

Cavity quantum Hall hydrodynamics

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Motivated by recent experiments, we study the coupling of quantum Hall (QH) hydrodynamics to quantum electrodynamics within a resonance cavity. In agreement with experimental observations, we find that the Hall conductivity remains unchanged. However, the coupling to the cavity induces a second-order quantum reactance effect, contributing distinctly to the longitudinal AC conductivity. This effect arises from the exchange of energy between the QH fluid and cavity photons. Beyond the topological response, we show that the cavity couples to collective excitations, resulting in a shift of the Kohn mode frequency. Our methods are broadly applicable to both integral and fractional QH liquids, and our results offer a universal perspective on the protection of topological properties against long-range interactions induced by electromagnetic cavity modes.

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I. INTRODUCTION

Recent advances in optical cavities provide a new platform for investigating the response of matter to vacuum fluctuations of the electromagnetic field in both weak- and strong-coupling regimes [1–8]. These developments have spurred numerous proposals for the cavity engineering of novel material properties, without the need for external pumping [9–25]. Within this context, important experiments have measured deviations from the quantized value of the Hall conductance due to cavity fields [26–29], with notable observations of the breakdown of topological protection at high filling factors. In sharp contrast, recent experiments at low filling factors have confirmed that there is no modification of the quantized Hall conductance [30]. These findings raise fundamental questions regarding the robustness of topological material properties in the presence of fluctuating electromagnetic cavity fields, which can induce long-range interactions. In this work, we approach this issue from the perspective of quantum Hall (QH) hydrodynamics, specifically considering a Hall bar embedded within a cavity, as shown in Fig. 1. The hydrodynamic framework applies in the regime where cavity photons do not cause inter-Landau level transitions, but is equally applicable to interacting and noninteracting systems. It follows that

our conclusions are universal features of the QH states in a cavity, and are valid at both integer and fractional fillings. We demonstrate that while a linearly polarized cavity mode does not alter the Hall conductivity, it does induce a quantum reactance effect that significantly contributes to the AC longitudinal conductance, most strongly in the direction transverse to the cavity polarization. Furthermore, we show that the cavity mode couples to collective excitations of the QH liquid, resulting in a shift in the Kohn mode frequency, which agrees with the finding of [29] for the integer case.

The integer QH effect is observed when n Landau levels are fully occupied, with the Hall conductivity $\sigma^{xy} = n\sigma_0 = ne^2/2\pi$ being measured to an extraordinary precision of less than 10^{-9} in sufficiently clean macroscopic samples at low temperatures [31–34]. The precise quantization is fundamentally rooted in electron band topology [35–37], and an integer invariant of the corresponding electronic wave functions [38,39]. In strongly interacting systems, electron correlations can lead to fractional values of the Hall conductance. The first explanation of this remarkable phenomenon was in terms of explicit model wave functions for states in a partially filled Landau level, [35], and later by mean-field theories based on composite bosons [40–42], and composite fermions [43,44].

In parallel with these microscopic approaches, there exists a well-developed hydrodynamic theory for QH liquids, which provides a unified description of the topological properties of both integer and fractional states [41,45]. This theory can be derived, for instance, from a theory of composite bosons, and to the lowest order in a derivative expansion, it is the theory proposed by Wen and Zee [45,46]. By including higher-order derivative terms, this theory can also describe nontopological effects, such as collective excitations [44]. Although it describes the coupling of QH liquids to external electromag-

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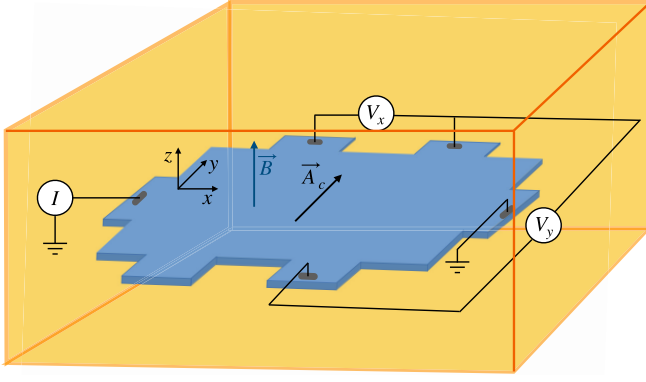


FIG. 1. Schematic view of an experiment to measure σ^{xy} in a cavity. The Hall bar is in the x - y plane, and we take the polarization of the lowest energy cavity mode as the y direction. While the electromagnetic fluctuations in the cavity do not modify σ^{xy} , they do modify the AC longitudinal conductivity in the x direction, i.e., transverse to the polarization of the dominant mode.

netic fields, it neglects the effects of electromagnetic field fluctuations. In this paper, we extend this approach to investigate the universal impact of cavity electromagnetic fluctuations on QH liquids, and explain how in the topological scaling limit these essential signatures can be separated from dynamical effects. We show that our universal hydrodynamic results are consistent with previous microscopic calculations that used filled Landau levels of noninteracting electrons to study cavity effects on integer QH states. Finally, we discuss various possibilities to experimentally verify our predictions.

II. HYDRODYNAMIC DESCRIPTION

We shall use the hydrodynamic action,

$$\mathcal{L}_H = -\frac{m}{4\pi}\epsilon^{\mu\nu\rho}b_\mu\partial_\nu b_\rho - \frac{e}{2\pi}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu b_\rho + \frac{m}{4\pi\omega_B}\vec{E}_b^2 - \frac{u}{2}B_b^2, \quad (1)$$

where $E_b^i = -\partial_i b^i - \partial^i b^0$, $B_b = \epsilon^{ij}\partial_i b_j$, and $j^\mu = \epsilon^{\mu\nu\rho}\partial_\nu b_\rho$ is the electric current that couples to the external electromagnetic field A_μ . The first two terms give the topological Wen-Zee action for a $\nu = 1/m$ QH liquid [45], and the last two are higher derivative, nontopological terms [47]. Here, $\omega_B = eB/m_e$ is the cyclotron frequency, and the constant u is the strength of a repulsive delta function potential. The generalization to an arbitrary potential $u(q)$ is straightforward [41]. We use units where $\hbar = c = \epsilon_0 = 1$.

For a single linearly polarized cavity mode,

$$\vec{A}_c(t, \vec{x}) = q(t)\phi_{k_c}(\vec{x})\hat{e}_y, \quad (2)$$

where $\hat{e}_y$ is a unit polarization vector (see Fig. 1), ϕ_{k_c} is the normalized mode function satisfying $\Delta\phi_{k_c} = -k_c^2\phi_{k_c}$, and $q(t)$ is the fluctuating amplitude. For this single mode, the usual Maxwell Lagrangian $\mathcal{L}_M = \frac{\epsilon}{2}\vec{E}^2 - \frac{1}{2\mu}\vec{B}^2$ reduces to

$$\mathcal{L}_c = \frac{\epsilon}{2}(\dot{q}^2 - \omega_c^2 q^2), \quad (3)$$

where $\omega_c = k_c/\sqrt{\epsilon\mu}$ is the cavity frequency. For a rectangular cavity, as shown in Fig. 1, k_c is the momentum of the lowest

TE mode. In relevant experimental settings, one expects the cavity effects on the QH liquid to be dominated by a single lowest energy mode [27,30]. We note that the frequency and amplitude of this mode are determined both by the cavity geometry, which specifies k_c , and by the effective values of the relative permittivity ϵ and permeability μ .

III. TOPOLOGICAL RESPONSE

We first calculate the topological response by dropping the higher-order terms in (1),

$$S = \int (\mathcal{L}_M + \mathcal{L}_H) = \int dt \left[\frac{\epsilon}{2}(\dot{q}^2 - \omega_c^2 q^2) - \int d^2x \left(\frac{m}{4\pi}\epsilon^{\mu\nu\rho}b_\mu\partial_\nu b_\rho + \frac{e}{2\pi}\epsilon^{\mu\nu\rho}(A + A_c)_\mu\partial_\nu b_\rho \right) \right], \quad (4)$$

where A_c is the projection of the (three-dimensional) cavity field (2) on the (two-dimensional) plane of the Hall bar, and for simplicity we assume that the polarization vector of the cavity mode lies in this plane. Also, A is a nondynamical background probe field. With this, the coupling term becomes

$$-\frac{e}{2\pi} \int dt q(t) [\partial^t b_{k_c}^x - (\partial^x b^t)_{k_c}], \quad (5)$$

where $f_{k_c}(t) = \int d^2x \phi_{k_c}(\vec{x})|_H f(t, \vec{x})$ denotes the average of the function $f(t, \vec{x})$, weighted by the cavity mode function, evaluated on the Hall bar.

Since (4) is quadratic in both b_μ and q , we can integrate these to get the effective response action

$$S_{\text{eff}}[A] = -\frac{e^2}{4\pi m} \int \frac{d\omega}{2\pi} \left\{ \int d^2x i\omega\epsilon^{ij}A_iA_j + \frac{2}{\epsilon} \left(\frac{e^2}{4\pi m} \right) \frac{\omega^2}{\omega^2 - \omega_c^2} A_{x,k_c}A_{x,k_c} \right\}, \quad (6)$$

where we simplified the expressions by using the radiation gauge, $A^0 = \vec{\nabla} \cdot \vec{A} = 0$, for the probe field. The corresponding electric current is

$$j^k(\omega, \vec{x}) = -\frac{1}{m} \frac{e^2}{2\pi} i\omega\epsilon^{kl}A_l(\omega, \vec{x}) + \frac{1}{m} \frac{e^2}{2\pi} \frac{2\alpha}{m\epsilon} \frac{\omega^2}{\omega^2 - \omega_c^2} \delta_x^k A_{x,k_c}(\omega)\phi_{k_c}(\vec{x}). \quad (7)$$

We recognize the leading term as the Hall conductivity. The correction is parametrically suppressed by the ratio α/ϵ , where $\alpha = \frac{e^2}{4\pi}$ is the fine-structure constant, and is spatially modulated by the cavity mode function. Moreover, only the component of the probe field, which is transverse to the cavity polarization, A_x , gives an additional current, $\propto \delta_x^k$, which is also transverse to the cavity polarization. It follows that only the longitudinal conductivity is modified.

Let us assume that the Hall bar is small compared to the cavity dimensions and approximate both A and A_c as homogeneous (long-wavelength approximation). Then the current (7) simplifies to

$$j^x(\omega) = \frac{e^2}{2\pi} \frac{1}{m} E_y(\omega) + \frac{e^2}{2\pi} \frac{1}{m^2} \frac{2\alpha}{L_{\text{eff}}} \frac{i\omega}{\omega^2 - \omega_c^2} E_x(\omega), \quad (8)$$

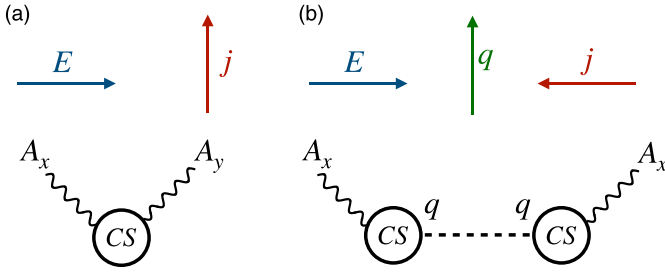


FIG. 2. Pictorial representation of the effective response action of the cavity-Hall system. (a) The Chern-Simons two-point vertex, which gives a transverse current response to an applied field. (b) The longitudinal term, which results from connecting two Chern-Simons vertices by a cavity propagator. This is a second-order effect where an applied field leads to the exchange of cavity photons with transverse polarization, which in turn results in a longitudinal current. The dissipationless periodic exchange of energy between the QH liquid and the cavity generates an effective quantum reactance.

$$j^y(\omega) = -\frac{e^2}{2\pi} \frac{1}{m} E_x(\omega), \quad (9)$$

where the length scale $L_{\text{eff}} = \varepsilon/(\phi^2 A_H)$ is proportional to the ratio of the quantization volume of the cavity V_c to the area of the Hall bar A_H , since ϕ is normalized to unity. We believe that this simplified description captures the essence of the experiment in [30]. We see that the Hall conductivity is not modified, in agreement with the experimental findings. Interestingly, we find an additional longitudinal contribution to the AC conductivity. It vanishes in the DC limit $\omega \rightarrow 0$, consistent with the quantum Hall phenomenology. Its magnitude is controlled by the ratio α/L_{eff} , which is related to the energy scale of the light-matter coupling. As expected, this effect is enhanced for smaller cavities, where quantum fluctuations are stronger.

IV. QUANTUM REACTANCE

To understand the nature of the longitudinal response σ_{xx} in (8), it is useful to recall the current response in an LC circuit,

$$I(\omega) = -\frac{1}{L} \frac{i\omega}{\omega^2 - \omega_0^2} V(\omega), \quad (10)$$

where $\omega_0 = 1/\sqrt{LC}$ is the resonant frequency. The parallel to (8) is obvious, and in particular, the current and voltage (or in our case, j_x and E_x) are out of phase by an angle $\pi/2$. In the LC circuit energy is transferred without loss between the capacitor and the inductor, while in our case it is transferred between the electrons in the QH liquid and the cavity field. In neither case is there any dissipation due to the phase lag; the impedance is purely reactive. Note that for an ideal (i.e., very high Q) cavity, we find that the Hall-cavity setup will act as a perfectly dissipationless bandpass filter at the cavity frequency. For a lossy cavity (i.e., finite Q), the longitudinal impedance will also have a resistive component proportional to the cavity loss (see our accompanying paper [48]). The interplay between the QH liquid and the cavity field is illustrated in Fig. 2, where panel (a) shows the usual Hall effect: An applied field generates a current in the transverse

direction. Panel (b) shows the following second-order process: First an applied field leads to the emission of a transverse cavity photon, then the absorption of a cavity photon generates a transverse current, so that finally the current response is parallel to the applied field. This is the quantum reactance effect. Note that this picture also explains why the effect is induced only in the direction transverse to the polarization of the cavity mode.

V. THE KOHN MODE

We now go beyond the topological response and reintroduce the higher-order terms in (1). The resulting dynamics describes the Kohn mode at high frequency $\omega = \omega_B$. The effect of the cavity can be seen by taking a homogeneous $b_i(t)$. Going to the frequency domain and using the radiation gauge, the effective action becomes

$$\int \frac{d\omega}{2\pi} \left\{ A_H \left[\frac{m}{4\pi} i\omega \epsilon^{ij} + \frac{m}{4\pi \omega_B} \omega^2 \delta^{ij} \right] b_i b_j - \frac{1}{2\varepsilon} \left(\frac{e}{2\pi} \right)^2 \frac{\omega^2}{\omega^2 - \omega_c^2} b_{k_c}^* b_{k_c} \right\}. \quad (11)$$

The Euler-Lagrange equations have a solution corresponding to the Kohn mode, with the modified frequency

$$\omega = \omega_B + \frac{1}{m} \frac{\alpha}{L_{\text{eff}}}. \quad (12)$$

Thus the coupling to the cavity mode gives an $O(\alpha/L_{\text{eff}})$ correction to the Kohn mode frequency. Similarly, one can compute the modification of the conductivities due to the higher-order terms in the hydrodynamic action. The corrections are suppressed at small frequencies by powers of (ω/ω_B) , which vanish in the topological scaling limit $\omega_B \rightarrow \infty$. From the microscopic point of view they arise from the transitions between Landau levels, as we now discuss (see also [48]).

VI. COMPARISON WITH PREVIOUS WORK

While the above derivation captures the universal aspects of the response of the QH liquid to the probe and cavity fields, it does not capture any details about the microscopics behind the phenomenology. For the integer effect, however, one can address this by coupling a free $2d$ electron gas filling an integer number of Landau levels to the cavity mode. This problem was studied in Ref. [29] by Rokaj, *et al.* in terms of the Hamiltonian

$$H_{\text{c.m.}} = \frac{(\vec{\pi} + e\sqrt{N}\vec{A}_c)^2}{2m_e} + \omega_c \left(a^\dagger a + \frac{1}{2} \right), \quad (13)$$

for the (renormalized) center of mass (c.m.) coordinate of the electrons with an effective mass m_e , and the cavity mode $\vec{A}_c = A(a + a^\dagger)\hat{e}_y$, which is taken to be constant over the Hall sample.

Starting from the commutation relations $[\pi^x, \pi^y] = -ieB$ and $[a, a^\dagger] = 1$, one can diagonalize $H_{\text{c.m.}}$ in terms of new

canonical raising and lowering operators $b_{\pm}^{\dagger}, b_{\pm}$ [29],

$$H_{\text{c.m.}} = \sum_{j=\pm} \omega_j \left(b_j^{\dagger} b_j + \frac{1}{2} \right). \quad (14)$$

The frequencies $\omega_{\pm}$ are given in the Supplemental Material [49] in terms of ω_B , and $\tilde{\omega}_c = \sqrt{\omega_c^2 + \omega_d^2}$, where the diamagnetic frequency is $\omega_d^2 = e^2 N / (m_e \epsilon V_c) = 2\alpha v \omega_B / L_{\text{eff}}$ for the case of ν fully occupied Landau levels, corresponding to the center of an integer Hall plateau. We recognize the characteristic energy scale $\omega_d^2 / \omega_B = 2\nu\alpha / L_{\text{eff}}$ of the light-matter coupling in the cavity. The higher-frequency mode ω_+ is adiabatically connected to the Kohn mode in the topological scaling limit of large ω_B , while the lower frequency ω_- is connected to the cavity mode. Expanding in ω_c / ω_B we find, to leading order,

$$\omega_+ = \omega_B + \frac{\alpha v}{L_{\text{eff}}}, \quad (15)$$

which for integer filling fractions ν ($1/m$) agrees with the value (12) obtained from the hydrodynamics.

For a linearly polarized cavity, Ref. [29] gives the results for the AC conductivities (or, more precisely, the conductances, although we shall not make this distinction). For $\nu = 1/m = 1$, we can now compare these with the results (8) and (9), obtained from the topological part of the hydrodynamic action, in the parameter regime $\omega_B \gg \tilde{\omega}_c > \omega_c > \omega_d$. Expanding the results from Ref. [29] to second order in ω_d / ω_B , we find

$$\frac{\sigma^{xx}(\omega)}{e^2 \nu / (2\pi)} = \frac{i\omega\omega_B}{\omega_B^2 - \omega^2} + \frac{2\alpha v}{L_{\text{eff}}} \frac{i\omega}{\left(1 - \frac{\omega^2}{\omega_B^2}\right)^2 (\omega_c^2 - \omega^2)}, \quad (16)$$

$$\frac{\sigma^{yy}(\omega)}{e^2 \nu / (2\pi)} = \frac{i\omega\omega_B}{\omega_B^2 - \omega^2} + \frac{2\alpha v}{L_{\text{eff}}} \frac{i\omega_B^{-2} \omega^3}{\left(1 - \frac{\omega^2}{\omega_B^2}\right)^2 (\omega_c^2 - \omega^2)}, \quad (17)$$

$$\frac{\sigma^{xy}(\omega)}{e^2 \nu / (2\pi)} = \frac{\omega_B^2}{\omega_B^2 - \omega^2} + \frac{2\alpha v}{L_{\text{eff}}} \frac{\omega_B^{-1} \omega^2}{\left(1 - \frac{\omega^2}{\omega_B^2}\right)^2 (\omega_c^2 - \omega^2)}. \quad (18)$$

In the topological scaling limit $\omega / \omega_B \rightarrow 0$, the only modification due to the cavity is to the σ^{xx} in (16), which agrees with the quantum reactance term in (8). Therefore, the formalism of [29] is consistent with our results, and we both find that the Hall conductivity is robust at low filling factors, as was measured in [30]. On the other hand, for a finite ω / ω_B , the remaining terms with poles at $\pm\omega_B$ account for transitions between the Landau levels. At low frequencies they give a linear contribution to both σ^{xx} and σ^{yy} . In the experiments [27,30], these terms can be comparable to the quantum reactance term (see the supplemental material). To properly investigate the topological properties, we propose that the experiments should consider smaller cavities, where we predict that the quantum reactance term dominates the full longitudinal AC conductivity (see Fig. 3). For the Hall conductivity, the leading correction appears only in quadratic order in ω , and it vanishes in the topological scaling limit $\omega / \omega_B \rightarrow 0$.

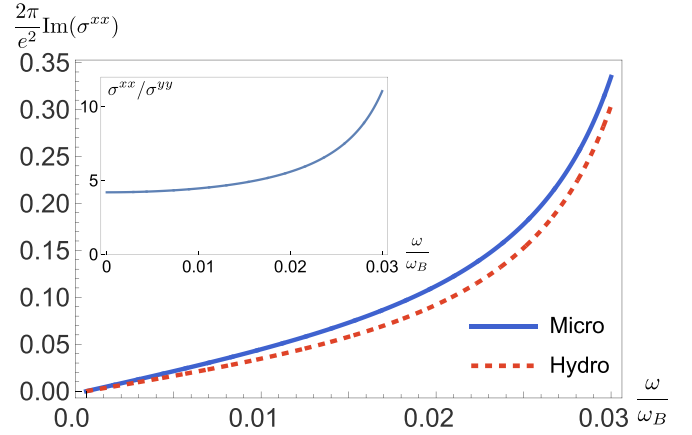


FIG. 3. Full AC conductivity $\sigma^{xx}(\omega)$ obtained from the microscopic calculation, Eqs. (16) and (17) (Micro), and the quantum reactance contribution obtained from the hydrodynamic calculation (Hydro). The conductivity is shown in units of $e^2 \nu / (2\pi)$ and the frequency is in units of the cyclotron frequency ω_B . Here we consider a Hall bar in a cavity with the parameters from recent experiments [27,30]: a sample area of $A_H = 40 \times 200 \mu\text{m}^2$, a filling ratio of $\nu = 1$, a cyclotron frequency of $\omega_B = 24.3 \text{ THz}$, and a cavity frequency of $\omega_c = 0.88 \text{ THz}$, while we set the effective length $L_{\text{eff}} = 42.8 \mu\text{m}$, corresponding to an effective cavity volume reduced to 10% of the original experimental size $V_c = 2.7 \times 10^5 \mu\text{m}^3$. We see that for these parameters the quantum reactance contribution from Hydro dominates σ^{xx} . The inset shows that the ratio $\sigma^{xx} / \sigma^{yy}$ from Micro is large, which demonstrates the anisotropic effect from the y -polarized cavity mode. Note that by the Hydro prediction, this ratio would be infinite as σ^{yy} is zero.

In the zero-frequency limit, $\sigma^{xx} = \sigma^{yy} \rightarrow 0$, and it is instructive to express the Hall conductivity as

$$\sigma^{xy}(\omega) = \frac{n_e}{B^2} \langle \mathbf{0} | i[\pi^x, \pi^y] | \mathbf{0} \rangle + O(\omega), \quad (19)$$

where n_e is the electron density and $|\mathbf{0}\rangle$ is the ground state of $H_{\text{c.m.}}$. For the fully occupied Landau levels with filling ratio ν , this gives the quantized DC Hall conductivity as $e^2 \nu / (2\pi)$, independent of the polarization of the cavity mode (see supplemental material [49], which includes Ref. [50]). Equation (19) also illustrates an important limitation of the calculations based on using the c.m. degree of freedom only, as it is precisely the expression for the response of a single charge Ne particle moving in a magnetic field, and has no relation to topology. It follows directly from a classical argument based on Galilean invariance [51], and cannot explain the presence of a plateau [52]. The hydrodynamic theory, on the other hand, incorporates the topological properties of both the integer and the fractional quantum Hall states [53], and can be used to calculate the finite-momentum response in the case of inhomogeneous fields.

VII. CONCLUSIONS AND OUTLOOK

Our results give another perspective on recent experimental and theoretical efforts to understand the effect of cavity fluctuations on quantum Hall liquids in the zero-temperature limit. In the $\nu = 1$ the experiment reported in [30], $\omega_B = 24.3 \text{ THz}$, which is much greater than the cavity frequency

$\omega_c = 0.88$ THz, which is again greater than the diamagnetic frequency $\omega_d = 0.50$ THz for a $40 \times 200 \mu\text{m}^2$ Hall bar. The experiment was performed at temperatures lower than 50 mK, where thermal fluctuations are smaller than 6.5 GHz, so finite-temperature effects can be neglected. Moreover, the spectrum of the cavity is such that the single-mode approximation is applicable, with the other modes lying at much higher energies. As we have shown in this work, this setup can be studied in terms of the quantum Hall hydrodynamics coupled to a single linearly polarized cavity mode.

We found that the cavity mode gives a contribution to the longitudinal conductivity in the direction transverse to the polarization through a second-order quantum reactance effect. This effect is mediated by the dissipationless exchange of energy between the QH liquid and the cavity photons which, to our knowledge, has not been reported before. We note that, while this effect reaches resonance at the cavity frequency, which is typically in the terahertz range, the effective impedance can still be measured at small probe frequencies, in which case it is purely reactive for an ideal cavity. The resonance can in principle be used as a bandpass filter that selects the cavity frequency. For a cavity formed by movable mirrors, the output frequency can be tuned. We compared our findings with previous calculations for the integer case in terms of the center-of-mass Hamiltonian [29]. We also showed that this approach predicts that corrections to the Hall conductance at the center of the plateau are absent for an arbitrary cavity polarization, at least in the case of a homogeneous cavity field. In addition, going beyond this limitation, the hydrodynamic formalism also by construction incorporates all topological aspects of the Hall response. It leads to universal predictions that are also relevant to the newly realized fractional states in cavities [54], and gives a clear analysis of the cavity signatures in the topological scaling limit. More broadly, hydrodynamics is expected to play a central role in exploring the cavity re-

sponse of other strongly correlated phases, where microscopic approaches are often limited.

The work in this paper can be extended in a number of interesting directions. The hydrodynamic calculation of the AC conductivities can be extended to higher frequencies by including higher-order contributions such as the Maxwell terms in (1), and terms related to the Hall viscosity [55]. Another extension is to finite temperature by using the Keldysh formalism, and to lossy cavities by having a finite Q number. Chiral cavities, that support only a single circularly polarized mode, could introduce time-reversal symmetry-breaking fluctuations, potentially giving rise to novel phenomena as suggested by previous studies in chiral quantum electrodynamics [7,9,13,20,56–61]. Several of these ideas will be explored in a future paper [48], where we also investigate the particle nature of the charge carriers in the coupled system.

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DATA AVAILABILITY

No data were created or analyzed in this study.

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