

Spin and pair density waves in two-dimensional altermagnetic metals

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Altermagnetism, a recently proposed and experimentally confirmed class of magnetic order, features collinear compensated magnetism with unconventional spin-split bands. Here, we show that in a metallic 2D d -wave altermagnet with $[C_2||C_4]$ symmetry, secondary instabilities can arise. Using an unbiased functional renormalization group approach, we analyze the weak-coupling instabilities of a 2D Hubbard model with a preexisting altermagnetic order inspired by our *ab initio* electronic structure calculations of realistic material candidates from V_2X_2O ($X = \text{Te, Se}$) family. We identify two distinct spin-density-wave (SDW) states that break the underlying altermagnetic $[C_2||C_4]$ symmetry. Additionally, we find spin-fluctuation-induced instabilities leading to a singlet d -wave superconducting state and an unconventional commensurate pair-density-wave (PDW) state with extended s -wave and spin-triplet symmetry. We analyze the pairing mechanism and characterize the excitation spectrum, which exhibits Bogoliubov Fermi surfaces or nodal points depending on the gap size.

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I. INTRODUCTION

An unconventional form of collinear magnetic order, dubbed altermagnetism, was recently theoretically proposed to be ubiquitous in many materials [1–3] and experimentally confirmed in MnTe and CrSb [4–10]. This particular state differs from ferro- and antiferromagnets, as it breaks time-reversal symmetry (TRS) in the form of d -, g -, or i -wave magnetization in direct space with zero-net magnetization [1,11]. In momentum space, the altermagnetic order also exhibits the corresponding even-parity-wave spin splitting. The experimentally confirmed altermagnets exhibit two spin-sublattices, which are not mapped onto each other by translation or inversion as in conventional antiferromagnets, but instead by a real-space rotation. As a consequence, altermagnets possess distinct electronic, magnetic, and topological properties, which manifest in unconventional spin, charge, and optical responses [2,3,11–24].

The most common mechanism that captures altermagnetism arises from anisotropies in the crystal lattice [1,2,19,25] and can be readily captured by *ab initio* methods

such as density functional theory (DFT). In this scenario, the local anisotropy of the crystal induces local anisotropic spin densities in the magnetic phase.

The altermagnetic (AM) transition can happen at high temperatures and reach spin splittings on the order of eV similar to ferromagnets because of its exchange origin. This offers the exciting opportunity to realize quantum many-body states as secondary instabilities of AM metals. They can be expected to be of unconventional type because they arise out of an unconventional parent state.

While many theoretical studies focused on the mechanism behind the origins of an AM instability [26–28,28–30,30–33], analyses of secondary instabilities are limited. Possible superconducting states in AM metals were discussed, including such interesting options as finite-momentum or topological superconductors [4,24,34–40] because of the lifted Kramers spin degeneracy. Yet, an understanding of their pairing mechanisms possibly from repulsive interactions remains an open problem, despite the importance of the relation between magnetism and superconductivity. Interestingly, AM has also been theorized to appear as possible interaction-induced instability in cuprates [41,42].

A paradigmatic model for the study of competing magnetic and superconducting orders is the square-lattice Hubbard model. It highlights the importance of single-particle features such as Van Hove (VH) singularities and Fermi-surface nesting for Fermi-surface instabilities and a weak-coupling pairing scenario, as well as the importance of spin

fluctuations for moderate couplings. Realization of an analogous model in an AM promises a similarly rich phase diagram with additional, unexplored effects due to the AM splitting. For example, a secondary spin-density wave was indicated in the AM $\text{KV}_2\text{Se}_2\text{O}$ [8].

In this paper, we explore the effects of electronic correlations in a square lattice Hubbard model where an AM phase has already been formed. We employ DFT and functional renormalization group (FRG) calculations, which allow us to connect the search for suitable candidate materials and the unbiased determination of many-body ground states. We present a comprehensive phase diagram in terms of model parameters and make connections with material realizations. We further draw parallels with the well-studied Hubbard model on the square lattice without altermagnetism to understand the qualitative impact on the nature and mechanisms of interaction-driven instabilities. We find instabilities towards distinct density waves arising from spin splitting or nesting of the Fermi surfaces. In addition, we identify a fluctuation-induced instability to a singlet d -wave superconducting state, as well as a commensurate triplet pair-density wave enabled by the unconventional altermagnetic spin splitting.

II. 2D ALTERMAGNETIC METAL MATERIAL CANDIDATES

We are interested in candidate materials with favorable properties for a secondary transition inside the AM phase. Typically, Stoner-like instabilities in any ordered channel can arise in a Fermi liquid for strong enough interactions when the dimensionless interaction $u\rho_0 \gtrsim 1$. Here, u is the coupling in the ordering channel and ρ_0 the density of states (DOS) at the Fermi level. While such Stoner-like transitions are straightforward routes towards secondary instabilities, we see that transition temperatures can be boosted when the DOS exhibits a peak. Thus, quasi-two-dimensional (2D) systems are particularly interesting for us because they inherently possess a Van Hove singularity with a logarithmic divergence of the DOS. We note in passing that high-order Van Hove singularities can potentially increase transition temperatures even more [43].

Canonical or even approximate Fermi surface nesting, where the dispersion $\xi_{\mathbf{k}+\mathbf{q}} \approx -\xi_{\mathbf{k}}$ for a finite $\mathbf{q} \neq 0$, is another feature beneficial for interaction-induced instabilities, leading to spin or charge density waves.

The presence of strong spin or charge fluctuations— independent of proximity to nesting or Van Hove singularities—is also advantageous for electronic pairing mechanisms, such as a Kohn-Luttinger or spin-fluctuation mechanism. If the interaction becomes attractive, a pairing instability arises whose type depends on the mediating fluctuations and the Fermi surface. Because of the unconventional spin splitting in an altermagnet, these systems bear the potential for triplet and finite-momentum pairing. The latter can be boosted by nesting in the Cooper channel $\xi_{-\mathbf{k}+\mathbf{q}} = \xi_{\mathbf{k}}$, $\mathbf{q} \neq 0$.

As feasible candidates, we have identified the monolayer $\text{V}_2\text{Te}_2\text{O}$ [44] and the three-dimensional (3D) compound with quasi-2D electronic structure $\text{KV}_2\text{Se}_2\text{O}$ [8]. Both materials have parent nonmagnetic space group $P4/mmm$ (123). We have obtained the AM configuration that is the thermody-

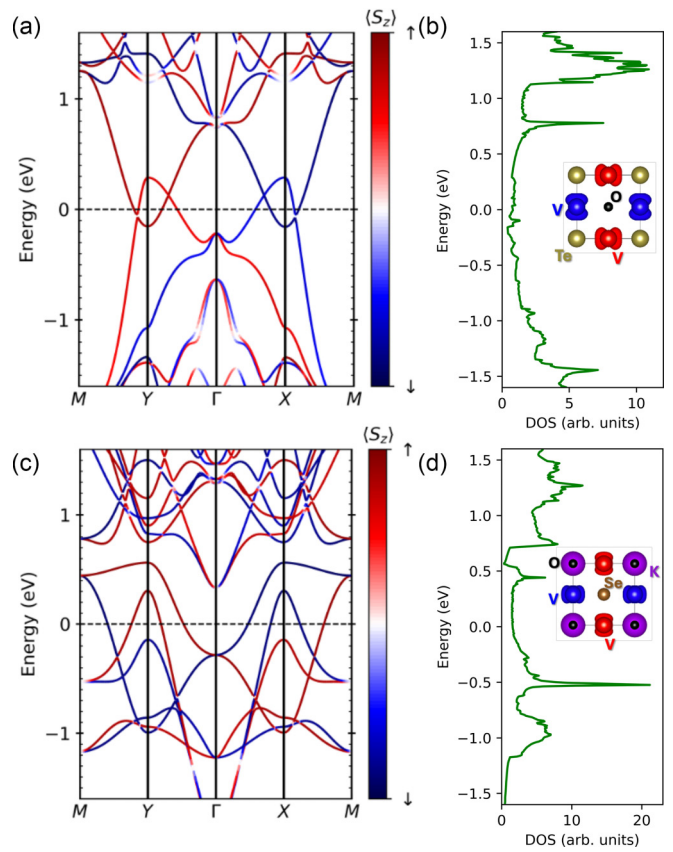


FIG. 1. Electronic structures of AM metals. (a) The predicted 2D band structure of $\text{V}_2\text{Te}_2\text{O}$ with $f_{x^2-y^2}$ altermagnetic splitting and (b) its density of states. The inset corresponds to the atom arrangement in the unit cell. (c) Quasi-2D electronic band structure of $\text{KV}_2\text{Se}_2\text{O}$ with altermagnetic splitting and (d) its density of states. The inset shows the top view of the crystal structure along with the magnetization density of the atoms. The red-blue color map represents the z component of the spin expectation value.

namically favored for both $\text{V}_2\text{X}_2\text{O}$ ($X = \text{Te}, \text{Se}$) monolayers via total energy minimization using DFT calculations (see Sec. VI). In addition, we have also verified its stability under isotropic in-plane strain (see the Supplemental Material, SM [45]). While we focus on the $\text{V}_2\text{Te}_2\text{O}$ monolayer, we remark that metallic altermagnetism was experimentally indicated in $\text{KV}_2\text{Se}_2\text{O}$ [8].

$\text{V}_2\text{Te}_2\text{O}$ exhibits the electronic structure shown in Figs. 1(a) and 1(b), favorable for the generation of secondary instabilities. We show the calculated magnetization density in the unit cell of $\text{V}_2\text{Te}_2\text{O}$ in the inset of Fig. 1(b). Similarly, in Figs. 1(c) and 1(d), we show the electronic structure of $\text{KV}_2\text{Se}_2\text{O}$, along with the magnetization density in the inset. In both compounds, the opposite local magnetizations of the V atoms are related by a combined $[C_2][C_4z]$ spin symmetry operation, which consists of a twofold spin rotation combined with a fourfold lattice rotational symmetry. This symmetry leads to an altermagnetic $d_{x^2-y^2}$ -wave type momentum-dependent spin-splitting in the band structure [1,2].

Our DFT band structure calculations, shown in Figs. 1(a) and 1(c) confirm the presence of the $d_{x^2-y^2}$ -wave type

momentum-dependent spin-splitting and reveal a large magnitude reaching ~ 1.5 eV in the valence bands of both V_2Te_2O and KV_2Se_2O . The spin-orbit coupling is included in Figs. 1(a) and 1(c), and we see that it introduces only weak corrections to the nonrelativistic altermagnetic band structures (see the SM [45] for a comparison). Consistent with previous reports, we obtain a metallic electronic structure for both the V_2Te_2O monolayer [44,46,47] and KV_2Se_2O [8] as shown in the DOS calculations in Figs. 1(b) and 1(d).

The total DOS for the V_2Te_2O monolayers also reveals distinct electronic singularities near the Fermi level. Notably, a peak associated with a Van Hove singularity (VHS) appears approximately 0.7 eV above the Fermi level and around -1.4 eV below it. Additionally, weaker DOS singularities are also observed directly near the Fermi level. These VHS features can be tuned via electronic (hole) doping or via strain, as demonstrated for V_2Se_2 (see the SM [45]). In the DOS of KV_2Se_2O [Fig. 1(d)], we obtain a pronounced Van Hove singularity at -0.5 eV below the Fermi level.

We also point out that the V_2Te_2O lattice is conducive to a large AM spin splitting due to the Te and O atoms generating strong AM sublattice anisotropies, which are around the two V sublattices related by a 90° rotation. There exist, in principle, also 2D altermagnets with a smaller AM spin splitting. In the SM [45], we show the electronic structure of CoS_2 (parent nonmagnetic space group $P2_1/c$), where we observe much smaller AM spin splitting, related to the weak rotations of the S polyhedra in opposite sense around the two Co atoms in the unit cell. The spin group symmetry analysis predicts that the momentum-dependent spin splitting in CoS_2 is of d_{xy} type.

III. EFFECTIVE MODEL FOR A 2D AM METAL

To capture the essential features of the unconventional spin splitting of the material candidates in Sec. II, we set up an effective, minimal tight-binding model on the square lattice. The corresponding Hamiltonian is given by

$$H_0 = \sum_{\mathbf{k}, \sigma} \xi_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma}, \quad (1)$$

where $c_{\mathbf{k}\sigma}^\dagger$ ($c_{\mathbf{k}\sigma}$) is the Fourier transform of the operator $c_{i\sigma}^\dagger$ ($c_{i\sigma}$), creating (annihilating) an electron with spin projection $\sigma = \pm$ for $\uparrow, \downarrow$ at lattice site i . Because the system is inside the AM phase, the dispersion $\xi_{\mathbf{k}\sigma}$ is modified by a momentum- and spin-dependent splitting

$$\xi_{\mathbf{k}\sigma} = \xi_{\mathbf{k}}^0 - \lambda \sigma f(\mathbf{k}), \quad (2)$$

$$\xi_{\mathbf{k}}^0 = -2t(\cos k_x + \cos k_y) - 4t' \cos k_x \cos k_y - \mu. \quad (3)$$

Here, t labels nearest- and t' second-nearest-neighbor-hopping amplitudes, μ the chemical potential, and λ the magnitude of the AM splitting. Based on possible symmetry configurations [2], we consider two types of AM form factors $f(\mathbf{k})$, which fall into the B_1 and B_2 irreducible representations of the C_{4v} point group, described by the following basis functions:

$$f_{x^2-y^2}(\mathbf{k}) = \cos k_x - \cos k_y, \quad (4)$$

$$f_{xy}(\mathbf{k}) = \sin k_x \sin k_y. \quad (5)$$

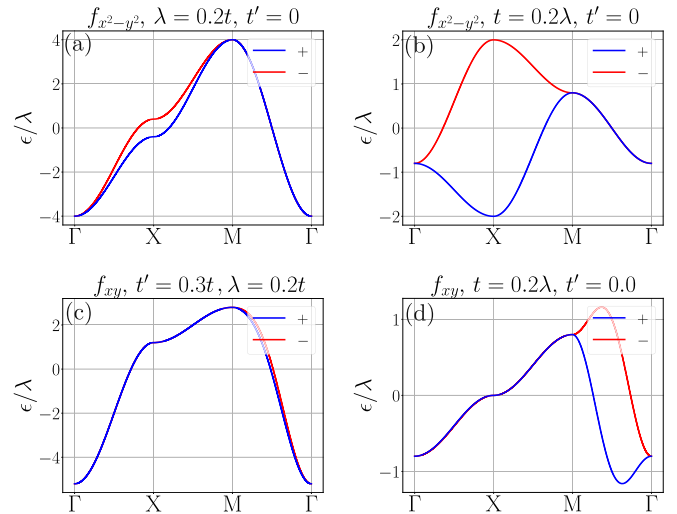


FIG. 2. Effective models of an AM metal, according to the bare dispersion for the two AM form factors studied [Eqs. (4) and (5)]. The spin splitting is tuned by λ . The relative ratio t/λ tunes between the small (a), (c) and large (b), (d) altermagnetic regime and affects the location of saddle points and extrema in the BZ.

The effective model for quasi-2D AM metals Eqs. (3)–(5) is rooted in tetragonal symmetries. Thus, it can be expected to describe other AM metals with appropriately modified parameters. We take this as a motivation to analyze a comprehensive parameter regime beyond the specific material realizations. Our findings can guide additional material searches for further promising candidates. The single-particle dispersion Eq. (1), exhibits two distinct behaviors in terms of the ratio $t/\lambda \lesssim 1/2$ (see the SM [45]), which we dub as small or large AM regime. We focus on the case with $t' = 0$ and remark on the effect of finite t' when necessary. These two regimes dictate where extrema and saddle points of the dispersion lie in the Brillouin zone (BZ). Both regimes are relevant for realistic material candidates as noted in Sec. II. The small AM regime $t/\lambda < 1/2$ connects to the well-studied square-lattice dispersion for $\lambda = 0$, where perfect nesting $\xi_{\mathbf{k}+\mathbf{Q}}^0 = -\xi_{\mathbf{k}}^0$ with $\mathbf{Q} = (\pi, \pi)$ occurs in combination with VH singularities at the Fermi level for $\mu = 0, t' = 0$. The VH points occur at the X = $(\pi, 0)$ and Y = $(0, \pi)$ points of the BZ. When t' is increased nesting is destroyed and the VH singularities move in energy according to $\mu_{\text{VH}}^0 = 4t'$.

Adding $\lambda \neq 0$ modifies this scenario differently for the two form factors $f_{x^2-y^2}$ and f_{xy} . For the $f_{x^2-y^2}$ symmetry of the AM splitting Eq. (4), nesting between equal spins remains intact $\xi_{\mathbf{k}+\mathbf{Q}\sigma} = -\xi_{\mathbf{k}\sigma}$ at $\mu = 0$, but is destroyed for opposite spins. The VH singularities are split in energy to chemical potential values $\mu_{\text{VH}} = 4t' \pm 2\lambda$, Fig. 2(a). The AM splitting is opposite at X and Y so that VH points become spin polarized at VH fillings. In the f_{xy} case Eq. (5), nesting is destroyed for equal spins but remains intact for opposite spins $\xi_{\mathbf{k}+\mathbf{Q}\sigma} = -\xi_{\mathbf{k}-\sigma}$ at $\mu = 0$. Spin-degenerate VH singularities still occur at X and Y. At a generic chemical potential, away from half filling or Van Hove filling, Fermi surfaces form spin-split ellipses.

In the large AM regime, when the dominant energy scale is the AM splitting $\lambda > 2t$, the dispersion changes considerably. In the $f_{x^2-y^2}$ case, the minimum of the dispersion is found

at X , while $M = (\pi, \pi)$ and $\Gamma = (0, 0)$ points become spin-degenerate Van Hove singularities located at $\mu_{\text{VH}} = \pm 4t$, Fig. 2(b). In the f_{xy} case, a VH singularity is still found at $\mu = 0$ and an additional one occurs for $\mu_{\text{VH},\uparrow} = 4t$ at M and for $\mu_{\text{VH},\downarrow} = -4t$ at Γ [Figs. 2(c) and 2(d)].

We further equip the effective AM model with an interaction term $H = H_0 + H_U$ given by an on-site Hubbard repulsion

$$H_U = U \sum_i n_{i\uparrow} n_{i\downarrow}, \quad (6)$$

where $U > 0$, and $n_{i\sigma} = c_{i\sigma}^\dagger c_{i\sigma}$ labels the density operator. This form of (effective) on-site Hubbard repulsion of the altermagnetic quasiparticles is relevant for a strongly screened Coulomb repulsion. We assume that renormalizations from high-energy modes are already included in the parameters of H , i.e., we consider not only the hopping amplitudes t, t' and the AM splitting λ , but also U as an effective parameter.

IV. INTERACTION-INDUCED INSTABILITIES

Our aim is to understand the effects of electronic correlations in the presence of altermagnetism. To this end, we study the different instabilities of an AM metal as a function of μ , t , and λ of the model given by Eqs. (1) and (6). Instabilities are signaled by singularities in the static, effective interaction, which can be defined as the amputated, one-particle irreducible (1PI) part of the two-particle Green's function $\Gamma_{\alpha\beta\gamma\delta}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) = \langle c_{\mathbf{k}_1\alpha}^\dagger c_{\mathbf{k}_2\beta}^\dagger c_{\mathbf{k}_3\gamma} c_{\mathbf{k}_4\delta} \rangle_{\text{1PI}}$. We employ random phase approximation (RPA) [45] and truncated-unity functional renormalization group (TUFRG) [48–56] to calculate Γ and extract the critical temperature, type, and symmetry of an instability from its singular part, see Sec. VI. Within the TUFRG, fluctuation corrections of particle-particle and particle-hole type are summed up to infinite order on equal footing. Keeping just one of these channels amounts to a RPA or the Cooper ladder. Since the TUFRG takes their mutual feedback into account, it allows us to not only analyze RPA-type instabilities, but also the suppression or cooperation of different ordered channels. For example, this includes mechanisms to obtain charge orders or superconductivity from repulsive interactions [55–57]. We refer to Sec. VI for more details on our TUFRG implementation. For our calculations, we use an intermediate value of the bare Hubbard U . Since both the altermagnetic form factor and the ratio t/λ affect the bandwidth, we choose $U = 3t$ for both $f_{x^2-y^2}$ and f_{xy} in the small AM regime. For the large AM, we choose $U = 3\lambda$ for $f_{x^2-y^2}$ and $U = 0.8\lambda$ for f_{xy} . All relevant quantities are given in units of the dominant energy scale t (λ) in the small (large) AM regime. We note that similar models have been used in the context of spin-Pomeranchuk physics; however, they consider either the mechanism of the spin Pomeranchuk transition itself, or assume an attractive interaction as a starting point to study secondary instabilities [58–60].

Density waves. The ordering tendencies we find can be understood as Fermi surface instabilities. It is therefore instructive to analyze the Fermi surface geometry. Two important ingredients in this regard are the presence of nesting and Van Hove singularities near the Fermi level in different

parameter regimes of the effective model for an AM metal, as detailed in Sec. III. When at least one of these features is present, RPA predicts an instability at sufficiently low temperatures. This is well known from the SU(2) symmetric Van Hove scenario on the square lattice, where a SDW appears as a result of nesting around Van Hove filling for $t' = 0$ [55,61]. In the AM case, we need to distinguish scattering processes of the same or opposite spins. Thus, we define the RPA susceptibility

$$\chi_{\perp/\parallel}(\mathbf{q}) = \chi_{\perp/\parallel}^0(\mathbf{q}) [1 - U \chi_{\perp/\parallel}^0(\mathbf{q})]^{-1}, \quad (7)$$

where $\chi_{\perp/\parallel}^0$ denote the bare susceptibilities for opposite/same spin. Nesting or the presence of a VH singularity ensures that they will at least diverge $\propto \log T$ (see the SM [45] for details) in the limit of weak coupling.

In general, we find density-wave instabilities, which we label as longitudinal (SDW $_{\parallel}$) and transverse (SDW $_{\perp}$) spin density wave (SDW). They correspond to the order parameters

$$\text{SDW}_{\parallel}: \phi^z(\mathbf{Q}) = \sum_{\mathbf{k}} \langle c_{\mathbf{k}+\mathbf{Q}}^\dagger \sigma^z c_{\mathbf{k}} \rangle, \quad (8)$$

$$\text{SDW}_{\perp}: \phi^{x,y}(\mathbf{Q}) = \sum_{\mathbf{k}} \langle c_{\mathbf{k}+\mathbf{Q}}^\dagger \sigma^{x,y} c_{\mathbf{k}} \rangle, \quad (9)$$

with Pauli matrices σ^a ($a = x, y, z$) and wave vector $\mathbf{Q} = (\pi, \pi)$, which can also turn incommensurate [$\mathbf{Q} = (\pm(\pi - \delta), \pi)$ or $\mathbf{Q} = (\pi, \pm(\pi - \delta))$, $\delta \ll 1$]. These describe ordered states that spontaneously break translation symmetry, as well as the AM symmetry of simultaneous spin-flip and fourfold lattice rotation [$C_2 \parallel C_4$]. As such the corresponding phase transition constitutes a novel type of symmetry breaking. The transverse SDW additionally breaks the remaining U(1) spin symmetry (while the longitudinal does not). Let us note that for a generic Fermi surface typical for the small AM regime such as at $\mu = -2t$ as shown in Fig. 3(a), we find a transverse SDW. It arises when $U \gtrsim 4.95$ for $T \rightarrow 0$ and its wave vector is dictated by momentum transfers connecting segments that lie around the major and minor axes of the two ellipses.

A. Competing orders in the small AM regime

1. f_{xy} splitting

We start the discussion with the f_{xy} form factor with $t' = 0$, as it provides a clear analogy to the well-studied Hubbard model on the square lattice [50,55,61–66]. As noted before, a finite λ breaks the spin SU(2) symmetry, but the VH singularity remains degenerate for the two spin species. Furthermore, there is only opposite spin nesting at $\mu = 0$. This results in the antiferromagnetic order present in the SU(2) symmetric case becoming a SDW $_{\perp}$ for the same values of μ [Fig. 3(b)].

To investigate the effect of t' for finite λ we look at the instabilities at $\mu = \mu_{\text{VH}}$ as a function of t' , which follow a similar trend to the SU(2) symmetric Hubbard model on the square lattice. As t' is increased, the critical temperature T_c of the spin instabilities decreases because nesting is also destroyed. Additionally, there is a transition from SDW $_{\perp}$ to SDW $_{\parallel}$, because exactly at $t' = \lambda/4$ (keeping $\mu = \mu_{\text{VH}} = 4t'$) nesting between equal spins is restored promoting SDW $_{\parallel}$ as the dominant instability. After a threshold value ($t' \sim 0.2t$), the

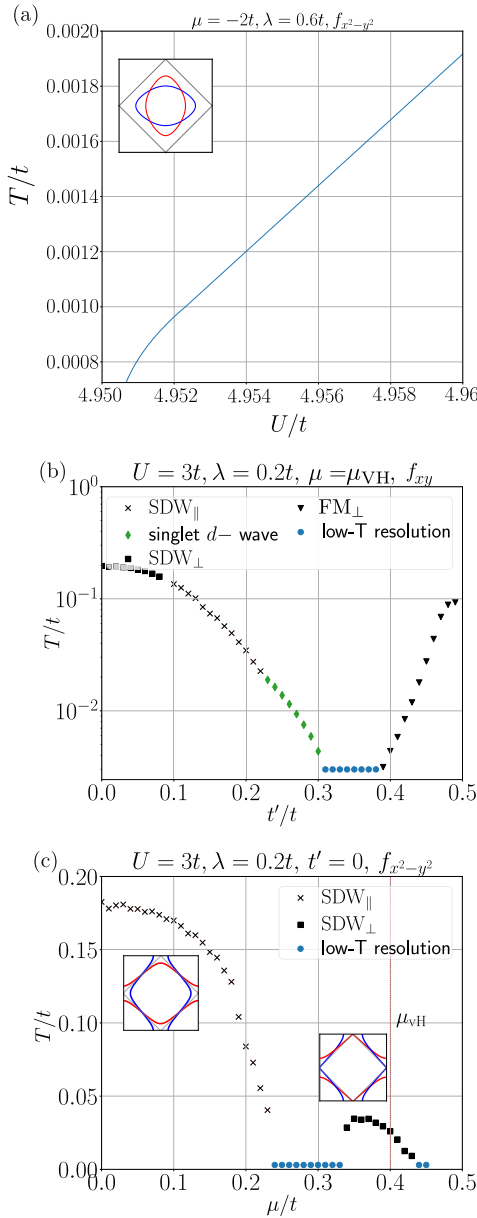


FIG. 3. Phase diagrams for the small AM regime. (a) Critical temperature for the instability towards an $\text{SDW}_\perp$ away from a VH singularity and nesting as function of the bare Hubbard interaction computed within RPA. The inset corresponds to the FS for the value of μ , where the calculation was performed. In order for the system to exhibit an instability at a finite T , a critical value of the bare interaction U_c must be exceeded. (b) Critical temperatures T of FRG instabilities as function of next-nearest-neighbor hopping t' at $\mu_{\text{VH}} = 4t'$ for the f_{xy} AM case. The results are qualitatively similar to the ones obtained in the $\text{SU}(2)$ symmetric Hubbard model where for $t' \sim 0.2t$ a transition from a SDW to d -wave pairing is observed. For larger values of t' the system displays a ferromagnetic instability. In the f_{xy} AM case, a transition between $\text{SDW}_\perp$ and $\text{SDW}_\parallel$ occurs. (c) For $f_{x^2-y^2}$ the observed instabilities $\text{SDW}_\parallel$ and $\text{SDW}_\perp$ are separated into two domes as a function of μ within our temperature resolution. The former is dominated by equal spin nesting at $\mu = 0$ and the latter because of the spin split VH singularities occurring at X and Y. The different symbols correspond to the instabilities identified with the vertex divergence terminating the FRG flow. The insets correspond to relevant FS for the instabilities identified.

$\text{SDW}_\parallel$ instability gives way to a singlet pairing instability in the d -wave channel. This pairing instability arises despite the AM-induced mismatch $\xi_{\mathbf{k}\uparrow} \neq \xi_{-\mathbf{k}\downarrow}$, which, however, is zero at the VH points driving it. Further increase of t' leads to an upturn of T_c in a ferromagnetically dominated regime along the transverse direction ($\text{FM}_\perp$).

2. $f_{x^2-y^2}$ splitting

We now turn to the $f_{x^2-y^2}$ form factor and $t' = 0$. Our analysis yields a longitudinal SDW close to $\mu = 0$, where a condition of perfect equal-spin nesting is satisfied. This is signaled by diverging peaks at $\mathbf{Q} = (\pi, \pi)$ in the longitudinal particle-hole vertex. The longitudinal SDW order can become incommensurate as the chemical potential is tuned away from $\mu = 0$. In doing so, the critical temperature decreases and drops to zero within our numerical resolution for $\mu \gtrsim 0.2t$. Close to Van Hove filling $\mu_{\text{VH}} = \pm 2\lambda$, the AM metal becomes unstable to a transverse SDW . This is because for this value of μ , the spin up band has a Van Hove singularity only at the Y point, while the spin-down band has one only at X, as shown in the right inset in Fig. 3(c). Scattering processes between these two points in the BZ combined with spin flips favor a transverse SDW with $\mathbf{Q} = (\pi, \pi)$. The two instabilities are thus separated into two domes: one around $\mu = 0$, dominated by $\text{SDW}_\parallel$, and the other around μ_{VH} , dominated by $\text{SDW}_\perp$. This effect becomes more pronounced as λ is tuned to larger values. Based on the previous discussion, we expect that the material candidate $\text{V}_2\text{Te}_2\text{O}$ would fall into the small AM regime. As mentioned in Sec. III, a generalized Stoner instability can also occur if the value of the Hubbard interaction U exceeds a critical threshold. We find that our model can exhibit an instability to an $\text{SDW}_\perp$ at a generic filling, where there is no nesting or the presence of a VH singularity [Fig. 3(a)]. In this case, the absence of the logarithmic temperature scaling, precludes strong competition between different orders and the critical scales can be adequately captured within a simple RPA calculation (see the SM [45]). No superconducting instability is identified within our temperature resolution. This can be understood as the suppression of singlet pairing, in particular at the VH points because of the AM splitting, which violates the condition $\xi_{\mathbf{k}\uparrow} = \xi_{-\mathbf{k}\downarrow}$, thereby suppressing the so-called “Cooper logarithm”. We note, however, equal spin pairing states cannot be excluded on the basis of the above argument, and we cannot rule out that they occur at temperatures lower than our numerical calculations permit.

B. Large AM regime and pair density wave

We now turn to the regime $t/\lambda < 1/2$, where the AM splitting sets the energy scale.

1. f_{xy} splitting

In the f_{xy} case for $U = 0.8\lambda$ and small values of $\mu \lesssim 0.05\lambda$, we find that the instability landscape as function of μ is dominated by the $\text{SDW}_\perp$ as t is tuned to higher values because of opposite spin nesting for both finite t and λ occurring at $\mu = 0$. In a small region in the phase diagram, we also note the occurrence of instabilities towards a longitudinal stripe order, where the divergent channel develops peaks at X and Y. This is

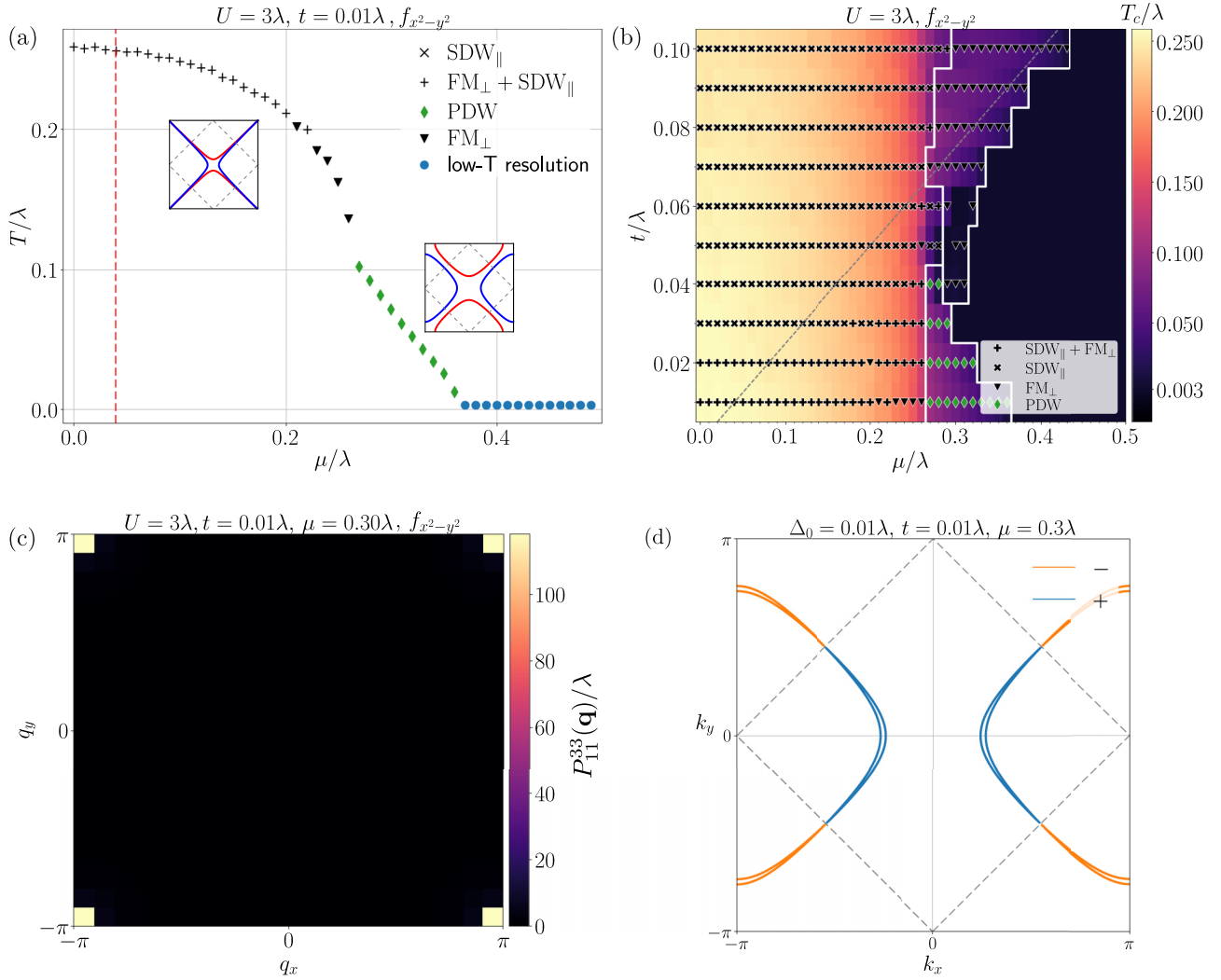


FIG. 4. FRG phase diagram and PDW in the large $f_{x^2-y^2}$ antiferromagnetic regime. (a), (b) Critical temperature T_c of FRG instabilities as function of chemical potential μ for fixed $t = 0.01\lambda$ and varying t . We only show $\mu \geq 0$ caused by particle-hole symmetry. The dashed lines correspond $\mu_{\text{VH}} = 4t$. The insets depict example FSs for the observed instabilities. For $t \lesssim 0.04\lambda$ and for $\mu > 0.27\lambda$, an instability towards a PDW arises. (c) The singular part of the vertex for $t = 0.01\lambda$, $\mu = 0.30\lambda$ at the critical temperature. The singular contribution arises in the $m = 0$ pairing vertex with extended s -wave symmetry $P_{11}^{33}(\mathbf{q})$ and exhibits sharp peaks at $\mathbf{Q} = (\pi, \pi)$. The subscript labels the form factor ($f_{\mathbf{k}}^1 = \cos k_x + \cos k_y$) and superscript the spin vertex component ($t^3 = i\tau^2\tau^3$). See Sec. VI for additional information. (d) The PDW Bogoliubov quasiparticle bands indexed by $\pm$ exhibit a FS at $E = 0$, for small values of the maximal gap magnitude $\Delta_0 \lesssim t$. For $\Delta_0 \gtrsim t$, the Bogoliubov spectrum only possesses nodes at $(k_x^*, k_y^*) = (\pm \arccos \mu/2\lambda, \pm \arccos \mu/2\lambda)$, which is where the two quasiparticle bands touch because of the s' -wave form factor symmetry. These nodes are pinned at $E = 0$ for any $\mu \leq 2\lambda$.

the result of the perfect equal-spin nesting in the particle-hole channel in the limit of $t = 0$, for $\mathbf{Q} = \mathbf{X}$ or $\mathbf{Q} = \mathbf{Y}$.

2. $f_{x^2-y^2}$ splitting

For the $f_{x^2-y^2}$ form factor and small values of t , the system displays a strong competition between a transverse ferromagnetic (FM $_{\perp}$) and a $\mathbf{Q} = \mathbf{M}$ SDW $_{\parallel}$ instability for several values of the chemical potential. In this case, for $\mu = 0$, the Fermi surfaces are given by two almost “flat” (they are perfectly flat if $t = 0$) diamonds centered around the X and Y points and both containing a Van Hove singularity at the M point [see inset of Fig. 4(a)]. For small values of nearest-neighbor hopping $t \leq 0.04\lambda$, we find that the two channels are nearly

degenerate at the level of our FRG analysis. This is the result of a relatively large coupling $U/W = 3/4$, which facilitates a Stoner instability towards FM $_{\perp}$. Furthermore, the transverse ferromagnetic instability is boosted as a result of the interchannel feedback from SDW $_{\parallel}$ enabled by the FRG. For smaller values of U/W , the combination of nesting with a Van Hove singularity strongly favors the formation of a longitudinal SDW over a transverse ferromagnetic instability. We confirm this behavior for smaller values of U , when the effects of the double logarithmic scaling of the particle-hole bubble for $\mathbf{Q} = \mathbf{M}$ are allowed to set in. For $U = 2\lambda$, we find that the dominant instability for a wide range of values of μ is indeed the longitudinal SDW (see the SM [45]). As the chemical potential is tuned further away from zero $\mu \gtrsim 0.3\lambda$

and for larger values of $t \gtrsim 0.05\lambda$, where the VH singularity is found away from $\mu = 0$ and perfect equal-spin nesting is no longer present, $\text{FM}_\perp$ becomes the dominant instability [Fig. 4(b)]. Based on the DFT input, $\text{KV}_2\text{Se}_2\text{O}$ is expected to lie in this regime. We have performed FRG calculations for the two bands closest to Fermi level, which lead to an $\text{SDW}_\parallel$ instability for the relevant values of the chemical potential (see the SM [45]).

Finally, we show that the large AM regime facilitates an exotic pairing state mediated by magnetic fluctuations. For $\mu \gtrsim 0.3\lambda$ and $t \lesssim 0.05\lambda$ [Fig. 4(b)], we observe an instability towards a pair density wave (PDW) in the $m = 0$ triplet pairing channel with a commensurate wave vector $\mathbf{Q} = \mathbf{M}$ [Fig. 4(c)] and an extended s -wave form factor. Thus, we extract the order parameter as

$$\Delta_{\mathbf{Q}} \equiv \sum_{\mathbf{k} \in \text{BZ}} \Delta_0 f_{\mathbf{k}}^s \langle c_{\mathbf{k}+\mathbf{Q}} (i\sigma^y \sigma^z) c_{-\mathbf{k}} \rangle, \quad (10)$$

with $f_{\mathbf{k}}^s = \cos k_x + \cos k_y$.

The unusual combination of extended s -wave symmetry with a triplet pair is a special case for the general exchange symmetry of such a PDW gap function. We find that for unconventional finite- q pairing the gap function must satisfy

$$\Delta_{\mathbf{Q}}^{\sigma\sigma'}(\mathbf{k}) = \langle c_{\mathbf{k}+\mathbf{Q}\sigma} c_{-\mathbf{k}\sigma'} \rangle = -\Delta_{\mathbf{Q}}^{\sigma'\sigma}(-\mathbf{k} + \mathbf{Q}), \quad (11)$$

owing to fermion statistics, i.e., the gap is odd under the combination of spin exchange and momentum inversion plus shift $\mathbf{k} \rightarrow -\mathbf{k} + \mathbf{Q}$. Similar to the SDW states discussed above, the PDW also breaks the lattice translation symmetry, as well as $[C_2 \parallel C_4]$. The unconventional PDW state is similar to η pairing [67–72] or π -triplet pairing [73–76]. However, a crucial difference is the presence of the AM term in the parent Hamiltonian and the unconventional mechanism generating the PDW state as a ground state.

The Bogoliubov spectrum of the PDW state contains nodal points at $(k_x^*, k_y^*) = (\arccos(-\mu/2\lambda), \arccos(\mu/2\lambda))$ and C_4 -symmetry equivalent locations. Since their location depends on the ratio $\mu/2\lambda$, they are always present in the BZ so long as $-1 \leq \mu/2\lambda \leq 1$. For $\Delta_{\mathbf{Q}} \gtrsim t$, these are the only nodal points in the excitation spectrum. However, for $\Delta_{\mathbf{Q}} \lesssim t$, Bogoliubov Fermi surfaces arise [Fig. 4(d)].

The structure of the FS is again insightful in understanding the mechanism driving this particular instability. For $t = 0$, there is perfect nesting in the pairing channel $\xi_{\mathbf{k}+\mathbf{Q},\sigma} = \xi_{-\mathbf{k},-\sigma}$ for any μ . For the values of μ where the dominant instability is towards the PDW, the FS occur in spin-split pockets centered around the X,Y points [Fig. 4(a) inset], which are connected by the high-symmetry wave vector $\mathbf{Q} = \mathbf{M}$. This means that if the electron repulsion is turned attractive, there will be an instability in the Cooper channel for center-of-mass momentum $q = Q$. For a finite t , $\xi_{\mathbf{k}+\mathbf{Q},\sigma} = \xi_{-\mathbf{k},-\sigma}$ no longer holds. However, we expect this effect to still persist for a finite range of $t \lesssim T_c$. Indeed, as can be seen in Fig. 4(d), the PDW still occurs until $t \sim 0.04\lambda$. Analyzing the interchannel feedback within FRG, we find the main mechanism for the attraction to arise from both, $\text{SDW}_\parallel$ and $\text{FM}_\perp$ fluctuations. Thus, it does not necessarily rely on the vicinity to a SDW induced by nesting and a Van Hove singularity because FM fluctuations can also do the job. This can be understood in a spin-fluctuation

approach restricted to projecting the spin vertex into the pairing vertex (see [45] for how this arises within FRG). Close to a magnetic instability, the effective interaction is of the form $V_{\text{spin}} = \sum_{a=1}^3 V_a(\mathbf{q}) S_{\mathbf{q}}^a S_{-\mathbf{q}}^a$ with $S_{\mathbf{q}}^a = \sum_{\mathbf{k}} c_{\mathbf{k}+\mathbf{q}}^\dagger \sigma^a c_{\mathbf{k}}$. This leads to the properly antisymmetrized pairing interaction in the $m = 0$ triplet channel,

$$\begin{aligned} V_{t,m=0}^{\text{pair}}(k, -k+q, k', -k'+q) \\ = \frac{1}{4} [V_x(k-k') + V_y(k-k') - V_z(k-k')] \\ - (k \leftrightarrow -k+q), \end{aligned} \quad (12)$$

which we project into the form factor basis

$$V_{t,m=0}^{\text{pair},l}(q) = \sum_{k,k'} f_l(k) f_l(k') V_{t,m=0}^{\text{pair}}(k, -k+q, k', -k'+q). \quad (13)$$

Furthermore, we can use that for $\text{FM}_\perp$ fluctuations $V_x(q) = V_y(q) < 0$ peaks around $q = 0$ while $V_z \approx 0$, and for $\text{SDW}_\parallel$ fluctuations $V_z(q) < 0$ peaks around $q = Q$ while $V_{x,y} \approx 0$, i.e., we can approximate $V_{x,y}(q) \approx -J\delta_\epsilon(q)$ in the FM case and $V_z(q) \approx -J\delta_\epsilon(q - Q)$ in the SDW case, where $\delta_\epsilon(q)$ is a function that peaks at $q = 0$ with some finite width ϵ . Then, the main contributions in the sum over k, k' come from $k - k' \approx 0$ or $-k + q - k' \approx 0$ in $V_{x,y}$ for FM fluctuations, and from $k - k' \approx Q$ or $-k + q - k' \approx Q$ in V_z for $\text{SDW}_\parallel$ fluctuations. Therefore, the interaction in the PDW channel with wave vector Q and angular momentum l is approximately given by

$$V_{t,m=0}^{\text{pair},l}(Q) \approx J \sum_k f_l(\pm k) [f_l(k \mp Q) - f_l(-k)] \leq 0, \quad (14)$$

where the upper sign is for the SDW and the lower for the FM case. This PDW vertex is nonzero and attractive $V_{t,m=0}^{\text{pair},l} \leq 0$ for even lattice harmonics with $f_l(k \pm Q) = -f_l(k)$. Apart from the unusual pairing symmetry, the mechanisms for inducing a pairing attraction are analogous to the well-known cases of spin-fluctuation mediated pairing. (Transverse) FM fluctuations induce an attraction in the triplet channel. Longitudinal SDW fluctuations need to flip an extra sign [$-V_z$ in Eq. (12)] and do so because the SDW wave vector connects regions of opposite sign of the gap function $f_{\mathbf{k}+\mathbf{Q}}^s = -f_{\mathbf{k}}^s$. We note that other pairing symmetries for the PDW triplet, such as d wave, are subleading (see the SM [45]). Furthermore, a PDW singlet would only be mediated by $\text{SDW}_\parallel$ not by $\text{FM}_\perp$ fluctuations and thus suffers from a smaller attraction. Finally, in the case of large f_{xy} AM, there is also nesting in the pairing channel $\xi_{\mathbf{k}+\mathbf{Q},\sigma} = \xi_{-\mathbf{k},\sigma}$, now for equal spin. However, fluctuations of the $\text{SDW}_\perp$ in the f_{xy} AM phases diagram do not mediate an attraction for PDW triplet pairing, which explains why it is absent in the FRG phase diagram.

V. DISCUSSION

We studied the phase diagram of the 2D Hubbard model on the square lattice in the presence of a d -wave AM as a function of several model parameters within a TUFGR approach. Remarkably, the inclusion of the AM splitting introduces two distinct energy regimes corresponding to small and large AM spin splitting.

Both regimes are feasible in realistic material candidates. The large AM spin splitting in metals was indicated experimentally in 3D RuO₂ [77] or CrSb [7] and in quasi-2D KV₂Se₂O [8]. We note that the tendency to secondary spin density wave described in our work, could be a starting point for understanding the experimental indication of the spin density wave in KV₂Se₂O [8]. The small AM splitting regime discussed for CoS in SI is also experimentally feasible as it can be achieved by various different means including functionalization of more conventional antiferromagnetic monolayers such as antiferromagnetic FeSe [78].

In both regimes, we found that the system as a function of the chemical potential, displays instabilities towards distinct SDW states either along the direction imposed by the AM splitting or perpendicular to it. The SDW states break the background AM $[C_2||C_{4z}]$ symmetry of twofold spin and fourfold spatial rotation. In addition, correlations can induce pairing instabilities akin to the already studied mechanism in the SU(2)-symmetric Hubbard model in the small AM regime.

When the AM dominates over the hopping term, we observe that the system can become unstable to an unconventional triplet PDW with commensurate wave vector $\mathbf{Q}$. This finite-momentum pairing instability is mediated by spin fluctuations and boosted by approximate nesting in the pairing channel $\xi_{-\mathbf{k}+\mathbf{Q},-\sigma} = \xi_{\mathbf{k},\sigma}$ of the $f_{x^2-y^2}$ -wave AM term. Thus, the pairing state is stronger than for other finite-momentum states that rely on small splittings of the FS [34,36,38,79] and a larger critical temperature can be expected. We also showed that for such an unconventional PDW, the antisymmetry of the gap function needs to be adapted to include a momentum shift making spin triplet (singlet), even (odd) angular harmonics possible for appropriate lattice harmonics.

The secondary instabilities we find are induced by the interaction in the AM parent state, which we have included in the form of a Hubbard U . We use it to model the renormalized interaction that remains after the system undergoes the AM transition. The specific choice of the value of U depends on the concrete material to model. Its value affects non-universal quantities such as critical temperatures and phase boundaries with respect to chemical potential μ and the relative strength of AM splitting λ/t . However, most of the instabilities we identified are rooted in universal mechanisms: spin density wave instabilities arise as a result of Fermi surface nesting and/or the presence of Van Hove singularities in the single-particle dispersion. Similarly, the pairing instabilities rely on the Cooper instability either from (approximate) inversion symmetry $\xi_{\mathbf{k},\sigma} \approx \xi_{-\mathbf{k},\sigma'}$ or from (approximate) nesting in the pairing channel $\xi_{\mathbf{k},\sigma} \approx \xi_{-\mathbf{k}+\mathbf{Q},-\sigma}$ with increased critical temperatures due to the vicinity to a VH singularity. As such, these instabilities survive at arbitrarily weak coupling. Yet, we also demonstrated that an instability to an SDW_⊥ can generically be observed at arbitrary filling, because of the spin split FS, provided that the bare value of the interaction is large enough. The effect of longer-range components in the Coulomb interaction is an interesting topic for future studies.

Since the SDWs break translational symmetry, they can be experimentally probed by spin-resolved scanning tunneling microscopy (STM). Furthermore, Fermi surface reconstruction can be observed in ARPES measurements. Unfortunately, the specific PDW we find cannot be seen in STM, as the

superconducting gap only changes sign from one sublattice to the other, there is no spatial modulation. However, signatures for its nodal points or Bogoliubov Fermi surfaces can, e.g., be observed via specific heat, thermal conductivity, or NMR measurements, while its specific nodal structure can be visualized via ARPES. In contrast to a zero-momentum superconducting state, an anisotropy occurs in thermal transport because of the breaking of $[C_2||C_4]$.

Our analysis also suggests several further directions for studying the effects of correlations in altermagnetic systems. It would be interesting to explore how the inclusion of sublattices and the symmetries of the underlying lattice could modify the results presented here. Starting from such a microscopic description would not only be relevant for other material candidates [2], but also for “meta-altermagnets”; a recent theoretical proposal suggested twisted van der Waals materials as a platform for realizing altermagnetism with a high degree of tunability [80]. Furthermore, our approach can be generalized to include other symmetries of AM splitting, such as the g -wave type predicted in Nb₂FeB₂ and Ta₂FeB₂ [81]. In such systems, the coexistence of same-spin Fermi surface nesting and a spin-degenerate VH singularity at $\mu = 0$ suggests a strong competition among different SDW states. Furthermore, a finite t' , leading to FS frustration, could facilitate the emergence of a SC instability close to the VH singularity. In addition, secondary instabilities of odd-partial-wave unconventional spin splitting such as p or f wave [82] could be studied as well within this effective picture.

Generally, our results show that altermagnetic metals can become unstable towards interaction-induced secondary phases. Thus, they can serve as a platform for the realization of exotic many-body states enabled by the unconventional spin splitting of the single-particle bands. It will be particularly interesting to find materials in the large AM regime, which can facilitate the investigation of the commensurate PDW.

VI. METHODS

A. DFT

The electronic structure calculations for KV₂Se₂O, V₂Te₂O and V₂Se₂O were performed using the Vienna *Ab Initio* Simulation Package (VASP) [83,84] with the generalized gradient approximation (GGA) as formulated by Perdew, Burke, and Ernzerhof (PBE) [85]. A plane-wave basis set with a 500-eV kinetic energy cutoff was employed to ensure result accuracy. To account for the strong electron correlations in the V-3d orbitals, the effective Hubbard method was employed with a $U = 4.0$ eV, consistent with previous *ab initio* studies [86–88]. The Brillouin zone was sampled using a Γ -centered $13 \times 13 \times 1$ $\mathbf{k}$ -mesh in the Monkhorst-Pack special scheme [89].

B. TUFRG

To resolve competition between ordering tendencies, we employ a truncated unity FRG approach (TUFRG) [48,49] using a temperature cutoff [50]. The key object of our computations is the so-called vertex function V , which is the one-particle irreducible (1PI), amputated part of the

two-particle Green's function. It corresponds to the sum of all 1PI diagrams with two incoming and two outgoing legs [55,90,91]. We neglect all frequency dependencies in V [92], and parametrize it as

$$V^{\sigma_1\sigma_2\sigma_3\sigma_4}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3; T), \quad (15)$$

where T is the temperature, and $\sigma_i, \mathbf{k}_i$ label the spins and momenta of the incoming and outgoing fermions, respectively. We drop the temperature dependence for brevity, unless otherwise required. The momentum dependence can be further reduced to a channel decomposition [48,65,66,93]

$$\begin{aligned} V^{\sigma_1\sigma_2\sigma_3\sigma_4}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \\ = U(\delta_{\sigma_1\sigma_3}\delta_{\sigma_2\sigma_4} - \delta_{\sigma_1\sigma_4}\delta_{\sigma_2\sigma_3}) + P_{\mathbf{k}_1\mathbf{k}_3}^{\sigma_1\sigma_2\sigma_3\sigma_4}(\mathbf{k}_1 + \mathbf{k}_2) \\ + D_{\mathbf{k}_1\mathbf{k}_4}^{\sigma_1\sigma_2\sigma_3\sigma_4}(\mathbf{k}_3 - \mathbf{k}_1) - D_{\mathbf{k}_1\mathbf{k}_3}^{\sigma_1\sigma_2\sigma_4\sigma_3}(\mathbf{k}_2 - \mathbf{k}_3). \end{aligned} \quad (16)$$

Here, P, D correspond to the particle-particle and particle-hole channels. Notice the momenta appearing both as indices and as arguments in the channels. The latter, often characterized as a “strong” dependence, provides information about the wave vector at which the instability occurs. The former dependence, we expand in form factors. Because of the broken SU(2) symmetry at the single-particle level, we also expand the channels in terms of Pauli matrices. This leads to

$$\begin{aligned} P_{\mathbf{k}\mathbf{k}'}^{\sigma_1\sigma_2\sigma_3\sigma_4}(\mathbf{q}) &= \frac{1}{2} \sum_{\alpha, \beta=0}^3 \sum_{\ell, \ell'} P_{\ell\ell'}^{\alpha\beta}(\mathbf{q}) t_{\sigma_1\sigma_2}^{\alpha} (t_{\sigma_3\sigma_4}^{\beta})^{\dagger} f_{\mathbf{k}}^{\ell} (f_{\mathbf{k}'}^{\ell'})^*, \\ D_{\mathbf{k}\mathbf{k}'}^{\sigma_1\sigma_2\sigma_3\sigma_4}(\mathbf{q}) &= \frac{1}{2} \sum_{\alpha, \beta=0}^3 \sum_{\ell, \ell'} P_{\ell\ell'}^{\alpha\beta}(\mathbf{q}) \tau_{\sigma_1\sigma_2}^{\alpha} \tau_{\sigma_3\sigma_4}^{\beta} f_{\mathbf{k}}^{\ell} (f_{\mathbf{k}'}^{\ell'})^*, \end{aligned} \quad (17)$$

where τ^{α} label the Pauli matrices (including unity), $t^{\alpha} = i\tau^2\tau^{\alpha}$ and $\{f_{\mathbf{k}}^{\ell}\}$ the set of form factors. They constitute a complete basis, i.e.,

$$\sum_{\ell, \ell'} f_{\mathbf{k}}^{\ell} (f_{\mathbf{k}'}^{\ell'})^* = \delta_{\mathbf{k}\mathbf{k}'} \quad (18)$$

We choose $\{f_{\mathbf{k}}^{\ell}\}$ such that they transform under the irreducible representations of C_{4v} and truncate at the first shell of nearest neighbors for our flow equations. That is, we only consider the following basis functions:

$$f_{\mathbf{k}}^0 = 1, \quad (19)$$

$$f_{\mathbf{k}}^1 = \cos k_x + \cos k_y, \quad (20)$$

$$f_{\mathbf{k}}^2 = \cos k_x - \cos k_y, \quad (21)$$

$$f_{\mathbf{k}}^3 = \sqrt{2} \sin k_x, \quad (22)$$

$$f_{\mathbf{k}}^4 = \sqrt{2} \sin k_y. \quad (23)$$

Within the one-loop truncation [50,55,56], we get the following flow equations:

$$\dot{X}_{\ell\ell'}^{\alpha\beta}(\mathbf{q}) = \frac{1}{2} \zeta_X \sum_{\alpha', \beta'} \sum_{m, m'} X[V]_{\ell m'}^{\alpha\beta'}(\mathbf{q}) \dot{\Gamma}_{X, mm'}^{\beta'\alpha'}(\mathbf{q}) X[V]_{m'\ell}^{\alpha'\beta}(\mathbf{q}), \quad (24)$$

where $X = P, D$, $\zeta_D = 2$, $\zeta_P = -1$, and $\dot{X} = \frac{d}{dT}$. $X[V]$ defines the projection of the full vertex onto channel X and reads

$$\begin{aligned} P_{\ell\ell'}^{\alpha\beta}[V](\mathbf{q}) \\ = \frac{1}{2} \int_{\mathbf{k}\mathbf{k}'} \sum_{\substack{\sigma_1, \sigma_2 \\ \sigma_3, \sigma_4}} V^{\sigma_1\sigma_2\sigma_3\sigma_4}(\mathbf{k}, \mathbf{q} - \mathbf{k}, \mathbf{k}') (t_{\sigma_1\sigma_2}^{\alpha})^{\dagger} t_{\sigma_3\sigma_4}^{\beta} f_{\mathbf{k}}^{\ell} f_{\mathbf{k}'}^{\ell'}, \end{aligned} \quad (25)$$

$$\begin{aligned} D_{\ell\ell'}^{\alpha\beta}[V](\mathbf{q}) \\ = \frac{1}{2} \int_{\mathbf{k}\mathbf{k}'} \sum_{\substack{\sigma_1, \sigma_2 \\ \sigma_3, \sigma_4}} V^{\sigma_1\sigma_2\sigma_3\sigma_4}(\mathbf{k}, \mathbf{k}' + \mathbf{q}, \mathbf{k}) \tau_{\sigma_1\sigma_2}^{\alpha} \tau_{\sigma_3\sigma_4}^{\beta} f_{\mathbf{k}}^{\ell} f_{\mathbf{k}'}^{\ell'}. \end{aligned} \quad (26)$$

Furthermore, the fermion loop integrals are given by

$$\Pi_D^{\alpha\beta}(\mathbf{q}) = -\frac{1}{2\beta} \sum_{iv} \int_{\mathbf{k}} \text{tr}[\tau^{\alpha} G(\mathbf{k} + \mathbf{q}, iv; T) \tau^{\beta} G(\mathbf{k}, iv; T)], \quad (27)$$

$$\begin{aligned} \Pi_P^{\alpha\beta}(\mathbf{q}) \\ = -\frac{1}{2\beta} \sum_{iv} \int_{\mathbf{k}} \text{tr}[t^{\alpha\dagger} G(\mathbf{q} - \mathbf{k}, -iv; T) t^{\beta} G(\mathbf{k}, iv; T)], \end{aligned} \quad (28)$$

where G labels the free propagator, renormalized by the temperature

$$G(\mathbf{k}, iv; T) = T^{1/2}(iv - \xi_{\mathbf{k}\sigma})^{-1}. \quad (29)$$

We integrate the flow equations given by Eq. (24), until a temperature T_c where the divergence condition $\max\{X\} \geq 100$ is met (given in units of t or λ depending on the regime studied) or until the numerical integration of the bubbles becomes unstable, which for our calculations is $T_s = 3 \times 10^{-3}(t \text{ or } \lambda)$. For the RPA calculations, the integration (nonadaptive) was performed on a 2401×2401 grid, with a temperature resolution of $T_s = 8 \times 10^{-3}(t \text{ or } \lambda)$.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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