

# Normalized Andreev conductance peak in metal-superconductor point contacts can be enhanced above its ballistic maximum by disorder in the superconductor

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The metal-superconductor point contact (PC) is ballistic, if the coherence length  $\xi_0$  in the superconductor, the electron mean free path  $l_0$  in the metal, and the electron mean free path  $l$  in the superconductor are much larger than the PC radius  $a$ . The differential conductance  $G(V)$  of the ballistic PC at low temperatures and at voltages  $|V| \leq \Delta/e$ , where  $\Delta$  is the superconducting gap, exhibits the Andreev peak caused by Andreev reflection. In the ballistic PC involving an isotropic superconductor, the Andreev peak at  $|V| \leq \Delta/e$  normalized by the background conductance  $G(V)$  at  $|V| \gg \Delta/e$ , is equal to two. It is believed that in a disordered PC, where conditions  $l_0, l \gg a$  fail, the normalized Andreev peak is smaller than two. Here we study the PC between a clean metal ( $l_0 \gg a$ ) and a weakly disordered ( $l \gtrsim a$ ) isotropic superconductor in the limit  $l \ll \xi_0$ . We show theoretically and experimentally that, surprisingly, disorder on the superconducting side, however weak, raises the normalized Andreev peak above the ballistic value of two. Such disorder does not affect the Andreev reflection and  $G(V)$  at  $|V| < \Delta/e$ . However, it scatters the quasiparticles injected into the superconductor above the gap and reflects a part of them back to the PC before they join the condensate. As a result,  $G(V)$  at  $|V| > \Delta/e$  is lowered and the Andreev peak-to-background ratio becomes larger than two. In the normal state, the backscattering modifies the Sharvin-Maxwell PC resistance by adding a coherent resistance term  $\propto (a/l)^2$ .

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## I. INTRODUCTION

The point contact (PC) between a metal and a superconductor is ballistic, if the coherence length  $\xi_0$  in the superconductor, the electron mean free path  $l_0$  in the metal, and the electron mean free path  $l$  in the superconductor are much larger than the PC radius  $a$  [1–4]. The differential conductance  $G(V)$  of a ballistic PC at temperatures  $T$  well below the critical temperature of the superconductor  $T_C$  and at voltages  $V$  below the superconducting gap  $\Delta$  shows a pronounced conductance peak. Specifically, in the ballistic PC involving an isotropic superconductor, the peak is two times larger than the Sharvin conductance of the ballistic PC in the normal state,

$$G_N = \frac{2e^2}{h} \frac{a^2 k_F^2}{4}, \quad (1)$$

where  $k_F$  is the electron Fermi wave vector [5]. This so-called Andreev peak is due to the Andreev reflection at the metal-superconductor interface [6]. If an electron impinges on the PC from the metal side with energy  $|eV| \leq \Delta$ , it forms a Cooper pair to enter the superconductor. Therefore a hole with the same energy but opposite spin and momentum as

the incident electron is reflected into the metal. Due to this Andreev reflection, the charge crossing the PC is  $-2e$  and  $G(V)$  at  $|V| \leq \Delta/e$  is two times larger than  $G_N$  [1–4]. The background conductance of the PC, defined by

$$G_B \equiv G(|V| \gg \Delta/e), \quad (2)$$

is due to the electronlike quasiparticles that enter the superconductor with energies  $\gg \Delta$ . Since the electronlike quasiparticle states at high energies coincide with the normal-electron states, for the ballistic PC  $G_B = G_N$ . Thus, in the ballistic metal-superconductor PC, the height of the Andreev peak normalized by  $G_B$  is just

$$\frac{G(|V| \leq \Delta/e)}{G_B} = 2, \quad (3)$$

where the width of the Andreev peak,  $2\Delta$ , reflects  $\Delta$  of the bulk superconductor. If the experiment shows the Andreev peak of height  $2G_B$  and width  $2\Delta$ , the peak can be ascribed to the Andreev reflection in the ballistic PC and the value of  $G_B$  can be identified as  $G_N$  [3,4].

Blonder, Tinkham, and Klapwijk (BTK) analysed the conductance of the ballistic metal-superconductor PC using the Bogoliubov equations [1]. The BTK theory confirmed the ballistic Andreev conductance peak of height  $G(|V| \leq \Delta/e) = 2G_N$  and width  $2\Delta$ , as well as the ballistic  $G_B$  equal to  $G_N$ . Later, Zaitsev [2] rederived the BTK results using the quasiclassical Eilenberger equations. He concluded (see also Ref. [7]), that the BTK theory holds for the PCs obeying  $l_0, l, \xi_0 \gg a$ , no matter whether the superconductor is clean ( $l \gg \xi_0$ ) or dirty ( $l \ll \xi_0$ ). Indeed, the BTK theory [1]

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(modified to model an imperfect metal-superconductor interface [1,8,9]) fits well the  $G(V)$  spectra of various PCs including those with a dirty ( $l \ll \xi_0$ ) superconducting side [3,4,10–13]. For instance, the ballistic Andreev peak of height  $2G_B$  and half-width  $\Delta$  coinciding with the bulk value of  $\Delta$  was observed in the PC between a clean normal Ti tip and a dirty superconducting Al film with  $l \simeq 3.4$  nm and  $\xi_0 \sim 10^3$  nm [13].

The BTK theory [1] assumes that  $\Delta$  exhibits at the metal-superconductor interface a sudden increase from zero to its bulk superconductor value, which is only possible if the PC radius  $a$  is small enough to prevent the proximity effects between the superconductor and the metal. According to the semiclassical theory [2] the mentioned BTK assumption is valid if the superconducting side obeys both  $l \gg a$  and  $\xi_0 \gg a$ . However, according to the review [11] the dirty superconductor only needs to obey  $\xi \gg a$ , where  $\xi \simeq \sqrt{\xi_0 l}$  is the coherence length in the dirty limit.

Experiment shows that the condition  $\xi = \sqrt{\xi_0 l} \gg a$  [11] suffices because the BTK theory fits well the Andreev PC spectra of the superconductors that obey  $\xi \gg a$  but fail to obey  $l \gg a$  [3,10–13]. For instance, the ballistic Andreev peak of height  $2G_B$ , reported in Ref. [13], was observed for the superconducting Al film with  $\xi \gg a$  and  $l \simeq 1.36a$ , probed by the clean Ti-metal tip with  $a \simeq 2.5$  nm. Remarkably, the half-width of the observed peak,  $\Delta$ , coincided with the expected bulk value of  $\Delta$ , thus supporting the criterion  $\xi \gg a$ . Also other authors have observed that once the superconductor obeys the limit  $\xi \gg a$ , the BTK theory works even if  $l \ll a$  (see Table II in Ref. [10]).

We think that the question whether the BTK theory holds for  $l, \xi_0 \gg a$  [2] or for  $\xi = \sqrt{\xi_0 l} \gg a$  [11] can be answered empirically. The proximity effect between the superconductor and the metal would suppress  $\Delta$  on the superconducting side of the PC and induce a nonzero  $\Delta$  on the metal side. Since the BTK theory ignores the proximity effect by assuming a zero  $\Delta$  in the metal and a bulk value of  $\Delta$  on the superconducting side, a natural criterion for applicability of the BTK theory is the observation of the Andreev peak of half-width  $\Delta$  equal to the bulk value of  $\Delta$  [13]. In addition, if the observed peak height is  $2G_B$  [13] as predicts Eq. (3), one can also believe that the Andreev reflections at energies below  $\Delta$  are ballistic.

The works [14–17] analyzed the PC with the disordered metal side in the diffusive limit  $l_0 \ll a$ . Disorder on the superconducting side was ignored as in the works [3,10–12] and disorder on the metal side was incorporated into the BTK theory. In the model [15,16], the electron approaching the PC from the metal was first scattered by a diffusive metal region much larger than  $l_0$  and then by the PC interface, while the Andreev reflected hole was first scattered by the PC interface and then by the diffusive metal region. It was found [14–16] that (in accord with the earlier but different approach [18]) the Andreev conductance at  $|V| \ll \Delta/e$  reduces to the normal-state diffusive PC conductance.

Disorder on the superconducting PC side was added into the diffusive PC model [15,16] only at  $T > T_C$  [17], when it takes the finite resistivity ( $\rho = 3\pi^2 \hbar / e^2 k_F^2 l$ ) and dissipative Maxwell resistance ( $R_M = \rho/4a$ ) of the no longer superconducting electrode [3,4,10,13,19]. The PC resistance in the normal state reads  $R(T > T_C) = 1/G_N + R_M + R_0$ , where

$R_0 = \rho_0/4a$  is the Maxwell resistance of the disordered metal side, with  $\rho_0 = 3\pi^2 \hbar / e^2 k_F^2 l_0$  being the metal side resistivity. In the experiment,  $R_M$  manifests itself as a sudden drop of  $R(T)$  at  $T = T_C$ , when  $R_M$  turns to zero due to the onset of superconductivity on the superconducting PC side (see, for instance, Ref. [13] and also Fig. 5 in this work).

Does disorder on the superconducting PC side play any role when  $T < T_C$ ? In this work, we study theoretically and experimentally the PC between a clean ( $l_0 \gg a$ ) metal and a conventional isotropic superconductor which is weakly disordered in the sense  $l \gtrsim a$  but obeys the limit  $\xi = \sqrt{\xi_0 l} \gg a$  [3,10,11] justifying the applicability of the ballistic BTK theory. Assuming that the electrons impinging on the metal-superconductor interface are transmitted through the interface with transmission probability close to unity, the ballistic regime is perturbed solely by weak disorder on the superconducting PC side. Incorporating this weak perturbation into the ballistic BTK theory, we find that disorder on the superconducting side enhances the Andreev peak to background conductance ratio above the ballistic value of two.

We rely on two assumptions. First, since  $\xi \gg a$ , we assume that weak disorder on the superconducting side does not affect the Andreev reflections and supercurrent flowing below the gap. In other words, we assume that the Andreev conductance  $G(V)$  at  $|V| \leq \Delta/e$  can be described by the ballistic BTK theory as in the previous works [3,4,11–17,19]. We verify the applicability of the BTK theory at  $|V| \leq \Delta/e$  experimentally using the empirical criterion discussed above.

Second, unlike the previous works, we do take into account that disorder on the superconducting side, however weak, scatters coherently the quasiparticles injected from the PC to the superconductor above the gap. Before joining the condensate, a part of the injected quasiparticles is backscattered to the PC and the background conductance  $G_B \equiv G(|V| \gg \Delta/e)$  is reduced from  $G_B = G_N$  to  $G_B = G_N(1 - r_C)$ , where  $r_C \propto (a/l)^2$  is the probability of the quasiparticle backscattering. As a result, the normalized Andreev peak  $G(V=0)/G_B$  becomes larger than two. If both PC sides are disordered,  $G(V=0)/G_B$  remains larger than two as long as disorder on the superconducting side prevails. In the normal state (at  $T > T_C$ ) the coherent backscattering raises the PC resistance  $1/G_N + \rho/4a$  by adding an extra resistance term  $\propto (a/l)^2$ . We verify these results by Andreev PC spectroscopy [13] of superconducting Al films.

## II. THEORY

We first review the BTK theory [1] which describes the PC between a clean metal and a clean isotropic BCS-type superconductor. Fig. 1 depicts the quasiparticle excitation energy above the Fermi level,  $E(k)$ , as a function of the wave vector  $k$  for the metal-superconductor interface. It also depicts the allowed processes for an electron impinging on the interface from the metal, occurring with probabilities  $A$ ,  $B$ ,  $C$ , and  $D$ . The BTK results for  $A$ ,  $B$ ,  $C$ , and  $D$  are shown in Table I. They hold for the ballistic PC with a  $\delta$ -function-like energy barrier of strength  $Z$  at the interface. The parameter  $Z$  can model the electron reflection due to the interfacial barrier [1] or due to the Fermi velocity mismatch at the interface [9]. Moreover, it can model the electron/hole scattering by

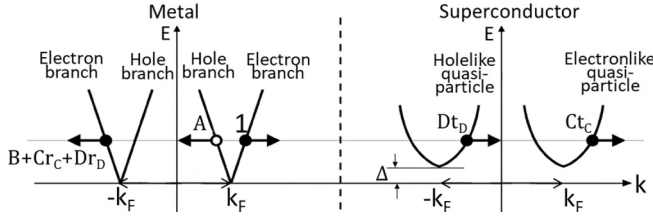


FIG. 1.  $E(k)$  diagram for the metal-superconductor interface in equilibrium (adapted from Fig. 4 of Ref. [1]). The open circles denote holes and closed circles electrons. The arrows point in the direction of the group velocity. Depicted are the processes for an electron incident at 1: the Andreev reflection of the hole with probability  $A$ , the normal electron reflection with probability  $B$ , the transmission of the electronlike quasiparticle with probability  $C$ , and the transmission of the holelike quasiparticle with probability  $D$ . Due to disorder in the superconductor (see this text), the electron- and holelike flows  $C$  and  $D$  are reflected with probabilities  $r_C$  and  $r_D$  and transmitted with probabilities  $t_C = 1 - r_C$  and  $t_D = 1 - r_D$ , respectively. This modifies  $C$  to  $Ct_C$ ,  $D$  to  $Dt_D$ , and (see text)  $B$  to  $B + Cr_C + Dr_D$ .

disorder on the metal side of the diffusive PC [14–16] (see also Appendix B). The current  $I$  through the PC is expressed [1] on the metal side as  $I(V, T) = (G_N/e) \int_{-\infty}^{\infty} dE [f_{\rightarrow}(E) - f_{\leftarrow}(E)]$ , where  $f_{\rightarrow}(E)$  and  $f_{\leftarrow}(E)$  are the electron energy distributions for  $k > 0$  and  $k < 0$ . Namely,  $f_{\rightarrow}(E) = f_0(E - eV)$ , where  $f_0(E) = 1/[\exp(E/k_B T) + 1]$  is the Fermi function, and

$$f_{\leftarrow}(E) = A(E)[1 - f_{\rightarrow}(-E)] + B(E)f_{\rightarrow}(E) + [C(E) + D(E)]f_0(E). \quad (4)$$

Here the term  $A(E)[1 - f_{\rightarrow}(-E)]$  is due to the Andreev reflection of the hole in state  $(\rightarrow, -E)$ ,  $B(E)f_{\rightarrow}(E)$  is due to the normal electron reflection, and  $[C(E) + D(E)]f_0(E)$  represents the electron flow coming from the superconductor. Note the origin of the last term [1]. If  $V = 0$ , the electron flow transmitted from the metal to the superconductor is  $[C(E) + D(E)]f_0(E)$  and this flow has to be balanced by the same electron flow transmitted from the superconductor to the metal. Thus the flow transmitted from the superconductor to the metal at  $V = 0$  is just  $[C(E) + D(E)]f_0(E)$ , and this flow remains unchanged for  $V \neq 0$  [1]. After some algebra [1], one

TABLE I. The BTK results [1] for  $A(E)$ ,  $B(E)$ ,  $C(E)$ , and  $D(E)$ , valid for the ballistic PC with the  $\delta$ -function-like energy barrier of strength  $Z$  positioned at the interface.  $\gamma^2 = [u_0^2 + Z^2(u_0^2 - v_0^2)]^2$  and  $u_0^2 = 1 - v_0^2 = \frac{1}{2}[1 + (E^2 - \Delta^2)/E^2]^{1/2}$ .

|   | Normal state          | Superconducting state                                 |                                                    |
|---|-----------------------|-------------------------------------------------------|----------------------------------------------------|
|   |                       | $E < \Delta$                                          | $E > \Delta$                                       |
| A | 0                     | $\frac{\Delta^2}{E^2 + (\Delta^2 - E^2)(1 + 2Z^2)^2}$ | $\frac{u_0^2 v_0^2}{\gamma^2}$                     |
| B | $\frac{Z^2}{1 + Z^2}$ | $1 - A$                                               | $\frac{(u_0^2 - v_0^2)^2 Z^2 (1 + Z^2)}{\gamma^2}$ |
| C | $\frac{1}{1 + Z^2}$   | 0                                                     | $\frac{u_0^2 (u_0^2 - v_0^2)(1 + Z^2)}{\gamma^2}$  |
| D | 0                     | 0                                                     | $\frac{v_0^2 (u_0^2 - v_0^2) Z^2}{\gamma^2}$       |

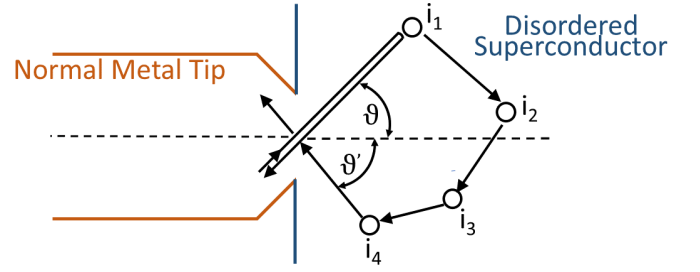


FIG. 2. Particle entering the superconductor can be backscattered to the PC by a single impurity ( $i_1$ ) or by many impurities ( $i_1, i_2, i_3, i_4$ ).

gets the BTK conductance

$$G(V, T) \equiv \frac{\partial I(V, T)}{\partial V} = \int_{-\infty}^{\infty} dE \frac{\partial f_0(E - eV)}{\partial (eV)} g(E), \quad (5)$$

where  $g(E)$  is the zero-temperature conductance at energy  $E$ :

$$g(E) = G_N[1 + A(E) - B(E)]. \quad (6)$$

Using the BTK equations (5) and (6) combined with Table I one obtains the Andreev conductance at  $V = 0$  and  $T = 0$ ,

$$G(V = 0) = \frac{2G_N}{(1 + 2Z^2)^2}, \quad (7)$$

the background conductance (valid for any  $T$ ),

$$G_B \equiv G(|V| \gg \Delta/e) = \frac{G_N}{1 + Z^2}, \quad (8)$$

and the Andreev-peak-to-background ratio  $G(0)/G_B \leq 2$ .

Consider now the PC between a clean metal and a dirty ( $\xi_0 \gg l$ ) superconductor which is weakly disordered in the sense  $l \gtrsim a$  but obeys the empirical limit  $\xi = \sqrt{\xi_0 l} \gg a$  [3,10,11] justifying the use of the ballistic BTK theory. This means, that we could, in principle, ignore disorder in the superconductor and describe the PC by the BTK theory [1] similarly as the works [10–13] which ignored disorder on the superconducting side even for  $l \ll a$  [10]. Nevertheless, we think that disorder on the superconducting side, however weak, cannot be ignored entirely if it is the only perturbation of the ballistic regime. We therefore proceed as follows.

We assume that weak disorder on the superconducting side does not affect the Andreev reflection term  $A(E)$ . Thus, at energies  $|E| < \Delta$ , the supercurrent injected into the superconductor remains unchanged and  $g(E)$  does not change either [it is given by Eq. (6)]. At this point, our approach is the same as in works [10–13,15,16], and we will verify it experimentally.

However, the current injected into the superconductor at energies  $|E| > \Delta$  is initially carried by the transmitted quasiparticles. Before joining the condensate, the quasiparticles can be scattered by disorder in the superconductor back to the metal (Fig. 2). This scattering effect, however weak, has to lower the quasiparticle current [and thus  $g(E)$  at  $|E| > \Delta$ ] below the value in the clean superconductor. In order to incorporate this physics into the BTK theory, we modify

Eq. (4) to

$$f_{\leftarrow}(E) = A(E)[1 - f_{\rightarrow}(-E)] + [B(E) + \beta(E)]f_{\rightarrow}(E) + [C(E)t_C + D(E)t_D]f_0(E). \quad (9)$$

The last term in equation (9) has the same origin as the term  $[C(E) + D(E)]f_0(E)$  in equation (4), except that it is modified by transmissions  $t_C = 1 - r_C$  and  $t_D = 1 - r_D$  (see Fig. 1). Assuming  $r_C \ll 1$ ,  $r_D \ll 1$  and  $Z^2 \ll 1$ , we can keep the BTK coefficients in Table I intact and we can also neglect the multiple reflections between the PC and impurities (see Fig. 2). The term  $\beta(E)f_{\rightarrow}(E)$  represents the electron flow first transmitted into the superconductor and then scattered back to the metal. Taking into account that  $f_{\leftarrow}(E) = f_0(E)$  for  $V = 0$ , we find from Eq. (9) that  $\beta(E) = C(E)r_C + D(E)r_D$ . Finally, repeating the above BTK derivation of  $G(V, T)$  with equation (9) instead of equation (4), we get equation (5) with

$$g(E) = G_N[1 + A(E) - B(E) - C(E)r_C - D(E)r_D]. \quad (10)$$

Equation (10) coincides with the BTK equation (6) at energies  $|E| < \Delta$ , where  $C = D = 0$ . For  $|E| > \Delta$ , however,  $C$  and  $D$  are not zero and the coefficients  $r_C$  and  $r_D$  suppress  $g(E)$ .

Coefficient  $r_C$  is estimated in Appendix A. We find that

$$r_C \simeq \frac{\pi}{4} \left( \frac{3}{2\pi} \right)^{\frac{3}{2}} \left( \frac{a}{l} \right)^2 \quad \text{where} \quad \frac{a}{l} \lesssim 1, \quad (11)$$

if we consider the multiple-impurity backscattering (Fig. 2), while for the single-impurity backscattering (Fig. 2) we get about two times smaller  $r_C$ ,  $r_C \simeq a^2/8l^2$ . Thus about one half of the collisions contributing to Eq. (11) are the single-impurity collisions, implying that the backscattered quasiparticles penetrate nearly to the depth  $l$ . The transmitted part  $t_C$  penetrates into the superconductor as a diffusive quasiparticle current, decaying into the condensate within time  $\tau_c$ . Equation (11) holds for sufficiently large  $\tau_c$  (see Appendix A).

For  $T = 0$  and  $V = 0$ , we obtain from equations (5), (10), and Table I the BTK result for  $G(V = 0)$ , given by Eq. (7). However, for the background conductance, we get the result

$$G_B \equiv G(|V| \gg \Delta/e) = \frac{G_N(1 - r_C)}{1 + Z^2}, \quad (12)$$

which, unlike the BTK result (8), is lowered by factor  $1 - r_C$ . In effect, the Andreev-peak-to-background ratio

$$\frac{G(V = 0)}{G_B} = \frac{2(1 + Z^2)}{(1 - r_C)(1 + 2Z^2)^2} \quad (13)$$

is larger than two for large enough  $r_C$  even if  $Z > 0$ . As discussed in the Appendix B,  $Z$  can also model electron scattering by disorder on the metal side. In such case Eq. (13) holds for the PC disordered on both sides. Note that Eq. (12) holds for any  $T$ , while Eq. (13) holds only for  $T \ll \Delta/k_B$ . Note also that  $G_B$  is  $r_D$ -independent. The estimate (11) holds also for  $r_D$ , but with a different  $l$ . Below we assume  $r_D = r_C$  and we neglect their dependence on  $E$  (see Appendix A).

In Fig. 3, we show  $G(V)$  calculated using Eqs. (5), (10), (11), and  $r_D = r_C$ , assuming  $T = 4.2$  K and  $\Delta = 2.4$  meV as in our experiment. Panel (a) shows the data for  $Z = 0$  and various  $a/l$ , normalized by  $G_N$ . Panel (b) shows the same data normalized by  $G_B$  as is usual in experiment. Clearly, the Andreev peak normalized by  $G_B$  exceeds two for  $a/l = 0.62$

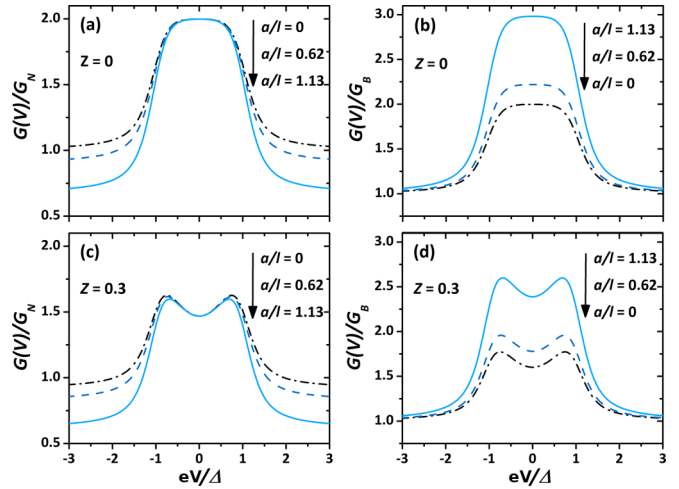


FIG. 3.  $G(V)$  spectrum calculated for  $T = 4.2$  K,  $\Delta = 2.4$  meV, and for various  $a/l$  and various  $Z$ . The  $G(V)$  data for  $Z = 0$  are presented in (a) and (b), in (c) and (d) we present the  $G(V)$  data for  $Z = 0.3$ . In (a) and (c) the  $G(V)$  data are normalized by  $G_N$  while in (b) and (d) they are normalized by  $G_B$ .

and 1.13. Observing result  $G(0)/G_B > 2$  experimentally without knowing its origin, one might think that  $G(0) > 2G_N$ . However, the correct interpretation rather is that  $G(0) = 2G_N$  and  $G_B < G_N$  due to nonzero  $r_C$ .

Panels (c) and (d) show the data for  $Z = 0.3$ . Finite  $Z$  lowers  $G(V)$ , but  $G(0)/G_B$  still exceeds two for large  $r_C$ . Data for  $a/l = 0$  are the usual BTK results showing  $G(0)/G_B \leq 2$ .

### III. EXPERIMENTAL METHOD

Now we introduce our experimental method [13]. Table II shows the list of our devices. In device 1, the superconducting electrode is the 15 nm/80 nm thick Al/NbN bilayer, where Al and NbN are isotropic superconductors. The NbN layer becomes superconducting at  $T_C = 15.3$  K and its low- $T$  gap is  $\Delta \simeq 2.4$ – $2.5$  meV. Disorder in this layer, weak in terms of  $k_F l \simeq 10$ , affects  $T_C$  and  $\Delta$  of pure NbN only slightly [20]. In the Al layer  $k_F \simeq 1.75 \times 10^{10} \text{ m}^{-1}$  and  $l \simeq 3.4$  nm, that means, disorder is even weaker ( $k_F l \simeq 60$ ). The Al layer becomes superconducting due to the proximity effect caused by the NbN film and its  $T_C$  and  $\Delta$  perfectly mimic those of the NbN film [13]. The role of the isotropic superconductor probed by the metal tip (Fig. 2) is played by the Al film.

Our superconducting Al films, though clean in the sense  $k_F l \simeq 60$ , are dirty in the sense  $\xi_0 \gg l$ , because  $\xi_0$  in the

TABLE II. List of properties and sketch of our PC devices.

| Device | Tip   | Superconducting multilayer (nm) |
|--------|-------|---------------------------------|
| 1      | Ti    | Al(15)NbN(80)                   |
| 2      | Pt-Ir | Al(15)NiCu(5)NbN(80)            |
| 3      | Ti    | Al(15)Ni(5)NbN(80)              |
| 4      | Ti    | Al(15)NiCu(50)NbN(80)           |
| 5      | Ti    | Al(15)NiCu(20)NbN(80)           |
| 6      | Ti    | Al(15)NiCu(5)NbN(80)            |

clean Al material is about  $1.5 \times 10^3$  nm. Though in our Al film  $\xi_0$  is reduced to  $\xi = \sqrt{\xi_0 l} \sim 2 \times 10^2$  nm, the value of  $\xi$  still exceeds the Al film thickness more than ten times. This is consistent with the fact that the proximity-effect-induced superconductivity in the Al film perfectly mimics  $T_C$  and  $\Delta$  of the underlying superconducting NbN film.

In devices 2, 3, 4, 5, and 6, the Al and NbN films are separated by a thin Ni (device 3) or NiCu film (devices 2, 4, 5, 6). This extra film allows to control  $\Delta$  in the Al film by reducing the proximity effect caused by the NbN film. Moreover, the ferromagnetic nature of the NiCu (or Ni) film is expected to introduce a nonzero Dynes parameter  $\Gamma$  (see Sec. V) which tends to reduce [8] the Andreev peak. Thus PC devices 2, 3, 4, 5, and 6 are designed to show the result  $G(0)/G_B < 2$  even if they are ballistic on both PC sides ( $l_0, l \gg a$ ). However, contrary to naive expectations, we will see that most of them show the result  $G(0)/G_B > 2$ , predicted for the PC between the ballistic ( $l_0 \gg a$ ) metal and the weakly-disordered ( $l \gtrsim a$ ) superconductor. Nonzero  $\Gamma$ , if any, weakens the  $G(0)/G_B > 2$  effect, but does not suppress it entirely.

We cannot use a pure Al electrode, since it would be superconducting at  $\sim 2$  K while our setup works for  $T \geq 4.2$  K. A pure NbN electrode is unsuitable due to oxide on its top [13].

The Ti tip in devices 1, 3, 4, 5, and 6 was formed as in the memristor [13] usable for PC spectroscopy [13,21]. The Al film was covered by a 50 nm insulating  $\text{TiO}_2$  film followed by a Pt film serving as the upper electrode. The  $\text{TiO}_2$  film was subjected to soft breakdown that connected the Pt and Al films by one conical Ti filament with tip radius  $a \sim 2\text{--}3$  nm and electron mean free path  $\gg a$ . The sketch in Table II shows the tip, while the  $\text{TiO}_2$  matrix and Pt film are skipped. The Pt-Ir tip in device 2 is a needle with tip radius  $a \sim 6$  nm, pressed to the Al surface.

In all our devices, the superconducting Al side of the PC obeys condition  $\xi \gg a$  justifying the use of the BTK theory. We recall that the condition  $\xi \gg a$  is expected to suppress the proximity effect between the superconductor and the metal, in our case between the Al film and the Ti (or Pt-Ir) tip. In our  $G(V)$  measurements, absence of the proximity effect between the Al film and the tip will be manifested by observation of the Andreev conductance peak of half-width  $\Delta \simeq 2.4\text{--}2.5$  meV, corresponding to  $\Delta$  in the NbN film underneath the Al film.

The film growth, characterisation, and device processing are described in Refs. [13, 22]. Transport measurements were performed in a LHe Dewar container in four-contact configuration. Our setup consists of the Keithley 220 DC current source, Keithley 2002 multimeter, and SRS SR830 lock-in amplifier [13]. In this text we present a selected set of our transport data, the complete set is presented in Ref. [22].

#### IV. DISCUSSION OF MEASURED $G(V)$ SPECTRA

In Fig. 4 (panels in the left column) we show the  $G(V)$  spectra of devices 1, 2, 3, and 4, measured at 4.3 K. Also shown are the background fits  $G_B(V)$  which account for the fact that the  $G(V)$  spectra for  $V$  well above  $\Delta/e$  show a weak dependence on  $V$ . In the theory the background conductance is defined as  $G_B \equiv G(|V| \gg \Delta/e)$  while in the experiment  $G_B$  has to be identified as a point (full circles marked as  $G_B$ ) reasonably far from the Andreev peak. As is customary [4],

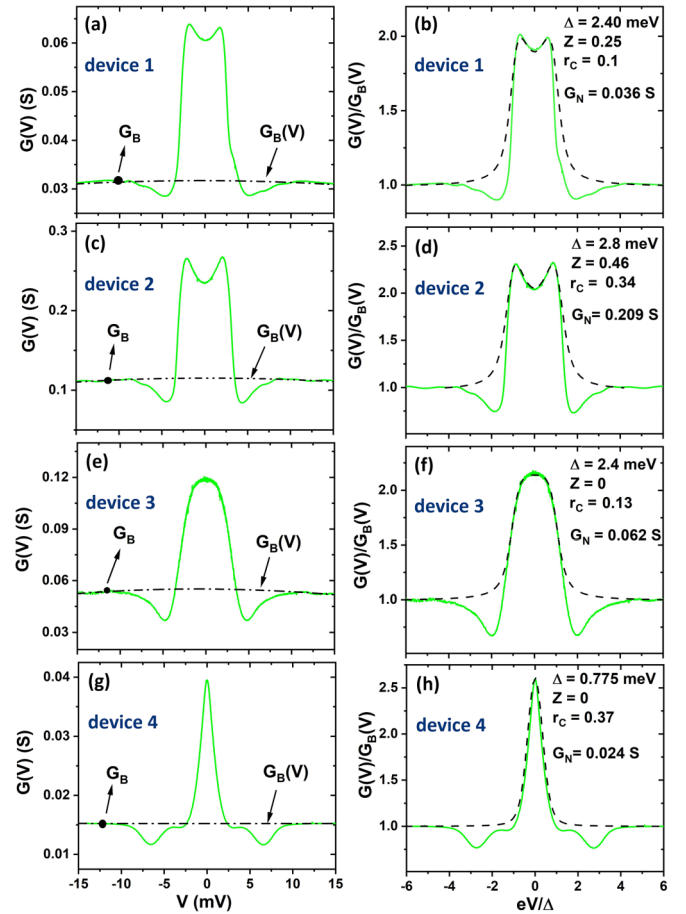


FIG. 4.  $G(V)$  spectra of devices 1, 2, 3, and 4 at 4.3 K. (a), (c), (e), and (g) show the measured data (solid lines). The  $G_B(V)$  curves (dotted-dashed lines) are polynomial fits of the background  $G(V)$ . (b), (d), (f), and (h) show the measured data normalized as  $G(V)/G_B(V)$  (solid lines) and fits  $G(V)/G_B$  (dashed lines) calculated (see text) by fitting  $\Delta$ ,  $Z$ , and  $r_C$ . Values of  $G_N$  were obtained from Eq. (12) using the  $G_B$  values shown by circles.

in the right column of panels the experimental  $G(V)$  spectra normalized as  $G(V)/G_B(V)$  (solid lines) are fitted by the theoretical  $G(V)$  spectra normalized as  $G(V)/G_B$  (dashed lines).

While the BTK theory relies on Eqs. (5) and (6) with fitting parameters  $\Delta$  and  $Z$ , the theoretical fits in Fig. 4 are calculated using the equations (5), (10), (11), and  $r_D = r_C$ , where  $r_C$  is another fitting parameter in addition to  $\Delta$  and  $Z$ . Clearly, the spectra measured for devices 2, 3, and 4 exhibit the Andreev peak-to-background ratio  $G(0)/G_B$  larger than the ballistic value of two, just as predicted by Eq. (13) and Fig. 3. Presented theoretical curves fit the anomalously high Andreev peaks quite well, while the BTK curves (not shown) only give  $G(0)/G_B < 2$  because the quasiparticle reflections described by  $r_C$  and  $r_D$  are not taken into account.

In device 1, the ratio  $G(0)/G_B$  is slightly smaller than two, but still high in comparison with the usual BTK value, as already noticed in Ref. [13]. If we insert  $Z$  and  $r_C$  for device 1 into Eq. (13), we get  $G(0)/G_B = 1.9$  as in the experiment. However, if we put  $Z = 0$  and keep the same  $r_C$ , we get  $G(0)/G_B = 2.2$ . Thus, in device 1, the effect  $G(0)/G_B > 2$  is obscured by finite  $Z$ .

The width of the Andreev peak mirrors  $\Delta$  of the Al film if  $l$  in the tip is much larger than the tip radius  $a$ , otherwise, the peak is broadened and lowered by the tip resistance [13]. Devices 1 and 3 show the Andreev peak of halfwidth  $\Delta = 2.4$  meV equal to  $\Delta$  in the NbN film underneath the Al film. So the Ti tip in devices 1 and 3 is ballistic. Moreover, the fact that the Ti tip attached to the Al film sees the Andreev peak halfwidth corresponding to  $\Delta$  of the NbN film is clear evidence that there is no proximity effect between the Al film and the Ti tip. This fact, combined with observation  $G(0)/G_B > 2$ , justifies the use of our modified BTK theory.

Device 2 exhibits the Andreev peak with  $\Delta = 2.8$  meV. As shown in Appendix C, this peak is broadened and lowered by the resistance  $R_0$  of the Pt-Ir tip, implying that  $l$  in the tip is not large enough in comparison with  $a$ . After removing broadening, the ratio  $G(0)/G_B$  becomes even higher. Finally, the Andreev peak of device 4 mirrors  $\Delta$  in the Al film, lowered to 0.77 meV by the thick NiCu film beneath (see Appendix C). This peak does not obey  $k_B T \ll \Delta$  as documented by the  $T$ -dependent peaks in Fig. 8 of Appendix D.

Using the  $G_N$  values in Fig. 4 and  $k_F = 1.75 \times 10^{10} \text{ m}^{-1}$  [13] in equation  $G_N = (2e^2/h)(a^2 k_F^2/4)$ , we find for devices 1, 2, 3, and 4, respectively,  $a = 2.5, 6, 3.2$ , and  $2$  nm. Then, using Eq. (11) and values of  $r_C$  in Fig. 4, we get  $l = 4, 5.2, 4.5$ , and  $1.7$  nm. These values are scattered around  $l = 3.4$  nm extracted from the resistivity of the reference Al film grown on sapphire,  $\rho_{\text{ref}} = 1.18 \times 10^{-7} \Omega\text{m}$  [13]. As noted in Appendix A,  $r_C$  reflects a local value of  $l$  fluctuating from PC to PC.

We have seen that in our Al films  $l \sim a$ , where both  $l$  and  $a$  are much smaller than the Al film thickness. This means that the quasiparticles entering the Al film from the PC are backscattered only by disorder in the Al film, without any additional backscattering by the interface between the Al film and the film beneath. Indeed, the  $G(0)/G_B > 2$  effect exists no matter whether the Al and NbN films are separated by another film (devices 2, 3, 4) or not (device 1), and no matter whether the separating film is NiCu or Ni. The existence of the effect depends neither on the tip material (Ti, Pt-Ir), only the interface tip-Al superconductor (parameter  $Z$ ) plays a role.

Superconductivity of a proximity-induced state is, in principle, not of simple BCS-type [25]. To capture the gross features of such a state, it is usual to modify the BCS description by the Dynes substitution  $E \rightarrow E + i\Gamma$ , where  $\Gamma$  is an empirical parameter [26]. A further justification of such a procedure is provided by the presence of ferromagnetic interlayers NiCu and Ni [27]. When necessary, we include the Dynes substitution into the BTK theory following Ref. [8]. Although the fits in Fig. 4 do not need a finite  $\Gamma$ , the  $G(V)$  data for devices 5 and 6 (Appendix C) cannot be fitted without it.

Finally, the measured  $G(V)$  data in Fig. 4 show dips above the gap, ignored in our theory. Such dips do not affect  $G(0)/G_B$ . Recently, their origin was ascribed to the PC critical current, and it was proposed that they may lower the peak width [28]. However, the Andreev peaks of devices 1, 3, 6, and the device in Fig. 7 of Ref. [13] show the same gap (2.4–2.5 meV) as obtained from tunneling spectra [29,30] of NbN films with similar  $T_C$  ( $\simeq 15$  K) and  $\rho$  ( $\simeq 10^{-6} \Omega\text{m}$ ).

## V. DISCUSSION OF $R(T)$ MEASUREMENTS

Now we turn to the PC resistance  $R(T)$ . If  $T > T_C$ , the supercurrent is replaced by a dissipative current carried by the transmitted electrons. In effect, the originally superconducting electrode shows the Maxwell resistance  $R_M \equiv \rho/4a = R_N(3\pi/16)(a/l)$  [3], where  $\rho = 3\pi^2\hbar/e^2k_F^2l$  is the Drude resistivity and  $R_N \equiv 1/G_N$  the Sharvin resistance. Thus

$$R(T) = \frac{1}{G(V \rightarrow 0, T)} + \Theta(T - T_C)R_M, \quad (14)$$

where  $\Theta \equiv 1$  if  $T > T_C$  and  $0$  if  $T \leq T_C$ . For  $V \rightarrow 0$ , Eq. (5) can be written in the form

$$G(V = 0, T) = \frac{1}{2k_B T} \int_0^\infty dE \frac{g(E)}{\cosh^2\left(\frac{E}{2k_B T}\right)}, \quad (15)$$

where  $g(E)$  is given by Eq. (10) combined with Table I. Evaluating Eq. (15) for  $T = 0$ , we reproduce the BTK result for  $G(0, 0)$  given by Eq. (7). Thus  $R(0) \equiv 1/G(0, 0) = R_N(1 + 2Z^2)^2/2$ , where  $R_N = 1/G_N$ . Further, we evaluate Eq. (15) for  $T < T_C$  in the limit  $T \rightarrow T_C$ , when  $\Delta(T \rightarrow T_C) \rightarrow 0$ . In this limit,  $g(E)$  in the integrand can be replaced by  $g(E) = G_N(1 - r_C)/(1 + Z^2)$  valid for  $E > \Delta \rightarrow 0$ . We get

$$G(V = 0, T = T_C) = \frac{G_N(1 - r_C)}{1 + Z^2} \quad (16)$$

and  $R(T_C) = 1/G(0, T_C)$ , where  $r_C$  can depend on  $T$  and has to be evaluated at  $T = T_C$ . If  $r_C$  is  $T$ -independent, then

$$R(T_C) = \frac{1}{G_B}, \quad (17)$$

where  $G_B \equiv G(|V| \gg \Delta/e, T = 0)$  is given by Eq. (12). If confirmed experimentally, result  $R(T_C) = 1/G_B$  would imply that  $r_C$  does not depend on  $T$ , which is the assumption made in Appendix A. Moreover, it is obvious that

$$\frac{R(T_C)}{R(0)} = \frac{G(0, 0)}{G_B}. \quad (18)$$

This means that the result  $G(0, 0)/G_B > 2$ , already observed by  $G(V)$  measurements, should coexist with result  $R(T_C)/R(0) > 2$ , observable by usual  $R(T)$  measurements.

For  $T > T_C$ , we obtain from Eq. (14) [remembering that Eq. (10) holds for  $r_C \ll 1$  and  $Z^2 \ll 1$ ] the normal-state PC resistance in the form

$$R(T > T_C) \simeq R_N(1 + Z^2) + R_M + R_{CB}, \quad (19)$$

where  $R_{CB} = R_N r_C/(1 - r_C) \simeq R_N r_C$ . The term  $R_{CB}$  is absent in the usual theory [3,19,31]. Both  $R_{CB}$  and  $R_M$  are due to disorder in the (no longer superconducting) electrode. However, while  $R_M$  is the dissipative spreading resistance described by semiclassical transport theory [31],  $R_{CB}$  is the resistance caused by coherent backscattering (CB) by disorder near the PC (Fig. 2), absent in the semiclassical theory. Moreover,  $R_M$  disappears below  $T_C$  while CB still affects Eq. (10). Note that values of  $l$  entering  $r_C$  and  $R_M$  may differ as the former  $l$  fluctuates from PC to PC. In Appendix B, we modify Eq. (19) for the PC disordered on both sides [see Eq. (B2)].

If  $Z = 0$ , then  $R_N$ ,  $R_M$ , and  $R_{CB}$  can be expressed by the equations listed in Fig. 5(a). Figure 5(a) illustrates how to extract  $R_N$ ,  $R_M$ , and  $R_{CB}$  from the measured  $R(T)$  curve, here

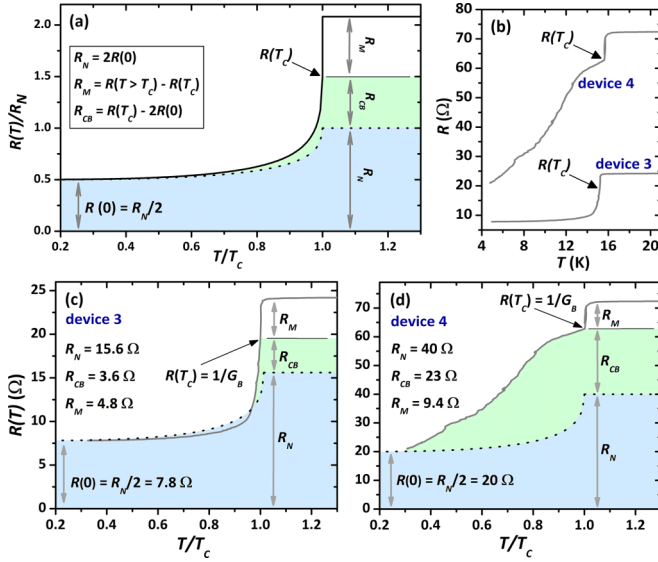


FIG. 5. (a) Theoretical  $R(T)$  curve obtained from Eq. (14) (see text for details) for  $Z = 0$  and  $T_c = 15.3$  K. Solid line:  $a/l = 1.13$ . Dotted line: the BTK result ( $a/l = 0$ ). The marked values of  $R_N$ ,  $R_M$ , and  $R_{CB}$  follow from the equations in the legend. (b) shows the measured  $R(T)$  data for devices 3 and 4. In (c) and (d), the measured  $R(T)$  data (full lines) are analyzed following the theory in (a). The dotted lines show the  $R(T)$  data calculated using the usual BTK theory ( $r_C = r_D = 0$ ) with  $Z = 0$ ,  $R_N = 2R(0)$ , and  $T_c = 15.3$  K, and  $\Delta(T)$  specified in the text. The green areas visualize the CB contributions to the usual BTK result.

modelled by Eqs. (14), (5), (10), (11),  $r_D = r_C$ , using  $a/l = 1.13$ , and  $Z = 0$ . The  $T$  dependence of the gap is modelled by the simplified [39] BCS-like formula

$$\Delta(T) = 1.764k_B T_c \tanh \left( 1.74 \sqrt{\frac{T_c}{T}} - 1 \right). \quad (20)$$

Figure 5(a) visualizes theoretical prediction  $R(T_c)/R(0) > 2$ .

Figures 5(b)–5(d) present the measured  $R(T)$  data for devices 3 and 4. Both devices exhibit the results  $R(T_c)/R(4.2K) = G(0)/G_B > 2$ , which confirm our theoretical prediction  $R(T_c)/R(0) = G(0)/G_B > 2$ . Note that  $R(T_c)$  determined from the ohmic resistance measurements is equal to  $1/G_B$  determined from the low-temperature  $G(V)$  spectra in Fig. 4, as predicted by Eq. (17). This implies that  $r_C$  does not depend on  $T$ . The values of  $R_M$  and  $R_{CB}$  are determined in Figs. 5(c) and 5(d) following the legend in Fig. 5(a).

Remarkably, Figs. 5(c) and 5(d) also show that the CB contribution (green area) for device 3 is similar to the theoretical one in Fig. 5(a), while for device 4 it is much larger. This is due to the 50-nm separating NiCu film in device 4. It suppresses  $\Delta$  in the Al film of device 4 more than three times below the value  $\Delta = 2.4$  meV, observed for device 3 [cf. Figs. 4(h) and 4(f)]. Due to the suppressed  $\Delta(T)$ , the contribution of the quasiparticle conductance,  $g(E > \Delta)$ , to the total conductance [Eq. (15)] is increased and the contribution from the supercurrent-mediated conductance  $g(E < \Delta)$  is decreased. As a result, in device 4, the CB at energies  $E > \Delta$  contributes to the measured  $R(T)$  curve much more than in device 3. This experimental result is consistent with our theo-

retical assumption that disorder in the superconductor affects transport through the PC only at energies  $E > \Delta$ , without affecting the supercurrent at  $E < \Delta$ .

Devices 1, 3 and 4 exhibit  $R_M \simeq 12$  [13], 4.8, and 9.4  $\Omega$ , respectively, and  $R(T)$  for device 2 was not measured due to the thermal instability of the mechanical tip. If we estimate for devices 1, 3, and 4 the (effective) resistivity of the multifilm electrode  $\rho = 4aR_M$ , we get  $\rho = 1.2 \times 10^{-7}$ ,  $0.62 \times 10^{-7}$ , and  $0.75 \times 10^{-7} \Omega\text{m}$ , close to  $\rho_{\text{ref}}$ .

## VI. SUMMARY

We studied the  $G(V)$  and  $R(T)$  characteristics of the PC between a clean ( $l_0 \gg a$ ) metal and a conventional isotropic superconductor which was dirty in the sense  $\xi_0 \gg l$ , ballistic in the sense  $\xi_0 \gg a$ , and weakly disordered in the sense  $l \gtrsim a$ , but which obeyed the limit  $\xi = \sqrt{\xi_0 l} \gg a$  justifying the applicability of the ballistic BTK theory. Disorder on the superconducting PC side, being the only perturbation of the ballistic regime, was incorporated into the BTK theory as a weak perturbation. Specifically, due to the condition  $\xi \gg a$ , we could neglect the effect of disorder on the Andreev reflections and supercurrent flowing below the gap and we could describe the Andreev conductance  $G(V)$  at  $|V| \leq \Delta/e$  by the ballistic BTK theory.

However, we did incorporate into the BTK theory the fact that disorder in the superconductor, no matter how weak, scatters coherently the quasiparticles entering into the superconductor above the gap and reflects a small part of them back to the PC before they join the condensate. This weak backscattering effect lowers the background conductance  $G_B \equiv G(|V| \gg \Delta/e)$  from  $G_B = G_N$  to  $G_B = G_N(1 - r_C)$ , where  $r_C \propto (a/l)^2$  is the backscattering probability. As a result, our theory predicted the Andreev peak-to-background ratio  $G(0)/G_B$  larger than the ballistic value of two. Further, we showed that the  $G(0)/G_B > 2$  effect is accompanied by effect  $R(T_c)/R(0) > 2$ , where  $R(T_c)/R(0) = G(0)/G_B$ . Finally, for the normal state ( $T > T_c$ ) our theory predicted that the coherent backscattering raises the Sharvin-Maxwell PC resistance  $1/G_N + \rho/4a$  by adding an extra resistance term  $\propto (a/l)^2$ . All these results were demonstrated experimentally by PC spectroscopy of the superconducting Al films. According to our experimental data for device 2, the effect  $G(0)/G_B > 2$  persists also for the PCs with (weak) disorder on both sides, if disorder on the superconducting side prevails.

Though the increase of the ratio  $G(0)/G_B$  by disorder in the superconductor is a generic effect, the value larger than two occurs if a clean metal tip ( $l_0 \gg a$ ) probes a superconductor with disorder obeying  $l \sim a$  and  $\xi = \sqrt{\xi_0 l} \gg a$ , where the latter condition ensures a well defined  $\Delta$  assumed in the BTK theory. These conditions are met by our devices 1, 2, 3, and 4. However, they may be easily disrupted by factors  $Z$ ,  $R_0$ , and  $\Gamma$ , which might explain why the  $G(0)/G_B > 2$  effect was not noticed in the past. We stress that the effect should not be confused with various  $G(0) > 2G_N$  anomalies due to anisotropic pairing [32–38], resonant tunneling [39], and Kondo effect [40]. Neither should it be confused with the  $G(0)/G_B > 2$  anomaly caused by exceeding the PC critical current [41], exhibiting the anomalous peak of width  $\ll \Delta$ .

## ACKNOWLEDGMENTS

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## DATA AVAILABILITY

The data that support the findings of this article are openly available [42], embargo periods may apply.

APPENDIX A: ESTIMATE OF THE TYPICAL VALUE OF REFLECTION PROBABILITY  $r_C$ 

The quasiparticle injected into the superconducting electrode from the PC can be reflected by disorder in the electrode back to the PC as shown in Fig. 2. The reflection probability  $r_C$  can be easily estimated by considering the isotropic single-impurity collisions, such as the collision with impurity  $i_1$  in Fig. 2. In such case,  $r_C \simeq \pi a^2 \overline{\cos \vartheta} / 4\pi l^2 = a^2 / 8l^2$ , where the mean free path  $l$  represents the mean distance between the first single-impurity collision and PC center,  $\pi a^2 \cos \vartheta$  is the effective area of the PC seen by the quasiparticle reflected by the impurity  $i_1$  towards the PC (Fig. 2), and  $\overline{\cos \vartheta} = 1/2$ . Result  $r_C \simeq a^2 / 8l^2$  holds for  $a/l \lesssim 1$ .

Now we estimate  $r_C$  by considering the coherent multiple-impurity collisions (such as the collision with impurities  $i_1, i_2, i_3$ , and  $i_4$  in Fig. 2) which cause elastic quasiparticle diffusion. Consider the disordered halfspace  $z_0 > 0$  on the right hand side of the specularly reflecting plane at  $z_0 = 0$ . Assume that a particle starts its motion at time  $t = 0$  at position  $\vec{r}_0 = (0, 0, 0)$ . If disorder is weak ( $k_F l \gg 1$ ), the probability that the particle diffuses within time  $t$  to a position  $\vec{r}$  is

$$P(\vec{r}, \vec{r}_0, t) = 2(4\pi Kt)^{-3/2} \exp[-(\vec{r} - \vec{r}_0)^2 / 4Kt], \quad (\text{A1})$$

where  $K = lv/3$  is the diffusion constant and  $l(E)$  and  $v(E)$  are the mean free path and group velocity of the particle with energy  $E$ . Except for the factor of 2, equation (A1) coincides with the Green function valid for infinite system. The factor of 2 accounts for the presence of the reflecting plane.

Equation (A1) describes quantum diffusion at times  $\tau < t < \tau_c$ , where  $\tau = l/v$  is the elastic scattering time and  $\tau_c$  is the coherence time [43,44]. Thus  $P(\vec{r}, \vec{r}_0, t)$  represents the quantum single-particle density at point  $\vec{r}$  at time  $t$ . Assume that the PC is centered at  $\vec{r}_0 = (0, 0, 0)$  in the plane  $z_0 = 0$ . A quasiparticle that enters the superconducting halfspace through the PC at time  $t = 0$  can return back at any time  $t \geq 2\tau$ , where  $2\tau$  is the return time due to the single-impurity collision at time  $t = \tau$ . For  $a \lesssim l$  the probability that the quasiparticle returns at time  $t$  to a point  $\vec{r}$  at the PC is  $P(t) \simeq 2(4\pi Kt)^{-3/2}$ . The backscattered single-quasiparticle current density intersecting the PC at time  $t$  is  $(1/2)v \overline{\cos \vartheta} P(t)$ ,

where  $v \cos \vartheta'$  is the perpendicular component of  $v$  (Fig. 2), the factor  $1/2$  reflects that only one half of the quasiparticle density  $P(t)$  moves in the backward direction, and  $\overline{\cos \vartheta'} = 1/2$ . Finally,

$$\begin{aligned} r_C &= (1/2)\pi a^2 v \overline{\cos \vartheta'} \int_{2\tau}^{\tau_c} P(t) dt \\ &= (\pi/4)(3/2\pi)^{3/2} (a/l)^2 [1 - (2\tau/\tau_c)^{1/2}]. \end{aligned} \quad (\text{A2})$$

Though  $l(E)$ ,  $v(E)$ ,  $\tau(E)$ , and  $\tau_c$  should be derived for the Bogoliubov spectrum in Fig. 1, we approximate them by their normal-state values. In our samples  $l \simeq 3.4$  nm,  $v_F \simeq 2 \times 10^6$  m/s, and  $\tau = l/v_F \simeq 1.7$  fs. The coherence time  $\tau_c$  in the normal state reads  $\tau_c = \hbar/k_B T$  [43]. At  $T_C = 15.3$  K therefore  $\tau_c \simeq 0.5$  ps  $\gg 2\tau$ . Assuming  $\tau_c \gg 2\tau$  also below  $T_C$ , Eq. (A2) reduces to Eq. (11). Our estimate neglects interference effects studied in metallic PCs [45,46].

For  $T < T_C$ , the time  $\tau_c$  (no longer given by  $\tau_c = \hbar/k_B T$ ) might be approximated by the time at which the quasiparticle decays into the condensate. Thus  $\tau_c$  and  $r_C$  at  $T < T_C$  should depend on  $\Delta(T)$  and even on  $E$ . However, our experimental data indicate that this dependence can be neglected.

Our Al films contain also disorder due to the polycrystalline grains. The grain size in thin Al films is about the same as the film thickness, in our case 15 nm. This value is a few times larger than our tip size ( $2a \sim 5$  nm), and therefore the quasiparticle backscattering has to depend on the (random) position of the tip. In effect, the PC probes a local  $r_C$  mirroring the local  $l$  that fluctuates around  $l$  obtained from the film resistivity.

## APPENDIX B: METAL-SUPERCONDUCTOR PC WITH DISORDER ON BOTH SIDES

The standard way to incorporate electron scattering by disorder on the metal side into the BTK theory is to replace the parameter  $Z^2$  for each transport channel  $i$  by  $Z_{eff}^2 = Z^2 + r_i/(1 - r_i)$ , where  $r_i$  is the probability of reflection back to the metal in channel  $i$ . Afterwards, the formula for  $G(V)$  has to be averaged over the channels [15]. Here, staying at the same level of approximation as when treating the superconducting side, instead of averaging, we simply replace  $Z^2$  in the BTK results shown in Table I by  $Z_{eff}^2 = Z^2 + r_0/(1 - r_0)$ , where  $r_0$  is a typical value of  $r_i$ . Noticing that  $r_0 = r'_0$ , where  $r'_0$  is the probability that the electron entering the metal from the PC is reflected by disorder back to the PC,  $r'_0$  can be estimated in the same way as  $r_C$  [see Appendix A, Fig. 2, and Eq. (5)]. Thus  $r_0 = r'_0 \simeq (\pi/4)(3/2\pi)^{3/2} (a/l_0)^2$ , where  $l_0$  is the disorder-limited mean free path in the metal and  $a \lesssim l_0$ .

In addition, disorder on the metal side causes the dissipative Maxwell resistance  $R_0 = R_N(3\pi/16)(a/l_0)$  analogous to the Maxwell resistance  $R_M$ . Unlike  $R_M$ ,  $R_0$  is present also below  $T_C$ . In order to account for the fact that  $R_0$  reduces the voltage drop across the PC interface from  $V$  to  $V_i$ , thus changing the conductance from  $G(V)$  to  $G_i(V_i)$ , we write [13,47]

$$V = V_i + R_0 I(V), \quad 1/G(V) = 1/G_i(V_i) + R_0. \quad (\text{B1})$$

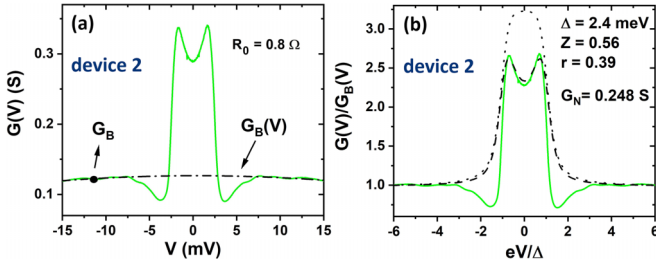


FIG. 6.  $G_i(V_i)$  data for device 2 (index  $i$  is skipped) obtained by transforming the original  $G(V)$  data for device 2 (Fig. 4) using Eq. (B1). Specifically, the data obtained by transforming the as-measured data from Fig. 4(c) are shown in (a) and the normalized data obtained by transforming the normalized data from Fig. 4(d) are presented in (b). All notations and symbols are the same as in Fig. 4. The dotted line in (b) shows the same calculation as the dashed line, but for  $Z = 0$ . The transformation reveals that the spreading resistance of the Pt-Ir tip is  $R_0 = 0.8 \, \Omega$  and slightly enhances  $Z$  and  $r_C$ .

To generalize Eq. (19), we add the term  $R_0$  and replace  $Z^2$  by  $Z^2 + r_0/(1 - r_0)$ . Assuming that  $Z^2$ ,  $r_0$ , and  $r_C$  are small,

$$R(T > T_C) \simeq R_N(1 + Z^2) + R_0 + R_M + R_N(r_0 + r_C). \quad (\text{B2})$$

In the standard theory [31], the term  $R_N(r_0 + r_C)$  is absent.

### APPENDIX C: OBSERVABILITY OF THE $G(0)/G_B > 2$ EFFECT IN COMPETITION WITH OTHER EFFECTS

(i) As one sees in Figs. 4(c) and 4(d), the Andreev peak of device 2 shows  $\Delta = 2.8 \, \text{meV}$  while the expected value is  $\Delta = 2.4 \, \text{meV}$ . It is known [47] that the Andreev peak may be broadened by the spreading resistance of the tip electrode,  $R_0$ . We eliminate this broadening by inserting into Eq. (B1) the experimental values of  $I(V)$ ,  $G(V)$ , and  $V$  for device 2 and we tune the value of  $R_0$  until  $G_i(V_i)$  shows the Andreev peak with  $\Delta = 2.4 \, \text{meV}$ . This transforms the as-measured  $G(V)$  data in panels (c) and (d) of Fig. 4 into the  $G_i(V_i)$  data (index  $i$  is skipped) in panels (a) and (b) of Fig. 6. The transformed Andreev peak is higher (more anomalous) than the as-measured one. Clearly, disorder on the metal side (responsible for  $R_0$ ) competes with disorder on the superconducting side by lowering the  $G(0)/G_B$  ratio, and thus acts against the  $G(0)/G_B > 2$  effect.

(ii) As can be seen from Eq. (12), another factor obscuring the  $G(0)/G_B > 2$  effect is finite value of  $Z$ . This is confirmed by Fig. 6. Finite  $Z$  can be caused by coherent electron backscattering by disorder on the metal side (see Appendix B), and by the oxide film [1] and/or Fermi velocity mismatch [4,9] at the PC interface. In addition to disorder on the metal side, in device 2, the Fermi velocity is certainly mismatched at the Pt-Ir/Al interface as the Fermi velocities in Pt [48,49] and Ir [49] are a few times smaller than in Al [50].

(iii) The Dynes parameter  $\Gamma$  is another factor acting against the  $G(0)/G_B > 2$  effect. We introduce substitution  $E \rightarrow E + i\Gamma$  into the BTK theory [1] as in Ref. [8]. Then, using Eqs. (5)

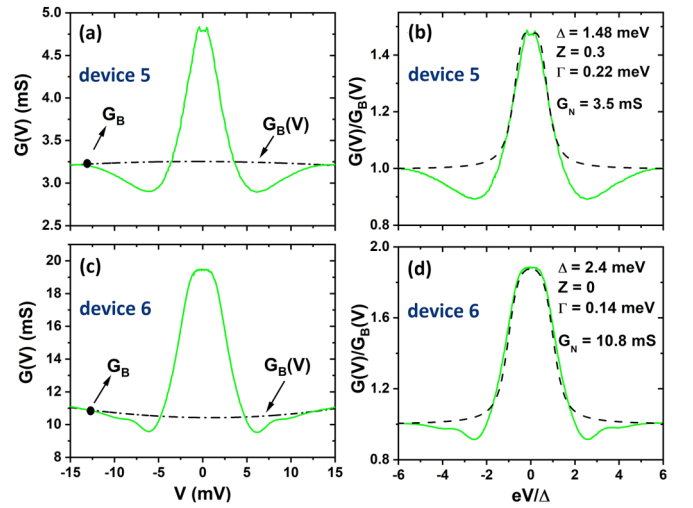


FIG. 7. Experimental and theoretical  $G(V)$  data for devices 5 and 6 at 4.3 K, presented in the same way as the  $G(V)$  data for devices 1, 2, 3, and 4 in Fig. 4. The as-measured data are shown in (a) and (c). The as-measured data normalized by  $G_B$  are shown in (b) and (d) together with theoretical fits (dashed lines). Both devices exhibit  $G(0)/G_B < 2$ . Therefore, the measured  $G(V)/G_B$  curves are fitted by the BTK theory with  $r_C = 0$  and with finite  $\Gamma$  ensuring that the theory reproduces the Andreev peak smaller than two. For  $r_C = 0$  and parameters in the figure, Eq. (C1), valid for  $T = 0$ , matches the measured  $G(0)/G_B$ .

and (10), we find that  $G_B \equiv G(|V| \gg \Delta/e)$  is still given by Eq. (12). However, at  $T = 0$  we also find the equation

$$\frac{G(0)}{G_N} = \frac{2[\Delta^2 + (1 - r_C)\Gamma[\Gamma + (1 + 2Z^2)\sqrt{\Delta^2 + \Gamma^2}]]}{[\Gamma + (1 + 2Z^2)\sqrt{\Delta^2 + \Gamma^2}]^2} \quad (\text{C1})$$

obtained using  $r_D = r_C$ . We see that  $\Gamma$  usually suppresses  $G(0)$  below the value  $G(0) = 2G_N/(1 + 2Z^2)^2$  and that also  $r_C$  lowers  $G(0)$ , but still  $G(0)/G_B > 2$  for large enough  $r_C$ .

In Fig. 7, we show the  $G(V)$  data for devices 5 and 6. These devices differ from device 4 by the thickness of the separating NiCu film, being 20 nm in device 5 and 5 nm in device 6. The increasing thickness of the NiCu film should reduce the proximity effect between the Al film and NbN film (this reduction will be discussed below). Moreover, due to the ferromagnetic nature of NiCu, it should increase  $\Gamma$  [27]. As one sees in Fig. 7, devices 5 and 6 do not show the  $G(0)/G_B > 2$  effect and their spectra can be fitted by the usual BTK theory ( $r_C = r_D = 0$ ) modified by the presence of  $\Gamma$  [8]. However, their spectra could also be fitted by using nonzero  $r_C$  and larger  $\Gamma$ . Similarly, it is likely that, owing to its large NiCu film thickness, the actual  $\Gamma$  of device 4 is even larger than that in devices 5 and 6. This could be again accounted for by combining nonzero  $\Gamma$  with an even larger value of  $r_C$ . The same is true for the fits of all devices in Fig. 4.

(iv) As shown in Ref. [13], the Al film grown on the top of the NbN film perfectly mimics  $T_C$  and  $\Delta$  of the NbN film ( $T_C = 15.3 \, \text{K}$ ,  $\Delta = 2.4 \, \text{meV}$ ). Indeed,  $\Delta = 2.4 \, \text{meV}$  is exhibited by the Andreev peak of device 1 in Fig. 4. However, if the Al film and NbN film are separated by an additional

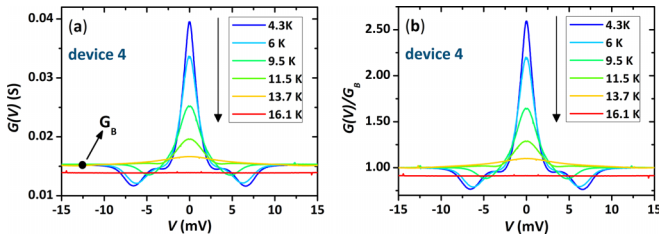


FIG. 8.  $G(V)$  spectra of device 4 measured at various temperatures. The as-measured spectra are presented in (a) while (b) shows the spectra normalized by the background conductance  $G_B$ .

NiCu (or Ni) film which is non-superconducting and even ferromagnetic, the value of  $\Delta$  in the Al film should be reduced, if the separating film is thick enough. Just this effect is manifested by the Andreev peak in device 4 (Fig. 4), which shows  $\Delta = 0.77$  meV. The fact that this small value is due to the 50-nm NiCu film, is confirmed in Fig. 7. In device 5 with 20-nm NiCu film  $\Delta$  is 1.48 meV, while in device 6 with 5-nm

NiCu film  $\Delta$  is 2.4 meV. Thus the 5-nm NiCu film in device 6 is thin enough not to affect  $\Delta$ . If the 5-nm NiCu film is replaced by the 5-nm Ni film, the value of  $\Delta$  remains 2.4 meV (device 3 in Fig. 4). Note also in Fig. 5 that neither the NiCu film nor the Ni film affect  $T_C$ , which is about 15.3 K in all our devices. In Ref. [22], we show how the NiCu (Ni) film controls  $\Delta(T)$ .

#### APPENDIX D: $T$ -DEPENDENT SPECTRA OF DEVICE 4

Fig. 8 shows the  $G(V)$  spectra of device 4 for  $T$  varying from 16.1 K down to 4.3 K. Notice the following features. (i) The peak at  $V = 0$  arises slightly below  $T_C \simeq 15.3$  (at 13.7 K). This indicates that it reflects the proximity-effect-induced superconductivity in the Al film and Andreev reflections at the tip-Al interface. (ii) The value of  $1/G(V, T = 16.1\text{ K}) - 1/G_B$  is  $9.4\ \Omega$ , in agreement with the Maxwell resistance  $R_M$  extracted from the  $R(T)$  data in Fig. 5(d). (iii) As  $T$  decreases,  $G(0)/G_B$  exceeds two, but does not show any saturation between 6 and 4.3 K, showing that in device 4 the limit  $k_B T \ll \Delta$  is not reached at our lowest  $T$ .

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