

Coupled topological interface states and phonon molecules in GaAs/AlAs superlattices

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Topological interface states in one-dimensional superlattices provide spatially localized phonon modes protected by the topology of the underlying band structure. In GaAs/AlAs distributed Bragg reflectors (DBRs), such states can be engineered through band inversion between superlattices with opposite Zak phases within the Su-Schrieffer-Heeger framework. Here, we demonstrate topological phonon molecules and extended chains formed by coupled nanophononic interface states. By concatenating three superlattices with alternating topology, we realize two coupled interface states that hybridize into symmetric and antisymmetric modes, whose splitting can be tuned over tens of gigahertz by varying the reflectivity of the central DBR. Extending this concept, we engineer chains of up to $N = 6$ coupled interface states that form narrow topological minibands while remaining strongly localized at the interfaces. We experimentally observe these coupled states in molecular-beam-epitaxy-grown GaAs/AlAs heterostructures using time-domain pump-probe transient reflectivity measurements, and reproduce their behavior using transfer-matrix calculations and a simple analytical model for the mode splitting. These results establish topological interface states as a robust platform for engineering coupled phononic systems and tunable nanophononic architectures in the GHz regime.

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I. INTRODUCTION

A one-dimensional periodic lattice containing two elements per unit cell can be described within the Su-Schrieffer-Heeger (SSH) tight-binding model, a paradigmatic framework for understanding topological phases and localized excitations in condensed matter [1,2]. This model, originally introduced to explain the electronic properties of polyacetylene [1], has since inspired the realization of topological states for a broad variety of excitations and phenomena [3], including photons [4–9], phonons [10–15], mechanical vibrations [16–25], polaritons [26–30], plasmons [31], and magnons [32]. More recently, advances in topological phononic crystals, including higher-order transport, valley-based edge modes, and gigahertz topological phononic waveguides and circuits, have been demonstrated [33–38]. In multilayered optical and acoustic structures, distributed Bragg reflectors exhibit high-reflectivity bands associated with band gaps in the underlying dispersion relation. The Bloch modes at the edges of these band gaps possess well-defined spatial symmetries, and concatenating two distributed Bragg reflectors (DBRs) whose band-edge modes have inverted symmetries around a common gap gives rise to a topological interface state within that gap [10,11,39,40]. The existence and robustness of this interface

state can be understood in terms of the Zak phase—the one-dimensional analog of the Berry phase—which changes by π across a band inversion.

Over the past two decades, the field of nanophononics [41–43], concerned with the control of acoustic nanowaves in solids, has developed a comprehensive toolbox of phonon mirrors, filters, and resonant cavities based on acoustic impedance engineering [11,44]. In contrast to optical or electronic systems, where the relevant dispersion is typically nonlinear [45–47], acoustic phonons exhibit a nearly linear dispersion [48], thereby enabling studies extending over a broad frequency range [10,12]. Moreover, the relatively slow speed of sound and long coherence lengths in these structures allow one to engineer effectively quasi-infinite lattices on the micron scale, while standard ultrafast optical techniques such as pump-probe spectroscopy and Brillouin scattering can provide direct access to their spectral and temporal response [12]. The control and manipulation of acoustic phonons, as well as their interactions, can have significant technological implications for thermal transport and heat management [49–51], quantum applications [52–54], and information exchange [55,56].

A key component to achieve phonon confinement and control is the distributed Bragg reflector. DBRs find a wide range of applications in both phononics and photonics, owing to their high reflectivities, which allow the formation of high- Q cavities, as well as their excellent material quality and reproducibility resulting from epitaxial growth by molecular beam epitaxy (MBE) [57]. In particular, GaAs/AlAs DBRs can confine photons and high-frequency acoustic phonons in a single structure thanks to the comparable refractive index and acoustic impedance contrasts between GaAs and AlAs,

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a situation often dubbed “double magic coincidence” in the literature [58,59].

In this context, topological concepts have recently been brought into nanophononics. Theoretical and experimental works have shown that GaAs/AlAs DBRs can be tailored to exhibit band inversion in a chosen acoustic gap by adjusting the unit-cell thickness ratio between the two constituent materials while preserving the total acoustic period [11]. The concatenation of two such band-inverted superlattices generates a symmetry-protected nanophononic interface state that is pinned to the center of the band gap and robust against thickness disorder that does not alter the underlying Zak phases [12]. These ideas have been used to design and observe topological acoustic interface modes in GaAs/AlAs structures via Raman spectroscopy and time-resolved pump-probe experiments [12], to realize interface states simultaneously for light and sound through coincident optical and acoustic band inversion [58], and to access higher-order acoustic band gaps to generate topological states across a broad frequency range [10,60]. Beyond their fundamental interest, such topological nanophononic resonators may offer a robust platform for enhanced optomechanical interactions and hybrid quantum devices. Topological interface states have been shown to exhibit exceptional robustness [10,58,61] against compositional fluctuations of the order typically encountered due to material intermixing or inhomogeneities during MBE growth [62], which is a highly desirable feature for transport and data communication applications [63,64].

Parallel to these developments, phonon engineering in multilayered structures has explored phonon molecules and more complex nanomechanical potentials in the conventional Fabry-Pérot picture [44,65]. In early work, double and multiple acoustic cavities coupled through DBRs were theoretically proposed as phononic analogues of diatomic molecules, exhibiting symmetric and antisymmetric normal modes whose splitting and lifetimes can be tuned by the reflectivity of the central and external mirrors [65]. These concepts were later generalized to adiabatically modulated superlattices implementing effective one-dimensional potentials such as parabolic wells, Morse potentials, and double wells for acoustic phonons [44]. These structures can support minibands and Wannier-Stark ladders of phonon states and can mimic Bloch oscillations, opening an avenue to simulate quantum mechanical potentials and transport using coherent acoustic phonons [66].

Coupling between topological interface states has been studied in detail in the optical domain. Photonic crystals composed of dielectric or metal-dielectric multilayers have been used to realize pairs of topological Tamm or Zak phase interface states; when two such states are brought into proximity by concatenating three photonic segments with alternating topological phases, they hybridize into molecular-like symmetric and antisymmetric modes with a characteristic mode splitting [67]. This splitting can be tuned by varying the thickness of the central segment and manifests in anticrossings and polarization-dependent resonances in reflectivity spectra [68]. Yet, despite the rapid progress on individual nanophononic topological states [10,11,58], a unified framework where topologically protected nanophononic interface modes play the role of the “atoms” forming

phonon molecules and extended chains has remained largely unexplored.

In this work, we bridge these two perspectives by introducing topological nanophononic molecules and multi-interface lattices built from SSH-type interface states in GaAs/AlAs superlattices. We first revisit the design of a single topological nanophononic interface mode in the acoustic band gap around 300 GHz, implemented by concatenating two DBRs with band-inverted centrosymmetric GaAs/AlAs unit cells centered on the AlAs layer. This structure provides the fundamental building block for our devices. We experimentally realize structures supporting topological phonon-molecule modes in a monolithic MBE-grown GaAs/AlAs heterostructure, by concatenating three superlattices with alternating Zak phases, yielding two symmetry-protected interface modes at the same frequency that couple through the central DBR. The hybridization of these modes results in a mode splitting between symmetric and antisymmetric combinations, whose magnitude we control by varying the number of periods—and hence the reflectivity—of the central mirror. We show that the splitting follows a simple expression in terms of an effective acoustic velocity of the structure, the central-mirror reflectivity, and an effective coupling length, providing an intuitive design rule for the coupling strength. Importantly, the resulting hybrid modes retain the interface localization associated with the underlying topological band inversion and show enhanced robustness against thickness-ratio perturbations compared with conventional Fabry-Pérot phonon molecules.

We further generalize this strategy to engineer devices hosting multiple coupled interface states by concatenating extended sequences of topologically distinct superlattices. This yields narrow topological minibands formed by $N = 6$ coupled interface modes that remain spatially pinned to their respective interfaces, with tunable bandwidth controlled by the separation between successive interfaces [65]. Building on established schemes for coherent phonon generation and detection in topological nanocavities [58] using time-domain pump-probe transient reflectivity, we resolve the 300 GHz interface resonance as well as its splitting into chain modes. Only one mode is observed in the molecular case; the other is symmetry forbidden, in agreement with transfer-matrix simulations. In contrast to previous studies focusing on isolated interface states, our approach enables controlled coupling of multiple topological phononic modes, forming molecule- and chainlike architectures with tunable interactions.

The paper is organized as follows. In Sec. II, we review the formation of topological nanophononic interface states by band inversion in GaAs/AlAs superlattices. In Sec. III, we introduce the concept of topological phonon molecules, analyze the coupling between interface states and the resulting mode splitting, and describe their experimental realization and observation. In Sec. VI, we investigate the robustness of the topological phonon molecules against thickness-ratio fluctuations and compare it with a Fabry-Pérot phonon molecule. Section V addresses multi-interface structures supporting chains of coupled interface states and their experimental observation. Section VI explores the band structure engineering in periodic topological phonon chains by changing the

coupling strengths between interface states. We conclude in Sec. VII with a discussion of future directions of the present work and prospects for nanophononic devices. The Appendix provides the calculation of the Zak phase.

II. TOPOLOGICAL PHONONIC INTERFACE STATES BY BAND INVERSION

The infinite periodic arrangement of two materials with contrasting acoustic impedances, defined as the product of sound velocity and mass density, can be described by a folded Brillouin zone in the acoustic dispersion relation, where band gaps emerge at both the zone center and the zone edge. The opening and closing of these gaps depend on the relative thickness ratio between the constituent layers [39,40]. In the present design, we consider centrosymmetric GaAs/AlAs unit cells centered on the AlAs layer. For a structure targeting an acoustic band gap around 300 GHz, the unit cell is composed of the layer sequence $(3\lambda_{\text{GaAs}}/8, \lambda_{\text{AlAs}}/4, 3\lambda_{\text{GaAs}}/8)$, where λ denotes the acoustic wavelength at the design frequency. This configuration opens a band gap at the Brillouin-zone center ($q = 0$), as shown in Fig. 1(a). The band-inverted counterpart [Fig. 1(b)] is obtained by exchanging the thickness distribution between the two materials, leading to the unit cell $(\lambda_{\text{GaAs}}/8, 3\lambda_{\text{AlAs}}/4, \lambda_{\text{GaAs}}/8)$. While both superlattices exhibit identical dispersion relations, the spatial symmetries of the band-edge modes are inverted. As a result, the two superlattices belong to distinct topological families.

In finite structures, the band gaps correspond to frequency regions of high acoustic reflectivity, characteristic of distributed Bragg reflectors. Band inversion arises when two DBRs that (i) share a common band gap and (ii) exhibit opposite spatial mode symmetries are concatenated [Figs. 1(a) and 1(b)]. When these conditions are satisfied, a topological interface mode forms at the junction between the two DBRs. This mode appears as a dip in the high-reflectivity region, as shown in Fig. 1(c) for a structure composed of eight periods in each superlattice. The spatial profile of the squared displacement $|u(z)|^2$ of the mode at 300 GHz is shown in Fig. 1(d), where the displacement reaches its maximum at the interface.

The linewidth of the interface mode decreases as the number of DBR periods in each superlattice increases, as shown in Fig. 1(e). This trend arises from the enhanced acoustic confinement provided by thicker Bragg mirrors, which increases the fraction of energy stored near the interface and reduces leakage into the surrounding layers. The mechanism is analogous to the linewidth narrowing observed in conventional Fabry-Pérot resonators when the mirror reflectivity is increased [65].

Nanophononic interface states achieved by band inversion in the SSH framework have been previously studied, both numerically and experimentally [10,12,40,58]. These works have shown that the interface mode remains robust against certain variations in the thickness of the layers constituting the DBRs, characteristic of topological protection. In the present context, this interface state serves as the fundamental building block for the engineering of more complex structures explored in the following sections.

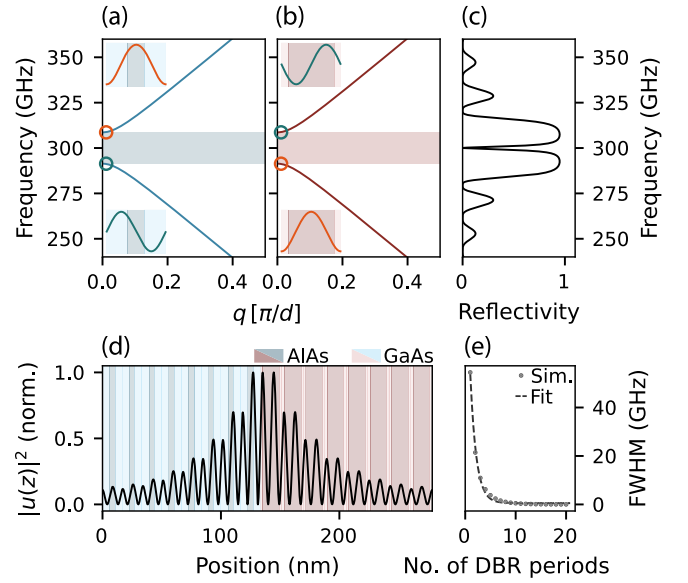


FIG. 1. Principle of a phononic interface state formed by band inversion. (a) Acoustic dispersion of an infinite GaAs/AlAs superlattice with centrosymmetric unit cell $(3\lambda_{\text{GaAs}}/8, \lambda_{\text{AlAs}}/4, 3\lambda_{\text{GaAs}}/8)$, designed for 300 GHz, showing a band gap at the Brillouin-zone center ($q = 0$). Insets: Displacement profiles of the band-edge modes, exhibiting symmetric and antisymmetric patterns with respect to the unit-cell center (green and orange curves). (b) Dispersion of the band-inverted unit cell $(\lambda_{\text{GaAs}}/8, 3\lambda_{\text{AlAs}}/4, \lambda_{\text{GaAs}}/8)$, which preserves the dispersion but reverses the band-edge symmetries. (c) Acoustic reflectivity of a finite structure formed by concatenating the two superlattices shown in (a) and (b), each consisting of eight periods. A localized resonance appears inside the common band gap (~ 300 GHz), indicating the formation of a confined interface mode. (d) Spatial profile of the squared displacement $|u(z)|^2$ at the interface-mode frequency, localized at the DBR junction and decaying evanescently into the mirrors. (e) Full width at half maximum (FWHM) of the interface resonance as a function of the number of DBR periods. The discrete data points are extracted from transfer-matrix simulations of the reflectivity spectra, and the dashed line is an exponential fit.

III. COUPLED INTERFACE STATES: TOPOLOGICAL PHONON MOLECULES

A topological phonon molecule can be understood as a pair of coupled topological bound states within an effective SSH-like framework, analogous to a double-cavity structure [65]. It can be realized by concatenating three superlattices with alternating topological invariants, resulting in two interfaces, as illustrated in Fig. 2(a). Each interface supports a localized mode designed at the same frequency. When these modes are coupled, they hybridize, producing a mode splitting that gives rise to symmetric and antisymmetric combinations of the original interface states.

The hybridized modes exhibit overlapping wave functions, as evidenced by the displacement fields in Fig. 2(b), which underlies the observed mode splitting. The coupling strength between the interface modes can be tuned by modifying the properties of the central mirror: increasing the number of layers enhances confinement, reducing the mode overlap and the resulting splitting, potentially producing nearly degener-

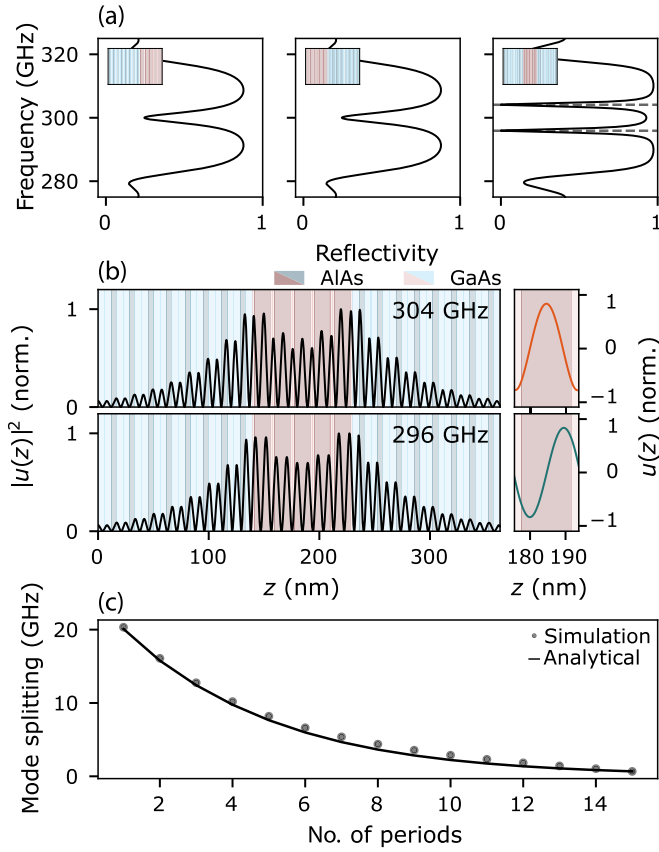


FIG. 2. Topological phonon molecules formed by coupling two interface states. (a) Acoustic reflectivity of structures illustrating the formation of coupled interface states: a single interface obtained by concatenating two band-inverted superlattices, the reverse concatenation producing an equivalent interface state, and a three-segment structure containing two interfaces. (b) Spatial profiles of the squared displacement of the two hybridized interface modes. The right panels show the corresponding normalized displacement profiles, $u(z)$, within the unit cell at the center of the structure, illustrating the symmetric (orange curve) and antisymmetric (green curve) character of the hybridized modes. (c) Mode splitting as a function of the number of periods in the central DBR. Increasing the mirror thickness reduces the coupling and therefore decreases the splitting. The discrete data points are obtained from transfer-matrix simulations of the reflectivity spectra, and the solid line is the analytical fit given by Eq. (1).

ate modes. Quantitatively, the coupling strength is given by $\kappa = \Delta\omega/2$, where $\Delta\omega$ is the frequency separation between the hybridized modes. The mode splitting can be estimated using a relation analogous to that of coupled double acoustic cavities [65],

$$\Delta\omega = v_{\text{eff}} \frac{\sqrt{1 - R_C}}{\pi L_{\text{eff}}}, \quad (1)$$

where v_{eff} is the effective longitudinal sound velocity, R_C denotes the reflectivity of the central DBR, and L_{eff} is the effective cavity length. The latter exceeds the physical thickness of the central DBR due to evanescent penetration of the acoustic mode into the adjacent reflectors. In the present case, L_{eff} is approximated as the central DBR thickness together

with an additional contribution corresponding to four unit-cell thicknesses, accounting for penetration on both sides [69]. The analytical expression is plotted in Fig. 2(c), together with transfer-matrix results showing the dependence of the mode splitting on the number of DBR periods.

Time-resolved pump-probe transient reflectivity measurements were performed to characterize the topological phonon molecule. The investigated sample consists of a GaAs/AlAs multilayer structure grown on a GaAs substrate by MBE. The structure contains three superlattices separated by two interfaces. Each of the outer superlattices contains eight periods of a centrosymmetric GaAs/AlAs/GaAs unit cell with layer thicknesses of 6/4.7/6 nm, respectively. The central superlattice consists of five periods of the band-inverted unit cell with layer thicknesses of 2/14.1/2 nm. The structure is terminated by a 6 nm GaAs cap layer.

The experiment was performed using a femtosecond Ti:sapphire laser (Coherent Chameleon Ultra II) with a pulse duration of ~ 140 fs and a repetition rate of 80 MHz. Pump and probe beams were cross polarized and focused onto the sample at normal incidence using a microscope objective, resulting in a spot diameter of ~ 10 μm . The reflected probe beam was detected using a fast photodetector, while residual pump light was suppressed using polarization filtering. Pump-induced reflectivity changes were measured using lock-in detection referenced to the 800 kHz pump modulation frequency provided by an acousto-optic modulator. The pump-probe delay was controlled using a multipath mechanical delay line. The average optical powers were approximately 5 mW for the pump and 2 mW for the probe. The excitation wavelength was 775 nm, above the electron-hole resonance energy of the thinnest quantum well in the structure. A schematic of the experimental setup is shown in Fig. 3(a).

The transient reflectivity signal is shown in Fig. 3(b). The signal exhibits a sharp change around zero time delay followed by a slow relaxation of the reflectivity, originating from electronic excitation and subsequent thermalization processes induced by the femtosecond pump pulse. To extract the acoustic contribution, this slowly varying background was removed using a Savitzky-Golay smoothing filter. The resulting acoustic contribution to the transient reflectivity signal is shown in the inset of Fig. 3(b). It consists of high-frequency oscillations embedded on a slower oscillatory background. The low-frequency oscillation at approximately 45 GHz corresponds to the Brillouin oscillation associated with the propagation of coherent acoustic phonons in the structure. Its frequency is given by

$$f_B = \frac{2n_{\text{eff}}v_{\text{eff}}}{\lambda_{\text{probe}}}, \quad (2)$$

where n_{eff} and v_{eff} are the effective refractive index and sound velocity of the structure, respectively.

The frequency spectrum of the high-frequency oscillations, obtained from the fast Fourier transform (FFT) of the acoustic signal, is shown in the inset of Fig. 3(c). A dominant spectral peak is observed near ~ 294 GHz, which coincides with a dip in the calculated acoustic reflectivity spectrum shown in Fig. 3(e). This peak corresponds to one of the hybridized interface modes of the topological phonon molecule. The measured peak at ~ 294 GHz is in good agreement with the

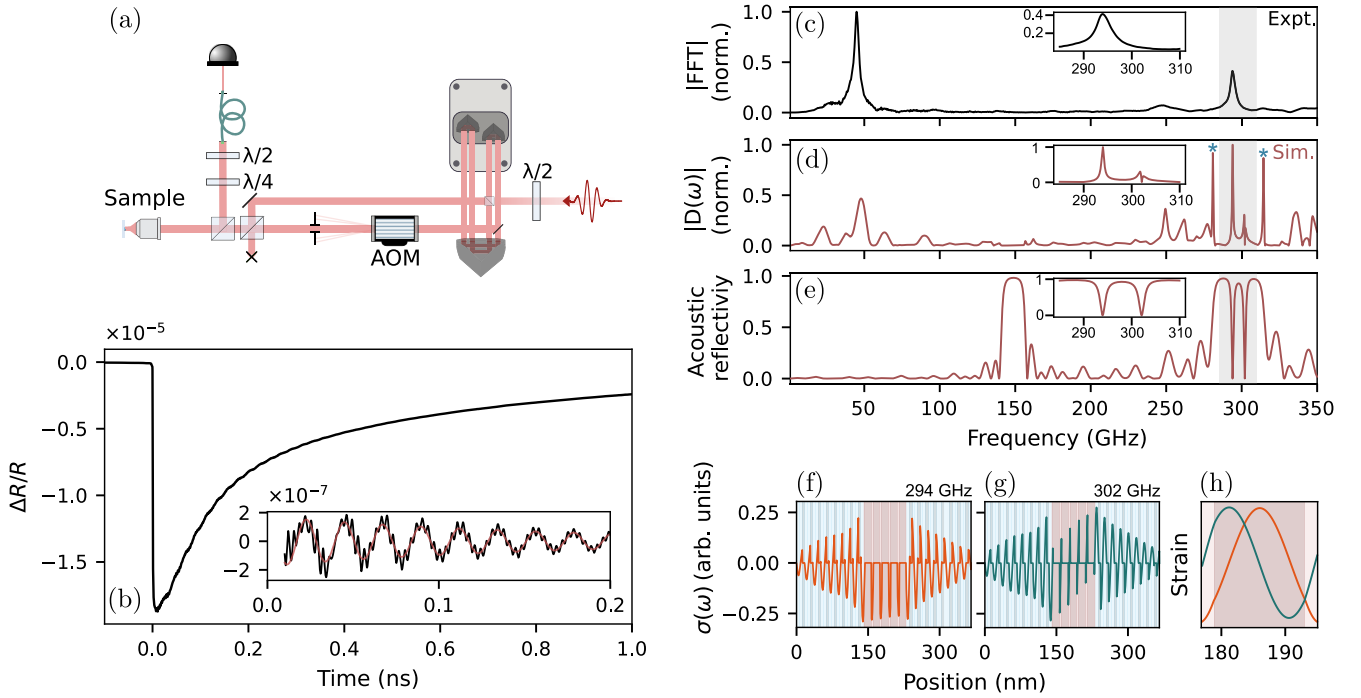


FIG. 3. Experimental observation and modeling of the topological phonon molecule. (a) Schematic of the transient reflectivity pump-probe setup. (b) Measured transient reflectivity signal. Inset: acoustic component after removal of the background. The red line shows a sinusoidal fit to the Brillouin oscillations, with $f_b = 45$ GHz. (c) Fourier spectrum of the acoustic component of the signal. Inset: zoom highlighting the dominant topological interface mode near 294 GHz. (d) Simulated $D(\omega)$ obtained from the photoelastic model. Star-marked peaks correspond to $q = 0$ modes of the individual superlattices and appear attenuated in the experimental spectra. Inset: zoom of the hybridized modes in the same spectral region as the experiment. (e) Calculated acoustic reflectivity of the multilayer structure. Inset: zoom around the band containing the interface states. (f),(g) Spatial profiles of the integrand of $\sigma(\omega)$ for the two interface modes. (h) Strain field within the unit cell of the central superlattice, showing symmetric and antisymmetric distributions for the two hybridized modes.

calculated resonance frequency within a few GHz. The second hybridized mode predicted by the acoustic reflectivity calculation is not observed. As we will demonstrate, this is related to the symmetry of the second peak of the studied modes. We performed additional simulations using a photoelastic model based on the transfer-matrix formalism for one-dimensional multilayer structures. The generation efficiency of coherent acoustic phonons is described by the Brillouin scattering cross section $\sigma(\omega)$, defined as [58,70,71]

$$\sigma(\omega) = \int |E_{\text{pump}}(z)|^2 k(z) \frac{\partial u}{\partial z} dz, \quad (3)$$

where $\partial u / \partial z$ represents the strain field calculated using the transfer-matrix method under stress-free boundary conditions at the surface of the sample. The term $|E_{\text{pump}}(z)|^2$ denotes the optical intensity distribution of the pump field inside the structure, while $k(z)$ is a material-dependent coefficient describing the efficiency of phonon generation.

The detection efficiency $D(\omega)$ of the probe beam can be approximated as [58,71]

$$D(\omega) = \int \sigma(\omega) E_{\text{probe}}(z)^2 p(z) \frac{\partial u}{\partial z} dz, \quad (4)$$

which depends on the strain field, the squared probe electric-field amplitude, and the photoelastic coefficient $p(z)$. Since the transduction efficiency and photoelastic response of GaAs are significantly larger than those of AlAs, a common approximation is to set both $k(z)$ and $p(z)$ equal to unity

in GaAs and zero in AlAs [58]. Although this simplified model neglects detailed material parameters, it captures the essential spectral features of the transient reflectivity signal.

The simulated $D(\omega)$ is shown in Fig. 3(d). The calculation reproduces the main experimental observation: a dominant peak associated with the first hybridized interface mode and a strongly suppressed contribution for the second mode. The absence of a second peak is consistent with the symmetry-induced suppression predicted by the photoelastic model. Because the photoelastic detection response depends on the optical electric-field distribution within the multilayer, changing the probe wavelength is expected to modify the visibility of the weak hybridized mode while preserving its underlying symmetry. Additionally, near electronic resonances, the photoelastic coefficient itself may acquire a strong spatial dependence, providing another mechanism to modify the optical selection rules for generation and detection [72]. The asymmetric line shape of the weakly hybridized mode shown in the inset of Fig. 3(d) arises from the spatial overlap between the optical electric field, the photoelastic coefficient, and the strain profile within the multilayer. Their relative spatial symmetries determine whether the spatial overlap integral vanishes or remains finite. Around the weak-mode resonance, changes in the strain profile modify this symmetry-dependent cancellation, resulting in the observed asymmetric line shape in the detection response.

To gain further physical insight, we analyze the spatial dependence of the integrand of $\sigma(\omega)$ for the two interface modes. The corresponding profiles are shown in Figs. 3(f) and 3(g). In the outer superlattices, the integrand contributions are similar for both modes and largely cancel due to symmetry. The main difference arises in the central superlattice region. For the 294 GHz mode, the integrand maintains the same sign throughout the central superlattice, resulting in efficient excitation of a “bright” mode. In contrast, for the 302 GHz mode, the integrand exhibits both positive and negative contributions, which partially cancel when integrated over the structure, leading to a strongly suppressed (“dark”) mode. The origin of this difference can be traced to the symmetry of the strain field within the unit cell of the central superlattice. As shown in Fig. 3(h), the strain field of the 294 GHz mode is symmetric with respect to the unit cell, whereas the 302 GHz mode exhibits an antisymmetric profile. This antisymmetric character reduces the overlap integral entering $\sigma(\omega)$, making this mode significantly less efficient to excite in the pump-probe experiment.

We note that the experimental FFT peak appears broader than the corresponding simulated spectral response. This difference may arise from several effects that are not included in the present model. In particular, attenuation of high-frequency phonons can contribute to spectral broadening in the experiment. In addition, the finite temporal window of the transient reflectivity measurement imposes a limited frequency resolution, which further broadens the experimental peak. Despite these differences, the simplified simulation reproduces the main spectral features and explains the dominant observation of a strong first hybridized mode and a suppressed second one.

IV. ROBUSTNESS AGAINST THICKNESS-RATIO FLUCTUATIONS

Single interface states produced by band inversion are known to be robust against perturbations that do not close the acoustic band gap or change the Zak phase associated to the perturbed unit cell [40,58]. Here, we examine whether this robustness is preserved when two such interface states are coupled to form a topological phonon molecule. To this end, we compare the response of the coupled topological interface states with that of a conventional Fabry-Pérot phonon molecule under controlled fluctuations of the GaAs/AlAs thickness ratio. The comparison focuses on two experimentally relevant quantities: the resonance frequencies and acoustic quality factors of the two coupled modes.

The centrosymmetric unit cell of the topological structure consists of three layers: $\frac{\lambda_{\text{GaAs}}}{4}(1 + \delta)$, $\frac{\lambda_{\text{AlAs}}}{2}(1 - \delta)$, and $\frac{\lambda_{\text{GaAs}}}{4}(1 + \delta)$. The structure is embedded in a GaAs medium and consists of two eight-period outer DBRs with $\delta = 0.5$ and a five-period center DBR with $\delta = -0.5$. The thickness-ratio fluctuation $\Delta\delta$ is varied from -0.5 to 0.5 and applied independently to each unit cell using a random factor r_n uniformly distributed between 0 and 0.99. Thus, the n th unit cell has layer thicknesses $\frac{\lambda_{\text{GaAs}}}{4}(1 + \delta + \Delta\delta r_n)$, $\frac{\lambda_{\text{AlAs}}}{2}(1 - \delta - \Delta\delta r_n)$, and $\frac{\lambda_{\text{GaAs}}}{4}(1 + \delta + \Delta\delta r_n)$. This perturbation changes the relative thickness between the GaAs and AlAs layers within one unit cell, while the total acoustic thickness remains con-

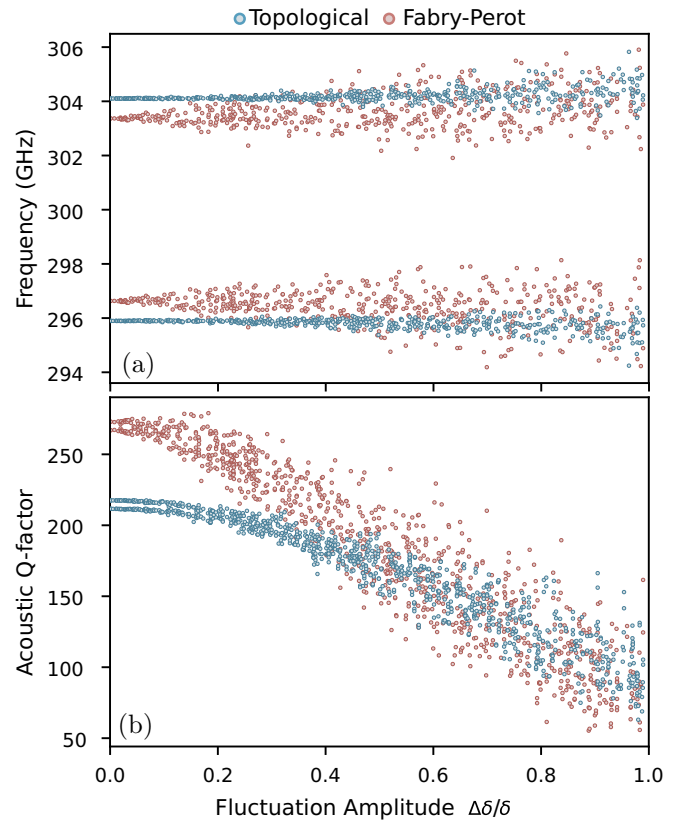


FIG. 4. Comparison of the stability of coupled modes in topological and Fabry-Perot phonon molecules under thickness-ratio fluctuations. The fluctuation amplitude $\Delta\delta$ modifies the GaAs/AlAs thickness ratio and is applied independently to each unit cell using a uniform distribution. Simulated (a) acoustic resonance frequencies and (b) quality factors of the two coupled phonon modes as a function of $\Delta\delta/\delta$. Frequency stability is significantly improved in the topological case, whereas Q -factor degradation remains governed mainly by disorder-induced scattering in both systems.

stant. In the Fabry-Perot case, the noncentrosymmetric unit cell consists of two layers: $\frac{\lambda_{\text{GaAs}}}{2}(1 + \delta)$ and $\frac{\lambda_{\text{AlAs}}}{2}(1 - \delta)$. The full structure is composed of three DBRs with eight, five, and eight periods ($\delta = 0.5$), respectively, separated by two λ -thick spacer layers. The spacer thicknesses are kept fixed, while the relative layer thicknesses within the DBRs are perturbed in the same way as in the topological structure.

The results are summarized in Fig. 4. For the topological molecule, the two resonance frequencies remain clustered around their unperturbed values over the full range of thickness-ratio fluctuations, whereas the Fabry-Pérot molecule exhibits a broader frequency dispersion. This behavior reflects the fact that the topological modes remain pinned to the interface-state frequency as long as the perturbation does not close the relevant band gap or remove the band inversion. On the contrary, the perturbations in the central superlattice affect its reflectivity, and thus the coupling factor between the two modes. This results in a perturbation-dependent coupling. It must be noted, however, that while not strictly robust against thickness-ratio fluctuations, the modes are more stable than in the case of the Fabry-Pérot molecule. The quality factors decrease with increasing dis-

order amplitude in both systems, as expected from enhanced scattering and imperfect confinement. However, the topological molecule preserves a more stable spectral response, indicating that the coupled modes retain the localization associated with the underlying Zak-phase contrast.

These simulations therefore support the interpretation of the hybridized modes as coupled topological interface states rather than conventional coupled cavity modes. The topology does not prevent all disorder-induced loss, but it stabilizes the resonance frequencies and preserves interface localization over a broad range of thickness-ratio perturbations.

V. FROM MOLECULES TO CHAINS: MULTIPLE COUPLED INTERFACE STATES

The two-interface system discussed above naturally generalizes to a chain of N coupled topological interface states [65], forming a one-dimensional lattice of interacting topologically localized modes. In analogy with coupled cavity arrays, a sequence of alternating topological superlattices generates a chain of localized interface states whose mutual coupling leads to the formation of collective modes. In this picture, each interface state plays the role of an effective phononic “site”, while the superlattices separating neighboring interfaces control the coupling strength between them.

This evolution is illustrated in Fig. 5(a), which shows the calculated acoustic reflectivity for structures containing different numbers of interfaces, N . Each interface is formed by joining two band-inverted superlattices; the structure comprises two outer DBRs with eight periods each and $N - 1$ inner DBRs of five periods, based on the same unit cells as described previously. For $N = 2$, the system corresponds to the phonon molecule discussed previously, characterized by two hybridized modes resulting from the coupling of the two interface states. Increasing the number of interfaces leads to the appearance of additional resonances inside the band gap corresponding to the collective modes of the coupled system. As N increases further, these discrete resonances progressively form a narrow miniband centered around the original interface-state frequency. In the infinite-chain limit, this miniband converges to the allowed band given by the phononic dispersion relation [Fig. 5(b)], arising from the periodic coupling of interface states. The dispersion is calculated for a periodic structure composed of band-inverted DBRs with a periodicity of five, by applying Bloch boundary conditions as described in the Appendix.

The spatial profiles of the displacement field for the case $N = 6$ are shown in Fig. 5(c). The six eigenmodes correspond to different collective combinations of the localized interface states. Although the modes extend over the entire structure, their amplitude remains strongly concentrated near the interfaces, reflecting the localized nature of the underlying topological states. The relative phase between neighboring interfaces determines the symmetry of the different modes, in analogy with the formation of molecular orbitals in coupled systems.

The bandwidth of the resulting miniband is controlled by the effective coupling between neighboring interface states, which depends on the reflectivity and thickness of the separating superlattices. Increasing the separation between interfaces

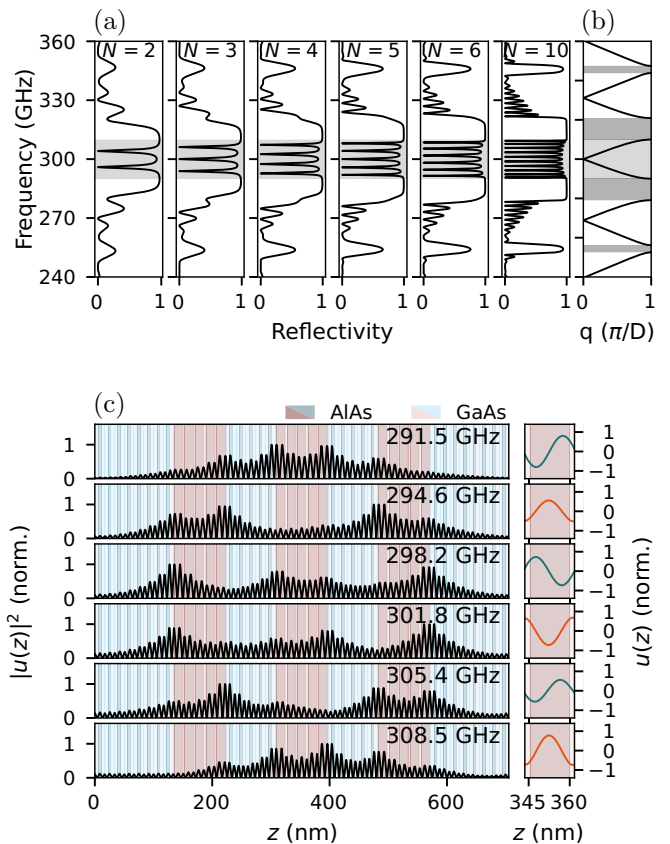


FIG. 5. Phononic chains formed by coupled topological interface states. (a) Acoustic reflectivity for structures (top-left) with different numbers of interfaces, N , illustrating the evolution from isolated interface modes to a miniband. (b) Phononic dispersion (top-right) for the infinite-chain limit, showing a miniband (light-shaded) arising from coupled interface states; band gaps are indicated by dark-shaded regions. (c) Spatial profiles of the squared displacement $|u(z)|^2$ for the case $N = 6$, showing the six hybridized interface modes localized at the interfaces. The right panels show the corresponding normalized displacement profiles, $u(z)$, in the unit cell located at the center of the structure, illustrating the alternating symmetric (orange curves) and antisymmetric (green curves) character of the hybridized modes.

reduces the overlap of the localized modes and therefore decreases the coupling strength, leading to a narrower miniband. In the limit of large separations, the modes recover the behavior of isolated interface states.

This system therefore provides a topological analog of phononic chains previously realized theoretically using coupled cavity spacers [65]. In contrast to conventional Fabry-Pérot cavities, the localized modes in the present structure originate from topological band inversion and remain pinned within the band gap of the surrounding superlattices. Consequently, the collective modes formed in the chain benefit from the spatial localization and robustness of the individual interface states, while allowing controllable coupling between them.

To experimentally investigate the topological phonon chain, transient reflectivity pump-probe measurements were performed on a multilayer structure containing six inter-

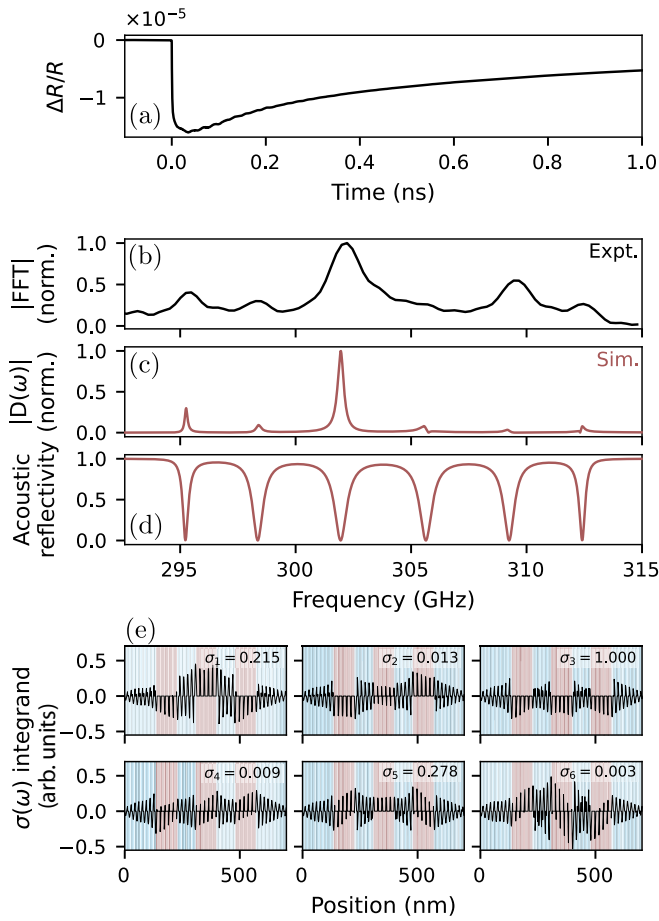


FIG. 6. Experimental observation and modeling of the topological phonon chain. (a) Transient reflectivity signal measured for the sample containing six interfaces. (b) Fourier spectrum of the acoustic component after removal of the electronic background and Brillouin oscillation. (c) Simulated $D(\omega)$ obtained from the photoelastic model. (d) Calculated acoustic reflectivity of the multilayer structure in the spectral region of the interface modes. (e) Spatial dependence of the integrand of $\sigma(\omega)$ for the different interface modes.

faces. The outer superlattices consist of eight periods of a centrosymmetric GaAs/AlAs/GaAs unit cell with layer thicknesses of 6/4.7/6 nm. The internal section of the structure is composed of alternating superlattices formed by five periods of the unit cell (2/14.1/2 nm) and five periods of the unit cell (6/4.7/6 nm), repeated three and two times, respectively. The structure is capped with a 6 nm GaAs layer. The experimental conditions are identical to those used for the topological phonon molecule sample described in the previous section. The measured transient reflectivity signal for the sample is shown in Fig. 6(a). After removing the slowly varying electronic and thermal background together with the low-frequency Brillouin oscillation, the acoustic component of the signal was analyzed in the frequency domain. The corresponding Fourier spectrum is shown in Fig. 6(b), where a dominant peak is observed near 302 GHz together with weaker side peaks. To interpret these observations, we calculated the $D(\omega)$ of the structure using the photoelastic model described previously. The simulated response in Fig. 6(c) shows a dominant peak near 302 GHz corresponding

to the third dip in the calculated acoustic reflectivity spectrum. The simulation also predicts weaker contributions from the remaining interface modes. The dominant experimental peak observed in Fig. 6(b), together with the weaker side peaks, aligns well with the reflectivity features in Fig. 6(d), confirming that the detected resonances originate from the coupled interface states of the topological chain. Although the relative amplitudes of the peaks in the experiment do not exactly match the simulated spectrum, this discrepancy can be attributed to the approximations used in the calculations. In these simulations, $\sigma(\omega)$ and $D(\omega)$ were evaluated using the same approach as in the two-interface sample. A more quantitative description would require explicit modeling of the photoelastic coefficients and phonon generation efficiency in each individual layer [73–76]. Such a detailed treatment was not pursued here since the simplified model already reproduces the main experimental observations.

Insight into the dominant excitation of the third interface mode is obtained by examining the spatial profile of the integrand of the Brillouin cross section shown in Fig. 6(e). The different panels correspond to the spatial distribution of the integrand of $\sigma(\omega)$ for the different interface modes. For the third mode [Fig. 6(e), top-right panel], the contributions from alternating inner superlattices of the structure have the same sign and therefore add constructively in the overlap integral, leading to efficient phonon generation. This behavior arises from the symmetric nature of the strain profile of this mode within the corresponding superlattice unit cells, similar to the mechanism discussed for the phonon-molecule case. In contrast, the other modes exhibit combinations of symmetric and antisymmetric strain profiles across the different superlattices, or symmetric strain profiles with alternating signs. As a result, the integrand contributions partially cancel in the overlap integral, leading to a reduced generation efficiency for these modes. The relative values of $\sigma(\omega)$ for the different modes are indicated in the figure. In the infinite-chain limit, the phononic dispersion [Fig. 5(b)] shows that the miniband is centered at the interface-state frequency, with the $q = 0$ mode located at the design frequency. Modes near $q = 0$ are preferentially generated and detected in pump-probe experiments [71].

We examine the stability of the coupled interface modes in the phonon chain structure when introducing fluctuations in the layer thickness ratio. Following the same procedure used for $N = 2$ in Fig. 4, we simulate the reflectivity for $N = 6$ and evaluate the frequencies and acoustic Q -factors of the six interface modes in both the topological and Fabry-Perot chain structures (see Fig. 7). Thickness-ratio fluctuations primarily modify the coupling strength and mode splitting in the phonon chain, similar to their effect on phonon molecules. For the topological phonon chain, the modes remain within the band gap, and their central frequency remains close to 300 GHz over the entire range of structural variations considered in the simulations. This behavior is consistent with the persistence of the interface states associated with the Zak-phase contrast. For the Fabry-Perot chain, interface defects modify both the mode splitting and the central frequency, resulting in larger fluctuations in the acoustic resonance frequency and acoustic Q -factor than in the topological phonon chain.

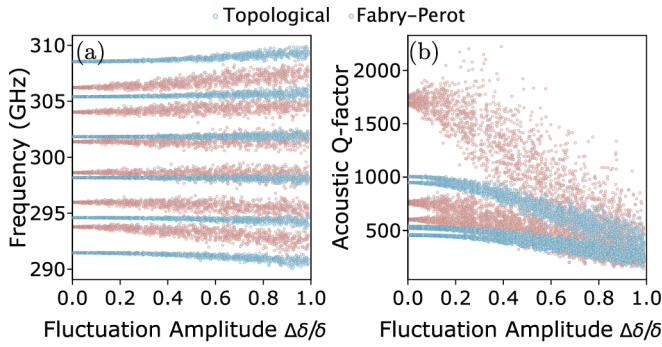


FIG. 7. Comparison of the stability of six coupled modes in topological and Fabry-Perot phonon chains in the presence of interface defects. (a) Acoustic resonance frequencies of the six hybridized interface modes and (b) their corresponding acoustic Q -factors.

VI. BAND STRUCTURE ENGINEERING IN PERIODIC TOPOLOGICAL PHONON CHAINS

The topological phonon chains introduced in the previous section show that coupling between topological interface states evolves from isolated resonances to a set of collective modes. In the infinite periodic limit, these collective modes merge into a continuous miniband with a well-defined dispersion relation. Experimentally, this miniband is observed as a finite set of hybridized interface-state resonances whose frequencies sample the dispersion of the corresponding infinite periodic chain. Their unequal amplitudes do not simply reflect the density of states, but are determined by the photoelastic generation and detection overlap integrals, leading to the preferential observation of the mode near $q = 0$. The phononic band structure of the periodic chain can be engineered by varying the number of periods in the two alternating DBRs, thereby controlling the coupling between neighboring interface states.

Figure 8 shows the calculated dispersion relations for periodic topological phonon chains with unit cells composed of m periods of DBR₁ and $(10 - m)$ periods of DBR₂, where DBR₁ and DBR₂ are the two band-inverted superlattices in-

troduced in Sec. II. The total number of DBR periods per unit cell is fixed at 10 and the relative number of DBR₁ periods is $m/10$. When the two DBRs contain the same number of periods ($m = 5$), the neighboring interface states couple equally, giving rise to a continuous miniband centered around the interface-state frequency, as shown in Fig. 8(b). Redistributing the fixed total number of periods between the two DBRs makes the two coupling strengths unequal and opens miniband gaps around 300 GHz, as shown in Figs. 8(a) and 8(c). Exchanging the relative numbers of periods of DBR₁ and DBR₂ (e.g., $m = 4$ and $m = 6$) interchanges the two coupling strengths, leading to a topological phase transition, as evidenced by the inversion of the Zak phases of the bands separated by the gap. The band-edge frequencies for different values of $m/10$ are shown in Fig. 8(d). The band-gap width is largest when the relative number of DBR periods is 0 or 1, corresponding to unit cells composed of only DBR₂ or only DBR₁, respectively. The opening and closing of the band gap in the band structure of the infinite phonon chain depend on the relative number of periods in the two DBRs. As the alternative coupling strengths are equal, the band gap closes; when they become unequal, the band gap reopens. This behavior is analogous to dispersion engineering in conventional phononic superlattices, where the band-gap opening and closing depend on the relative thickness of the constituent materials.

VII. CONCLUSION AND PERSPECTIVES

We have investigated the formation and coupling of topological phononic interface states in GaAs/AlAs superlattices engineered through band inversion of centrosymmetric unit cells. Two coupled interfaces give rise to hybridized modes forming a phononic molecule, characterized by symmetric and antisymmetric combinations of the localized interface states within the acoustic band gap. These hybridized states were experimentally observed using transient reflectivity pump-probe measurements, with resonance frequencies consistent with the calculated acoustic reflectivity spectrum and $D(\omega)$. We further extended this concept to structures containing multiple interfaces, forming chains of coupled topological

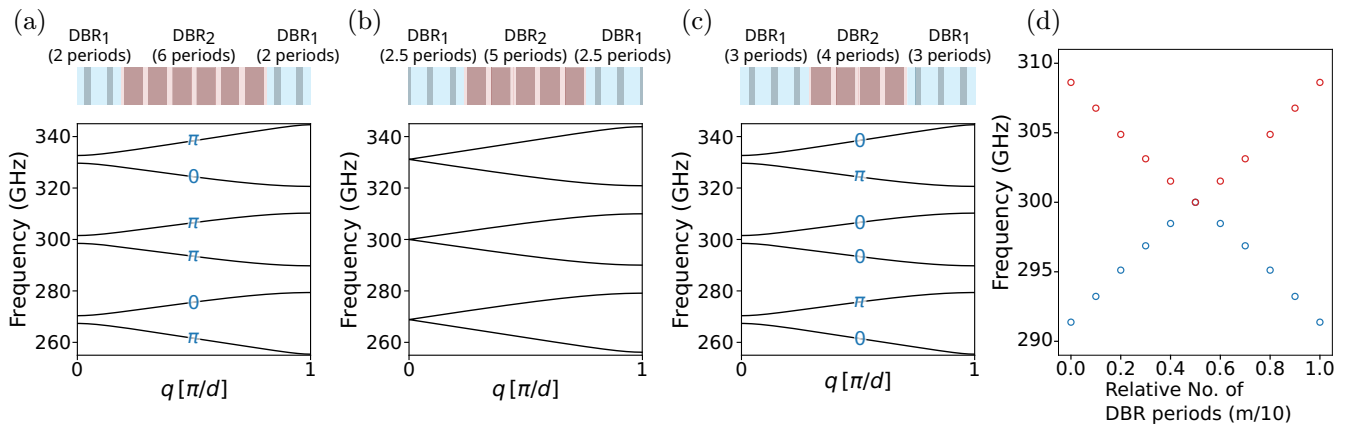


FIG. 8. Dispersion relations of phononic chains with a unit cell composed of m periods of DBR₁ and $(10 - m)$ periods of DBR₂ for (a) $m = 4$, (b) $m = 5$, and (c) $m = 6$. A band gap opens near 300 GHz when the two DBRs contain different numbers of periods and closes when their periods are equal. The numbers shown near the bands are the corresponding Zak phases. (d) Upper (red) and lower (blue) band-edge frequencies near 300 GHz as functions of the relative number of DBR periods, $m/10$.

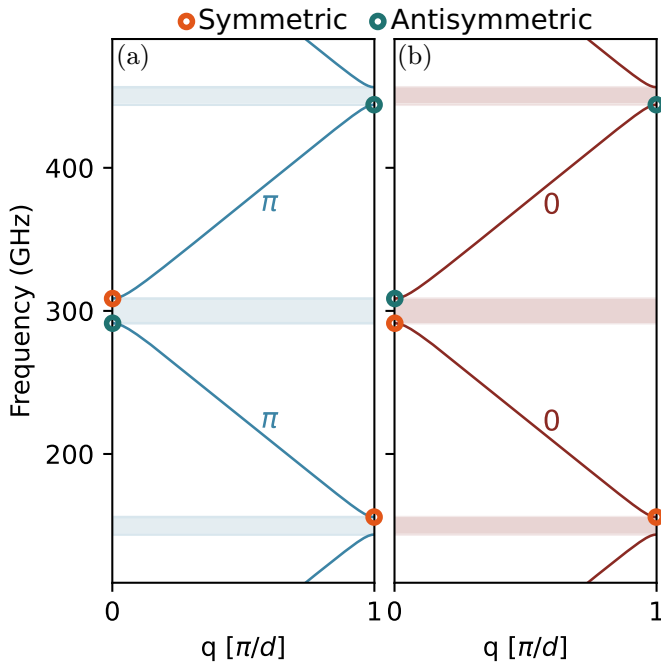


FIG. 9. Acoustic dispersion relations and corresponding Zak phases for two centrosymmetric superlattices: (a) ($3\lambda_{\text{GaAs}}/8$, $\lambda_{\text{AlAs}}/4$, $3\lambda_{\text{GaAs}}/8$) and (b) ($\lambda_{\text{GaAs}}/8$, $3\lambda_{\text{AlAs}}/4$, $\lambda_{\text{GaAs}}/8$). In (a), the eigenfunctions at $q = 0$ and $q = \pi/d$ have opposite symmetries in both branches around 300 GHz, giving $\theta^{\text{Zak}} = \pi$, whereas in (b), the symmetry is preserved, yielding $\theta^{\text{Zak}} = 0$. The resulting Zak-phase contrast leads to a localized interface state when two structures are joined. Markers indicate band-edge symmetries as in Fig. 1.

interface states. As the number of interfaces increases, the localized modes hybridize into a set of collective resonances that evolve toward a miniband structure. Experimental measurements on a chain sample containing six interfaces reveal a dominant resonance consistent with the calculated detection response and the corresponding acoustic reflectivity features of the structure.

Analysis of the spatial profile of the Brillouin generation integrand shows that the excitation efficiency of the different modes is governed by the symmetry of the strain field and the resulting constructive or destructive contributions in the overlap integral. In particular, the dominant excitation of one interface mode arises from constructive contributions across the multilayer structure, while other modes are suppressed due to partial cancellation of the generation integrand. These results demonstrate the controlled realization of phononic molecules and chains based on topological interface states in semiconductor superlattices. Altogether, these findings establish topological interface states in semiconductor superlattices as a versatile platform for engineering coupled phononic systems. The realization of controllable phononic molecules and chains opens new possibilities for manipulating coherent acoustic phonons and designing complex nanophononic structures.

Combined with the atomic-scale thickness control provided by MBE, these architectures can set a basis for constructing robust cavity modes using quasiperiodic arrays and for engineering complex phononic band structures span-

ning the gigahertz to terahertz frequency range. In particular, varying the relative number of periods of the constituent DBRs within the chain provides an additional degree of freedom for controlling miniband-gap opening and closing, as well as the associated topological phase transitions, thereby expanding the possibilities for phononic band engineering. Furthermore, introducing a controlled gradient in the interface-state frequencies through appropriate layer-thickness engineering could transform these minibands into Wannier-Stark ladders, providing a route toward acoustic Bloch oscillations [66]. Owing to the “magic” coincidence that GaAs/AlAs stacks with appropriately tuned layer thicknesses act as efficient Bragg mirrors for both near-infrared light and gigahertz acoustic phonons, the design principles developed here are directly translatable to optophononic structures, where coupled topological phonon molecules can be colocalized with optical cavity or interface modes. These results establish coupled topological interface states as a controllable route to engineer phononic molecules and minibands in semiconductor superlattices, with direct prospects for GHz optophononic devices.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [77].

APPENDIX: CALCULATION OF THE ZAK PHASE

Eigenvalue problem of 1D superlattice. The propagation of longitudinal acoustic waves in a periodic one-dimensional (1D) structure is described by the elastic wave equation. The acoustic field vector $[U(z), S(z)]$ at the left and right boundaries of layer i is related by the transfer matrix M_i [65,78],

$$M_i = \begin{pmatrix} \cos(k_i d_i) & \frac{\sin(k_i d_i)}{\omega Z_i} \\ -\omega Z_i \sin(k_i d_i) & \cos(k_i d_i) \end{pmatrix}, \quad (\text{A1})$$

where k_i is the wave vector, d_i is the layer thickness, ω is the angular frequency, and Z_i is the acoustic impedance. For a unit cell composed of multiple layers with total thickness d , the overall transfer matrix M is obtained by multiplying the individual layer matrices. The acoustic field across the unit cell can then be written as

$$\begin{pmatrix} U(d) \\ S(d) \end{pmatrix} = M \begin{pmatrix} U(0) \\ S(0) \end{pmatrix}. \quad (\text{A2})$$

For an infinite periodic structure, the dispersion relation is obtained by imposing the Bloch condition on the acoustic field,

$$\begin{pmatrix} U(d) \\ S(d) \end{pmatrix} = e^{iqd} \begin{pmatrix} U(0) \\ S(0) \end{pmatrix}, \quad (\text{A3})$$

where q is the Bloch wave vector. Combining Eqs. (A2) and (A3), one obtains an eigenvalue problem for M , whose eigenvalues are given by $\lambda_{\pm} = e^{\pm i q d}$. Solving this eigenvalue problem gives the dispersion relation and Bloch eigenvectors $[U(0), S(0)]$, which define the displacement and stress at the unit-cell boundary. The Bloch eigenfunctions are obtained by propagating the state vector through successive layers using the transfer matrix.

Calculating the Zak phase. The Zak phase is the geometric phase that characterizes the topological properties of bands in one-dimensional periodic systems. In inversion-symmetric systems, the Zak phase is quantized to 0 or π and is determined by the symmetry of the Bloch eigenfunctions at the band edges. When the eigenfunctions at the center ($q = 0$) and boundary ($q = \pi/d$) of the Brillouin zone have the same symmetry, the Zak phase of the n th band is $\theta_n^{\text{Zak}} = 0$. In contrast, when these modes have opposite symmetries, the Zak phase is $\theta_n^{\text{Zak}} = \pi$. For the n th band, the Zak phase is

defined as [19]

$$\theta_n^{\text{Zak}} = \int_{-\pi/d}^{\pi/d} \left[i \int_{\text{unit cell}} \frac{1}{2\rho(z)v^2(z)} u_{n,q}^*(z) \partial_q u_{n,q}(z) dz \right] dq. \quad (\text{A4})$$

Here, $u_{n,q}(z)$ is the normalized Bloch eigenfunction, and $\rho(z)$ and $v(z)$ are the position-dependent mass density and wave velocity within the unit cell, respectively.

For numerical calculation, the folded Brillouin zone is sampled at discrete wave vectors q_i , and the Zak phase is calculated using the discrete form [19]

$$\theta_n^{\text{Zak}} = -\text{Im} \sum_{i=1}^N \ln \left[\int_{\text{unit cell}} \frac{1}{2\rho(z)v^2(z)} u_{n,q_i}^*(z) u_{n,q_{i+1}}(z) dz \right]. \quad (\text{A5})$$

In this expression, q_i and q_{i+1} are two neighboring sampling points in the Brillouin zone, and the integral gives the overlap between adjacent Bloch eigenmodes within the same band. Figure 9 shows the acoustic dispersion relations and corresponding Zak phases for two centrosymmetric superlattices presented in the main text. The Zak phase of the bands changes from π to 0, reflecting a band inversion and the associated topological transition.

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