

Nonlinearities in the Hall resistance by retroreflective scattering

Frederik Bartels^{1,†}, Beate Horn-Cosfeld^{1,*}, Jakob Schluck^{1,†}, Mihai Cercez¹, Dominique Mailly², Klaus Pierz³, Hans W. Schumacher³, Benjamin Sanvee⁴, Jürgen Horbach⁴, and Thomas Heinzel^{1,‡}

¹*Institut für Experimentelle Physik der kondensierten Materie, Heinrich-Heine-Universität, Universitätsstraße 1, 40225 Düsseldorf, Germany*

²*Université Paris-Saclay, CNRS, Centre de Nanosciences et de Nanotechnologies, 91120 Palaiseau, France*

³*Physikalisch-Technische Bundesanstalt, Bundesallee 100, 38116 Braunschweig, Germany*

⁴*Institut für Theoretische Physik II, Heinrich-Heine-Universität Düsseldorf, Universitätsstraße 1, 40225 Düsseldorf, Germany*



(Received 2 December 2025; revised 11 March 2026; accepted 30 March 2026; published 21 April 2026)

Nonlinearities in the classical Hall resistance of two-dimensional electron gases exposed to retroreflective obstacles at random positions are reported. The Hall slope develops a nonmonotonic component, which increases strongly in amplitude and shifts to larger magnetic fields as the obstacle density increases. At small magnetic fields, the relation between the longitudinal and transverse conductivity remains in qualitative agreement with the Drude-Boltzmann equations, although the characteristic timescale can no longer be interpreted as a momentum relaxation time. The corresponding Hall conductivity develops a marked minimum at a characteristic magnetic field outside the interval where the data agree qualitatively with the Drude-Boltzmann model. With the help of classical event-driven simulations, this minimum is attributed to successive retroreflections which lead to inverted transverse deflection.

DOI: [10.1103/lg7z-nrjy](https://doi.org/10.1103/lg7z-nrjy)

I. INTRODUCTION

The Hall effect [1] is one of the hallmarks of electron transport in solids. Besides being used routinely to measure the density and the sign of the carriers as well as to detect magnetic fields, it has played a central role in the field of quantum electronics and continues to do so. Examples are the integer [2] and fractional [3] quantum Hall effects as well as spin Hall effects [4]. In the classical regime, i.e., in magnetic fields for which Landau quantization is negligible, the Hall slope is usually constant and $\propto n_e^{-1}$, where n_e denotes the electron density, with remarkable robustness. For example, neither the weak localization correction [5] nor the electron-electron interactions [6] or the temperature [7] cause strong deviations from this behavior. A notable exception are ferromagnetic materials which show the anomalous Hall effect, a quantum version of which is observed in magnetic topological insulators [8]. Some mechanisms leading to a nonlinear Hall slope have been reported, in particular, in systems close to a metal-insulator transition [9,10], the interaction of electrons with localized spins [11], or in antidot lattices [12], among others [13–15].

Here, we report a mechanism leading to a nonlinear Hall resistance which has hitherto remained unnoticed and originates from retroreflective electron scattering. We demonstrate it in a two-dimensional Lorentz gas (2DLG), realized experimentally in the form of a two-dimensional electron gas (2DEG) with a high density of lithographically defined Swiss-cross-shaped scattering centers. Recently, we studied the Hall effect in a 2DLG with randomly distributed obstacles of circular shape at different obstacle density, using a combination of classical molecular dynamics (MD) simulation and experiment [16]. In this work, we quantitatively identified the regime of very low obstacle densities and sufficiently small magnetic fields B where the classical Drude-Boltzmann (DB) theory holds, implying a Hall coefficient $R_H \propto n_e^{-1}$. We refer to the obstacle density via the dimensionless density n^* , defined as the fraction of the area covered by the obstacles in the absence of mutual overlaps. The deviations to DB theory at higher densities and magnetic fields could be attributed to classical memory effects, i.e., non-Markovian effects for which the assumption of successive collisions being uncorrelated and independent of each other no longer holds. However, as shown in Ref. [16] for circular obstacles, over a significant range of n^* and B a relationship between the magnetoconductivity σ_{xx} and the Hall conductivity σ_{xy} holds

$$\sigma_{xy} = \omega_{cy} \tau \sigma_{xx}, \quad (1)$$

with ω_{cy} the cyclotron frequency and τ a timescale. This relation is taken from DB theory but with an effective time τ that differs from the mean free time between collisions, τ_{Dr} , in DB theory. The time τ can be interpreted as a diffusive timescale since it has a similar dependence on density n^* as the conductivity $\sigma_{xx}(B=0)$ at zero magnetic field [16]. Moreover, the analysis in Ref. [16] showed that the Hall coefficient

*Present address: MWIKE des Landes Nordrhein-Westfalen, Berger Allee 25, 40213 Düsseldorf, Germany.

†Present address: II. Physikalisches Institut, Universität zu Köln, Zùlpicher Strasse 77, 50937 Köln, Germany.

‡Contact author: Thomas.Heinzel@hhu.de

was proportional to $\tau/\sigma_{xx}(B=0)$ and this ratio had a very weak dependence on n^* . This explains the robustness of the linear behavior of the Hall resistance ρ_{xy} as a function of the magnetic field $\rho_{xy} = R_H B$, albeit with increasing obstacle density n^* the Hall coefficient differs more and more from the one predicted by Drude theory and thus it does not provide a good estimate of the electron density n_e . The ansatz, given by Eq. (1), becomes questionable for a certain class of 2DLGs with random arrangements of noncircular obstacles where the conductivity σ_{xx} at $B=0$ is zero at any finite obstacle density n^* . This is the case in 2DLGs with overlapping square-shaped obstacles (Ehrenfests' wind-tree model) [17–20] as well as for 2DLGs with obstacles in the shape of Swiss crosses [21]. In these systems, the motion of the tracer particles is dominated by retroreflective paths where the particle frequently almost returns to points where it was already at an earlier time. As a consequence, the transport of the tracer particle is asymptotically subdiffusive. Therefore, its diffusion coefficient and conductivity σ_{xx} vanish. In the presence of a small magnetic field B , one observes an anomalous behavior of $\sigma_{xx}(B)$. As shown for overlapping obstacles with a Swiss cross shape [21], the conductivity $\sigma_{xx}(B)$ in the limit $B \rightarrow 0$ follows a power law $\sigma_{xx} \propto B^\delta$ with a small exponent $\delta < 0.22$, the precise value of which depends on the obstacle density and the orientation of the cross-shaped obstacles (i.e., whether they are equally or randomly oriented). Thus, especially for small B fields Eq. (1) fails—at least in the simulation of 2DLGs with Swiss crosses. The situation is different in the experiment where background scattering (e.g., due to impurities) cannot be avoided. As a consequence, the transport is no longer dominated by retroreflective paths for low densities of the artificial obstacles. The transport is diffusive at $B=0$ and, as shown below, does not show the aforementioned power-law behavior at small B fields. However, it is not clear to what extent the ansatz, as given by Eq. (1), can be used at all to analyze the Hall effect in experimental 2DLGs with Swiss crosses.

In the present work, we comprehensively describe the influence of retroreflectors on the transport properties. The same set of Lorentz arrays was already used in Ref. [21], but only in the range of small magnetic fields and in the limit $B \rightarrow 0$ as described above, and only with respect to the longitudinal conductivity σ_{xx} . The focus here is now on the influence of retroreflectors on the Hall conductivity σ_{xy} and the Hall resistance ρ_{xy} . We experimentally observe and numerically reproduce deviations of the Hall effect from the DB model in Lorentz gases with Swiss-cross-shaped scatterers and show the extent to which the approach from Eq. (1) is valid. These deviations are attributed to the retroreflective character of the scatterers, which is in line with the fact that they have not been observed in Lorentz gas arrays formed by circular scatterers [16] with comparable effective densities n^* . Using numerical simulations, we identify characteristic electron trajectories that lead to these effects.

In Sec. II, we present the sample layout and experimental details. Section III describes our simulation concept. In Sec. IV, we present the experimental results together with our numerical calculations and their comparison. Finally, the paper concludes with a summary and outlook in Sec. V.

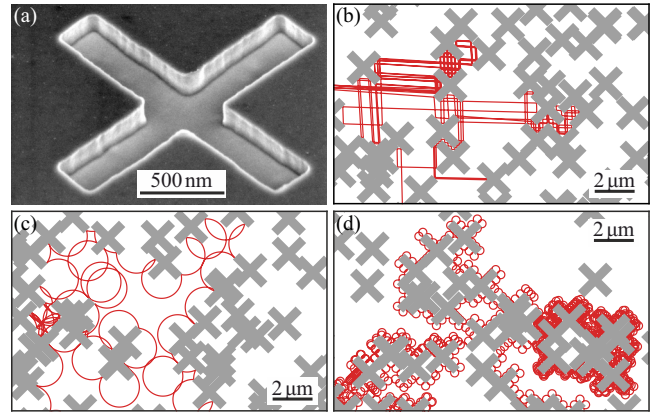


FIG. 1. In (a), a scanning electron microscopy picture of a single Swiss cross, taken under a tilt angle of $\pi/4$ is shown. (b)–(d) Visualizations of typical simulated electron trajectories (red) for $n^* = 0.4$, shown at magnetic fields B of (b) 6.9×10^{-5} T ($\tilde{B} = 8.23 \times 10^{-3}$), (c) 8.34×10^{-2} T ($\tilde{B} = 1$), and (d) 4.2×10^{-1} T ($\tilde{B} = 5$).

II. SAMPLE PREPARATION AND EXPERIMENTAL SETUP

The raw material is a GaAs/Al_{0.3}Ga_{0.7}As heterostructure with a two-dimensional electron gas (2DEG) 150 nm below the surface. At liquid helium temperatures, the pristine 2DEG has an electron density of $n_e = 2.55 \times 10^{15} \text{ m}^{-2}$, as determined from the Hall slope and the Shubnikov–de Haas (SdH) oscillations, see the Supplemental Material [22]. The electron mobility is slightly cooldown-dependent and varies somewhat across the wafer, with values close to $\mu \approx 700 \text{ m}^2/\text{Vs}$, corresponding to a Fermi velocity of $v_F \approx 2.19 \times 10^5 \text{ m/s}$ and an elastic mean free path of $\lambda \approx 60 \mu\text{m}$. A quantum scattering length of $\lambda_q = 436 \text{ nm}$ is obtained using the standard analysis of the envelope of the SdH oscillations [23]. The large ratio of $\lambda/\lambda_q = 135$ indicates that remote ionized impurities from the doping layer are the primary source of scattering, and that intrinsic strong scatterers are scarce.

The retroreflectors are defined by electron beam lithography and subsequent reactive ion etching, see Fig. 1(a) for a scanning electron microscopy picture. They are distributed at random positions in the x and y directions across a Hall bar, prepared by optical lithography, of $120 \mu\text{m}$ length and $100 \mu\text{m}$ width. Alloyed Au/AuGe Ohmic contacts allow electrical access to the 2DEG. Voltage probes of $10 \mu\text{m}$ width, with a spacing of $100 \mu\text{m}$ along the edge of the Hall bar, enable four-terminal measurements. The etching step generates nearly vertical walls of 100-nm depth. The retroreflectors are regarded as scatterers with hard walls of infinite height. By commensurability oscillation experiments on antidot lattices prepared by the same etching technique, the lateral depletion length at the edges has been estimated to $\approx 75 \text{ nm}$ [24], such that the electronic size of one retroreflector exceeds its lithographic size accordingly. One obstacle is composed of two rectangles of size $300 \text{ nm} \times 1850 \text{ nm}$ each, which are oriented perpendicular to each other. Taking the lateral depletion length into account, the effective length of a retroreflector amounts to $2 \mu\text{m}$ covering an area of approximately $1.6 \mu\text{m}^2$. Four separate arrays with number densities $n_s = 0.0625, 0.125, 0.1875$, and $0.25 \mu\text{m}^{-2}$ are studied, corresponding to dimensionless den-

ties $n^* = 0.1, 0.2, 0.3$, and 0.4 . The edges of the retroreflectors form an angle of $\pi/4$ with respect to the current (x) direction.

The transport measurements are carried out in a $^3\text{He}/^4\text{He}$ dilution refrigerator with a base temperature of 25 mK, which corresponds to an electron temperature of about 100 mK, as well as in a ^4He gas flow cryostat for temperatures above 1.2 K. Both cryostats are equipped with superconducting solenoids with maximum magnetic fields above $B = 8$ T. An AC current of amplitude 500 nA and a frequency of 17.7 Hz is applied, and the voltage drops across the arrays in longitudinal (x) and transverse (y) directions are recorded via the voltage probes using lock-in amplifiers (SR 830 DSP from Stanford Instruments). All data shown are symmetrized for ρ_{xx} and antisymmetrized for ρ_{xy} to eliminate possible mutual admixtures that may be generated, for example, by the obstacles close to the voltage probes. The numerical derivatives of the antisymmetrized measurements of ρ_{xy} were used to visualize the Hall slopes $d\rho_{xy}/dB$.

III. SIMULATION CONCEPT

In addition to the experiments, classical event-driven molecular dynamics simulations [16,25] were carried out. Point-like particles were placed randomly in a simulation square with periodic boundary conditions containing the randomly distributed obstacles with identical parameters as the experimental arrays. The number of obstacles is fixed to 10^5 , which results in a size of the simulation square that depends on the obstacle density. Possible effects due to the sample edges are thus not covered by the simulations. The particles are exposed to a uniform magnetic field $\vec{B} = B\hat{z}$ perpendicular to the plane of the 2DEG, resulting in trajectories composed of sections of cyclotron orbits with radius $r_{cy} = m^*v_F/eB$, with their ends determined by specular scattering at the obstacles. For each magnetic field, 10^5 independent electron trajectories are calculated in 200 different host structures. For the simulation, the elementary charge $e \equiv 1.0$, the effective mass $m^* \equiv 1.0$, and the Fermi velocity $v_F \equiv 1.0$ were defined as dimensionless quantities, leading to a reduced magnetic field $\tilde{B} = B/B_0$ with $B_0 = m^*v_F/ed_0 = 83.4$ mT with a length scale $d_0 = 1 \mu\text{m}$ and a timescale $t_0 = \frac{d_0}{v_F} = 4.57$ ps. Note that since all electrons move with the Fermi velocity, the simulated temperature equals zero. We compute the components $\sigma_{ij}(B)$, $i, j \in \{x, y\}$ of the magnetoconductivity tensor based on the Kubo expression

$$\sigma_{ij}(B) = \frac{m^*e^2}{\pi\hbar^2} \int_{t=0}^{\infty} \langle v_i(0)v_j(B, t) \rangle dt, \quad (2)$$

where $\langle v_i(0)v_j(B, t) \rangle$ denotes the velocity correlation function, averaged over an ensemble of trajectories as well as over the initial time within each trajectory. For comparison with the experiments, the values are converted to the experimental units using $v_F = 2.19 \times 10^5$ m/s and $m^* = 0.067m_e$. The components of the resistance tensor can be determined for an isotropic system via

$$\rho_{ij}(B) = \frac{\sigma_{ij}(B)}{\sigma_{xx}^2(B) + \sigma_{xy}^2(B)}. \quad (3)$$

Furthermore, we characterize the diffusive transport behavior of the particles using the mean-squared displacement (MSD),

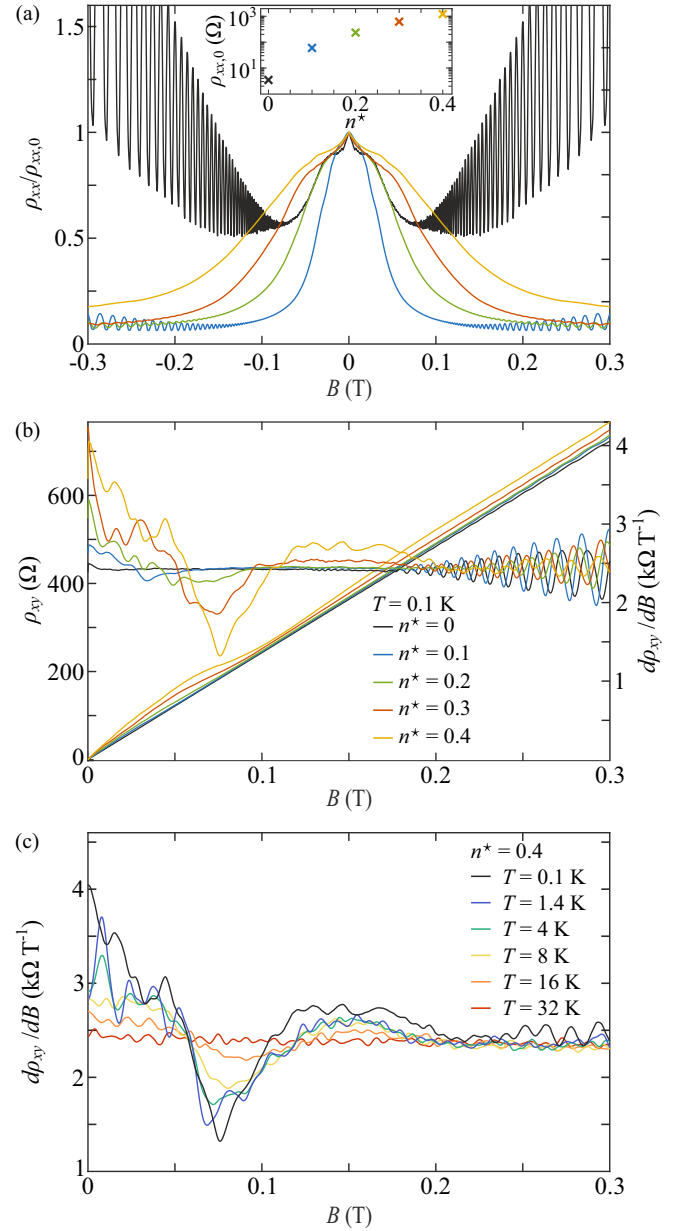


FIG. 2. In (a) the longitudinal magnetoresistances $\rho_{xx}(B)$, normalized to its value at $B = 0$, $\rho_{xx,0}$, is shown. The legend is given in (b). The inset shows the values of $\rho_{xx,0}$ for all effective obstacle densities n^* . The measurements are carried out at base temperature of a dilution refrigerator and the electron temperature is at approximately 100 mK. In (b), the corresponding Hall resistances $\rho_{xy}(B)$ and their numerical derivatives are presented. The temperature dependence of the Hall slope $d\rho_{xy}/dB(B)$ for the array with $n^* = 0.4$ is shown in (c).

defined as $\delta r^2 = \langle [\vec{r}(t) - \vec{r}(0)]^2 \rangle$, where $\vec{r}(t)$ is the time-dependent position of the electron.

IV. RESULTS AND INTERPRETATION

The measured magnetoresistances $\rho_{xx}(B)$ and $\rho_{xy}(B)$ at a temperature of $T = 0.1$ K are represented in Fig. 2. For the pristine 2DEG [black traces in Figs. 2(a) and 2(b)], $\rho_{xx}(B)$ shows the well-known negative magnetoresistance termed

Giant negative magnetoresistance (GNMR) with its two components that originate from electron-electron interactions in a smooth disorder potential landscape (the broad shoulder) and from strong scattering at sparse defects (the narrow peak on top) [26–32]. For $n_s > 0$, a negative magnetoresistance is still observed, albeit with a changed character since its properties are governed by the Lorentz obstacles. It is approximately linear for $B \rightarrow 0$, an effect attributed to retroreflection at individual obstacles [33]. A strong negative magnetoresistance takes over at larger B fields due to the Lorentz-gas-specific scattering discussed elsewhere [33–35]. In the quantum regime ($\omega_{cy}\tau > 1$), the SdH oscillations set in, damped by the Lorentz obstacles.

The Hall resistances at the focus of the present study are shown in Fig. 2(b). The 2DLG arrays show a pronounced kink in $\rho_{xy}(B)$ which increases in strength and shifts to larger B as n^* increases. The derivatives of $\rho_{xy}(B)$ with respect to B reveal additional details. Near $B = 0$, the Hall slopes are significantly enhanced, an effect that gets stronger with increasing n^* . A minimum of $d\rho_{xy}(B)/dB$ is found at magnetic fields where the cyclotron diameter ranges in between the size $2d_0$ of the obstacles and $\ell_s = n^{-1/2}$, the mean distance between two obstacle centers. At intermediate B close to 145 mT, the Hall slope develops a maximum, and it becomes independent of n^* , approaching the value expected for this electron density, at higher magnetic fields, i.e., for $r_{cy} \lesssim 0.4d_0$. Furthermore, the data reveal an additional fine-structure well above the noise level, which we interpret as a magnetofingerprint of our arrays due to their finite extension, characterizing the particular configuration of the obstacles inside the Hall bar. The nonlinearity is quite robust with respect to temperature, remaining visible up to 16 K in the array with $n^* = 0.4$, see Fig. 2(c). This indicates a classical origin. For $T = 32$ K and above, the validity of the DB model is recovered with respect to $\rho_{xy}(B)$ due to the momentum-relaxing scattering properties of the phonon bath [16].

In Fig. 3, we compare the conductivities for two arrays formed by circles and by the retroreflectors with identical $n^* = 0.4$ and obstacle size (the diameter of a circle and the length of one rectangle both equal $2\mu\text{m}$). The properties of $\sigma_{xx}(B)$, see Fig. 3(a), for both obstacle shapes have been discussed in detail elsewhere for comparable samples [21,36]. The Hall conductivity $\sigma_{xy}(B)$ of the retroreflective array, which is at the focus of the present study, shows a marked flattening as compared to that one of the Lorentz gas formed by circles [Fig. 3(b)], in qualitative disagreement with the DB model. To determine the range of magnetic fields in which the DB model is valid at least qualitatively, we use Eq. (1) as a criterion and fit the experimental data for $\sigma_{xx}(B)$ to those for $\sigma_{xy}(B)$ using τ as the fit parameter, see Fig. 4. For the pristine 2DEG, this plot shows that the DB model describes the experimental traces very well over the full classical range, i.e., up to the onset of the SdH oscillations. For the Lorentz arrays of all densities studied, the assumption of a B -independent scattering time holds only up to $\omega_{cy}\tau \approx 0.5$, well below the maximum of the Hall conductivity. Quantitatively, the fit value obtained for τ is strongly suppressed and decreases algebraically with increasing n^* , with $\tau \propto (n^*)^{-1.9}$. For $n^* = 0.4$, for example, $\tau = 1$ ps corresponds to a traveling distance of 250 nm, much smaller than $\ell_s = 2\mu\text{m}$.

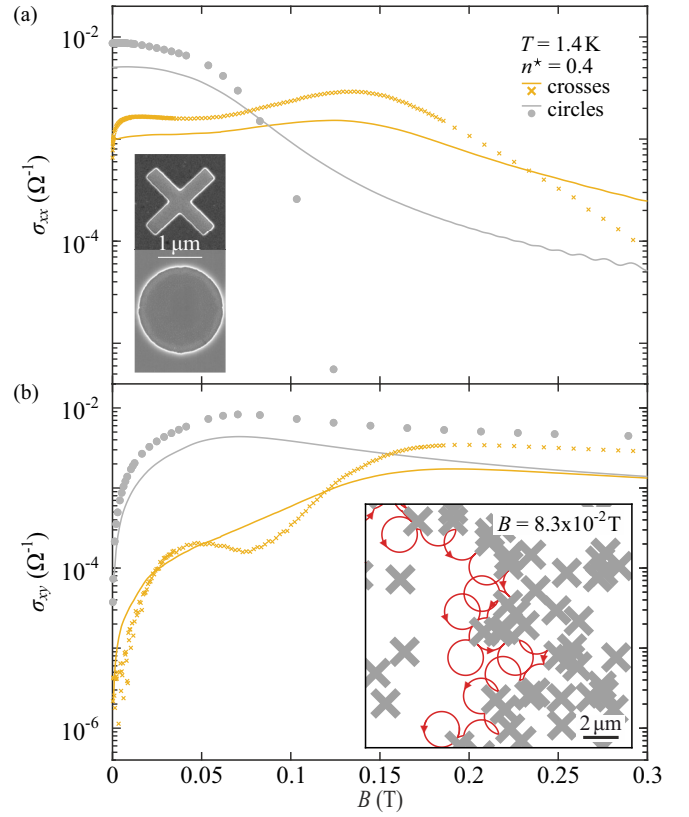


FIG. 3. Comparison of the conductivities of two different samples with $n^* = 0.4$ composed by obstacles in the shape of Swiss crosses (orange) and circles (gray). Both the experimental data (solid lines) and MD simulations (markers) are shown. Upper inset: Scanning electron microscope pictures of one single obstacle each. Lower inset: Exemplifying simulated electron trajectory (red) for $B = B_0 = 83.4$ mT. The magnetic field is directed out of the plane. The data for the circular obstacles are linked to a previous study [16], where the additional nonlinearity described here remained unnoticed.

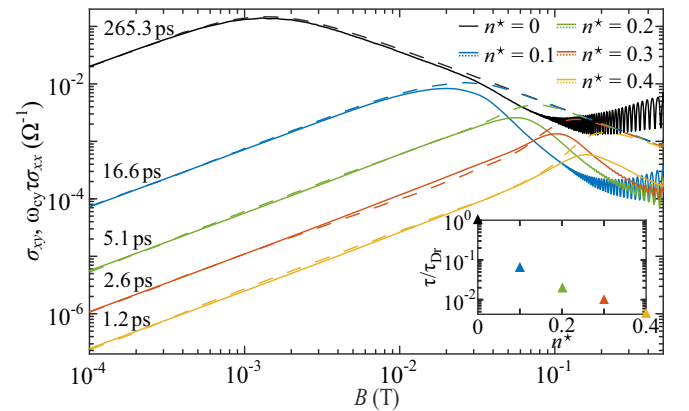


FIG. 4. Hall conductivities $\sigma_{xy}(B)$ (dashed lines) and $\omega_{cy}\tau\sigma_{xx}$ (solid lines) as a function of the magnetic field B for different obstacle densities for the experiment. The diffusive timescale τ , written above the respective trace, is extracted with Eq. (1). The inset shows the ratio τ/τ_{Dr} , where τ_{Dr} is the Drude scattering time for the pristine sample extracted from $\rho_{xx}(B = 0)$.

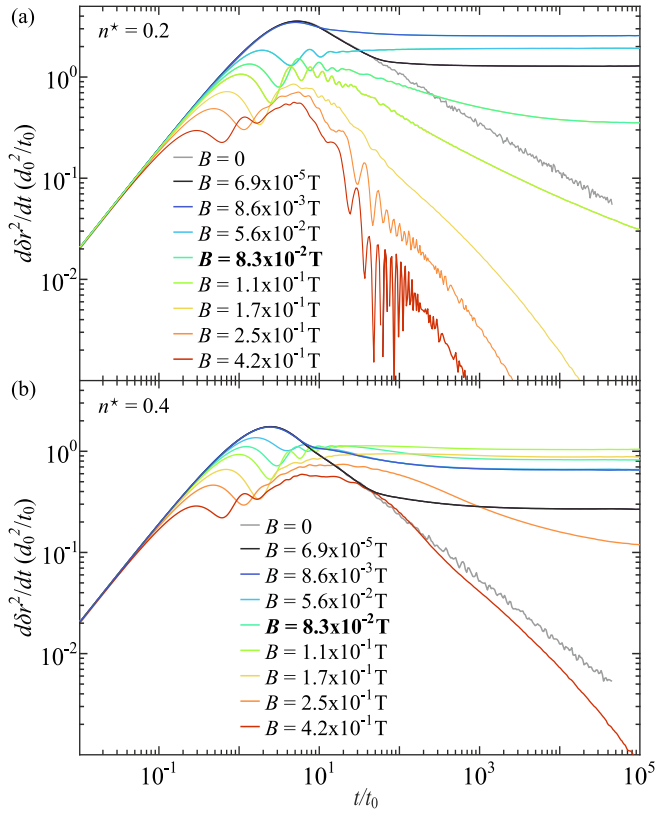


FIG. 5. Time derivative of the MSD, $d\delta r^2(t)/dt$, for (a) $n^* = 0.2$ and (b) $n^* = 0.4$ in different magnetic fields. The magnetic field printed in bold equals $B_0 = 1$. The time is normalized to the reduced timescale $t_0 = d_0/v_F = 4.57$ ps. The data for the case $B = 0$ are used from a previous publication [21].

For higher magnetic fields, the failure of Eq. (1) implies a B -dependent diffusive timescale [16]. It should be noted that in the Lorentz gases without background scattering as considered in the simulations, such a plot is impossible due to the anomalous magnetoconductivity mentioned above, and $\sigma_{xx} \rightarrow 0$ as B approaches zero [21]. Thus, the background scattering present in the experiments renders Eq. (1) to be valid qualitatively for small magnetic fields.

The interpretation of the qualitative failure of the DB model in terms of a B -dependent scattering time can be substantiated by MSD calculations. Figure 5 shows the time derivative $d\delta r^2/dt$ of the MSD as a function of time for $n^* = 0.2$ and $n^* = 0.4$ in various magnetic fields. In the case $B = 0$, $d\delta r^2/dt$ decays at long times which shows an asymptotic subdiffusive behavior as shown before [21]. For sufficiently low magnetic fields, $d\delta r^2/dt$ follows the trace at $B = 0$ and approaches, after passing through a global maximum, a constant value in the long time limit corresponding to the diffusion constant $D = \frac{1}{4} \lim_{t \rightarrow \infty} [d\delta r^2(t)/dt]$. For intermediate magnetic fields, $d\delta r^2/dt$ no longer reaches the global maximum. Rather, at timescales around t_0 , n^* -independent oscillations occur, before the convergence. At higher magnetic fields, $d\delta r^2/dt$ approaches zero in the long time limit, indicating localization of the electrons in the absence of sample edges and background scattering.

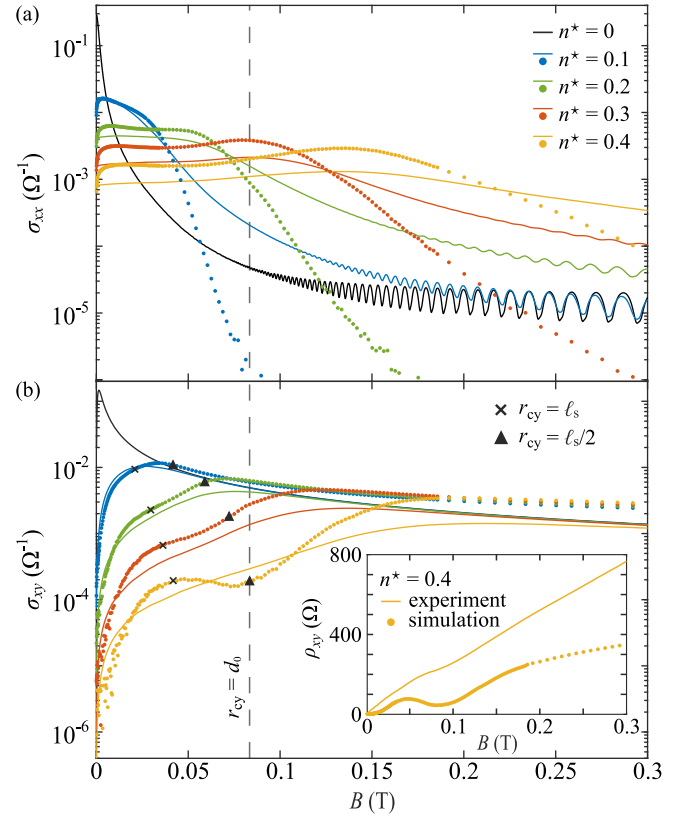


FIG. 6. (a) Magnetoconductivity σ_{xx} as a function of B from the measurements at base temperature (solid lines) and from the simulations (dashed lines). In (b) for the Hall conductivity $\sigma_{xy}(B)$. The markers indicate the positions where $r_{cy} = \ell_s$ and $r_{cy} = \ell_s/2$, respectively. The gray dashed vertical line shows the magnetic field where $r_{cy} = d_0$. Inset: Comparison between measured Hall resistance and resistance calculated by simulations for obstacle density $n^* = 0.4$.

We proceed by comparing the measurements to the classical transport simulations as summarized in Fig. 6. The simulations agree very well in the range of magnetic fields below the Landau quantization regime, and above very small magnetic fields where the background scattering becomes dominant, i.e., in the interval $1 \text{ mT} \lesssim B \lesssim 200 \text{ mT}$. This, in combination with the data at higher temperatures [see Fig. 2(c)], provides clear evidence of a classical origin. The two maxima visible in $\sigma_{xx}(B)$, as well as further properties of this function like their sharp drop towards zero as B approaches zero [Fig. 6(a)], have been reported and discussed in detail elsewhere [21], whereby the first maximum at small magnetic fields and the sharp drop near $B \rightarrow 0$ can be attributed to the retroreflective properties of the scatterers. For the pristine Hall bar (this case was not simulated), the measured $\sigma_{xy}(B)$ [black trace in Fig. 6(b)] agrees very well with the analytical solution of the Drude-Boltzmann model, showing the characteristic peak at $\omega_{cy}\tau = 1$ located at $B = 1.4 \text{ mT}$. As n^* is increased, this peak is suppressed and moves towards larger magnetic fields. In addition, the peak flattens, finally vanishes in the measurements, and evolves into a dip for large n^* in the simulated traces, respectively. In this magnetic field interval where the peak is suppressed, the cyclotron radius ranges in between $\approx \ell_s$ and $\approx d_0$. We attribute the attenuation

of this effect in the measurements as compared to the simulations to momentum-relaxing background scattering which smears out obstacle-specific memory effects, as well as to the inevitable smearing of the electric obstacle shape as compared to the lithographical one, due to lateral depletion in the 2DEG.

The characteristic B -field at which the obstacles' influence on the Hall effect is most pronounced deviates from the position of the main maximum in $\sigma_{xx}(B)$. This maximum corresponds to the most effective longitudinal transport of the electrons, composed of skipping along the edge of obstacles and clusters thereof and hops to obstacles in the neighborhood. For smaller magnetic fields, the skipping motion softens, while for larger B , it becomes less likely to change the obstacle cluster due to the smaller cyclotron radius [36]. Apparently, this particular efficient transfer mechanism of the electrons in the forward direction does not correspond directly to the suppression of the Hall conductivity. Furthermore, the flattened/indented interval of $\sigma_{xy}(B)$ broadens with increasing n^* and moves towards larger magnetic fields. At our highest obstacle density, where $r_{cy} = \ell_s/2 = d_0$, the fact that the dip in σ_{xy} is centered at this position appears to be a coincidence. This phenomenology suggests that this suppression of σ_{xy} is not solely a single-obstacle phenomenon, but also depends on the interobstacle distance.

To develop a picture of the underlying kinetics, we note that there must be a class of trajectories present that weakens the overall deflection of the electrons entering from the left towards the Hall (positive y) direction. Such trajectories can be found by inspecting the individual computed trajectories. Consider an electron injected from the left. In the case of $B = 0$, two successive reflections at an obstacle correspond to an exact retroreflection. In nonzero small and intermediate magnetic fields [see Fig. 1(b)–1(c)], this retroreflection develops into an enhanced backscattering with a corresponding offset of the electron guiding center towards the $-y$ direction by $\leq 2r_{cy}$. At small magnetic fields, such an electron flies away from the obstacle in an approximately straight line (on the scale of d_0), such that the probability of hitting the subsequent obstacle from the right is high, and the offsets of the guiding centers, will cancel out. At large magnetic fields [see Fig. 1(d)], on the other hand, the strong $\vec{E} \times \vec{B}$ drift guides the electrons along a contour of the obstacle and the effect vanishes as well. At intermediate magnetic fields with $r_{cy} \approx d_0$, the electron trajectories are bent towards the left edge of a subsequent obstacle and a series of downward shifts of the guiding center results. Since the probability for hitting the subsequent obstacle from the same side is larger than that one for reaching the opposite side, an average downward shift of the electrons emerges. A typical example is shown in the inset of Fig. 3(b). Likewise, electrons entering from the right experience an increased probability for being scattered in the $+y$ direction. This contributes to the suppression of σ_{xy} in the same way [recall that the Hall conductance is obtained from the velocity correlation function $\langle v_x(0)v_y(t) \rangle$, see Eq. (2)].

This picture is in qualitative agreement with all other related observations. First of all, this suppression of $\sigma_{xy}(B)$ looks similar and is only marginally weaker in retroreflector Lorentz gases with random orientation of the Swiss crosses (see Ref. [22]). This is expected considering that the retroreflective character of the obstacles is orientation independent.

Second, as n^* is increased, the density of such retroreflective cones increases superlinearly, since overlapping obstacles form additional cones albeit with a broad distribution of edge lengths. This may cause the strong increase of the σ_{xy} suppression with n^* , as well as the broadening of the structure and its shift towards larger B . The superposition of obstacles may also explain the weak visibility of this effect in Lorentz arrays featuring circular obstacles at large number densities, as can be seen in Fig. 8(b) of Ref. [16] for the highest density of circular obstacles. Regarding the transformation of these structures into $\rho_{xy}(B)$, please note that the suppression of σ_{xy} predominantly affects the numerator of Eq. (3) since $\sigma_{xy} \ll \sigma_{xx}$ holds in this interval and the denominator is therefore dominated by σ_{xx} , which is roughly independent of B in here. The kink measured in $\rho_{xy}(B)$ can thus be interpreted as a transformed representation of the suppression mechanism discussed above.

Finally, we note that the Hall resistance obtained from the simulations for B close zero is strongly suppressed and shows an approximately parabolic dependence on B with positive curvature. This suppression could not be verified experimentally, most likely due to background scattering. This behavior has not been observed in simulated arrays of the same obstacles with random orientation [22], and we therefore attribute it to electrons that get captured transiently in cavities formed by obstacles close to each other. This transient localization gets weaker with increasing B , when the curved trajectories reduce the capture times. Since this effect could not be observed experimentally, a more detailed discussion is beyond the scope of the present work.

V. SUMMARY AND OUTLOOK

We studied the influence of retroreflective, Swiss-cross-shaped scatterers on the Hall effect of a 2DEG using a combination of experiments and simulations. The Hall slope is not constant in the range of classical magnetic fields, rendering the extraction of the carrier density unreliable. Numerical simulations show that these deviations are caused by a suppression of $\sigma_{xy}(B)$ in the regime where the cyclotron radius is comparable to the obstacle size. The amplitude of the suppression increases superlinear with the obstacle density, and the center of the suppression interval shifts towards larger B . As a qualitative interpretation, we argued that the frequently occurring paired scattering events at the obstacles lead to a significant transverse shift of the electron guiding center in the direction opposite to the conventional Hall deflection. Within this picture, the superlinear amplitude of the suppression and the shift of the suppressed interval to larger B with increasing obstacle density follows naturally from the formation of additional retroreflective cones by overlapping obstacles. Future work may comprise a detailed study of the hitherto unobserved suppression of the Hall effect at very small magnetic fields, as well as possible amplification of the nonlinearities by an optimized size and/or the aspect ratio of the obstacles. In addition, the influence of such retroreflective scatterers on Hall field induced resistance oscillations (HIROs) [37,38] would be interesting, as they arise from backscattering at impurities. Moreover, the strict linearity of the Hall resistivity is the foundation of determining the carrier density in many materials and devices containing free electrons. Depending on the

density of such scatterers and the level of accuracy required for the carrier density, our work shows one possible origin of deviations from the strict linearity predicted by the DB model.

ACKNOWLEDGMENTS

We thank Dr. E. Neumann (HNF, Forschungszentrum Jülich) for providing the scanning electron microscopy pic-

tures of the obstacles. We are grateful to M. Hund for his support of the measurements in the dilution refrigerator.

DATA AVAILABILITY

The data that support the findings of this article are openly available [39].

-
- [1] E. H. Hall, On a new action of the magnet on electric currents, *American Journal of Mathematics* **2**, 287 (1879).
 - [2] K. v. Klitzing, G. Dorda, and M. Pepper, New method for high-accuracy determination of the fine-structure constant based on quantized Hall resistance, *Phys. Rev. Lett.* **45**, 494 (1980).
 - [3] D. C. Tsui, H. L. Stormer, and A. C. Gossard, Two-dimensional magnetotransport in the extreme quantum limit, *Phys. Rev. Lett.* **48**, 1559 (1982).
 - [4] J. Sinova, S. O. Valenzuela, J. Wunderlich, C. H. Back, and T. Jungwirth, Spin Hall effects, *Rev. Mod. Phys.* **87**, 1213 (2015).
 - [5] B. L. Altshuler, D. Khmel'nitskii, A. I. Larkin, and P. A. Lee, Magnetoresistance and Hall effect in a disordered two-dimensional electron gas, *Phys. Rev. B* **22**, 5142 (1980).
 - [6] M. A. Khodas and A. M. Finkel'stein, Hall coefficient in an interacting electron gas, *Phys. Rev. B* **68**, 155114 (2003).
 - [7] H. Fukuyama, Hall effect in two-dimensional disordered systems, *J. Phys. Soc. Jpn.* **49**, 644 (1980).
 - [8] C.-Z. Chang, J. Zhang, X. Feng, J. Shen, Z. Zhang, M. Guo, K. Li, Y. Ou, P. Wei, L.-L. Wang, Z.-Q. Ji, Y. Feng, S. Ji, X. Chen, J. Jia, X. Dai, Z. Fang, S.-C. Zhang, K. He, Y. Wang, *et al.*, Experimental observation of the quantum anomalous Hall effect in a magnetic topological insulator, *Science* **340**, 167 (2013).
 - [9] V. J. Goldman, M. Shayegan, and H. D. Drew, Anomalous Hall effect below the magnetic-field-induced metal-insulator transition in narrow-gap semiconductors, *Phys. Rev. Lett.* **57**, 1056 (1986).
 - [10] X. P. A. Gao, G. S. Boebinger, A. P. Mills, A. P. Ramirez, L. N. Pfeiffer, and K. W. West, Temperature and magnetic-field-enhanced Hall slope of a dilute 2D hole system in the ballistic regime, *Phys. Rev. Lett.* **93**, 256402 (2004).
 - [11] J. Jung, F. Zhang, and A. H. MacDonald, Lattice theory of pseudospin ferromagnetism in bilayer graphene: Competing interaction-induced quantum Hall states, *Phys. Rev. B* **83**, 115408 (2011).
 - [12] R. Fleischmann, T. Geisel, and R. Ketzmerick, Quenched and negative Hall effect in periodic media: Application to antidot superlattices, *Europhys. Lett.* **25**, 219 (1994).
 - [13] A. M. Ortiz de Zavallos, N. C. Mamani, G. M. Gusev, A. A. Quivy, T. E. Lamas, and J. C. Portal, Enhanced Hall slope in wide $\text{Al}_x\text{Ga}_{1-x}\text{As}$ parabolic wells, *Phys. Rev. B* **75**, 205324 (2007).
 - [14] A. Y. Kuntsevich, A. V. Shupletsov, and A. L. Rakhmanov, Simple mechanism that breaks the Hall-effect linearity at low temperatures, *Phys. Rev. B* **102**, 155426 (2020).
 - [15] Z. Z. Du, H.-Z. Lu, and X. C. Xie, Nonlinear Hall effects, *Nat. Rev. Phys.* **3**, 744 (2021).
 - [16] B. Sanvee, J. Schluck, M. Cerchez, D. Mailly, H. W. Schumacher, K. Pierz, T. Heinzel, and J. Horbach, Hall effect in a two-dimensional disordered Lorentz gas, *Phys. Rev. B* **108**, 035301 (2023).
 - [17] P. Ehrenfest and T. Ehrenfest, *The Conceptual Foundations of the Statistical Approach in Mechanics* (Courier, Mineola, NY, 1990).
 - [18] E. H. Hauge and E. G. D. Cohen, Normal and abnormal diffusion in Ehrenfests' wind-tree model, *J. Math. Phys.* **10**, 397 (1969).
 - [19] H. Van Beyerem and E. H. Hauge, Abnormal diffusion in Ehrenfest's wind-tree model, *Phys. Lett. A* **39**, 397 (1972).
 - [20] B. A. Sanvee, R. Lohmann, and J. Horbach, Normal and anomalous diffusion in the disordered wind-tree model, *Phys. Rev. E* **106**, 024104 (2022).
 - [21] R. Lohmann, B. Horn-Cosfeld, J. Schluck, D. Mailly, N. H. Siboni, H. W. Schumacher, K. Pierz, T. Heinzel, and J. Horbach, Anomalous transport due to retroreflection, *Phys. Rev. B* **102**, 081302(R) (2020).
 - [22] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/lg7z-nrjy> for the calculated Hall resistances from numerical simulations, which includes Refs. [40–43].
 - [23] T. Ando, Theory of quantum transport in a two-dimensional electron system under magnetic fields II. Single-site approximation under strong fields, *J. Phys. Soc. Jpn.* **36**, 1521 (1974).
 - [24] J. Schluck, S. Fasbender, T. Heinzel, K. Pierz, H. W. Schumacher, D. Kazazis, and U. Gennser, Commensurability resonances in two-dimensional magnetoelectric lateral superlattices, *Phys. Rev. B* **91**, 195303 (2015).
 - [25] M. P. Allen and D. J. Tildesley, *Computer Simulation of Liquids* (Oxford University Press, Oxford, 2017).
 - [26] L. Bockhorn, P. Barthold, D. Schuh, W. Wegscheider, and R. J. Haug, Magnetoresistance in a high-mobility two-dimensional electron gas, *Phys. Rev. B* **83**, 113301 (2011).
 - [27] A. T. Hatke, M. A. Zudov, J. L. Reno, L. N. Pfeiffer, and K. W. West, Giant negative magnetoresistance in high-mobility two-dimensional electron systems, *Phys. Rev. B* **85**, 081304(R) (2012).
 - [28] L. Bockhorn, A. Velieva, S. Hakim, T. Wagner, E. P. Rugeramigabo, D. Schuh, C. Reichl, W. Wegscheider, and R. J. Haug, Influence of oval defects on transport properties in high-mobility two-dimensional electron gases, *Appl. Phys. Lett.* **108**, 092103 (2016).
 - [29] L. Bockhorn, D. Schuh, C. Reichl, W. Wegscheider, and R. J. Haug, Influence of the electron density on the giant negative magnetoresistance in two-dimensional electron gases, *Phys. Rev. B* **109**, 205416 (2024).
 - [30] A. D. Mirlin, D. G. Polyakov, F. Evers, and P. Wölfle, Quasiclassical negative magnetoresistance of a 2d electron gas: Interplay of strong scatterers and smooth disorder, *Phys. Rev. Lett.* **87**, 126805 (2001).

- [31] I. V. Gornyi and D. G. Polyakov, Two-dimensional electron hydrodynamics in a random array of impenetrable obstacles: Magnetoresistivity, Hall viscosity, and the Landauer dipole, *Phys. Rev. B* **108**, 165429 (2023).
- [32] P. S. Alekseev and A. P. Dmitriev, Hydrodynamic magnetotransport in two-dimensional electron systems with macroscopic obstacles, *Phys. Rev. B* **108**, 205413 (2023).
- [33] J. Schluck, M. Hund, T. Heckenthaler, T. Heinzl, N. H. Siboni, J. Horbach, K. Pierz, H. W. Schumacher, D. Kazazis, U. Gennser, and D. Mailly, Linear negative magnetoresistance in two-dimensional Lorentz gases, *Phys. Rev. B* **97**, 115301 (2018).
- [34] A. V. Bobylev, F. A. Maaø, A. Hansen, and E. H. Hauge, Two-dimensional magnetotransport according to the classical Lorentz model, *Phys. Rev. Lett.* **75**, 197 (1995).
- [35] B. Horn-Cosfeld, J. Schluck, J. Lammert, M. Cercez, T. Heinzl, K. Pierz, H. W. Schumacher, and D. Mailly, Relevance of weak and strong classical scattering for the giant negative magnetoresistance in two-dimensional electron gases, *Phys. Rev. B* **104**, 045306 (2021).
- [36] N. H. Siboni, J. Schluck, K. Pierz, H. W. Schumacher, D. Kazazis, J. Horbach, and T. Heinzl, Nonmonotonic classical magnetoconductivity of a two-dimensional electron gas in a disordered array of obstacles, *Phys. Rev. Lett.* **120**, 056601 (2018).
- [37] C. L. Yang, J. Zhang, R. R. Du, J. A. Simmons, and J. L. Reno, Zener tunneling between Landau orbits in a high-mobility two-dimensional electron gas, *Phys. Rev. Lett.* **89**, 076801 (2002).
- [38] F. Bartels, J. Strobel, B. Horn-Cosfeld, M. Cercez, K. Pierz, H. W. Schumacher, D. Mailly, and T. Heinzl, Hall field induced resistance oscillations by designed random obstacle arrays, *Phys. Rev. B* **111**, 165301 (2025).
- [39] F. Bartels, B. Horn-Cosfeld, J. Schluck, M. Cercez, D. Mailly, K. Pierz, H. W. Schumacher, B. Sanvee, J. Horbach, and T. Heinzl, Nonlinearities in the Hall resistance by retroreflective scattering [data set], Zenodo (2025), [10.5281/zenodo.17702106](https://doi.org/10.5281/zenodo.17702106).
- [40] T. Ando, Theory of quantum transport in a two-dimensional electron system under magnetic fields. I. Characteristics of level broadening and transport under strong field, *J. Phys. Soc. Jpn.* **36**, 959 (1974).
- [41] T. Ando, Theory of oscillatory g factor in an MOS inversion layer under strong magnetic fields, *J. Phys. Soc. Jpn.* **37**, 1044 (1974).
- [42] T. Ando, Theory of quantum transport in a two-dimensional electron system under magnetic fields. IV. Oscillatory conductivity, *J. Phys. Soc. Jpn.* **37**, 1233 (1974).
- [43] T. Heinzl, *Mesoscopic Electronics in Solid State Nanostructures* (Wiley-VCH, New York, 2007).