batteries can harness collective effects [29,34,35]. Here we demonstrate that chargers exhibit superextensive charging rates due to many-body interactions, with the work per Floquet period W providing a natural measure of the effective charging speed. We emphasize, however, that the operational principles of the two devices differ in important ways: Batteries are transient systems that charge over a finite time. In contrast, the charger

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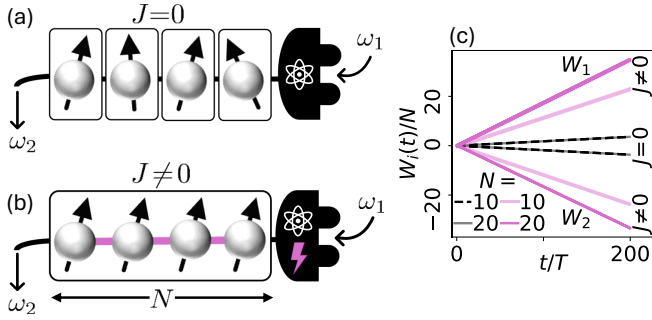


FIG. 1. Energy flow between drives with frequencies ω_1 and ω_2 through a many-body quantum charger. Arrows indicate the direction of the energy flow (sign of the work). (a) Parallel, noninteracting charger. (b) Collective, interacting charger. (c) Work per spin in a Floquet steady state. Long-range interactions enhance the charger's performance superlinearly in the system size N .

continuously pumps energy between the drives in a nonequilibrium Floquet steady state. As a consequence of its transient nature, batteries require optimized charging protocols [38,39]. In contrast, the steady-state charger functions without the need for a protocol. We note that our proposal also relates to fractional Thouless pumps [40–45] and interacting quantum motors [46–50].

In our analysis, we model a quantum charger as a long-range interacting [51,52] spin chain subject to two classically oscillating fields. For all-to-all interactions (Sec. II), the model realizes a driven Lipkin-Meshov-Glick (LMG) model [53], whose nonequilibrium dynamics has attracted significant attention [54–68]. We demonstrate that in its optimal operating modes, the charger converts energy superlinearly between the drives, both over a period and in the steady state [Fig. 1(c)]. For all-to-all coupling, the work scales quadratically up to a characteristic system size N^* , after which the scaling crosses over to linear. This crossover size can be estimated by comparing the interaction scale with the driving frequency, and marks the transition between the high-frequency and the low-frequency regime. We show that a high-frequency expansion of the Floquet operator accurately captures the work in the superlinear scaling regime (Sec. III). Superlinear charging is most efficient for comparable drive frequencies and decays exponentially with the driving period when they differ. We establish the effect for a wide range of realistic long-range interactions and provide corresponding scaling estimates. In Sec. IV, we identify accessible initial states to observe the scaling experimentally. Section V contains concluding remarks.

II. SUPERLINEAR CHARGING

We model a quantum many-body charger by the time-dependent Hamiltonian

$$H(t) = H_0 + V_1(\omega_1 t + \phi_1) + V_2(\omega_2 t + \phi_2). \quad (2)$$

The charger transfers energy between the two drives V_i with $i = 1, 2$ through the static part H_0 . Each drive oscillates periodically in time with frequency ω_i with associated individual period $T_i = 2\pi/\omega_i$ and phase ϕ_i . We focus on periodic driving

characterized by a commensurate ratio $\omega_1/\omega_2 = p/q$, where p and q are coprime integers. This choice fixes the overall period as $T = pT_1 = qT_2$, with fundamental frequency $\omega = 2\pi/T$. We treat the dynamics in a Floquet picture where the quasistationary Floquet states $|\psi_\alpha\rangle$ are defined through the eigenvalue equation $U(T)|\psi_\alpha\rangle = e^{-i\epsilon_\alpha T}|\psi_\alpha\rangle$ of the time-evolution operator $U(t) = \mathcal{T}e^{-i\int_0^t dt' H}$ for a single Floquet period T . The quasienergies can be restricted to the first Floquet zone $-\omega/2 < \epsilon_\alpha \leq \omega/2$.

The integrated pumping power (work) from each drive is given by

$$\begin{aligned} W_i(t) &= \int_0^t dt' \langle \psi(t') | \frac{dV_i(t')}{dt'} | \psi(t') \rangle \\ &= i\omega_i \langle \psi(0) | U^\dagger(t) \frac{\partial U(t)}{\partial \phi_i} | \psi(0) \rangle. \end{aligned} \quad (3)$$

Here, the state $|\psi(t)\rangle = U(t)|\psi(0)\rangle$ of the system is time evolved from the initial state $|\psi(0)\rangle$. The total work transferred over a single Floquet period, $W_i(T)$, thus provides a natural measure of the charging rate.

From the second line in Eq. (3), we identify the (hermitian) single-period work operator [22]

$$\mathcal{W}_i(T) = i\omega_i U^\dagger(T) \frac{\partial U(T)}{\partial \phi_i}, \quad (4)$$

whose expectation value for a given initial state determines the work output. It is useful to define the work spectrum $\mathcal{W}_i|w_{i,n}\rangle = w_{i,n}|w_{i,n}\rangle$. The maximum possible work over one period is obtained by initializing the system in the optimal work state $|w_{i,n}\rangle$ corresponding to the largest work eigenvalue, $\max_n w_{i,n} \equiv \|\mathcal{W}_i\|$, where $\|\cdot\|$ is the operator norm.

However, work states do not return to themselves after one driving period. Hence, their work output cannot be generated in the steady state. To get a quasistationary work output, we also consider initialization in a Floquet state. The work output per cycle for Floquet states is

$$W_{i,\alpha}(T) = \omega_i T \frac{\partial \epsilon_\alpha}{\partial \phi_i} = \langle \psi_\alpha | \mathcal{W}_i | \psi_\alpha \rangle. \quad (5)$$

Because steady states produce no net work on average, the energy from the first drive is fully transferred to the second, $W_{1,\alpha}(T) = -W_{2,\alpha}(T)$.

As a concrete example system, we consider the spin chain

$$\begin{aligned} H_0 &= \sum_{i<j}^N J_{ij} S_z^{(i)} S_z^{(j)} + \sum_{j=1}^N \mathbf{h}_0 \cdot \mathbf{S}^{(j)}, \\ V_i &= \cos(\omega_i t + \phi_i) \sum_{j=1}^N \mathbf{h}_i \cdot \mathbf{S}^{(j)}, \end{aligned} \quad (6)$$

with N spin- $\frac{1}{2}$ operators $S_{\alpha=x,y,z}^{(j)}$. The spins couple via static Ising couplings J_{ij} and are subject to a magnetic field with a static $\mathbf{h}_0$ and two driven components $\mathbf{h}_{1,2}$. The work can be expressed by integrating the spin dynamics modulated by the

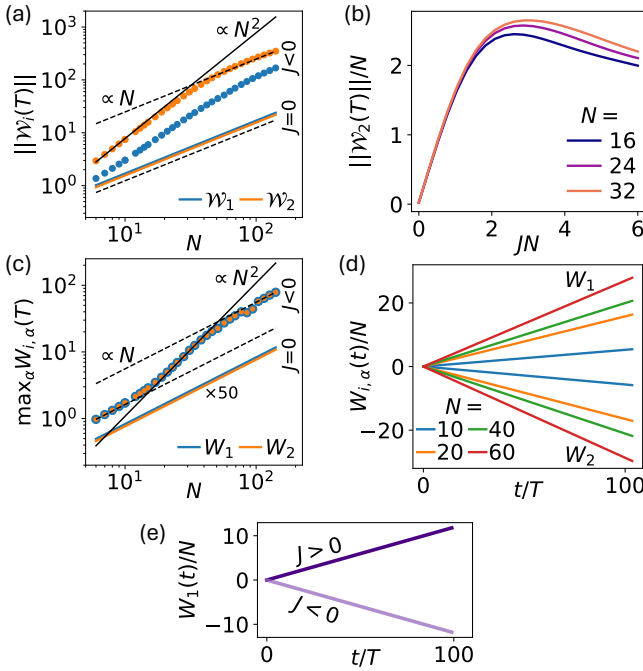


FIG. 2. Superlinear work scaling with N of an interacting quantum charger. (a) Maximum work over a period scales quadratically in N in the presence of interactions (dots), crossing over to linear scaling at large N . The noninteracting charger (solid) always scales linearly. (b) Same as (a) depending on the rescaled couplings. (c) Work of the optimal Floquet state also scales superlinearly for intermediate N . (d) Superlinear steady-state energy flow via dynamics of Floquet states (evaluated at multiples of the period). (e) The sign of the interaction controls the direction of energy flow for Floquet states. Parameters: $\omega_1 = 2\pi$, $\omega_2 = \pi$, $\phi_1 = 0$, $\phi_2 = \pi/4$, $\mathbf{h}_1 = 0.1\hat{\mathbf{x}} + \hat{\mathbf{z}}$, $\mathbf{h}_2 = \hat{\mathbf{x}} + 0.1\hat{\mathbf{x}}$; (a), (c) $J = -0.05$, $\mathbf{h}_0 = 0.01\hat{\mathbf{x}}$; (d), (e) $J = -0.02$, $\mathbf{h}_0 = 0.25\hat{\mathbf{x}}$, $N = 10$.

driving field,

$$W_i = -\omega_i \sum_{j=1}^N \int_0^t dt' \mathbf{h}_i \cdot \langle \psi(t') | \mathbf{S}^{(j)} | \psi(t') \rangle \sin(\omega_i t' + \phi_i). \quad (7)$$

We first focus on all-to-all couplings $J_{ij} = J$. In this case, it is useful to define collective spin operators, $\mathbf{S} = \sum_i \mathbf{S}^{(i)}$. The spin chain realizes a driven LMG model

$$H(t) = \frac{J}{2} S_z^2 + \mathbf{h}_0 \cdot \mathbf{S} + \sum_{i=1}^2 \cos(\omega_i t + \phi_i) \mathbf{h}_i \cdot \mathbf{S} \quad (8)$$

with conserved total spin $\mathbf{S}^2 = S_x^2 + S_y^2 + S_z^2$. We specify to the maximum total spin $S = N/2$ and calculate observables in the $|S_z\rangle$ basis with magnetic quantum number $S_z = -N/2, \dots, N/2$. The LMG model has been experimentally realized with Josephson junctions of Bose-Einstein condensates [69] as well as superconducting qubits [70].

Our main goal is to characterize the scaling of the work with the number of spins N . To calculate the work operator and its spectrum, we numerically time slice the Floquet operator and discretize the derivatives ∂_{ϕ_i} . Figure 2 shows the work performed by the individual drives for the all-to-all interacting

charger with driving ratio $p/q = 2$ (see legend for other parameters). In the presence of interactions, we observe that for small system sizes, the maximum work $\|\mathcal{W}_i(T)\|$ associated with the optimal work state shows a quadratic scaling $\propto N^2$ [Fig. 2(a)]. In this regime, the charger effectively transfers the interaction energy $I = \sum_{i,j} J_{ij} = JN^2$ from one drive to the other. For larger system sizes, we observe a crossover to linear scaling, $\propto N$. In contrast in the noninteracting case, the work always depends linearly on N . This is consistent with the expectation that each spin evolves independently from the others, so that the work of the drives is just the sum of contributions from the individual spins.

We stress that the nonlinear scaling with N is nontrivial in the all-to-all interacting model, Eq. (6), since the drives couple linearly in N to the charger. Note that the data in Fig. 2(a) are for small ferromagnetic couplings $J = -0.05$, but the effect is insensitive to the sign of J for work states. We further comment that the difference between $\|\mathcal{W}_1(T)\|$ and $\|\mathcal{W}_2(T)\|$ for $J < 0$ does not indicate any heating. Since the work states used to define the maxima are not quasistationary, there is no requirement that the total work over a single cycle sums to zero.

Figure 2(b) plots the work of the second drive versus the rescaled coupling strength JN . For small JN , we identify a linear scaling of the work per spin, $\|\mathcal{W}_2(T)/N\|$, with N , with data from different system sizes collapsing onto a single curve. For large JN , $\|\mathcal{W}_2(T)/N\|$ saturates to a constant, consistent with our findings in Fig. 2(a). This transition occurs when the available interaction energy becomes comparable to the energy scale set by the driving period, $JN^2 \sim 2\pi/T$.

We now analyze Floquet states to investigate whether superlinear pumping is possible in the steady state. We identify the optimal Floquet state by maximizing the expectation value of the work operator. Figure 2(d) shows its work dynamics for the first drive. Our results confirm that, even in the steady state, the work per period can depend superlinearly on N . As for work states, Floquet states undergo a transition from quadratic to linear N scaling [Fig. 2(c)], with similar crossover system size. We note, however, that work states can pump a substantially larger absolute amount of work per period (about an order of magnitude for the parameters in Fig. 2). Interestingly, the direction of energy flow for Floquet states can be controlled via the sign of the interaction J in the superlinear regime [Fig. 2(e)]. We have checked that this holds independent of the specific p, q , and follows from the linear dependence $\propto JN^2$ of the work on the interactions in the superlinear regime.

The superlinear pumping effect holds also for finite-range interactions. We consider the more realistic power-law interactions $J_{ij} = J/|i - j|^\gamma$, which can be realized in trapped-ion experiments [71–73] with tunable range $0 \leq \gamma \leq 3$. The results of our numerical calculation are shown in Fig. 3. The maximum work and the work of optimal Floquet states show a superlinear scaling effect for $\gamma < 1$. To get the N -scaling, we estimate the available interaction energy, which is

$$W \propto N \sum_{j>0}^N \frac{1}{|j|^\gamma} \propto \begin{cases} N^{2-\gamma}, & \gamma < 1 \\ N \ln N, & \gamma = 1 \\ N, & \gamma > 1. \end{cases} \quad (9)$$

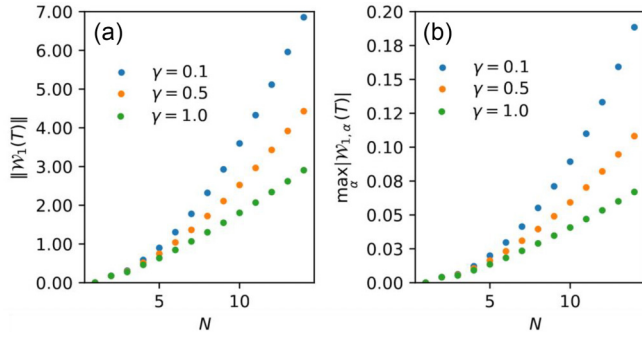


FIG. 3. (a) Maximum work, and (b) work of the optimal Floquet state, for drive 1 over a period as a function of system size for various power-law interactions with range γ . Parameters: $J = 1$, $T = 0.1$, $\mathbf{h}_0 = 0$, $\mathbf{h}_1 = 0.86\hat{x} + 0.25\hat{y} + 0.41\hat{z}$, $\mathbf{h}_2 = 0.18\hat{x} + 0.67\hat{y} + 0.72\hat{z}$, $\phi_1 = 0$, $\phi_2 = \pi/2$.

In Appendix A, we plot the results in logarithmic scale to compare with the scaling estimate, which is consistent with our data in Fig. 3. For the rest of the paper, we focus on all-to-all couplings.

The transition can be understood from the properties of the Floquet many-body spectrum. Since the work performed by a Floquet state is determined by its phase derivative, the scaling of the optimal work with N is governed by the scaling of the many-body bandwidth, provided that quasienergies depend smoothly on the phase. For $JN, h \ll \omega$, the Floquet spectrum covers only a fraction of the Floquet Brillouin zone. In this regime, the bandwidth is not constrained by the periodicity of the quasienergies and can therefore grow superlinearly with system size. This simple picture leads to the superlinear scaling of the work. In contrast, when $JN^2 \ll \omega$ or at $h \gtrsim \omega$, the many-body quasienergy spectrum extends across the entire Floquet Brillouin zone, $-\pi/T \lesssim \epsilon_\alpha \lesssim \pi/T$. Since quasienergies are defined modulo ω , the bandwidth cannot increase indefinitely. In other words, once the interaction energy exceeds the size of the Floquet Brillouin zone, the system can no longer distinguish energies separated by integer multiples of ω . Thus, the growth of the work slows down and eventually crosses over to linear scaling with N .

III. HIGH- AND LOW-FREQUENCY LIMITS

A. High-frequency limit

The superlinear scaling of the work at small system sizes can be understood in the high-frequency limit $JN, h \ll \omega$.

Figure 4(a) displays the maximum work of Floquet states when the two driving frequencies are equal ($p = q = 1$). In this case, the work scales quadratically, $W \propto N^2$, provided that $JN \ll \omega$. In contrast for $JN \gg \omega$, the scaling becomes linear $W \propto N$, indicating a crossover at $JN \sim \omega$. Interestingly, and in contrast to the intermediate-frequency regime shown in Fig. 2(c), the work can exceed quadratic scaling in this crossover regime. The system exits the superlinear regime also for large amplitudes $h \gtrsim \omega$ [Fig. 4(b)], reinforcing that the high-frequency limit must be satisfied.

A systematic comparison across various frequency ratios p/q is shown in Fig. 4(c). We find that the data collapses well for various p/q when rescaled according to

$$W_{i,\alpha}(T) \propto \frac{N^2}{T^{p+q}}. \quad (10)$$

The dependence on the period T indicates that efficient nonlinear pumping is achievable only for small p and q , but drops exponentially with increasing $p + q$.

While the static all-to-all interaction H_0 is already nonlinear in system size, it does not by itself guarantee nonlinear energy transfer. Instead, the relevant contribution arises from an effective high-frequency Hamiltonian, illustrated schematically in Fig. 5 and calculated in the following. We perform a van Vleck expansion in the high-frequency limit [74,75], derived from writing the Floquet operator as $U(T) = e^{-iK} e^{-iH_{\text{eff}}T} e^{iK}$ with effective Hamiltonian H_{eff} and kick operator K . For Floquet states, the work is given by the ∂_{ϕ_i} derivative of the effective Hamiltonian

$$W_{i,\alpha}(T) = \omega_i T \langle u_\alpha | \frac{\partial H_{\text{eff}}}{\partial \phi_i} | u_\alpha \rangle. \quad (11)$$

Here, $|u_\alpha\rangle = e^{iK} |\psi_\alpha\rangle$ are eigenstates of H_{eff} . Up to second order, the high-frequency expansion of H_{eff} in powers of $1/\omega$ yields [74,75]

$$\begin{aligned} H_{\text{eff}}^{(0)} &= H_0, & H_{\text{eff}}^{(1)} &= \sum_{n \neq 0} \frac{[H_{-n}, H_n]}{2n\omega}, \\ H_{\text{eff}}^{(2)} &= \sum_{n \neq 0} \frac{[[H_{-n}, H_0], H_n]}{2n^2\omega^2} + \sum_{n,m \neq 0} \frac{[[H_{-n}, H_{n-m}], H_m]}{3nm\omega^2}. \end{aligned} \quad (12)$$

The successive commutators contain the Fourier components in $H(t) = \sum_{n=-\infty}^{\infty} e^{-in\omega t} H_n$. Due to the all-to-all interactions, the bandwidth of the effective Hamiltonian scales

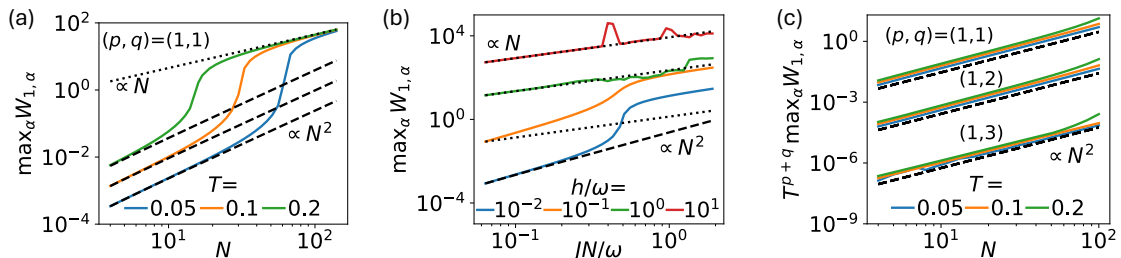


FIG. 4. Maximum work of Floquet states in the high-frequency regime. (a) $\omega_1 = \omega_2$. Analytical expression in Eq. (15) (dashed) captures data for small $JN \ll \omega$. (b) Same as in (a) but for different amplitudes h/ω (see legend). (c) Various frequency ratios $p/q = \omega_1/\omega_2$ as indicated. We collapse the data according to Eq. (10). Parameters: (a) $J = 1$, (b) $J = 1$, $T = 0.1$, (c) $J = 0.1$. Other parameters as in Fig. 3.

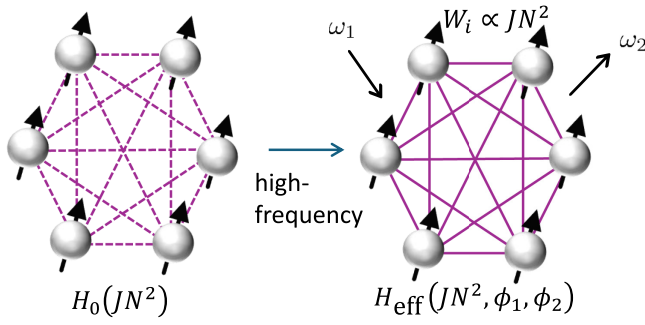


FIG. 5. Static all-to-all spin model (left) vs dynamic all-to-all charger (right). While the bare interactions H_0 (dashed lines) are all to all and thus intrinsically nonlinear, they do not directly contribute to the charging process and thus do not guarantee superlinear energy transfer. In contrast, the transferred energy in the driven system is governed by the effective high-frequency Hamiltonian H_{eff} , which depends on the drive phases ϕ_i and enables the nonlinear charging effect mediated via effective interactions (solid lines).

superlinearly with system size, $D_{\text{eff}} \propto N^2$. However, the work in Eq. (11) depends on the phase derivative, so a superlinear contribution can only be generated at higher orders in $1/\omega$. Specifically, H_0 is superlinear in N but independent of ϕ_i . In contrast, the drives (associated to H_n with $n \neq 0$) are linear in N and depend on ϕ_i . As a result, superlinear behavior can emerge at second order in $1/\omega$, provided that both drives have the same frequency ($p = q = 1$), in agreement with Fig. 4(a).

For other ratios $p \neq q$, the superlinearity only appears at higher orders in the effective Hamiltonian, consistent with the data collapse proposed in Eq. (10). Each term in the van Vleck expansion involves nested commutators of Fourier components H_n , with the constraint that the sum of all indices vanishes. Physically, this reflects conservation of the net number of absorbed and emitted “photons” from the drives. A nonlinear term in H_{eff} requires mixing p and q harmonics of the two drives, which combine H_0 with nontrivial ϕ_i dependence that survives the ∂_{ϕ_i} derivative. For a ratio p/q , the lowest-order process that generates such phase dependence occurs at order $p + q$ in the expansion. We explicitly provide further justification for the scaling in the case $p = 1$, $q = 2$ (see Appendix B).

We explicitly evaluate the work in the case $\omega = \omega_1 = \omega_2$. We find for the effective Hamiltonian

$$H_{\text{eff}} \approx JS_z^2 + \frac{2}{\omega} h_{\text{eff}}^{(1)} + \frac{J}{\omega^2} h_{\text{eff}}^{(2)} \quad (13)$$

up to second order in $1/\omega$. The first and second contributions with negative powers of ω involve

$$h_{\text{eff}}^{(1)} = \sin(\Delta\phi_{12}) \mathbf{S} \cdot (\mathbf{h}_1 \times \mathbf{h}_2) \quad (14)$$

and

$$h_{\text{eff}}^{(2)} = \sum_{i,j=1}^2 \cos(\Delta\phi_{ij}) [-(h_{i,y}h_{j,x} + h_{j,y}h_{i,x})\{S_y, S_x\} + (2h_{i,y}h_{j,y}(S_x^2 - S_z^2) + h_{i,z}h_{j,y}\{S_z, S_y\} + (x \leftrightarrow y))] \quad (15)$$

with phase difference $\Delta\phi_{ij} = \phi_i - \phi_j$. We take the expectation value in Eq. (11) in the unperturbed eigenstates $|u_\alpha\rangle \approx |S_z\rangle$. This yields for their work

$$W_{i,S_z}(T) \approx \pm \frac{2S_z}{\omega} (h_{1,x}h_{2,y} - h_{1,y}h_{2,x}) \cos \Delta\phi_{12} \mp \frac{2J}{\omega^2} \left[\frac{N}{2} \left(\frac{N}{2} + 1 \right) - 3S_z^2 \right] (h_{1,x}h_{2,x} + h_{1,y}h_{2,y}) \times \sin \Delta\phi_{12}, \quad (16)$$

where the positive (negative) sign appears for drive $i = 1$ ($i = 2$). To suppress the first order, one can choose a phase difference $\Delta\phi_{21} = \pi/2$, or the directions of the fields such that their cross product has no component parallel to the interaction. In that case, the maximum work is achieved for $S_z = N/2$. Our analytical calculations capture the numerical result in Fig. 4(a) in the high-frequency limit (small N). We reproduce the reversal of the energy flow by changing the sign of the interaction J at leading order in JN .

B. Low-frequency limit

In the large N limit, the work always becomes linear in system size (see Fig. 2). Here, we show this explicitly in the low-frequency limit $\omega \ll JN, h$, using a semiclassical argument. We also assume $h \ll JN$. We approximate the spin dynamics by replacing collective spin operators by numbers $\mathbf{S}_i \rightarrow \mathbf{s}_i$, leading to the Landau-Lifshitz equation of motion

$$\dot{\mathbf{s}} = \mathbf{h}_{\text{eff}}(t) \times \mathbf{s}. \quad (17)$$

The classical spin has length $|\mathbf{s}| = N/2$ and precesses in the effective field $\mathbf{h}_{\text{eff}}(t) = 2Js_z\hat{\mathbf{z}} + \mathbf{h}(t)$, which contains the exchange field alongside the external field $\mathbf{h}(t) = \mathbf{h}_0 + \sum_{i=1}^2 \cos(\omega_i t + \phi_i) \mathbf{h}_i$.

The work can then be computed from the spin trajectory

$$W_i(t) = \omega_i \int dt (\partial_{\phi_i} \mathbf{h}_{\text{eff}}) \cdot \mathbf{s}(t). \quad (18)$$

The phase derivative of the effective field contains two contributions, $\partial_{\phi_i} \mathbf{h}_{\text{eff}} = 2J(\partial_{\phi_i} s_z) \hat{\mathbf{z}} + \partial_{\phi_i} \mathbf{h}$, but only the exchange-field term depends on N . One finds for the nonlinear contribution to the work

$$W_{i,\text{NL}}(t) = 4J\omega_i \int dt \partial_{\phi_i} s_z^2(t). \quad (19)$$

For large N and at leading order in ω , one expects that the spin remains nearly locked in the direction of the exchange field. A self-consistent solution of the equations of motion yields (Appendix C)

$$\mathbf{s}(t) \simeq \frac{1}{2J} \{h_x(t)\hat{\mathbf{x}} + h_y(t)\hat{\mathbf{y}} + [JN + h_z(t)]\hat{\mathbf{z}}\} + \mathcal{O}(\omega). \quad (20)$$

The longitudinal component s_z becomes effectively constant in time (at leading order in N) and thus independent of ϕ_i . Hence, the nonlinear contribution $W_{i,\text{NL}}$ vanishes in the low-frequency limit. In Appendix C, we extend the self-consistent solution of the equation of motion to order ω and show that the nonlinear contribution to the work vanishes even at this order.

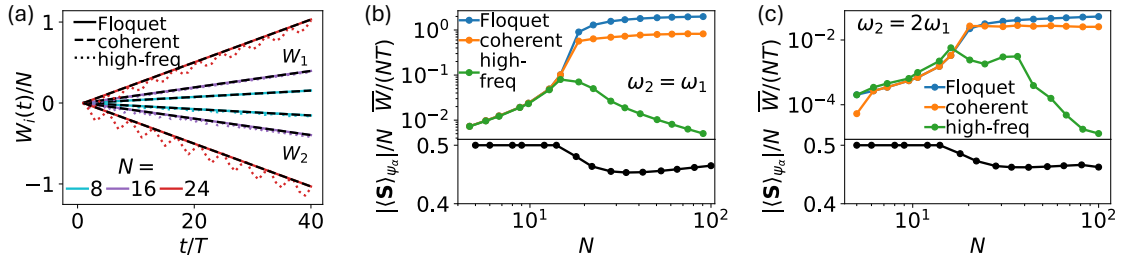


FIG. 6. Work for high-frequency and coherent state approximations of Floquet states (experimentally preparable). (a) High-frequency regime ($\omega_1 = \omega_2$): Superlinear steady-state pumping is observable both by the high-frequency approximation (dotted) and by the spin-coherent state approximation (dashed) of the optimal Floquet state (solid). (b),(c) Average work per cycle and per spin for different frequency ratios $\omega_1/\omega_2 = p/q$ as indicated (top panels). Bottom panels: Total polarization per spin in the Floquet state. Parameters as in Fig. 3.

IV. STABILIZATION OF SUPERLINEAR PUMPING

A. Superlinear steady-state pumping beyond Floquet states

In general, Floquet states are challenging to prepare experimentally, as they are typically highly fine-tuned. It is therefore important to ask whether more easily preparable states can also show superlinear pumping over many cycles. In the following, we consider high-frequency states and spin-coherent states as approximations to the optimal Floquet state. High-frequency eigenstates are eigenstates to the effective high-frequency Hamiltonian. For $\omega_1 = \omega_2$ and at lowest order in ω , they are just $|S_z\rangle$ states. Optimal pumping is then achieved for $|S_z\rangle = N/2$. Additionally, the mean-field spin nature of the LMG model motivates us to study spin-coherent states

$$|\phi, \theta\rangle = e^{-i\phi S_z} e^{-i\theta S_y} \left| S_z = \frac{N}{2} \right\rangle, \quad (21)$$

which are parametrized by the polar and azimuthal angles ϕ and θ . We choose the angles by matching the expectation values $\langle \dots \rangle_\alpha = \langle \psi_\alpha | \dots | \psi_\alpha \rangle$ to that of the optimal Floquet state, $\langle S_x \rangle_\alpha = |\langle \mathbf{S} \rangle_\alpha| \cos(\phi) \sin(\theta)$, $\langle S_y \rangle_\alpha = |\langle \mathbf{S} \rangle_\alpha| \sin(\phi) \sin(\theta)$, etc. For the spin-coherent states, the total polarization is maximal $|\langle \mathbf{S} \rangle| = \sqrt{\langle S_x \rangle^2 + \langle S_y \rangle^2 + \langle S_z \rangle^2} = N/2$ while for the optimal Floquet states $|\langle \mathbf{S} \rangle_\alpha| \leq N/2$. Both high-frequency states and spin-coherent states are easy to prepare experimentally.

In Fig. 6(a), we compare the work outputs of the high-frequency state and spin-coherent state to that of the Floquet state. For parameters in the high-frequency regime and $\omega_1 = \omega_2$, we find for both approximations that there is stable pumping over many periods, matching the superlinear scaling of the Floquet state.

The stability of the approximation is studied beyond the high-frequency regime in Figs. 6(b) and 6(c). We compute the time-averaged work per period and per spin $\bar{W}_i/(NT)$ over 100 cycles for $\omega_1 = \omega_2$ (b) and $\omega_2 = \omega_1$ (c). Both high-frequency state and coherent state agree with the superlinear increase of the Floquet state for $JN \lesssim \omega$. As N grows larger, pumping of the high-frequency state then goes to zero $\bar{W}_i/(NT) \rightarrow 0$. Conversely, the work output of the spin-coherent state has a qualitatively similar behavior as the Floquet state, settling to constant work per spin and per cycle (albeit lower than for the Floquet state).

B. Stabilizing work states by echo

We now show that superlinear pumping in the steady state can also be realized for work states. As previously noted, work states are not quasistationary, and thus require resetting after each driving period. Stabilization of work states is appealing because they also scale superlinearly, but generate substantially larger work compared to Floquet states.

Here, we propose a periodic protocol that approximately resets them in a unitary manner [Fig. 7(a)]. We first evolve the system initialized in a work state over one period. At the end of the period, we apply a fixed, instantaneous pulse

$$V_{\text{echo}} = e^{i\Delta\phi S_z} e^{i\Delta\theta S_y}, \quad (22)$$

which is parametrized by the polar and azimuthal angles $\Delta\phi$ and $\Delta\theta$. The protocol is motivated by the polarization dynamics of the optimal work state, $|\mathbf{S}(t)\rangle$ [Fig. 7(a), gray]. We observe that the total spin norm $|\langle \mathbf{S}(t) \rangle|$ remains approximately maximal and conserved throughout the evolution over one cycle (gray area). This indicates that in the superlinear regime, time-evolved work states are close to spin-coherent states, Eq. (21), with time-dependent angles $\phi(t)$ and $\theta(t)$. The purpose of the pulse in Eq. (22) is to time reverse the

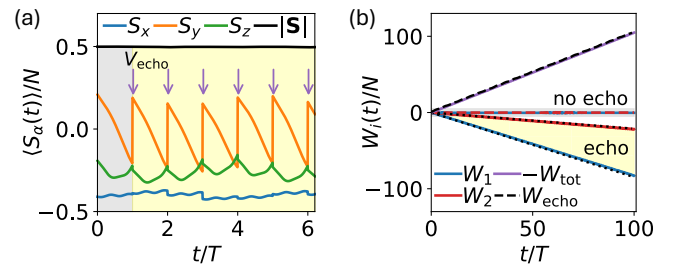


FIG. 7. Stabilization of work states via echo protocol: An instantaneous pulse V_{echo} is applied after each cycle [purple arrows in (a)]. (a) Polarization dynamics. The system remains in a spin-coherent state ($|\mathbf{S}| \simeq 1$) within the period (gray shaded). The pulse then reverses the evolution. (b) Work output. Without echo, no work is produced (gray shaded). With echo (yellow shaded), work states are stabilized in the steady state (dotted black). The energy added by the echo (dashed black) is fully converted to the net work of the drives (purple). Parameters: $N = 16$, $J = 0.25$, $\mathbf{h}_0 = 0$, $\mathbf{h}_1 = \hat{y}$, $\mathbf{h}_2 = \hat{x}$, $\omega_1 = 20\pi$, $\omega_2 = 10\pi$, $\phi_1 = 0$, $\phi_2 = \pi/2$, $\Delta\phi = -0.212\pi$, $\Delta\theta = 0.339\pi$.

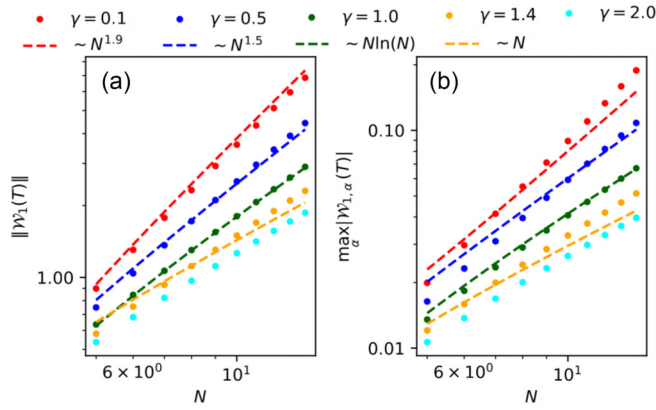


FIG. 8. (a) Maximum work, and (b) work of optimal Floquet state, for drive 1 over a period as a function of system size for different power-law interactions parametrized by γ , plotted at logarithmic scale, with dashed guided lines for comparison. The parameters are the same as in Figs. 3(a) and 3(b).

one-period dynamics of the drives. This can be achieved by choosing the rotation angles $\Delta\phi = \phi(T) - \phi(0)$ and $\Delta\theta = \theta(T) - \theta(0)$ in Eq. (22).

Computing the work of the individual drives, we find that the echo-induced stabilization of work states generates the optimal work output in the steady state [Fig. 7(b)]. In contrast to Floquet states, the work performed by the individual drives does not need to sum to zero. The work supplied by the pulse after N_c periods can be calculated as the instantaneous energy change

$$W_{\text{echo}} = \sum_{n=1}^{N_c} \langle \psi(nT) | [V_{\text{echo}}^\dagger H(T) V_{\text{echo}} - H(T)] | \psi(nT) \rangle. \quad (23)$$

The echo work W_{echo} exactly cancels the work by the two drives as shown in Fig. 7(b), restoring the energy balance $W_1 + W_2 + W_{\text{echo}} = 0$.

V. CONCLUSION

In this work, we introduced a quantum charger that generates a steady-state energy flow between periodic drives. Realizing the charger as a spin chain, interactions induce collective charging where energy pumping is superlinearly amplified with system size. Superlinear charging relies on cooperative motion of the constituent particles, converting the available interaction energy between the drives.

Remarkably, the effect holds for a wide range of realistic long-range interactions, realized, e.g., in trapped-ion experiments [71–73]. Qualitatively, it is insensitive to the sign or nature of the interactions, as long as they are longranged. We find that nonlinear pumping persists up to a critical system size N^* , where the drive becomes strong and the charger splits into independent subsystems. The existence of N^* suggests an efficient construction strategy. Beyond this size, one can partition a large charger into smaller subunits and then scale up, rather than increasing the system size of a single unit. This construction requires only finite-box interactions with range N^* .

The key quantity of our analysis is the work operator, whose largest eigenvalues set the maximum work output. Its eigenstates (work states) are superpositions of Floquet states and show superlinear charging, but pump significantly more than Floquet states. Thus, while not required for superlinear charging, interference can significantly enhance the work. We demonstrated that work states can be coherently stabilized by echo pulses.

In the steady state, the maximum performance is achieved in a Floquet eigenstate, and is given by the expectation value of the work operator in such a state. We find that classical spin-coherent states approximating the optimal Floquet states closely reproduce the quantum optimum in the collective pumping regime. Thus, the Floquet framework naturally allows for work optimization in the interacting case, which is a linear problem for a quantum spin chain but a nonlinear one in the classical case.

Superlinear charging occurs in the high-frequency limit and is most efficient for drives with small, commensurate frequency ratios. It allows substantial energy transfer between drives but does not enable efficient frequency conversion. Extending superlinear charging to schemes capable of frequency conversion remains an open theoretical challenge, complementing recent experimental advances in quantum transducers [76–81].

Future work should also explore the resilience of the proposed charger against dissipation and disorder, as well as treating the drives as quantized cavity fields [15]. Although we focused on the LMG model, other many-body models may further optimize the charger. Further, it would be interesting to relate the superlinear charging effect to time crystals [82–89]. Time crystals are characterized by periodic many-body oscillations and can be realized in long-range settings [58,63,90]. This suggests that they may serve as a useful resource for collective steady-state charging, in addition to existing practical applications in sensing [91–98].

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DATA AVAILABILITY

The data and code supporting this study are available on Zenodo [99].

APPENDIX A: FURTHER ANALYSIS ON THE SCALING OF W FOR REALISTIC INTERACTIONS

To evaluate how good our scaling estimates are for the more realistic power-law interactions, we plot the data similar to those in Fig. 3 at logarithmic scale in Fig. 8. Due to the limitation of the numerical methods, we are only able to simulate up to 14 spins. We expect to get a better scaling behavior in a much larger system.

APPENDIX B: HIGH-FREQUENCY SCALING OF THE WORK FOR $(p, q) = (1, 2)$

We elaborate on the scaling of the work for $\omega_1/\omega_2 = p/q = 1/2$. In this case, the scaling collapse, Eq. (10), predicts $W_{i,\alpha}(T) \propto \frac{N^2}{T^3}$. To recover this scaling from Eq. (11), we identify terms in the high-frequency expansion that (a) contain a factor of H_0 , yielding an N^2 scaling, and (b) exhibit dependence on ϕ_i , which generically remains finite after taking the derivative ∂_{ϕ_i} (except in fine-tuned cases).

Up to order $\mathcal{O}(1/\omega^2)$, the work vanishes. The zeroth- and first-order terms satisfy only one of the above conditions. At second order [Eq. (12)], the relevant commutators are of the form $[[H_p, H_0], H_{-p}]$ and $[[H_q, H_0], H_{-q}]$, but there are no mixed terms involving both p and q . Consequently, no ϕ_i dependence arises at this order.

At third order, the high-frequency Hamiltonian of the van Vleck expansion is [74,75]

$$\begin{aligned} H_{\text{eff}}^{(3)} = & \sum_{m \neq 0} \frac{[[[H_{-m}, H_0], H_0], H_m]}{2m^3\omega^3} + \sum_{m \neq 0} \sum_{\substack{n \neq 0 \\ n \neq m}} \frac{[[[H_{-m}, H_0], H_{m-n}], H_n]}{3m^2n\omega^3} + \sum_{m \neq 0} \sum_{\substack{n \neq 0 \\ n \neq m}} \frac{[[[H_{-m}, H_{m-n}], H_0], H_n]}{4mn^2\omega^3} \\ & - \sum_{m, n \neq 0} \frac{[[[H_{-m}, H_m], H_{-n}], H_n]}{12mn^2\omega^3} + \sum_{m \neq 0} \sum_{\substack{n \neq 0 \\ n \neq m}} \frac{[[[H_{-m}, H_0], [H_{m-n}, H_n]]]}{12m^2n\omega^3} + \sum_{m, n \neq 0} \sum_{\substack{l \neq 0 \\ l \neq m, n}} \frac{[[[H_{-m}, H_{m-l}], [H_{l-n}], H_n]]]}{6lmn\omega^3} \\ & + \sum_{m, n \neq 0} \sum_{\substack{l \neq 0 \\ l \neq m-n}} \frac{[[[H_{-m}, H_{m-n-l}], H_l], H_n]}{24lmn\omega^3} + \sum_{m, n \neq 0} \sum_{\substack{l \neq 0 \\ l \neq m, n}} \frac{[[[H_{-m}, H_{m-l}], [H_{l-n}], H_n]]]}{24lmn\omega^3}. \end{aligned} \quad (\text{B1})$$

The relevant commutator is $[[[H_{-q}, H_0], H_p], H_p] \propto e^{i(\phi_2 - 2\phi_1)}$, which satisfies both conditions (a) and (b), and therefore yields a nonvanishing contribution to the work.

While explicit expressions for the van Vleck expansion beyond third order are, to our knowledge, not available, its structure up to third order reveals a clear pattern: in each term, the indices appearing in the nested commutators sum to zero. Physically, this corresponds to conserving the net number of absorbed and emitted “photons” from the drives, such that $lp = mq$ with integers l, m , where $l + m = n$ is the perturbative order. This observation provides a natural justification for the scaling collapse in Eq. (10).

APPENDIX C: LOW-FREQUENCY LIMIT: SEMICLASSICAL DYNAMICS

We solve the equation of motion, Eq. (17), in the low-frequency limit $\omega \ll JN, h$. We also assume $h \ll JN$. We expand the spin in powers of ω ,

$$\mathbf{s}(t) = \mathbf{s}_a(t) + \mathbf{s}_d(t) + \mathcal{O}(\omega^2), \quad (\text{C1})$$

where $\mathbf{s}_a(t)$ is the adiabatic solution and $\mathbf{s}_d(t)$ is the first diabatic correction. The adiabatic solution fulfills

$$\mathbf{h}_{\text{eff},a}(t) \times \mathbf{s}_a = 0 \quad (\text{C2})$$

with $\mathbf{h}_{\text{eff},a}(t) = 2Js_{z,a}\hat{\mathbf{z}} + \mathbf{h}(t)$. The effective adiabatic field $\mathbf{h}_{\text{eff},a}$ differs from $\mathbf{h}_{\text{eff}}$ by s_z replaced with $s_{z,a}$. Solving self-consistently with the ansatz $\mathbf{s}_a(t) = \frac{N}{2}\hat{\mathbf{h}}_{\text{eff},a}(t)$ yields

$$\mathbf{s}_a(t) \simeq \frac{1}{2J}(h_x\hat{\mathbf{x}} + h_y\hat{\mathbf{y}} + (JN + h_z)\hat{\mathbf{z}}). \quad (\text{C3})$$

Thus, the phase derivative $\partial_{\phi_i}s_z$ vanishes at leading order in N and the nonlinear contribution to the work is zero.

Next, we show that also the first diabatic correction is phase independent:

$$\dot{\mathbf{s}}_a = \dot{\mathbf{h}}_{\text{eff},a} \times \mathbf{s}_d - \mathbf{s}_a \times (2Js_{d,z}\hat{\mathbf{z}}). \quad (\text{C4})$$

Plugging in $\mathbf{s}_a$ and using $\dot{\hat{\mathbf{a}}} = \hat{\mathbf{a}} \times (\dot{\hat{\mathbf{a}}} \times \hat{\mathbf{a}})$, we find the first diabatic correction (written in components)

$$(\mathbf{s}_d)_i(t) = \frac{N}{2}(\dot{\mathbf{h}}_{\text{eff},a} \times \mathbf{h}_{\text{eff},a})_i \begin{cases} 1, & i = x, y \\ \left(1 - \frac{NJ}{h_{\text{eff},a}}\right)^{-1}, & i = z. \end{cases} \quad (\text{C5})$$

The spin lags behind the field in a direction both perpendicular to $\mathbf{h}_{\text{eff},a}$ and $\dot{\mathbf{h}}_{\text{eff},a}$. To find the scaling, we note that in the transverse direction to the exchange field, the effective adiabatic field has strength $\mathcal{O}(1)$. In contrast in the longitudinal direction,

we have $h_{\text{eff},a} \sim \mathcal{O}(N)$. However, $s_{d,z}$ is suppressed by a factor $(1 - NJ/h_{\text{eff},a})^{-1} \simeq -h_z/(NJ)$. As a result, also the first diabatic correction s_d is linear in N in the low-frequency limit.

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