

Magnetic ordering in out-of-plane artificial spin systems based on Archimedean lattices

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Artificial spin systems, sometimes referred to as artificial spin ices, are arrays of coupled nanoscale magnets that order according to the lattice geometry, nanomagnet shape and magnetic anisotropy. Here, we characterize a family of artificial spin systems that are formed by placing arrays of out-of-plane nanomagnets on the vertices of the Archimedean lattices. On demagnetizing these lithographically fabricated nanomagnet arrays using a magnetic field protocol and subsequently imaging the magnetic configuration using magnetic force microscopy, we observe experimentally how these systems order. We compare our experimental results with those predicted by Monte Carlo simulations to assign an effective temperature to each lattice. We find that, for all of the lattices, the assigned effective temperature is above the transition temperature. This reflects the difficulty of obtaining system-spanning order in lattices with out-of-plane nanomagnets. We consider to what extent further-neighbor interactions affect the thermal properties and spin-spin correlations in each lattice, illustrating our results with four example lattices. We can divide the lattices into three main groups: bipartite lattices that admit a perfect antiferromagnetic ground state, frustrated lattices where ordering proceeds via a single step, and frustrated lattices with two-step ordering. Our work highlights the diversity of magnetic ordering that can be hosted in two-dimensional artificial spin systems with out-of-plane nanomagnets, and demonstrates the importance of including longer-range dipolar interactions to explain the magnetic ordering.

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I. INTRODUCTION

Artificial spin systems are arrays of dipolar-coupled nanomagnets [1,2] that exhibit interesting phenomena including spin liquid [3] and spin glass physics [4,5], vertex frustration [6], and emergent magnetic monopoles [7,8]. They were initially envisaged as a two-dimensional (2D) analog to frustrated rare earth spin-ice compounds [9], themselves a magnetic counterpart to water ice, in which incommensurate bonding distances between hydrogen ions and oxygen centres give rise to a residual entropy [10]. In this framework, artificial spin systems are often referred to as artificial spin ices because they demonstrate local constraints, short-range correlations and macroscopic ground state degeneracy. While not all artificial spin systems display ice-like physics, they

do manifest a wide variety of emergent behaviors that depend on the lattice geometry, nanomagnet shape and magnetic anisotropy.

Typically, artificial spin systems are 2D lattices of single-domain nanomagnets, whose magnetic configurations can be directly accessed in real space, with magnetic force microscopy (MFM) [1], magneto-optical Kerr effect (MOKE) microscopy [11], and synchrotron x-ray or electron microscopy [8,12–14], and also in reciprocal space, with x-ray scattering [15], for example. Analysis of these magnetic configurations is often simplified by assuming that each single-domain nanomagnet can be represented as a macrospin, with a direction set by the direction of the net moment of the nanomagnet.

Beyond their fascinating physics, artificial spin systems have also attracted interest as a way to perform logic operations [16,17], and for integration into unconventional computing schemes such as neuromorphic computing [18–21]. For such applications, arrays of nanomagnets with perpendicular magnetic anisotropy [11,12,22–25] are of particular interest because the magnetic state of the individual nanomagnets can be accessed via electrical means using the anomalous Hall effect [26–29]. This allows the direct reading or writing of the state of each nanomagnet using current, thus making it possible to interface artificial spin systems with existing CMOS architectures. However, before such out-of-plane artificial spin systems can be exploited in applications, it is

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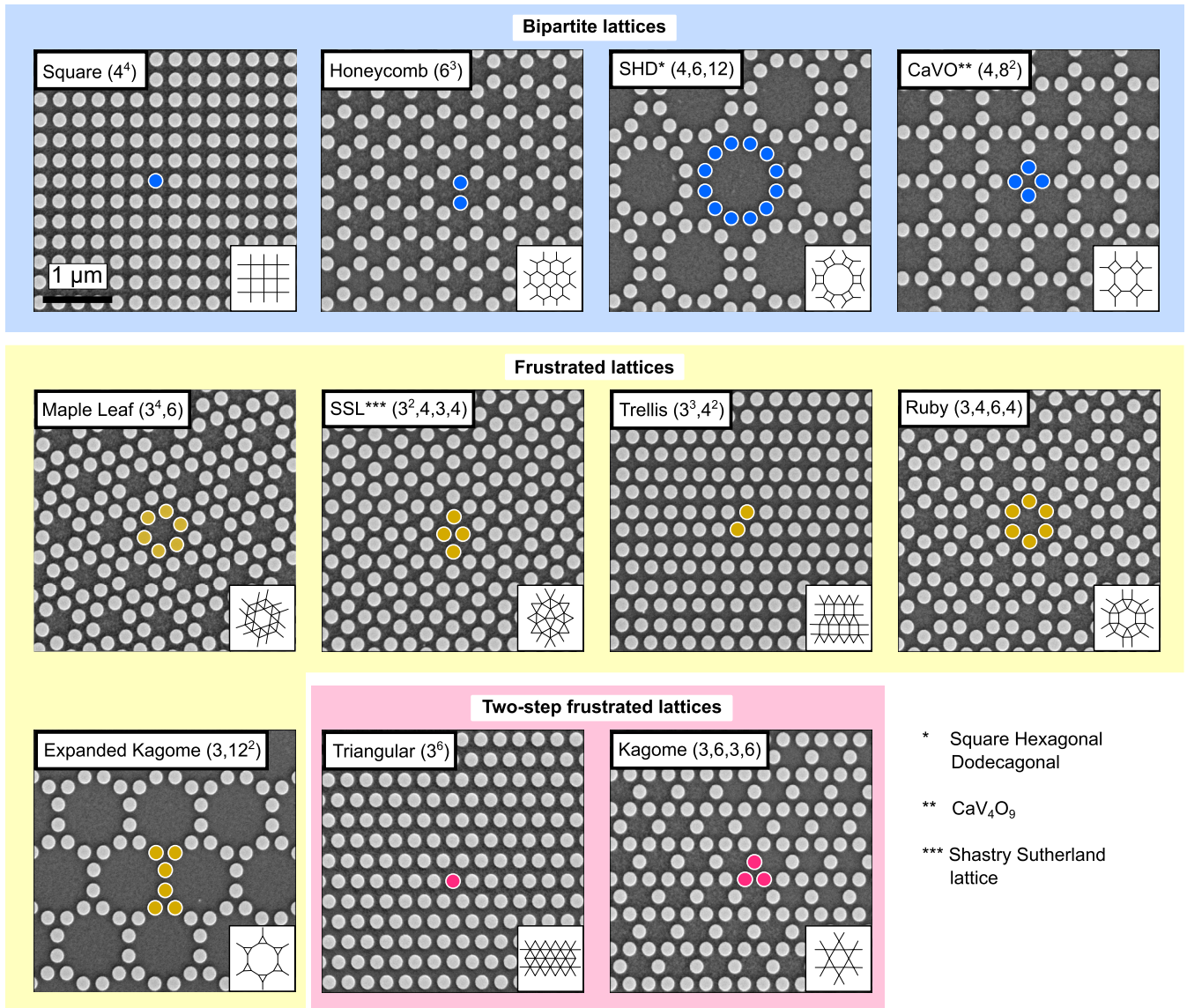


FIG. 1. SEM images of artificial spin systems with out-of-plane nanomagnets located on the vertices of the 11 Archimedean lattices. The name we give to each lattice, as well as the notation of Grünbaum and Shephard [35], which describes the shapes meeting at each lattice point, are given in the top left corner of each panel. The building block for all lattices is a circular Co/Pt nanomagnet with diameter $d = 200$ nm. Because of the strong perpendicular magnetic anisotropy, each nanomagnet behaves as a single macrospin with a magnetic moment either pointing out of or into the plane, which we refer to as up or down, respectively. While we have created arrays of nanomagnets with lattice parameters going from 220 nm to 300 nm, we show here arrays with lattice constant $a = 300$ nm, where the separation between adjacent nanomagnets $s = 100$ nm, because the largest lattice constant allows for better discrimination of each pattern. For each lattice, the nanomagnets highlighted in color indicate those in a single unit cell. Each panel contains a drawing of the underlying lattice structure in the bottom right corner. SEM images of the $a = 220$ nm arrays, which are the most densely packed arrays we fabricated with a separation between adjacent nanomagnets of 20 nm, can be found in Sec. I of the Supplemental Material [36]. In the main text, we present experimental results obtained with the $a = 225$ nm arrays (i.e., with separation $s = 25$ nm). These are the arrays of nanomagnets with the strongest dipolar coupling where we are confident that all of the magnetic material between adjacent nanomagnets has been removed by ion milling. Although the nanomagnet arrays with $a = 220$ nm should, in principle, exhibit an even stronger dipolar coupling, the incomplete removal of magnetic material results in some nanomagnets being exchange-coupled with their neighbors.

important to have a greater understanding of the effect of the lattice geometry and the long-range dipolar coupling on the resulting physics. The fine balance between these two factors sets the hierarchy of intermagnet interactions, or weights [30], which is interesting for designing neuromorphic networks [31–34].

Here, we present a complete experimental and computational classification of the behavior of out-of-plane artificial spin systems based on the Archimedean lattices [35], which are shown with the name we give them in Fig. 1. As can be seen in this figure and also in Sec. 1 of the Supplemental Material [36], the Archimedean lattices are the 11 uniform

2D lattices made from regular polygons, such that every vertex is topologically equivalent. In all of the lattices that we fabricated, the base nanomagnet is identical; it is circular with the same diameter, and is made of the same magnetic material. For a given set of 11 Archimedean lattices with the same nanomagnet separation, the lattices thus only differ in the number and positions of the neighboring out-of-plane macrospins. This allows us to compare the emergent magnetic order resulting from the dipolar interactions across the whole Archimedean lattice family by only changing the lattice geometry.

For each lattice, we determine the magnetic state following a magnetic field demagnetization, and compare the observed spin-spin correlations with those predicted by simulated anneals using Monte Carlo simulations. This comparison allows us to assign an effective temperature to each lattice, providing a measure of how closely the experimentally realized configurations approach the low-temperature states predicted by the Monte Carlo simulations. In the simulations, we define the characteristic temperature as the temperature at which the specific heat capacity exhibits a pronounced peak, marking a significant change in the thermodynamic behavior of the system, irrespective of whether this is a true phase transition or a crossover. Overall, we find that most lattices freeze at effective temperatures slightly above their characteristic temperature.

Of the 11 lattices, four are bipartite: the square, honeycomb, square hexagonal dodecagonal (SHD), and CaVO lattices. This means they admit a perfectly ordered antiferromagnetic ground state. The remaining seven exhibit varying degrees of frustration, which we quantify through their entropy as determined with Monte Carlo simulations based on single spin flips. Two of these seven lattices, the triangular and kagome lattices, are highly frustrated, and their ordering proceeds in two steps when all dipolar interactions are included.

Because of their strong geometric frustration, the triangular and kagome lattices are important model systems in highly frustrated magnetism. The triangular lattice with first-nearest-neighbor antiferromagnetic interactions is the archetypal frustrated lattice, as studied by Wannier [37]. This lattice was predicted to have a massively degenerate ground state, with a finite residual entropy of $0.323 k_B \text{ spin}^{-1}$, where k_B is the Boltzmann constant. In this context, the residual entropy S_0 is related to the logarithm of the number of ground states Ω through $S_0 = (k_B/N) \ln(\Omega)$, where N is the number of spins in the system. More recently, Smerald *et al.* [38] demonstrated that, in a triangular lattice with dipolar interactions, the system crosses over from a weakly coupled paramagnetic phase at high temperature to a spin-liquid phase as the temperature is decreased. Upon further cooling, the system then undergoes a first-order phase transition into a striped phase. The ground state for the triangular lattice with dipolar interactions is sixfold degenerate, which arises from the threefold degeneracy in terms of the possible stripe directions combined with the twofold Ising degeneracy associated with each of the spins [38]. In the spin-liquid phase, almost all of the three-nanomagnet triangular plaquettes have a spin configuration that is either “two-up/one-down” or “two-down/one-up”. However, because there are a large number of configurations that allow for such a local arrangement

of spins, long-range ordering is hindered. Just above the transition to the striped phase, the entropy of the triangular lattice with dipolar interactions is approximately $0.22 k_B \text{ spin}^{-1}$, which is lower than the residual entropy obtained from the first-nearest-neighbor model of $0.323 k_B \text{ spin}^{-1}$ [37]. In the ground state, the triangular lattice with dipolar interactions possesses no residual entropy.

Similarly, the thermodynamics of the kagome lattice has been investigated [23,39–42]. On this lattice, a first-nearest-neighbor antiferromagnetic model admits the existence of only short-range spin-spin correlations at low temperatures based on the formation of groups of low-energy triangular plaquettes [39]. The residual entropy for the kagome lattice with only first-nearest-neighbor antiferromagnetic interactions is $0.501 k_B \text{ spin}^{-1}$ [40]. When further-neighbor dipolar interactions are combined with an enhanced first-nearest-neighbor antiferromagnetic exchange, Monte Carlo simulations reveal a two-step ordering process in the kagome lattice. The resulting ground state is built from interlocking seven-shaped motifs [41], forming an alternating stripe pattern when described in terms of an emergent charge variable defined on each triangular plaquette [42].

We have characterized nanofabricated artificial spin systems based on all 11 Archimedean lattices with MFM and using full-dipolar Monte Carlo simulations, which we compare with first-nearest-neighbor Monte Carlo simulations. Our comprehensive work offers a foundation for future research, particularly since we realize several lattices experimentally for the first time. By fabricating all of the Archimedean lattices together on one substrate, we have acquired a complete data set with minimal differences in the intrinsic properties of the constituent nanomagnets. This allows us to compare the effect of different hierarchies of interactions across the family of lattices.

II. ARTIFICIAL SPIN SYSTEMS BASED ON THE ARCHIMEDEAN LATTICES: BACKGROUND AND EXPERIMENTAL SYSTEM

Archimedean lattices are uniform tilings of the plane in which all vertices are alike, which means that each vertex is surrounded by the same number and type of polygons. Kepler first proved that there were only 11 such lattices, and named them in homage to the Archimedean solids [35]. Eight of the Archimedean lattices feature two or more types of polygon, while the standard uniform tilings (square, triangular, and honeycomb) feature only one. Crystal planes in many real materials, such as alloys, resemble Archimedean lattices [43]. In addition, these tilings also appear in fields as diverse as template-directed assembly of eutectic structures, where diffusion of the fluid imposes symmetry constraints on the final structure [44], polymers [45] and other molecular complexes [46], and photonic crystals, where the Archimedean nature of the lattice can promote the formation of a photonic band gap [47,48]. In addition, the Archimedean lattices can be found in percolation problems, serving as models for the movement of fluid through a medium where the site percolation threshold for many of the lattices remains unsolved [49].

In terms of magnetic systems, different Hamiltonians on individual Archimedean lattices have been extensively

investigated, with perhaps the most famous being Onsager's solution of the 2D Ising model on the square lattice [50]. The prototypical frustrated magnetic system—a triangular lattice with first-nearest-neighbor antiferromagnetic exchange—is another famous example [37]. Complete catalogues of the magnetic behavior of all Archimedean lattices are somewhat rarer, although Ref. [51], which lists exact Curie temperatures for the case of a *ferromagnetic* first-nearest-neighbor interaction, is a notable exception. For the case of a Hamiltonian with *antiferromagnetic* first-nearest-neighbor interactions between Ising spins, Yu presented a comprehensive study of the different Archimedean lattices with Monte Carlo simulations and classified their ground states, finding that four of the lattices were unfrustrated while the remaining seven were frustrated to different degrees [52].

Taking inspiration from the works highlighted above, we explored the Archimedean lattices experimentally with artificial spin systems comprising arrays of *dipolar-coupled* nanomagnets where every nanomagnet is coupled to every other through the long-range dipolar interaction. To fabricate our systems, we patterned a Ta(4)/Pt(3)/[Co(1.1)/Pt(0.2)]₂/Co(1.1)/Ru(2) multilayer stack (numbers in brackets are thicknesses in nm) into arrays of circular nanomagnets placed on the vertices of the 11 Archimedean lattices, as shown in Fig. 1. In Sec. 2 of the Supplemental Material [36], we present magnetometry measurements of this multilayer stack. These confirm that the magnetization of the film lies out-of-plane at remanence, with a coercive field of approximately 25.5 mT. The circular nanomagnets in our arrays have a diameter $d = 200$ nm. The materials and layer thicknesses in the stack, as well as the diameter of the nanomagnets, have been carefully chosen so that the nanomagnets are single domain with a strong perpendicular magnetic anisotropy. There is an interfacial Dzyaloshinskii–Moriya interaction at the boundary between the platinum and the cobalt layers, which may contribute to this perpendicular magnetic anisotropy. However, the exact origin of the perpendicular magnetic anisotropy depends on the materials and the layer thicknesses in the multilayer stack, and can arise from both the broken symmetry at the interfaces and the interfacial Dzyaloshinskii–Moriya interaction [53]. By sputtering multiple layers of cobalt, we increase the total volume of magnetic material, leading to a stronger dipolar coupling between nanomagnets. Since each nanomagnet is single domain, it can be represented by an Ising macrospin, with a magnetic moment that points in one of the two out-of-plane directions, up or down.

In such out-of-plane artificial spin systems, the nature of the magnetostatic coupling means that the interaction between any two neighboring nanomagnets favors their antiparallel alignment. The total thickness of magnetic material in the stack is 3.3 nm, from which we estimate the magnetic moment of a single nanomagnet to be $1.5 \times 10^7 \mu_B$. This sets a lower limit for the interaction strength because it neglects any possible polarization of the Pt layers. Since the magnetic thickness is the same for all systems and lattice parameters, the strength of the interaction only depends on the distance between the macrospins. For the experimental work presented here, we focus on systems with a lattice constant $a = 225$ nm, giving a separation between first-nearest-neighbor nanomagnets of

25 nm. These were the most strongly coupled arrays that we fabricated, in which the magnetic material between adjacent nanomagnets was completely removed by ion milling. While we also patterned arrays of nanomagnets with an even smaller separation, $a = 220$ nm and $s = 20$ nm (images of which are shown in Sec. 1 of the Supplemental Material [36]), some magnetic material was present between adjacent nanomagnets in these arrays. This remaining material introduced an effective ferromagnetic exchange coupling between adjacent nanomagnets. This was evident in the MFM images because the moments of adjacent nanomagnets tended to align in the same direction, leading to an increase in the first-nearest-neighbor spin-spin correlation compared with the $a = 225$ nm arrays (see Sec. 4 of the Supplemental Material [36]).

Based on micromagnetic simulations performed using MuMax3 [54] (detailed in Sec. 2 of the Supplemental Material [36]), we estimate that the interaction strength between first-nearest-neighbor nanomagnets, separated by 25 nm, is $\approx 2.73 \times 10^{-19}$ J. In addition, we also show how this interaction strength changes as a function of lattice constant $225 \text{ nm} \leq a \leq 300 \text{ nm}$. The interaction strength falls to $\approx 0.78 \times 10^{-19}$ J for nanomagnet arrays with $a = 300$ nm, where the nanomagnets are separated by 100 nm. All of the data we present in the main text is for artificial spin systems comprising nanomagnets patterned from the stack with 1.1-nm-thick cobalt layers and with $a = 225$ nm.

Each array contains 1500 to 2500 nanomagnets, and spans an area of $10 \times 10 \mu\text{m}^2$. Each array was captured in a single MFM scan to determine the magnetic configuration. The exact number of nanomagnets in an array depends on the geometry of each lattice and thus the areal density. All nanomagnet lattices are fabricated on the same substrate, with five nominally identical copies of a given lattice with the same lattice constant. Further details regarding the fabrication are contained in Appendix A.

Regarding the naming of our lattices, which is given in Fig. 1, we tend to adopt the standard name where it is well-accepted, e.g., triangular, or where a spin system on that lattice has been studied already, as in the case of the Shastry–Sutherland lattice (SSL) [55], or where it mimics a certain crystal structure, as for the CaVO lattice. Otherwise, we use the nomenclature of Ref. [52], with two exceptions: “expanded kagome” instead of “star”, and “ruby” instead of “bounce”. It should also be pointed out that other names are in use, such as “briarwood” or “bathroom tiling” for the CaVO lattice [49]. In addition, there exists a general notation after Grünbaum and Shephard, which classifies each Archimedean lattice based on the polygons that surround a vertex [35]. For example, the trellis lattice is then denoted by $(3^3, 4^2)$ since *three* triangles (each with three edges) followed by *two* squares (each with four edges) are encountered going clockwise or anticlockwise around each vertex. This notation is also given in Fig. 1.

To make our results easier to classify, we have sorted the 11 Archimedean lattices into three groups, as indicated by the blue, yellow, and pink backgrounds in Fig. 1:

(1) *Bipartite lattices*, where the lattice points can be divided into two sublattices such that, for an arbitrary lattice point, its first-nearest-neighbors are all in the other sublattice. For our systems, in which the dipolar interaction favors

antiparallel alignment between any pair of nanomagnets, this means that this bipartite system can accommodate a Néel-like ground state. This group contains the square, honeycomb, SHD, and CaVO lattices.

(2) *Frustrated lattices*, where a perfect antiferromagnetic ground state cannot be accommodated. This group contains the maple leaf, SSL, trellis, ruby and expanded kagome lattices. These lattices are not bipartite because their vertices involve at least one triangle, which is an odd-sided polygon that cannot host a perfect alternating sequence of up and down spins.

(3) *Two-step frustrated lattices*, which are highly frustrated and where, as we will show, the magnetic ordering occurs in two stages. The remaining two lattices, triangular and kagome, belong to the two-step frustrated lattices group. One can consider the two-step frustrated lattices as a subset of the generally frustrated ones. However, we keep them in a separate group because they display different thermodynamic behavior for the full-dipolar model.

To justify the classification of these lattices into three groups, we now turn to Monte Carlo simulations to probe the behavior of their specific heat capacity. In particular, by examining the number of peaks that appear in the specific heat capacity as a function of temperature, along with the residual entropy, we provide evidence for our choice of groups.

III. DETERMINING THE THERMAL PROPERTIES OF THE ARCHIMEDEAN LATTICES WITH MONTE CARLO SIMULATIONS

To capture the thermal behavior of the Archimedean lattices, Monte Carlo simulations were performed using the Metropolis-Hastings algorithm [56,57] with single spin flip dynamics. Each nanomagnet was approximated as an Ising point dipole that can take one of the two possible out-of-plane directions. The full details of the Monte Carlo simulations can be found in Appendix B.

The two limiting cases that we consider are (i) the full-dipolar model and (ii) the first-nearest-neighbor model. The first case corresponds to the situation in which every nanomagnet is coupled to every other in the array through the point-dipolar interaction, irrespective of their separation. The second case corresponds to a severe truncation of the Hamiltonian, such that each spin is only coupled to its immediate neighbors. Determining the specific heat capacities and the spin-spin correlations as a function of temperature using these two models, and comparing the resulting data, allows us to determine the effect of long-range interactions.

To probe thermal properties of each lattice, we calculate the specific heat capacity c_V as a function of temperature T . Details on these calculations are given in Appendix B. Peaks in the specific heat capacity correspond to either crossovers or phase transitions [58], and are located at the corresponding crossover or critical temperature, respectively. The specific heat capacities versus temperature are shown for each lattice in Fig. 2 for the full-dipolar model (blue curves) and first-nearest-neighbor model (orange curves). In general, the inclusion of all interactions has the effect of shifting the peaks to lower temperatures, with the peaks in the blue curves located to the left of the peaks in the orange curves.

For the bipartite lattices, the specific heat capacity as a function of temperature exhibits a single peak [Figs. 2(a)–2(d)], indicating that the ordering proceeds in a single step. In addition, there is no difference between the ground state for the full-dipolar model and the first-nearest-neighbor model. In both cases, the ground state is a pattern of alternating up and down spins. Example Monte Carlo configurations at $T = 0$ for both the full-dipolar and first-nearest-neighbor models, confirming the nature of the ground state, are provided in Sec. 5 of the Supplemental Material [36]. This section also includes a broader discussion of features in the magnetic structure factor for all of the 11 Archimedean lattices.

The specific heat capacities of the frustrated lattices are also all single-peaked [Figs. 2(e)–2(i)]. In addition, the magnetic structure factors of the $T = 0$ magnetic configurations for both the full-dipolar model and the first-nearest-neighbor model look remarkably similar. These are shown in Sec. 5 of the Supplemental Material, [36]. Since the magnetic structure factor encodes the spin-spin correlations, it thus appears that the correlations in the frustrated lattices are practically the same irrespective of the model chosen.

We now make some comments about the real-space magnetic configurations of these lattices in the ground state. In our classification scheme, the SSL is included among the frustrated lattices because it is not bipartite. However, the frustration is only nominal, as a slight asymmetry in the bonds in the triangular plaquettes lifts the degeneracy that would otherwise render the motifs frustrated. More fully, in the SSL, one of the triangle bonds is shared between two neighboring triangles, giving it an effective value that is half that of the other two bonds in the triangle. This imbalance removes the degeneracy of the frustrated triangular motifs: each triangle has only two ground states rather than six. As a result, the system supports long-range antiferromagnetic order when the diagonal bond is equal to the other bonds, yielding thermodynamic behavior—in terms of both specific heat capacity and residual entropy—that is essentially indistinguishable from that of the bipartite group. In contrast, the trellis lattice retains a residual degeneracy that is associated with the ordering in one-dimensional rows, preventing the emergence of a fully ordered two-dimensional ground state, even though the entropy per site vanishes in the thermodynamic limit [52]. This highlights the variability in properties that can occur within a single group of lattices: the SSL displays signatures closer to bipartite order, while the trellis lattice is similar to the other frustrated lattices. We adopt the strategy of Yu in Ref. [52], who also classifies these two lattices as frustrated for the first-nearest-neighbor model, albeit with ground states that display some signatures of long-range order.

The other three frustrated lattices—the ruby, maple leaf, and expanded kagome—do not exhibit long-range order in either the first-nearest-neighbor model or the full-dipolar model, with regions of diffuse intensity appearing in their magnetic structure factor in the ground state. In the context of the first-nearest-neighbor model, Yu [52] described the ground states of these lattices as being similar to that of a spin ice, since the macrospins obey local constraints regarding the number of up and down spins on the triangular plaquettes. For the ruby, maple leaf, and expanded kagome

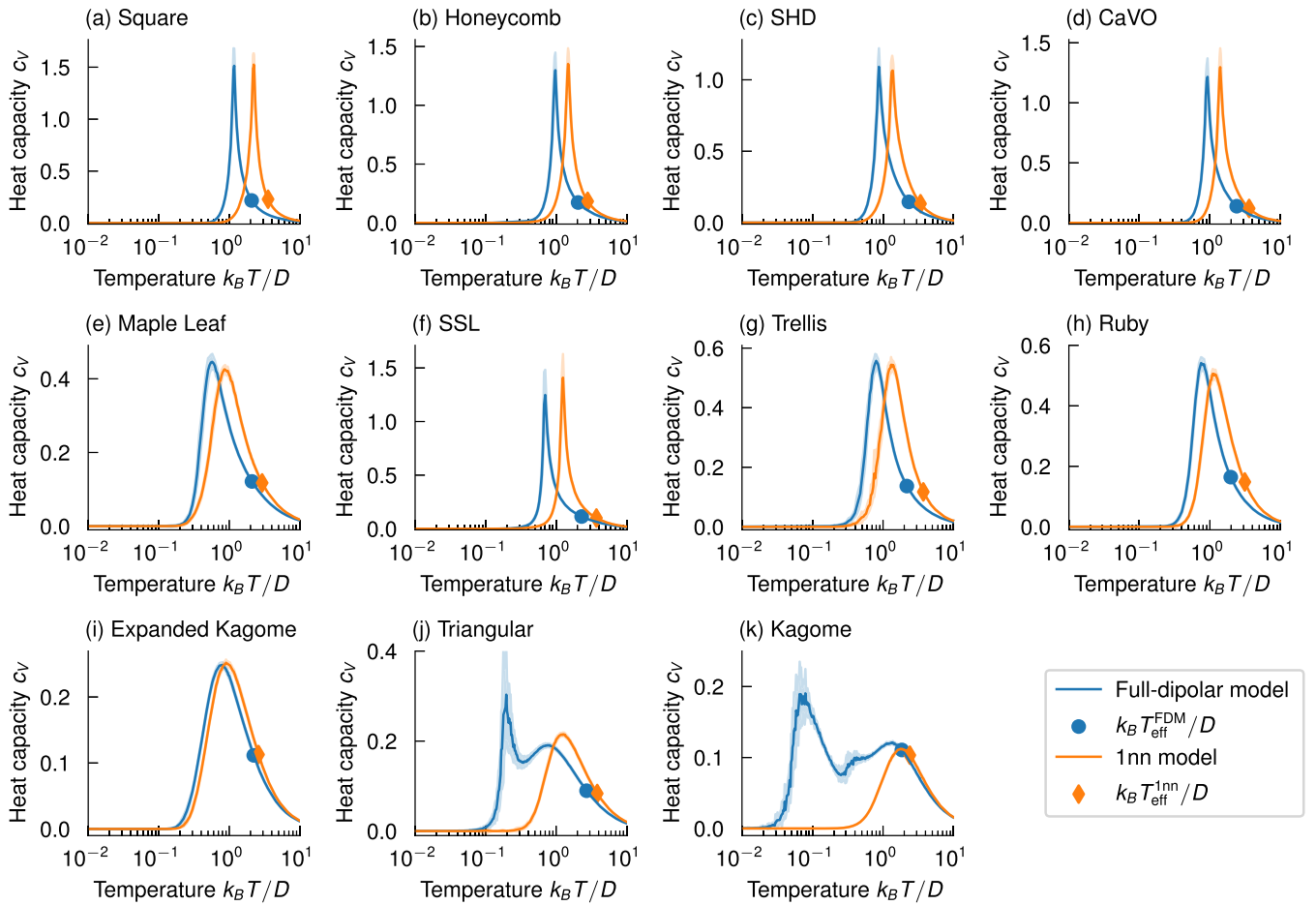


FIG. 2. Thermal properties of the 11 Archimedean lattices determined with Monte Carlo simulations. Specific heat capacities as a function of temperature for the bipartite lattices (a)–(d), frustrated lattices (e)–(i), and two-step frustrated lattices (j)–(k). In each panel, the blue and orange curves correspond to the full-dipolar model and the first-nearest-neighbor model, respectively. Peaks in the specific heat capacity represent phase transitions or crossovers between different types of magnetic order. On including all interactions (i.e., comparing the curves for the full-dipolar model with the curves for the first-nearest-neighbor model), the location of the peaks shifts to lower temperatures. For the two-step frustrated lattices, a low-temperature second peak appears on an energy scale well below that of the first-nearest-neighbor interaction in the system, $J_{1nn} = 1$. In general, the temperature at which a crossover or phase transition occurs is set by the energy of the dominant interaction that causes it, i.e., $k_B T_C \approx J$. This means that the lower-temperature peaks are controlled by spin-spin correlations between further-neighbor pairs, which are weaker in strength. For each panel, the blue circle and the orange diamond indicate the average effective temperature of each experimental nanomagnet array when calculated according to the spin-spin correlations of the full-dipolar model and the first-nearest-neighbor model, respectively. For space reasons, the full axis label “Specific heat capacity” and the units of c_v , $k_B \text{ spin}^{-1}$, are omitted from the axis labels in each panel. Shaded regions around each specific heat capacity curve represent ± 1 standard deviation from the mean, calculated over at least 50 independent Monte Carlo simulations.

lattices, the real-space magnetic configurations and magnetic structure factors in the ground state as obtained from Monte Carlo simulations are nearly identical in both the full-dipolar model and the first-nearest-neighbor model. This suggests that Yu’s [52] classification of these lattices as spin-ice-like for the first-nearest-neighbor model is also valid for the full-dipolar model.

Finally, the specific heat capacities of the two-step frustrated lattices, triangular and kagome, are shown in Figs. 2(j) and 2(k), respectively. Interestingly, both lattices exhibit two peaks for the full-dipolar model, while the second peak is absent when only first-nearest-neighbor interactions are included. For both lattices, and for both models, there is a broad peak close to $k_B T/D = 1$, which is on the order of the first-nearest-neighbor interaction $J_{1nn} = 1$. When using

the full-dipolar model, the second peak observed for both the triangular and the kagome lattices is much sharper and occurs at a significantly lower temperature of $k_B T/D \approx 0.1$. Local maxima in the specific heat capacity are suggestive of either a phase transition or, at least, a crossover in a system in the thermodynamic limit.

For the bipartite lattices, which attain a long-range ordered ground state in either model, the peaks in their heat capacity most likely reflect a second order Ising phase transition. This has been numerically confirmed for the square lattice [59], and probably holds for the other bipartite lattices.

For the two-step frustrated lattices, the single peak that appears when only first-nearest-neighbors are considered is a crossover, as established in Refs. [37,39]. When dipolar interactions are included, Smerald *et al.* [38] established for

the triangular lattice that the upper peak is a crossover and the lower peak is a first-order phase transition. The nature of the low-temperature phase transition in the kagome lattice for the full-dipolar model remains unresolved, though a plausible ground state has been proposed based on an interlocking seven-shaped spin motif in the case of enhanced first-nearest-neighbor antiferromagnetic coupling [41]. Whether this same ground state is realized when the nearest-neighbor interaction is governed solely by dipolar coupling remains an open question [42].

For the frustrated lattices, based on the results in Fig. 2, we do not explicitly define whether the peaks in the specific heat capacities are phase transitions or crossovers. Since the nature of the peaks cannot be unambiguously determined for all lattices, we refer to the temperature at which a peak occurs in the specific heat capacity as the *characteristic* temperature. Future work in this area could include undertaking a finite-size analysis of these five lattices to uncover their scaling behavior and, thus, the type of transitions. We present the first steps towards this endeavor in Sec. 3 of the Supplemental Material [36], where we display the results for a finite-size scaling of the expanded kagome lattice. These results demonstrate two key points: (1) the position of the peak in the specific heat capacity displays minimal shift in temperature for system sizes beyond 600 spins; and (2) at this size, the effective residual entropy is nearly equal to its extrapolated value in the thermodynamic limit. Our experimental nanomagnet arrays always contain at least 1500 nanomagnets, so with 1500 macrospins, indicating that the system sizes are likely to be large enough to avoid the need for finite-size corrections. Although beyond the scope of the current work, it would be useful to repeat this finite-size scaling for the remaining ten Archimedean lattices to determine the critical exponents associated with any possible phase transitions.

Having established the behavior of the specific heat capacity for each Archimedean lattice, we now compare the correlations in the experimental systems after demagnetization with those obtained with a Monte Carlo simulated anneal.

IV. MAGNETIC CONFIGURATIONS IN EXPERIMENTAL SYSTEMS FOLLOWING MAGNETIC FIELD DEMAGNETIZATION

The magnetic configuration in many of these Archimedean artificial spin lattices has not been imaged experimentally. While a few of the lattices—including the square, triangular, and kagome lattices—have been investigated [22,23], we have characterized the behavior of *all* 11 lattices experimentally as well as with Monte Carlo simulations. To probe the low-energy states of our Archimedean spin lattices, we subjected them to a demagnetization protocol involving an alternating magnetic field, with a maximum applied field of 1 T and reducing the field in steps of 0.1 mT to zero field. At each step, the magnetic field was applied for at least 3 seconds. We do not rotate the sample during demagnetization; instead, we change the sign of the magnetic field each time the field is stepped down in magnitude, a strategy that is used to produce lower-energy states in artificial spin ices [1,11,60]. The total demagnetization procedure took around 17 hours to complete. One could also consider performing a

thermal anneal [61,62]. However, we avoid applying heat to our arrays of out-of-plane nanomagnets because the interfaces between the layers, and therefore the magnetic properties, may be modified on heating.

Of course, a demagnetization protocol using a magnetic field may result in the formation of metastable states due to quenched disorder resulting from the distribution in the nanomagnet switching fields, which themselves arise from variations in microstructure and nanomagnet shape [24,63]. Ideally, the switching field distribution could be measured directly—for example, via the magneto-optical Kerr effect magnetometry on non-interacting arrays—but the Kerr signal was too weak for reliable data when we attempted this. Although an alternating magnetic field is unlikely to produce equilibrium configurations, it offers a practical compromise in systems such as these where thermal annealing may lead to irreversible interfacial intermixing and therefore a significant change in the magnetic properties with a lowering of both the saturation magnetization and the out-of-plane anisotropy.

After demagnetization, the magnetic configuration of each lattice was measured using MFM. With MFM, each nanomagnet presents with a uniform dark or bright contrast, which allows for unambiguous assignment of the direction of the associated macrospin, which points up or down, respectively. We find that the residual magnetization after demagnetization is low (typically less than 10% of the saturation magnetization). Any residual magnetization is likely to be caused by residual stray magnetic fields present in the experimental setup. In particular, during demagnetization, the sample is located between the poles of an electromagnet. To obtain the required maximum magnetic field, the gap between the poles is set to ≈ 2 cm and the magnetic field supplied by the electromagnet is calibrated prior to each protocol. Even at zero current, there is a small residual magnetic field from the cores of the electromagnet, which may slightly bias the magnetic state of our lattice. This is likely to be responsible for the small residual magnetization even after the demagnetization protocol was applied. To minimize the effects of the experimental setup and obtain more data points used for averaging, the demagnetization protocol was repeated three times. Each lattice appears five times on the sample and thus the spin-spin correlation values determined from the MFM images represent the average of 15 (3×5) measurements.

We now focus on the magnetic configurations following demagnetization in a magnetic field for four representative lattices: the bipartite CaVO lattice, the frustrated ruby lattice, and both two-step frustrated lattices; the triangular and kagome lattices. Neither the CaVO lattice nor the ruby lattice have been experimentally investigated in an artificial spin system context. The full results for all 11 lattices are given in Sec. 4 of the Supplemental Material [36].

For each of the four representative lattices, we display in Fig. 3 a digitized MFM image of the nanomagnet array obtained after applying a demagnetization protocol (left column), the low-energy motif that constitutes the ground state (middle column), and the spin-spin correlations $\langle \mathbf{s}_i \cdot \mathbf{s}_j \rangle$ as a function of temperature as given by the full-dipolar model (right column). The process by which the spin configuration is extracted from the MFM images is described in Appendix C. Examples of original MFM images for all lattices following

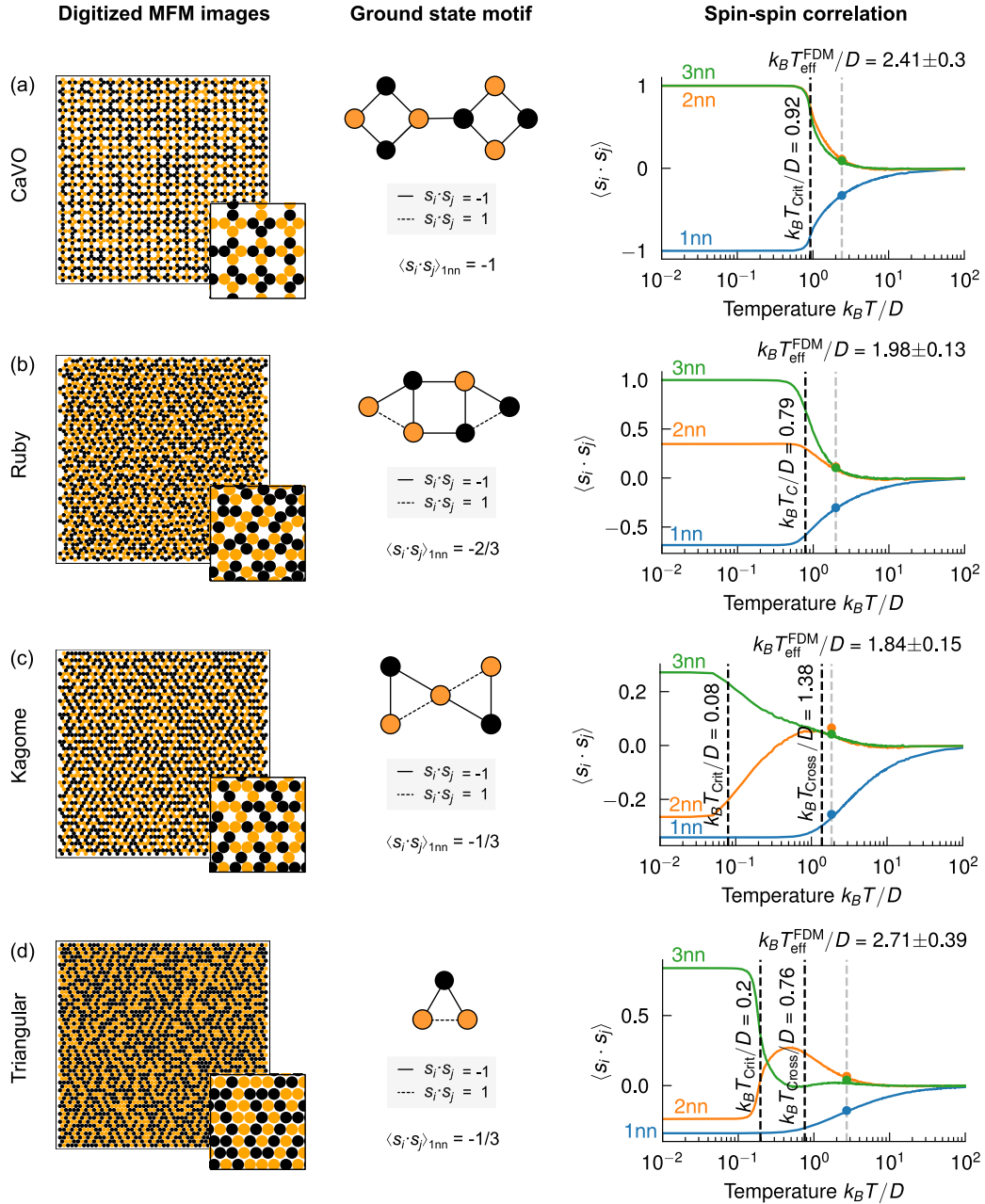


FIG. 3. Ordering in four representative artificial spin systems following demagnetization in a magnetic field. Shown in the left-hand column is the magnetic configuration in each lattice following magnetic field demagnetization for (a) the CaVO lattice, (b) the ruby lattice, (c) the kagome lattice, and (d) the triangular lattice. Here, the MFM images were digitized and the nanomagnets colored, with orange (black) representing the magnetic moment pointing out of the plane (into the plane). In the central column, we display the ground state motif that is expected at low temperature for each lattice along with the corresponding value of $\langle \mathbf{s}_i \cdot \mathbf{s}_j \rangle_{1nn}$ given below each plaquette. For the unfrustrated CaVO lattice, it is possible to have $\langle \mathbf{s}_i \cdot \mathbf{s}_j \rangle_{1nn} = -1$. For the frustrated ruby lattice, the ground-state motif consists of six antiferromagnetically aligned pairs and two ferromagnetically aligned pairs resulting in a higher $\langle \mathbf{s}_i \cdot \mathbf{s}_j \rangle_{1nn} = -2/3$. For the two-step ordered kagome and triangular lattice, which feature only triangular plaquettes, we expect $\langle \mathbf{s}_i \cdot \mathbf{s}_j \rangle_{1nn} = -1/3$. Shown in the right-hand column are the spin-spin correlation curves for up to three nearest neighbors based on the Monte Carlo simulation results for the full-dipolar model. The black dashed lines indicate the characteristic temperature $k_B T_C/D$, and the gray dashed lines indicate the average value of the effective temperature $k_B T_{eff}/D$ following demagnetization in a magnetic field, based on the 15 individual MFM measurements for each lattice type. The location of the characteristic temperatures is determined in each case from the temperature at which peaks occur in the specific heat capacity in Fig. 2. For these lattices, we have also indicated whether the characteristic temperature is more likely to be associated with a phase transition or a crossover. The circular markers represent the value of the average experimental spin-spin correlation. The error bars corresponding to the standard deviation are smaller than the marker size. The definitions of the nearest-neighbor spin pairs, up to the third-nearest-neighbor, are given for each of the Archimedean lattices in Sec. 1 of the Supplemental Material [36].

the magnetic field demagnetization protocol can be found in Sec. 4 of the Supplemental Material [36]. For the spin-spin correlations, the blue, orange, and green curves are the first-, second-, and third-nearest-neighbor spin-spin correlations as a function of temperature extracted from the full-dipolar model Monte Carlo simulations. The average spin-spin correlations extracted from the MFM images are indicated in each case with circular markers with the corresponding colors. For each lattice, the black dashed vertical lines indicate the location of the characteristic temperature $k_B T_C/D$, while the grey dashed vertical lines indicate the average effective temperature for the full-dipolar model $k_B T_{\text{eff}}^{\text{FDM}}/D$ that each artificial spin system reaches following demagnetization in a magnetic field. As described in Appendix D, the effective temperature is defined as the temperature for which the mean square difference between the experimentally measured spin-spin correlations and those obtained from Monte Carlo simulations for the first-, second-, and third-nearest-neighbour pairs are minimized.

The critical temperature is defined as the temperature at which the system transitions to a nonzero order parameter and is usually associated with the appearance of a sharp peak in the specific heat capacity at that temperature. Equivalently, at a phase transition, there is some breaking of a global symmetry, which might be associated with the appearance of long-range order in the magnetic configuration. As emphasized in Sec. III, it remains uncertain whether the peak in each heat capacity corresponds to a true phase transition or a crossover for every lattice, either for the full-dipolar model or the first-nearest-neighbor model. For this reason, we refer to the temperatures at which peaks occur as characteristic temperatures. Nonetheless, for the four representative lattices shown in Fig. 3, we can make the following comments. For the CaVO lattice, which belongs to the bipartite group, our Monte Carlo simulations suggest the emergence of long-range order at the transition temperature, indicating this is likely to be a critical temperature. For the ruby lattice, which belongs to the frustrated group, the ground state does not exhibit long-range order and so we are not able to definitively characterize the behavior in Fig. 2(h) as either a phase transition or crossover. For the two-step frustrated lattices for the full-dipolar model, Smerald *et al.* [38] established that the high-temperature peak in the triangular lattice is a crossover, while the low-temperature peak marks a first-order phase transition. Similarly, Chioar *et al.* [41] concluded for the kagome lattice that the high-temperature peak corresponds to a crossover, which is followed by what looks like a low-temperature phase transition. To reflect these distinctions, we have labeled the black dashed vertical lines in Fig. 3 as either T_{crit} for critical temperatures or T_{cross} for crossover temperatures, where there is a strong indication that this is the case. Otherwise, we simply label them characteristic temperature T_c , as is the case for the ruby lattice.

Looking at the graphs of spin-spin correlation versus temperature in Fig. 3, in particular the gray dashed vertical lines, it becomes immediately apparent that none of the lattices displayed reach an effective temperature lower than $k_B T/D \approx 1$. The implication of this is twofold: First, we mainly work in a regime where the first-nearest-neighbor interaction is the dominant interaction. This is because all the effective temperatures are on the scale of the first-nearest-

neighbor interaction, $J_{\text{1nn}} = 1$. Second, we never encounter the phase transition in any of the lattices because this always occurs at $k_B T/D < 1$. Interestingly, for the triangular lattice, the configurations obtained after demagnetization in a magnetic field have an effective temperature, $k_B T_{\text{eff}}^{\text{FDM}}/D \approx 2.71$, that is higher than that of the other lattices where $k_B T_{\text{eff}}^{\text{FDM}}/D < 2.41$. On inspection of the digitized MFM images in Fig. 3, we do not appear to achieve system-spanning order for any of the lattices. At least, it is not possible to pick out a regular, repeating pattern over the entire measured region, though some crystallites of magnetic order are present in some of the lattices.

We now make a brief comment on each lattice in turn. For the bipartite CaVO lattice [Fig. 3(a)], there are small patches of the predicted antiferromagnetic ground state in the MFM image, which is built by tiling the motif shown in the middle column of Fig. 3. The experimental spin-spin correlations agree extraordinarily well with those predicted by the Monte Carlo simulation at its effective temperature. Since the lattice admits a perfect antiferromagnetic ground state, the first-nearest-neighbor spin-spin correlation determined by Monte Carlo simulations reaches -1 shortly after the characteristic temperature is traversed. For the frustrated ruby lattice [Fig. 3(b)], we also observe small patches of antiferromagnetic ordering in the MFM image. The agreement between the experimental and simulated spin-spin correlations at the effective temperature is also excellent with minimal differences between the two. However, as a result of the frustrated nature of the artificial ruby lattice, the first-nearest-neighbor spin-spin correlation is suppressed and the low-temperature value that is reached is $-2/3$ as opposed to the value of -1 for the CaVO lattice. This comes about because of the unsatisfied interactions within the triangular plaquettes that cannot be fully mitigated through the presence of the square plaquettes in the ruby lattice.

For the two-step frustrated lattices, the situation is more nuanced. For the kagome lattice [Fig. 3(c)], the tiling is such that the first-nearest-neighbor spin-spin correlation only reaches $-1/3$ at low temperatures indicating that the interactions on a triangular plaquette cannot be fully satisfied and the system will never be fully ordered. The experimental spin-spin correlations match well with those predicted by Monte Carlo simulations and the effective temperature reached is $k_B T_{\text{eff}}^{\text{FDM}}/D = 1.84$, which is the lowest T_{eff} out of the four example lattices. In fact, in terms of the specific heat capacity, this effective temperature of the kagome lattice is very close to the temperature at which the upper peak occurs for the full-dipolar model, which is indicated with the higher-temperature black dashed line in Fig. 3(c).

In comparison with the kagome lattice, the triangular lattice [Fig. 3(d)] freezes out at an effective temperature (gray dashed line) that is further above the temperature of the upper peak in the specific heat capacity (higher-temperature black dashed line). At this stage, it is important to note that all of the nanomagnet arrays based on the triangular lattice were affected by a small dilation along the vertical direction, as seen in Fig. 1 and Fig. S1, which was introduced during the electron beam lithography. This dilation affected only the triangular lattice while the sizes of the nanomagnets themselves were unaffected. A detailed analysis of its impact on the

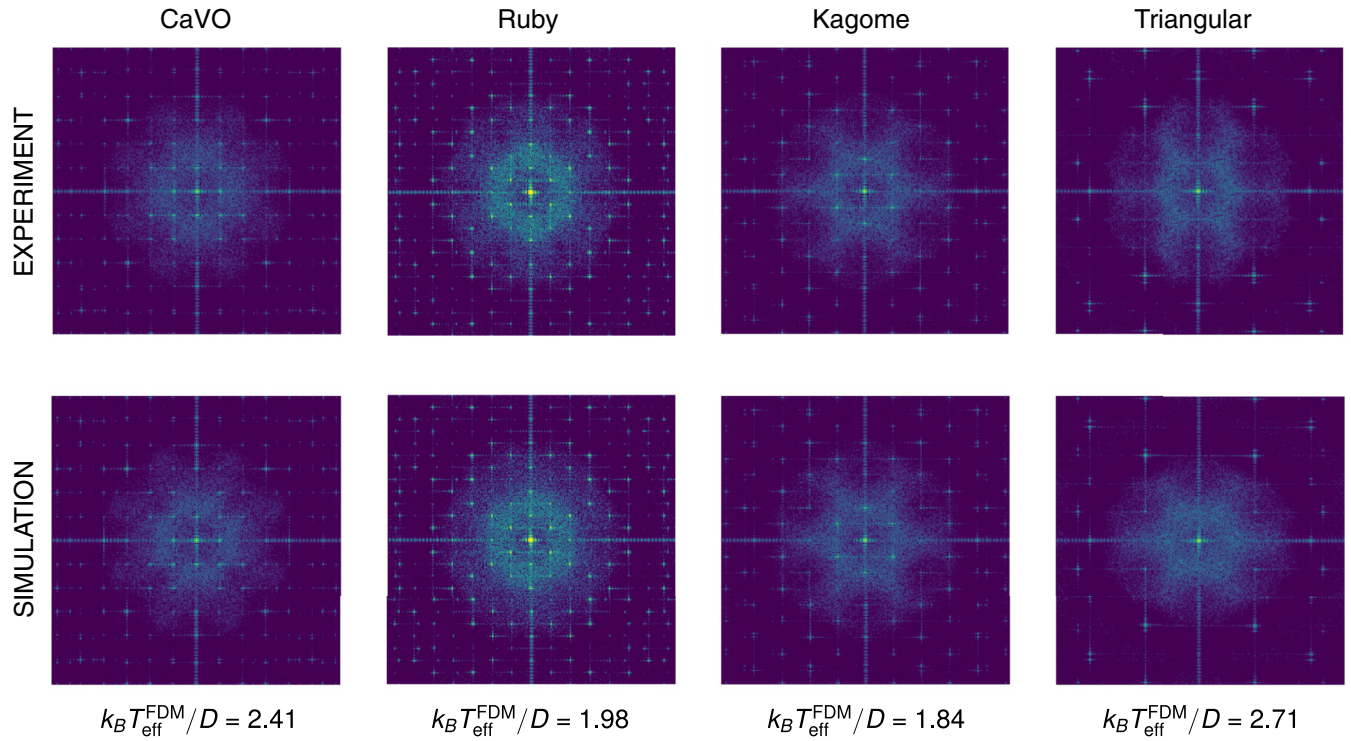


FIG. 4. Comparison of experimental and simulated magnetic structure factors for the four representative artificial spin lattices. Top row: Magnetic structure factors calculated from the experimental magnetic configurations following demagnetization in a magnetic field. Bottom row: Corresponding magnetic structure factor calculated from the simulated configurations at the assigned effective temperature, as obtained using the full-dipolar model. The effective temperatures $k_B T_{\text{eff}}^{\text{FDM}}/D$ are indicated for each lattice. These are obtained by fitting the experimental data to the spin-spin correlations (see Fig. 3 and associated discussion).

specific heat capacity, spin-spin correlations, and the resulting effective temperatures is provided in Sec. 7 of the Supplemental Material [36]. Here, we note that the effective temperatures obtained from such a dilated lattice remain within the standard deviation of those obtained for an ideal lattice. For the rest of this paper, we assume a perfect triangular lattice.

In Sec. 2 of the Supplemental Material [36], we compare the first-nearest-neighbor spin-spin correlations of the nanomagnet arrays presented in the main text, where each cobalt layer has a thickness of 1.1 nm, to those of nanomagnet arrays fabricated from a stack where each cobalt layer has a thickness of 0.9 nm. The same magnetic field protocol is applied to demagnetize both samples. Since the interaction strength between nanomagnets scales approximately with the square of their saturation magnetization, the nanomagnets made from a multilayer stack with 0.9-nm-thick cobalt layers exhibit weaker coupling to their neighbors. As a consequence, the magnitudes of the first-nearest-neighbor spin-spin correlations are smaller for this stack.

In addition, in Sec. 3 of the Supplemental Material [36], we present the spin-spin correlations as a function of temperature up to the fifth-nearest neighbor for all 11 Archimedean lattices, separately for both the full-dipolar model and the first-nearest-neighbor model, as taken from Monte Carlo simulations. For both bipartite and frustrated lattices, the two models exhibit little qualitative difference in the behavior of the spin-spin correlations except that the characteristic temperatures given by the full-dipolar model are consistently

lower than those given by the first-nearest-neighbor model. However, the nearest-neighbor spin-spin correlations behave differently between the two models for the two-step frustrated lattices. Specifically, for the triangular and kagome lattices, the behavior of the second- and third-nearest-neighbour spin-spin correlations behave differently as a function of temperature. For the triangular lattice, this can be explained by the fact that a striped phase emerges for the full-dipolar model, which introduces a degree of long-range order.

In Fig. 4, we display the magnetic structure factors for the four representative lattices, determined from the experimental images (top row) and from a spin configuration from the Monte Carlo simulations incorporating the full-dipolar model at the calculated effective temperature (bottom row). Further details about the calculation of the magnetic structure factor are given in Appendix E. A good agreement is seen for all lattices except the triangular lattice, for which the magnetic structure factor obtained with Monte Carlo simulations appears slightly elongated along the horizontal axis while the magnetic structure factor of a typical experimental configuration appears elongated along the vertical axis. This discrepancy may result from a difference in the orientations of the stripes between the experimental and simulated configurations, along with a slight distortion in the experimental triangular lattice arrays.

It should be noted that the magnetic configurations that were used as the basis for the experimental magnetic structure factor are taken from a single measurement, which is not pre-

cisely at the average effective temperature. This experimental magnetic structure factor is then compared with a single magnetic configuration from a Monte Carlo simulation taken exactly at $k_B T_{\text{eff}}^{\text{DM}}$. However, the true effective temperature of the experimental configuration still falls within one standard deviation of the average value. This may explain any small differences between the magnetic structure factors determined from MFM images and from Monte Carlo simulations.

V. IMPORTANCE OF INCLUDING FURTHER NEIGHBOR INTERACTIONS

We now seek to understand how long-range interactions and geometric frustration determine the ordering in artificial spin systems based on the Archimedean lattices. Since this ordering is manifested experimentally through the effective temperatures reached after demagnetization in a magnetic field, we examine how these effective temperatures relate to the effective residual entropies predicted by our Monte Carlo simulations.

First, we define a reduced effective temperature $t_{\text{eff}} = (T_{\text{eff}} - T_C)/T_C$, where T_{eff} is the effective temperature and T_C is the characteristic temperature at which there is a peak, either sharp or broad, in the specific heat capacity for a given lattice. In effect, the reduced effective temperature is a measure of how close each experimentally measured magnetic configuration is in temperature to that particular peak. The effective residual entropy S_0^{eff} quantifies the number of degenerate ground states, and is obtained through an appropriate integration of the area under the curves of specific heat capacity versus temperature shown in Fig. 2 (see Appendix B). We describe this quantity as an *effective* residual entropy because our Monte Carlo simulations use single spin flips only, so we expect the system to drop out of equilibrium at some point. A different approach would employ some appropriate form of cluster [64] or worm [38] update in the Monte Carlo simulations that is tailored to each lattice. Such nonlocal updates may enable a frustrated system to access pathways in phase space that are inaccessible to single-spin-flip dynamics, allowing it to reach lower-energy states more efficiently and, in doing so, to maintain ergodicity. This would allow more precise sampling of the relevant low-temperature configurations, leading to improved averages for observables such as the specific heat capacity and spin-spin correlations. Despite this, single-spin flips are likely to be the most appropriate representation of the fluctuations during our athermal demagnetization using a magnetic field, and the values of S_0^{eff} we report reflect the accessible low-temperature configurations in these single-spin-flip Monte Carlo simulations.

To explore whether the low-temperature degeneracy of states for different lattices influences the effective temperatures reached in our experimental nanomagnet arrays, we plot in Fig. 5 the reduced effective temperature t_{eff} against the effective residual entropy S_0^{eff} for all Archimedean lattices. This is shown for both the first-nearest-neighbor model and the full-dipolar model. For convenience, the numerical values for the reduced effective temperatures and effective residual entropies for both models for all the 11 Archimedean lat-

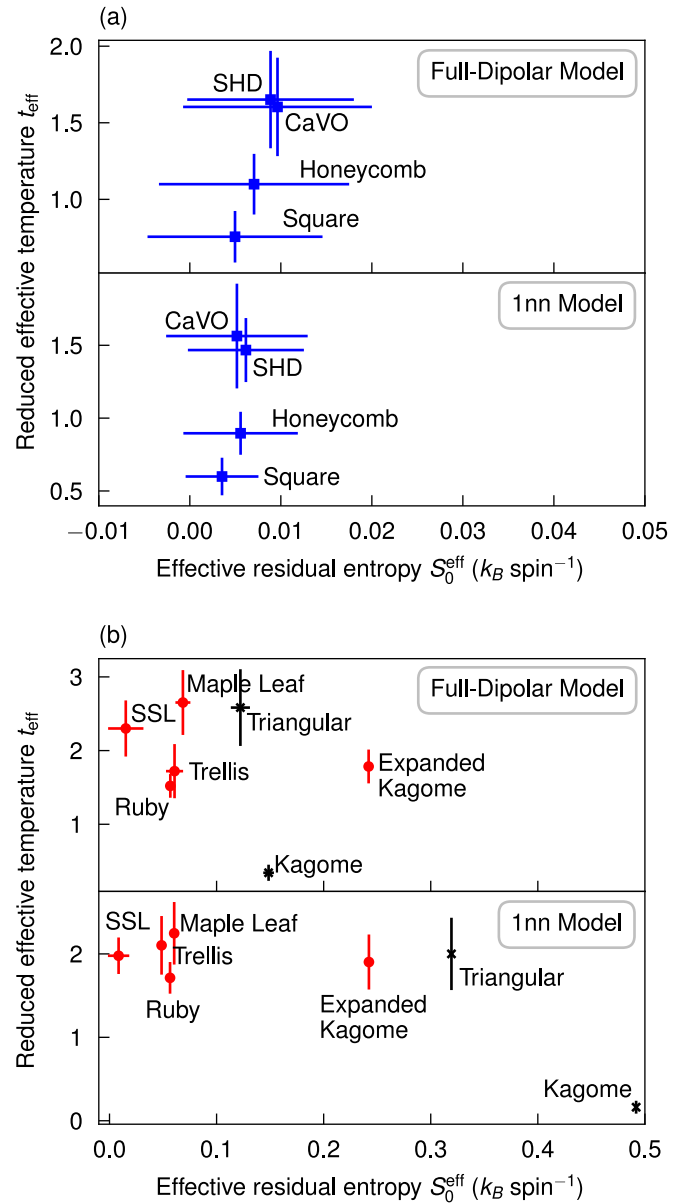


FIG. 5. Reduced effective temperature as a function of effective residual entropy for the bipartite and frustrated lattices. The effective residual entropy was obtained using Monte Carlo simulations for the full-dipolar model and the first-nearest-neighbor model. The reduced effective temperature was calculated using the effective temperature taken from our experimental data and the characteristic temperature was obtained from the simulated heat capacity curves. Panel (a) is for the bipartite lattices, while (b) includes all frustrated lattices, encompassing the two-step frustrated ones. In (b), the frustrated lattices are shown as red circles and the two-step frustrated lattices as black crosses. For the two-step frustrated lattices, the temperature of the upper peak (the crossover) is used to define the reduced effective temperature. Error bars in the effective residual entropy indicate a ± 1 standard deviation around the mean, as determined from at least 50 independent Monte Carlo simulations. Error bars in the reduced effective temperature correspond to ± 1 standard deviation from 15 measurements on experimental nanomagnet arrays.

tics are given in Table II within Sec. 6 of the Supplemental Material [36].

The bipartite lattices all have an S_0^{eff} close to zero [Fig. 5(a)] as a result of the finite (twofold) degeneracy of their ground states. Considered as a group, they also display a lower t_{eff} than the remaining seven lattices. It seems likely that this reflects their lack of frustration. However, demagnetization in a magnetic field still does not drive the bipartite lattices to their antiferromagnetic ground state, and there is little difference between the results for the full-dipolar model and the first-nearest-neighbor model.

In Fig. 5(b), we display t_{eff} versus S_0^{eff} for the frustrated lattices, including the two-step frustrated lattices. To determine the reduced effective temperature for the two-step lattices for the full-dipolar model, we take T_C as the temperature of the upper peak in the specific heat capacity because it is closest in temperature to the experimentally accessible regime.

On average, the frustrated lattices have a higher t_{eff} than that of the bipartite lattices, as well as a nonzero S_0^{eff} . The only exception is the SSL that, although classified as frustrated in our scheme, exhibits an S_0^{eff} close to zero, resembling the behavior of the bipartite lattices. The nonzero S_0^{eff} can be explained by the fact that, for both the frustrated and two-step frustrated lattices, not all of the interactions can be satisfied simultaneously, and thus there are many low-energy states. Nevertheless, while frustration plays an important role in these lattices, we reach reduced effective temperatures of the same order of magnitude as those of the bipartite lattices irrespective of whether these are calculated according to the full-dipolar model or first-nearest-neighbor model.

Interestingly, the kagome lattice exhibits the lowest reduced effective temperature among all lattices, even lower than those in the bipartite group. This is because, for the kagome lattice, the effective temperature lies very close to the temperature of the upper peak for the full-dipolar model and to that of the single peak for the first-nearest-neighbor model, leaving some ambiguity as to whether this is the correct reference temperature. It is also worth noting that the high-temperature peak of the kagome lattice (near $k_B T/D \sim 1$) occurs at the highest temperature among all lattices for the first-nearest-neighbor model, and at the second-highest temperature for the full-dipolar model.

In addition, within the frustrated group, the expanded kagome lattice stands out because of its large effective residual entropy, which remains high for both the full-dipolar model and the first-nearest-neighbor model [see upper and lower panels of Fig. 5(b)]. This can be understood by viewing the expanded kagome lattice as a kagome lattice of spin dimers, which leads naturally to a residual entropy that is half that of the kagome lattice for the first-nearest-neighbor model. It seems likely that the entropy of the expanded kagome lattice remains high for the full-dipolar model because the sparseness of the lattice strongly suppresses the influence of longer-range interactions.

We have also explored plotting t_{eff} against the specific entropy calculated from Monte Carlo simulations at the experimentally determined effective temperature $s(T_{\text{eff}})$ (see Sec. 8 of the Supplemental Material [36]). While this gives an additional perspective on the data, it does not reveal a clear-cut grouping of the lattices. In particular, the different groups do not cluster systematically at distinct entropies; instead,

most lattices reach states with comparable specific entropy, approximately $0.6k_B \text{ spin}^{-1}$, at their respective T_{eff} , despite their differing frustration and connectivity.

In summary, we have adopted a pragmatic classification scheme that first distinguishes bipartite from nonbipartite lattices, and then, for the remaining lattices, classifies them according to whether the specific heat capacity exhibits one or two peaks for the full-dipolar model from our single-spin-flip Monte Carlo simulations. While t_{eff} , S_0^{eff} , and $s(T_{\text{eff}})$ each capture different aspects of the experimental configurations and the physics of the underlying lattice, they do not provide a rationale for this classification, nor do they explain why particular lattices settle at particular effective temperatures. This absence of a clear correspondence is still informative: It suggests that the effective temperatures reached experimentally are governed by a common energy scale, which is likely to be associated with the energy barrier to reversal of the macrospin, and less by the degree of geometric frustration in the lattice.

VI. CONCLUSIONS

A. Thermal behavior

We have engineered artificial spin systems based on the 11 Archimedean lattices comprising arrays of Co/Pt circular nanomagnets with perpendicular magnetic anisotropy. The nanomagnet arrays were prepared in low-energy states with a demagnetization protocol using an alternating magnetic field. From images of the exact microstate of each lattice recorded with MFM, we calculated the experimental spin-spin correlations and compared them to the correlations predicted using single-spin-flip Monte Carlo simulations. This enabled us to assign effective temperatures to each lattice, demonstrating that, irrespective of the lattice, the system freezes just above its characteristic temperature, with effective temperatures of the order of the first-nearest-neighbor interaction, $k_B T_{\text{eff}}/D \sim 1$. The fact that this lower bound in effective temperature is essentially identical for every lattice indicates that it is not necessarily governed by the residual frustration. Instead, it appears to reflect a common energy scale, likely to be that of the intrinsic energy barrier to reversal of the macrospin of an individual nanomagnet.

Near to these effective temperatures, quenched disorder is not expected to alter the thermal properties of the lattices. This is supported by analysis we present in Sec. 3 of the Supplemental Material [36], where we perform Monte Carlo simulations for the square, trellis, triangular, and kagome lattices, introducing a Gaussian distribution of intermagnet interactions around their nominal value. The amount of disorder used in a given Monte Carlo simulation is then reflected by the standard deviation of the Gaussian distribution. We find that, for all lattices, the high-temperature portion of c_V , i.e., for $k_B T/D > 1$, appears relatively immune to the presence of even significant amounts of disorder, e.g., up to 20%. Instead, we find that the presence of disorder affects only the low-temperature peaks in the triangular and kagome lattices, causing them to disappear (broaden) in the triangular (kagome) lattice.

Despite their similar effective temperatures, we were able to sort the lattices into three groups:

- (1) *Bipartite lattices* that host a perfect antiferromagnetic ground state, and where the specific heat capacity features a single peak;
- (2) *Frustrated lattices*, with a single peak in the specific heat capacity; and
- (3) *Two-step frustrated lattices*, where the specific heat capacity has two peaks for the full-dipolar model.

To further distinguish the degree of frustration within these groups, we quantified the *effective* residual entropy of each lattice using single-spin-flip Monte Carlo simulations.

Building on this classification, our analysis also establishes whether further-neighbor interactions are important when modeling the Archimedean lattices. For all of the lattices, including all interactions does not substantially affect the thermal properties in the vicinity of effective temperatures relevant to our experiment. The two-step frustrated lattices—the triangular and kagome lattices—are an exception, but only at low temperatures where, in both cases, the specific heat capacity for the full-dipolar model features an additional low-temperature peak, which is likely to be a first-order phase transition. The existence of this extra low-temperature peak in the specific heat capacity for the full-dipolar model was previously noted in Refs. [38,41].

In summary, we provide the most complete classification of artificial spin systems with out-of-plane magnets arranged on the Archimedean lattices, demonstrating how long-range interactions influence the ordering in systems with different levels of frustration, going from the bipartite to the frustrated to the two-step highly frustrated lattices.

B. Outlook

Looking ahead, Archimedean artificial spin systems offer a rich platform for investigating different forms of magnetic ordering. For this, it would be beneficial to reach lower effective temperatures experimentally, which will require precise tuning of the material parameters to control both the energy barrier required for switching and the intermagnet coupling [21]. In addition to achieving lower-energy states, this will provide a way to observe thermal relaxation at room temperature, which is something already achieved for in-plane systems [65–68].

In this vein, it has recently been demonstrated that small reductions in the cobalt thickness within the Pt/Co multilayers can significantly lower the energy barriers to switching of the nanomagnets, while maintaining strong dipolar coupling [21]. This development offers a promising route towards using thermal annealing in artificial spin systems with out-of-plane nanomagnets to obtain low-energy states at temperatures that do not alter the material microstructure. Unlike demagnetization in a magnetic field—where the resulting magnetic state is sensitive to exact details of the magnetic field protocol—a real thermal anneal should allow the system to explore a larger proportion of its phase space and, in doing so, approach a low-energy state.

Future observations of the thermodynamics, in particular how the Archimedean artificial spin systems order through a crossover or first-order phase transition, will provide ample

scope for future research. This would be particularly interesting for those Archimedean lattices that feature both even- and odd-sided polygons within a single lattice, which results in a mixture of frustrated and unfrustrated motifs. The triangular and kagome lattices are likely to continue to fascinate, since the precise nature of their low-temperature states remains a topic of ongoing research, with some promising candidates identified [41].

Finally, the insights gained from these systems into the delicate interplay of lattice geometry and long-range interactions are critical for applications exploiting novel computing schemes. By tailoring the lattice geometry, artificial spin systems can be engineered to respond to particular inputs, such as magnetic field cycles, in ways that harness their computational capacity [19,21,69,70], offering a pathway to low-power reservoir computing.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [71].

APPENDIX A: FABRICATION

Artificial spin systems with circular nanomagnets arranged on the vertices of the 11 Archimedean lattices were fabricated with electron beam lithography on the same chip for a given Co thickness. This minimized potential sources of variation from the fabrication process and demagnetization protocol. The arrays of nanomagnets were small enough ($10 \times 10 \mu\text{m}^2$) to fit within a single MFM image (scan size $15 \times 15 \mu\text{m}^2$), allowing us to capture the magnetic configuration of all of the nanomagnets within a single image. The artificial spin systems were fabricated with electron beam lithography from a Co/Pt multilayer deposited on a silicon oxide substrate with UHV sputtering, with the exact film composition being Ta(4)/Pt(3)/[Co(1.1)/Pt(0.2)]₂/Co(1.1)/Ru(2). The numbers in brackets correspond to the thickness of the layers in nm. As discussed in the main text, we have also fabricated the artificial spin systems from a multilayer film with the same composition but with a reduced thickness of magnetic material; 0.9 nm instead of 1.1 nm of Co.

Using a superconducting quantum interference vibrating sample magnetometer (SQUID-VSM), we have measured the bulk magnetic properties of the film for magnetic fields applied parallel and perpendicular to the sample plane, confirming the PMA nature of the stack. The hysteresis curves from the SQUID-VSM measurements can be found in Sec. 2 of the Supplemental Material [36].

The nanomagnet diameter was chosen to be 200 nm to ensure that the nanomagnets were small enough to be single domain. While arrays of different lattice constant were fabricated (see Sec. 1 of the Supplemental Material [36]), we have focused here on the arrays with $a = 225$ nm, which have an intermagnet separation of 25 nm. One can increase the dipolar coupling by decreasing the separation between the nanomagnets. However, this is the smallest nanomagnet separation for which we can be sure that adjacent nanomagnets are completely separated.

In more detail, the nanomagnet arrays were fabricated by first depositing the magnetic multilayer onto a silicon substrate using DC magnetron sputtering at a base pressure of 2.5×10^{-8} mbar. The film was prebaked at 125°C for 5 minutes to help with the adhesion between the top surface of the film and the electron-beam resist. Once cooled down, the film was spin coated with hydrogen silsesquioxane (HSQ) electron-beam resist (XR-1541 2%) at 4000 rpm for 60 s to obtain a total resist thickness of ≈ 50 nm. This was followed by electron beam exposure of the HSQ resist with the desired patterns of arrays of dots using an electron beam writer (Raith EBPF5000Plus). Because of the negative polarity of the resist, the developed resist was an array of HSQ dots that was subsequently used as a mask for milling the Co/Pt multilayer with Ar ions, leaving behind the arrays of circular nanomagnets. The ion milling took 23 s, during which the sample was rotated at 20 rpm, with the sample normal tilted at 15° to the Ar beam. The Ar beam voltage was 400 V, the flow of the gas was kept to 8 sccm, and the ion current was 122 mA. Finally, the resist on top of the nanomagnet arrays was removed using a buffered oxide etch solution of hydrofluoric acid. The circular nanomagnets appeared with a uniform dark or bright contrast in the MFM images, indicating that they are single-domain and magnetized up or down, respectively. Using scanning electron microscopy, we confirmed the diameter and separation of the nanomagnets for each lattice.

APPENDIX B: MONTE CARLO SIMULATIONS

To capture the thermal behavior of each of the Archimedean spin systems, Monte Carlo simulations were performed using the Metropolis-Hastings algorithm with single spin flip dynamics. Each nanomagnet was approximated as an Ising point dipole that can take one of the two out-of-plane directions. The Hamiltonian of the system was

$$\mathcal{H} = D \sum_{i \neq j} \frac{\mathbf{s}_i \cdot \mathbf{s}_j}{r_{ij}^3}, \quad (\text{B1})$$

which corresponds to the simplified form of the dipolar interaction appropriate for out-of-plane spins. In Eq. (B1), the spin i at position $\mathbf{r}_i$ has magnetic moment $\mathbf{s}_i$. The connecting vector $\mathbf{r}_{ij} = \mathbf{r}_j - \mathbf{r}_i$ points from spin i to spin j . The energy scale of the interactions is set through the dipolar constant, $D = \mu_0(M_S V)^2/a^3$, where M_S and V are the saturation magnetization and volume of each nanomagnet, respectively. We work in reduced units throughout this paper so that the first-nearest-neighbor interaction strength, $J_{1nn} = D = 1$. The temperature is then quoted in dimensionless units, i.e., $k_B T/D$.

In this work, we also consider the effect of truncating the dipolar interaction to the first-nearest-neighbors. This cutoff allows us to map from the full-dipolar model—without a cutoff—to a first-nearest-neighbor antiferromagnetic Ising model. Such a cutoff has been shown to affect the critical behavior of an artificial spin system, especially at low temperatures [23].

For a system of N spins, a single Monte Carlo step involves N attempted single spin flips. At each temperature, 10^4 Monte Carlo steps are used to equilibrate the system before 10^4 steps are used for averaging. The configuration from the previous temperature is then used as the starting state for the next temperature point. The specific heat capacity c_V is determined from the first and second moments of the energy E using

$$c_V = \frac{1}{N} \frac{\langle E^2 \rangle - \langle E \rangle^2}{k_B T^2}, \quad (\text{B2})$$

which has units of $k_B \text{ spin}^{-1}$.

To obtain a measure of the level of frustration in the lattices, we define an *effective* residual entropy S_0^{eff} , which is the residual entropy of the state reached when only single spin flips are used in the Monte Carlo simulations. This gives a measure of the degeneracy of states at zero temperature. The quantity S_0^{eff} is obtained by integrating c_V/T with respect to T , and subtracting this value from the entropy of a spin in the high-temperature limit $S = k_B \ln(2)$ as follows

$$S_0^{\text{eff}} = k_B \ln(2) - \int_0^{100} \frac{c_V}{T} dT. \quad (\text{B3})$$

where the temperature limits for the integral correspond to the final and starting temperatures (0 and 100, respectively) in our Monte Carlo simulated anneal. The greater the value of S_0^{eff} , the higher the degeneracy of the state reached and thus the higher the level of frustration.

The data points from the Monte Carlo simulations reflect the average of at least 50 independent simulation runs. Error bars are taken as the standard deviation of this average. In order to perform a simulated anneal of the Archimedean lattices, the temperature is first decreased linearly from $k_B T/D = 100$ to $k_B T/D = 1$, and is then decreased logarithmically. The final temperature point is at $k_B T/D = 0$. For all lattices, the specific heat capacity evaluated at the highest and lowest temperatures is very small, with any transitions happening somewhere within the temperature range $0 < k_B T/D < 100$. This integration over a wide temperature range of the specific heat curve, which includes all peaks, where each peak is associated with either a phase transition or a crossover, ensures that our estimate of the effective residual entropy is accurate. In principle, a small correction arises from the behavior of the specific heat between the highest simulated temperature and infinity. However, by selecting the highest annealing temperature $k_B T/D = 100$, where the specific heat is already small, any such correction becomes negligible. In total, 254 temperature points are used. Periodic boundary conditions were implemented by creating periodic copies of the system out to 15 times the system size.

For those lattices that are highly frustrated, e.g., the triangular or kagome lattice, it is likely that single spin flips are not sufficient to equilibrate the system. This problem will

be especially acute in the vicinity of the critical temperature, where the autocorrelation time is large. The autocorrelation time in the context of artificial spin systems is a measure of the timescale over which the spin configuration of a system remains correlated with itself. A short autocorrelation time indicates that the system evolves rapidly and configurations become decorrelated quickly, whereas a long autocorrelation time suggests slower dynamics and more persistent correlations. Such diverging time scales reflect the fact that large numbers of spins need to re-orientate to effect change in the diverging length-scales. In terms of Monte Carlo simulations, performing only single spin flips will result in poorly sampled observables, e.g., heat capacity. As we mentioned in the main text, a more sophisticated approach would be to implement nonlocal updates, such as cluster or worm algorithms. However, as we have demonstrated, all experimental systems appear to freeze-out at a temperature where single spin flips still capture the essential physics, and so this does not affect our assignment of effective temperatures.

APPENDIX C: EXTRACTING THE SPIN CONFIGURATIONS FROM MFM MEASUREMENTS

MFM measurements were performed using a Bruker Veeco Dimension 3100 with commercially available low-moment Bruker MESP-LM-V2 tips. The scan size was $15\text{ }\mu\text{m} \times 15\text{ }\mu\text{m}$, with a typical lift height of 40 nm and a scan frequency of 0.5 Hz. There were either 256 or 512 pixels per line, and each image took approximately 20 minutes to acquire. Since MFM is sensitive to the out-of-plane component of the stray magnetic field gradient, each single-domain circular nanomagnet appears as either a bright or a dark disk. Bright (dark) contrast within a nanomagnet means that there is a repulsive (attractive) interaction between the magnetic moment of the nanomagnet and the magnetic moment of the tip. Equivalently, regions of bright or dark contrast correspond to the macrospin moment and the tip moment being antiparallel or parallel, respectively. Without *a priori* knowledge of the direction of the magnetic moment of the tip, an absolute direction for every macrospin cannot be assigned. Instead, the magnetic configuration of the nanomagnet array can be determined up to a global spin flip. This is possible because the magnetic moment of the tip does not change during the course of a scan. We adopt the convention that bright contrast corresponds to an up macrospin, i.e., $s = +1$.

Each MFM image was postprocessed using a custom Python code. First, a Savitzky-Golay filter was applied to remove the nonlinear background from each scanned line in both the topography and magnetic phase channels. Then, the position of the lattice in each image was identified, which enabled classification of all pixels in the image as part of either a nanomagnet or the background. A local threshold, based on the average intensity of nearby background pixels, was then applied to the nanomagnet pixels in the magnetic phase channel in order to discriminate the nanomagnets in the image. To automatically read in the orientation of each macrospin, our code calculated the average intensity of the pixels within each nanomagnet and, when compared to the overall mean, used this as a figure of merit to classify the nanomagnet as bright (intensity greater than the overall mean,

corresponding to an up macrospin) or a dark (intensity less than the overall mean, corresponding to a down macrospin).

APPENDIX D: ASSIGNING THE EFFECTIVE TEMPERATURE

For each Archimedean lattice, the effective temperature for the full-dipolar and first-nearest-neighbor models is determined by comparing the experimental spin-spin correlations with those obtained from the Monte Carlo simulations. More precisely, the effective temperature for a given model is defined as the temperature that minimizes the sum of the mean-squared deviations between the experimental correlations and the Monte Carlo predictions for the first three nearest-neighbor spin-spin correlations. The effective temperatures as a function of nanomagnet separation, computed for both models, are provided in Sec. 4 of the Supplemental Material [36].

APPENDIX E: MAGNETIC STRUCTURE FACTORS

The intensity of the magnetic structure factor $I(\mathbf{q})$ is determined by applying a Fourier transform to map the spin configuration in real space to that in reciprocal space. For a system of spins $\mathbf{s}_i$ at lattice positions $\mathbf{r}_i$, the intensity of the magnetic structure factor is given by

$$I(\mathbf{q}) = \sum_{i,j} \mathbf{s}_i \cdot \mathbf{s}_j e^{i\mathbf{q} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \quad (\text{E1})$$

where $\mathbf{q}$ is the wave vector at a point in reciprocal space, and $\mathbf{s}_i$ and $\mathbf{s}_j$ are the normalized moments of two spins at positions $\mathbf{r}_i$ and $\mathbf{r}_j$, respectively. The double sum is taken over all pairs of spins in the system.

The magnetic structure factor gives a visual signature of the spatial distribution and correlations of magnetic moments. For ordered states, the magnetic structure factor contains Bragg peaks that appear for specific wavevectors and represent dominant periodicities and correlations in the spin configuration. The magnetic structure factor can therefore be used to indicate the presence of long-range correlations. For disordered systems, the magnetic structure factor becomes diffuse, and thus can be used to indicate a reduction of long-range ordering. This can help to determine which lattices have long-range or short-range magnetic order. The magnetic structure factors determined from the MFM images and Monte Carlo simulations are discussed in more detail in Sec. 5 of the Supplemental Material [36]. There we compare the experimental magnetic structure factor to that of a configuration taken from Monte Carlo simulations at T_{eff} . We also use the simulated magnetic structure factors to highlight the differences in magnetic ordering when using the full-dipolar model and the first-nearest-neighbor model. The differences are most apparent when comparing the magnetic structure factors for the ground state taken from Monte Carlo simulations of the triangular and kagome lattices. In particular, for these two-step frustrated lattices, the peaks in the magnetic structure factor are sharper for the full-dipolar model than those for the first-nearest-neighbor model.

Finally, in Sec. 5 of the Supplemental Material [36], we compare the simulated real-space magnetic configura-

tions and magnetic structure factors at $T = 0$, which is the final temperature in our Monte Carlo simulated anneal, with those obtained from the Monte Carlo simulations at the experimentally measured effective temperature. In general, the simulated real-space configurations at $T = 0$ exhibit

more long-range order than those at T_{eff} . This is most obviously seen across all 11 lattices by the reduction in diffuse intensity in the $T = 0$ magnetic structure factors compared with those simulated at the experimentally measured effective temperature.

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- [1] R. F. Wang, C. Nisoli, R. S. Freitas, J. Li, W. McConville, B. J. Cooley, M. S. Lund, N. Samarth, C. Leighton, V. H. Crespi, and P. Schiffer, Artificial ‘spin ice’ in a geometrically frustrated lattice of nanoscale ferromagnetic islands, *Nature (London)* **446**, 102 (2007).
 - [2] S. H. Skjærvø, C. H. Marrows, R. L. Stamps, and L. J. Heyderman, Advances in artificial spin ice, *Nat. Rev. Phys.* **2**, 13 (2020).
 - [3] B. Canals, I.-A. Chioar, V.-D. Nguyen, M. Hehn, D. Lacour, F. Montaigne, A. Locatelli, T. O. Menteş, B. S. Burgos, and N. Rougemaille, Fragmentation of magnetism in artificial kagome dipolar spin ice, *Nat. Commun.* **7**, 11446 (2016).
 - [4] M. Saccone, A. Scholl, S. Veltens, S. Dhuey, K. Hofhuis, C. Wuth, Y.-L. Huang, Z. Chen, R. V. Chopdekar, and A. Farhan, Towards artificial Ising spin glasses: Thermal ordering in randomized arrays of Ising-type nanomagnets, *Phys. Rev. B* **99**, 224403 (2019).
 - [5] M. Saccone, F. Caravelli, K. Hofhuis, S. Parchenko, Y. A. Birkhölzer, S. Dhuey, A. Kleibert, S. van Dijken, C. Nisoli, and A. Farhan, Direct observation of a dynamical glass transition in a nanomagnetic artificial Hopfield network, *Nat. Phys.* **18**, 517 (2022).
 - [6] I. Gilbert, Y. Lao, I. Carrasquillo, L. O’Brien, J. D. Watts, M. Manno, C. Leighton, A. Scholl, C. Nisoli, and P. Schiffer, Emergent reduced dimensionality by vertex frustration in artificial spin ice, *Nat. Phys.* **12**, 162 (2016).
 - [7] S. Ladak, D. Read, G. Perkins, L. Cohen, and W. Branford, Direct observation of magnetic monopole defects in an artificial spin-ice system, *Nat. Phys.* **6**, 359 (2010).
 - [8] E. Mengotti, L. J. Heyderman, A. F. Rodríguez, F. Nolting, R. V. Hügli, and H.-B. Braun, Real-space observation of emergent magnetic monopoles and associated Dirac strings in artificial kagome spin ice, *Nat. Phys.* **7**, 68 (2011).
 - [9] M. J. Harris, S. T. Bramwell, D. F. McMorrow, T. Zeiske, and K. W. Godfrey, Geometrical frustration in the ferromagnetic pyrochlore $\text{Ho}_2\text{Ti}_2\text{O}_7$, *Phys. Rev. Lett.* **79**, 2554 (1997).
 - [10] L. Pauling, The structure and entropy of ice and of other crystals with some randomness of atomic arrangement, *J. Am. Chem. Soc.* **57**, 2680 (1935).
 - [11] S. Kempinger, R. D. Fraleigh, P. E. Lammert, S. Zhang, V. H. Crespi, P. Schiffer, and N. Samarth, Imaging the stochastic microstructure and dynamic development of correlations in perpendicular artificial spin ice, *Phys. Rev. Res.* **2**, 012001(R) (2020).
 - [12] E. Mengotti, L. J. Heyderman, A. Bisig, A. Fraile Rodríguez, L. Le Guyader, F. Nolting, and H. B. Braun, Dipolar energy states in clusters of perpendicular magnetic nanoislands, *J. Appl. Phys.* **105**, 113113 (2009).
 - [13] S. A. Daunheimer, O. Petrova, O. Tchernyshyov, and J. Cumings, Reducing disorder in artificial kagome ice, *Phys. Rev. Lett.* **107**, 167201 (2011).
 - [14] Y. Li, G. W. Paterson, G. M. Macauley, F. S. Nascimento, C. Ferguson, S. A. Morley, M. C. Rosamond, E. H. Linfield, D. A. MacLaren, R. Macêdo, *et al.*, Superferromagnetism and domain-wall topologies in artificial “pinwheel” spin ice, *ACS Nano* **13**, 2213 (2019).
 - [15] O. Sendetskyi, V. Scagnoli, N. Leo, L. Anghinolfi, A. Alberca, J. Lüning, U. Staub, P. M. Derlet, and L. J. Heyderman, Continuous magnetic phase transition in artificial square ice, *Phys. Rev. B* **99**, 214430 (2019).
 - [16] A. Imre, G. Csaba, L. Ji, A. Orlov, G. H. Bernstein, and W. Porod, Majority logic gate for magnetic quantum-dot cellular automata, *Science* **311**, 205 (2006).
 - [17] H. Arava, P. M. Derlet, J. Vijayakumar, J. Cui, N. S. Bingham, A. Kleibert, and L. J. Heyderman, Computational logic with square rings of nanomagnets, *Nanotechnology* **29**, 265205 (2018).
 - [18] L. J. Heyderman, Spin ice devices from nanomagnets, *Nat. Nanotechnol.* **17**, 435 (2022).
 - [19] J. C. Gartside, K. D. Stenning, A. Vanstone, H. H. Holder, D. M. Arroo, T. Dion, F. Caravelli, H. Kurebayashi, and W. R. Branford, Reconfigurable training and reservoir computing in an artificial spin-vortex ice via spin-wave fingerprinting, *Nat. Nanotechnol.* **17**, 460 (2022).
 - [20] C. Yun, Z. Liang, A. Hrabec, Z. Liu, M. Huang, L. Wang, Y. Xiao, Y. Fang, W. Li, W. Yang, *et al.*, Electrically programmable magnetic coupling in an Ising network exploiting solid-state ionic gating, *Nat. Commun.* **14**, 6367 (2023).
 - [21] A. Kurenkov, J. Maes, A. Pac, G. M. Macauley, B. Van Waeyenberge, A. Hrabec, and L. J. Heyderman, Perpendicular-anisotropy artificial spin ice with spontaneous ordering: A platform for reservoir computing with flexible timescales, *Commun. Eng.* **4**, 183 (2025).
 - [22] S. Zhang, J. Li, I. Gilbert, J. Bartell, M. J. Erickson, Y. Pan, P. E. Lammert, C. Nisoli, K. K. Kohli, R. Misra, V. H. Crespi, N. Samarth, C. Leighton, and P. Schiffer, Perpendicular magnetization and generic realization of the Ising model in artificial spin ice, *Phys. Rev. Lett.* **109**, 087201 (2012).
 - [23] I. A. Chioar, N. Rougemaille, A. Grimm, O. Fruchart, E. Wagner, M. Hehn, D. Lacour, F. Montaigne, and B. Canals, Nonuniversality of artificial frustrated spin systems, *Phys. Rev. B* **90**, 064411 (2014).
 - [24] R. D. Fraleigh, S. Kempinger, P. E. Lammert, S. Zhang, V. H. Crespi, P. Schiffer, and N. Samarth, Characterization of switching field distributions in Ising-like magnetic arrays, *Phys. Rev. B* **95**, 144416 (2017).
 - [25] S. Kempinger, Y.-S. Huang, P. Lammert, M. Vogel, A. Hoffmann, V. H. Crespi, P. Schiffer, and N. Samarth, Field-tunable interactions and frustration in underlayer-mediated artificial spin ice, *Phys. Rev. Lett.* **127**, 117203 (2021).
 - [26] E. Hall, XVIII. On the “Rotational coefficient” in nickel and cobalt, *Philos. Mag.* **12**, 157 (1881).

- [27] N. Nagaosa, J. Sinova, S. Onoda, A. H. MacDonald, and N. P. Ong, Anomalous Hall effect, *Rev. Mod. Phys.* **82**, 1539 (2010).
- [28] G. Sala, V. Krizakova, E. Grimaldi, C.-H. Lambert, T. Devolder, and P. Gambardella, Real-time Hall-effect detection of current-induced magnetization dynamics in ferrimagnets, *Nat. Commun.* **12**, 656 (2021).
- [29] G. Sala, J. Meyer, A. Flechsig, L. Gabriel, and P. Gambardella, Deterministic and stochastic aspects of current-induced magnetization reversal in perpendicular nanomagnets, *Phys. Rev. B* **107**, 214447 (2023).
- [30] S. Chakraverty, D. M. Sahoo, and N. R. Mahato, McCulloch-Pitts neural network model, in *Concepts of Soft Computing* (Springer, Singapore, 2019), pp 167–173.
- [31] H. Arava, N. Leo, D. Schildknecht, J. Cui, J. Vijayakumar, P. M. Derlet, A. Kleibert, and L. J. Heyderman, Engineering relaxation pathways in building blocks of artificial spin ice for computation, *Phys. Rev. Appl.* **11**, 054086 (2019).
- [32] J. Grollier, D. Querlioz, and M. D. Stiles, Spintronic nanodevices for bioinspired computing, *Proc. IEEE* **104**, 2024 (2016).
- [33] S. Yang, J. Shin, T. Kim, K.-W. Moon, J. Kim, G. Jang, D. S. Hyeon, J. Yang, C. Hwang, Y. Jeong, *et al.*, Integrated neuromorphic computing networks by artificial spin synapses and spin neurons, *NPG Asia Mater.* **13**, 11 (2021).
- [34] K. M. Song, J.-S. Jeong, B. Pan, X. Zhang, J. Xia, S. Cha, T.-E. Park, K. Kim, S. Finizio, J. Raabe, *et al.*, Skyrmion-based artificial synapses for neuromorphic computing, *Nat. Electron.* **3**, 148 (2020).
- [35] B. Grünbaum and G. C. Shephard, *Tilings and Patterns* (Courier Dover Publications, 1987).
- [36] See Supplemental Material at <https://10.1103/n2ns-j2hp> for further information on: the definition of nearest-neighbors in each of the 11 Archimedean lattices; SEM images of more closely packed arrays; the effect on the spin-spin correlations of varying the cobalt thickness; MFM images of all 11 Archimedean lattices following demagnetization; spin-spin correlations extracted from Monte Carlo simulations for all 11 lattices, along with a finite-size scaling analysis to classify their transitions and crossovers; the effect of introducing disorder on the correlations; further experimental results, including on the effect of varying the lattice constant for all 11 lattices on the spin-spin correlations after demagnetization; a comparison between the experimental and simulated magnetic structure factors; further notes on the residual and effective residual entropies; and the effect of a slight dilation on the phase diagram of the triangular lattice with dipolar interactions.
- [37] G. H. Wannier, Antiferromagnetism. The triangular Ising net, *Phys. Rev.* **79**, 357 (1950).
- [38] A. Smerald and F. Mila, Spin-liquid behaviour and the interplay between Pokrovsky-Talapov and Ising criticality in the distorted, triangular-lattice, dipolar Ising antiferromagnet, *SciPost Phys.* **5**, 030 (2018).
- [39] I. Syôzi, Statistics of kagomé lattice, *Prog. Theor. Phys.* **6**, 306 (1951).
- [40] K. Kanô and S. Naya, Antiferromagnetism. The kagomé Ising net, *Prog. Theor. Phys.* **10**, 158 (1953).
- [41] I. A. Chioar, N. Rougemaille, and B. Canals, Ground-state candidate for the classical dipolar kagome Ising antiferromagnet, *Phys. Rev. B* **93**, 214410 (2016).
- [42] J. Hamp, R. Moessner, and C. Castelnovo, Supercooling and fragile glassiness in a dipolar kagome Ising magnet, *Phys. Rev. B* **98**, 144439 (2018).
- [43] M. O’Keefe and B. G. Hyde, Plane nets in crystal chemistry, *Philos. Trans. A Math. Phys. Eng. Sci.* **295**, 553 (1980).
- [44] A. A. Kulkarni, E. Hanson, R. Zhang, K. Thornton, and P. V. Braun, Archimedean lattices emerge in template-directed eutectic solidification, *Nature (London)* **577**, 355 (2020).
- [45] A. Takano, W. Kawashima, A. Noro, Y. Isono, N. Tanaka, T. Dotera, and Y. Matsushita, A mesoscopic Archimedean tiling having a new complexity in an ABC star polymer, *J. Pol. Sci. B* **43**, 2427 (2005).
- [46] L. Voigt, M. Kubus, and K. S. Pedersen, Chemical engineering of quasicrystal approximants in lanthanide-based coordination solids, *Nat. Commun.* **11**, 4705 (2020).
- [47] S. David, A. Chelnikov, and J.-M. Lourtioz, Isotropic photonic structures: Archimedean-like tilings and quasi-crystals, *IEEE J. Quantum Electron.* **37**, 1427 (2001).
- [48] K. Ueda, T. Dotera, and T. Gemma, Photonic band structure calculations of two-dimensional Archimedean tiling patterns, *Phys. Rev. B* **75**, 195122 (2007).
- [49] P. N. Suding and R. M. Ziff, Site percolation thresholds for Archimedean lattices, *Phys. Rev. E* **60**, 275 (1999).
- [50] L. Onsager, Crystal statistics. I. A two-dimensional model with an order-disorder transition, *Phys. Rev.* **65**, 117 (1944).
- [51] A. Codello, Exact Curie temperature for the Ising model on Archimedean and Laves lattices, *J. Phys. A: Math. Theor.* **43**, 399801 (2010).
- [52] U. Yu, Ising antiferromagnet on the Archimedean lattices, *Phys. Rev. E* **91**, 062121 (2015).
- [53] S. Bandiera, R. Sousa, B. Rodmacq, and B. Dieny, Asymmetric interfacial perpendicular magnetic anisotropy in Pt/Co/Pt trilayers, *IEEE Magn. Lett.* **2**, 3000504 (2011).
- [54] A. Vansteenkiste, J. Leliaert, M. Dvornik, M. Helsen, F. Garcia-Sanchez, and B. Van Waeyenberge, The design and verification of MuMax3, *AIP Adv.* **4**, 107133 (2014).
- [55] B. S. Shastry and B. Sutherland, Exact ground state of a quantum mechanical antiferromagnet, *Physica B+C* **108**, 1069 (1981).
- [56] N. Metropolis and S. Ulam, The Monte Carlo method, *J. Am. Stat. Assoc.* **44**, 335 (1949).
- [57] N. Metropolis, A. W. Rosenbluth, M. N. Rosenbluth, A. H. Teller, and E. Teller, Equation of state calculations by fast computing machines, *J. Chem. Phys.* **21**, 1087 (1953).
- [58] P. M. Chaikin, T. C. Lubensky, and T. A. Witten, *Principles of Condensed Matter Physics* (Cambridge University Press, Cambridge, 1995), Vol. 10.
- [59] Z. Racz, Phase boundary of Ising antiferromagnets near $H = H_C$ and $T = 0$: Results from hard-core lattice gas calculations, *Phys. Rev. B* **21**, 4012 (1980).
- [60] C. Nisoli, R. Moessner, and P. Schiffer, *Colloquium: Artificial spin ice: Designing and imaging magnetic frustration*, *Rev. Mod. Phys.* **85**, 1473 (2013).
- [61] J. M. Porro, A. Bedoya-Pinto, A. Berger, and P. Vavassori, Exploring thermally induced states in square artificial spin-ice arrays, *New J. Phys.* **15**, 055012 (2013).
- [62] S. Zhang, I. Gilbert, C. Nisoli, G.-W. Chern, M. J. Erickson, L. O’Brien, C. Leighton, P. E. Lammert, V. H. Crespi, and P. Schiffer, Crystallites of magnetic charges in artificial spin ice, *Nature (London)* **500**, 553 (2013).

- [63] J. W. Lau, R. D. McMichael, S.-H. Chung, J. Rantschler, V. Parekh, and D. Litvinov, Microstructural origin of switching field distribution in patterned Co/Pd multilayer nanodots, *Appl. Phys. Lett.* **92**, 012506 (2008).
- [64] U. Wolff, Collective Monte Carlo updating for spin systems, *Phys. Rev. Lett.* **62**, 361 (1989).
- [65] A. Farhan, P. M. Derlet, A. Kleibert, A. Balan, R. V. Chopdekar, M. Wyss, J. Perron, A. Scholl, F. Nolting, and L. J. Heyderman, Direct observation of thermal relaxation in artificial spin ice, *Phys. Rev. Lett.* **111**, 057204 (2013).
- [66] S. Gliga, G. Hrkac, C. Donnelly, J. Büchi, A. Kleibert, J. Cui, A. Farhan, E. Kirk, R. V. Chopdekar, Y. Masaki, *et al.*, Emergent dynamic chirality in a thermally driven artificial spin ratchet, *Nat. Mater.* **16**, 1106 (2017).
- [67] K. Hofhuis, A. Hrabec, H. Arava, N. Leo, Y.-L. Huang, R. V. Chopdekar, S. Parchenko, A. Kleibert, S. Koraltan, C. Abert, *et al.*, Thermally superactive artificial kagome spin ice structures obtained with the interfacial Dzyaloshinskii-Moriya interaction, *Phys. Rev. B* **102**, 180405(R) (2020).
- [68] K. Hofhuis, S. H. Skjærvø, S. Parchenko, H. Arava, Z. Luo, A. Kleibert, P. M. Derlet, and L. J. Heyderman, Real-space imaging of phase transitions in bridged artificial kagome spin ice, *Nat. Phys.* **18**, 699 (2022).
- [69] G. W. Paterson, G. M. Macauley, and R. Macêdo, Field-driven reversal models in artificial spin ice, *Adv. Theory Simul.* **4**, 2100109 (2021).
- [70] J. H. Jensen, A. Strømberg, I. Breivik, A. Penty, M. A. Niño, M. W. Khaliq, M. Foerster, G. Tufte, and E. Folven, Clocked dynamics in artificial spin ice, *Nat. Commun.* **15**, 964 (2024).
- [71] G. Macauley, A. Pac, and V. Scagnoli, Magnetic ordering in out-of-plane artificial spin systems based on the Archimedean lattices [Data set], Zenodo (2026), <https://doi.org/10.5281/zenodo.18319236>.