

Correlations between superconducting and resistive anisotropies

Sayan Banerjee ¹, Harley D. Scammell ², and Mathias S. Scheurer ¹¹*Institute for Theoretical Physics III, University of Stuttgart, 70550 Stuttgart, Germany*²*School of Mathematical and Physical Sciences, University of Technology Sydney, Ultimo NSW 2007, Australia*

(Received 4 December 2025; accepted 29 April 2026; published 22 May 2026)

There are multiple possible origins of transport anisotropies in metals and superconductors. For instance, rotational symmetry can be spontaneously broken in the normal state as a result of electronic nematic order inducing anisotropies in an otherwise s -wave superconducting state. Another possibility is that the dominant source of rotational symmetry breaking is the superconductor itself and its vestiges that may survive in the normal state. We here theoretically analyze the correlations of transport anisotropies in the normal and the corresponding superconducting phase for different scenarios of broken symmetry, either coming solely from the normal state, solely from the superconductor and its vestiges in the metallic regime, or simultaneously from both. We further include both zero-momentum and finite-momentum pairing; we develop a theory of vestigial order for the latter, characterized by broken rotational and translational symmetry. Our findings reveal that the relative transport anisotropies in the normal and superconducting phases sensitively depend on the scenario, including the form of vestigial order and, in some cases, the parity of the superconducting order parameter. As such, measuring the directional dependence of the critical current and resistivity can provide strong constraints on the origin of rotational symmetry breaking. We demonstrate our findings in minimal models relevant to twisted multilayer graphene, rhombohedral graphene, and twisted transition-metal dichalcogenides.

DOI: [10.1103/kjyl-rxg7](https://doi.org/10.1103/kjyl-rxg7)

I. INTRODUCTION

Symmetries are a key guiding principle for our understanding of both metallic and superconducting states [1]. In weakly interacting metals, transport properties primarily reflect the symmetries of the crystalline lattice. In contrast, for strongly interacting systems, electronic correlations can drive additional symmetry breaking transitions that do not stem from the lattice itself. In the case of electronic nematic order [2–6], only rotational symmetry is broken while translational symmetry is preserved. However, rotational symmetries can also be spontaneously broken by the formation of charge-density waves (CDWs) where the charge is modulated only along one direction. When a superconductor (SC) emerges from such a symmetry-reduced electron liquid, the superconducting condensate will inherit the same (broken) symmetries, resulting in anisotropic superconducting properties. However, the symmetry reduction in the normal state can also stabilize pairing states that would otherwise be not favored energetically, since, e.g., a nematic superconducting order parameter can couple to the first power of the nematic order parameter of the normal state. Beyond spatial symmetries, also the breaking of time-reversal symmetry (TRS) in the normal state has profound consequences for pairing; the degeneracy of states with momenta $\mathbf{k}$ and $-\mathbf{k}$ is lifted, which can lead to

nonzero momentum of Cooper pairs [7,8], possibly lowering the system's symmetries further [9–11].

Conversely, superconductors themselves can also spontaneously break symmetries [1], and, upon melting of the superconducting order, these broken symmetries can imprint on the normal-state responses through “vestigial orders” [12,13]—a very actively studied field at the moment [14–42]. All of this suggests a deep and intricate relationship among symmetries in the normal states and superconducting phases.

One of the central goals in the field of superconductivity is to identify the form and symmetries of the superconducting order parameter in a given material—in most cases, however, this is very challenging. For instance, ever since the discovery of superconductivity in two-dimensional (2D) twisted-graphene superlattices [43–47], which can exhibit a plethora of conventional and unconventional states, the mechanism and type of pairing have been subjects of intense debate, see e.g., [48–65] and many more. Recently, other (untwisted) rhombohedral graphene stacks [66,67] have also surged in interest, and multiple exotic candidate pairing states have been discussed [68–81].

One particular avenue to probe the pairing symmetry is based on transport anisotropies in both the normal (resistivity) and superconducting states (critical current). For instance, in magic-angle twisted bilayer graphene, a twofold anisotropy in the resistivity and critical current was observed and attributed to be a signature of nematic pairing [82]. However, rotational symmetry breaking in the form of external strain [83–85] or nematicity has also been shown to exist in the normal state [82,86–89], which might or might not be related to superconductivity. It is quite hard to distinguish whether the SC breaks

Published by the American Physical Society under the terms of the Creative Commons Attribution 4.0 International license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

rotational symmetry explicitly or inherits it from the normal state [52,58,61,65,90–92]. What is more, the point group often allows for more than one possible nematic superconducting state, and it would be interesting to determine whether, and under which conditions, transport anisotropies can be used to distinguish even-parity and odd-parity nematic superconductors.

Another interesting experimental finding in twisted graphene superlattices was reported in Ref. [93], where an atypical behavior was observed in the directionalities of the superconducting and normal states. Specifically, a transport anisotropy was observed where the direction of the maximum critical current in the superconducting state coincided with that of the maximum resistivity in the normal state. A Ginzburg-Landau analysis was performed, which revealed that if one takes an anisotropic normal state and a uniform gap, one obtains an opposite relationship between the two, which is in stark contrast to the experimental observation. While this places constraints on the order parameter, one can also imagine a situation where fluctuation-driven vestigial orders play a role, going beyond strain-dominated effects. Additional important open aspects are the role of trigonal warping and that additional anisotropies in the normal state, which are unrelated to superconductivity, will also affect the superconducting energetics itself and select specific order-parameter configurations.

Motivated by the rich physics and sensing capabilities of correlations between normal and superconducting angular-dependent transport, we here study various possible cases and sources—in particular, those relevant to graphene and transition-metal dichalcogenide (TMD) heterostructures—of transport anisotropies in both normal and superconducting phases. We demonstrate the interplay of various primary and vestigial orders and their transport signatures, both in mean-field theory and by taking fluctuations into account.

The presence of $U(1)_v$ valley symmetry in these systems allows us to discuss the sources of rotational symmetry breaking for two kinds of pairing states separately: inter- and intravalley superconductivity. We perform an in-depth analysis for both cases. For the intervalley case (see Sec. II), we first outline the general formalism to evaluate the angular dependence of critical currents and resistivities. We then show the dependencies for k -independent superconducting order parameters and for nematic pairing, considering also the case where the superconducting order parameter or fluctuations thereof are the only sources of rotational symmetry breaking. By comparing the interplay of the anisotropies, we show that there are features that can be used to extract the parity of the order parameter and gain crucial insights into the nature of superconductivity. In Sec. III, we shall consider the intravalley pairing case. We develop a theory of vestigial orders emerging out of finite-momentum superconductivity in a single valley, which is particularly relevant to multilayer rhombohedral graphene. We will see how this can lead to striped and nematic phases [94–96], emerging out of the primary order parameter provided by superconductivity. Our analysis will show how different forms of orders (all breaking the same symmetries) can manifest differently in transport. While we broadly base our analysis on van der Waals heterostructures, we emphasize that our formalism and results are also directly applicable

to other systems such as kagome materials; in particular, an anticorrelation between normal state and superconducting anisotropic transport behavior has been reported in the kagome superconductor CsV_3Sb_5 [97].

II. INTERVALLEY PAIRING

A. General formalism

Before presenting explicit calculations, we start by discussing the general methodology to extract normal-state and resistive properties for intervalley-paired SCs. We shall first focus on the case where the normal state has TRS and the band dispersions in the two valleys, $v = \pm$, are just mirror images of one another, related by $\mathbf{k} \rightarrow -\mathbf{k}$. Although the generalization to include spin is straightforward, we will, for the simplicity of the presentation, assume that either the Fermi surfaces are spin polarized or spin and valley are locked. This applies to twisted TMDs, such as twisted WSe_2 [98,99], as a result of the strong spin-orbit coupling and is further natural for the spin polarized half-metal phases in rhombohedral graphene [67], twisted MoTe_2 [100], or for pairing around filling fraction 2 in twisted multilayer graphene [101,102]. As a result of TRS, pairing between opposite valleys and momenta $\mathbf{k}$ and $-\mathbf{k}$ is favored, and the superconducting order parameter has to transform under an irreducible representation (IR) of the normal-state point group. While interband pairing can lead to interesting superconducting properties [54,103], here we will focus on IRs with band-diagonal order parameters. Then the relevant active low-energy electrons can be described by the creation operators $c_{\mathbf{k},v}^\dagger$, and the pairing Hamiltonian is given as

$$\mathcal{H}_c = g_c \sum_{\mathbf{k}} \sum_{j=1}^{d_\Gamma} [\eta_j(\mathbf{q}) f_j(\mathbf{k}) c_{\mathbf{k}+\mathbf{q},+}^\dagger c_{-\mathbf{k}+\mathbf{q},-}^\dagger + \text{H.c.}]. \quad (1)$$

Here, the pairing depends on center-of-mass momentum $\mathbf{q}$ and j runs over d_Γ components of the IR Γ with superconducting order parameter $\eta_j(\mathbf{q})$ and the associated form factors $f_j(\mathbf{k})$. For the point groups relevant to us here, only $d_\Gamma = 1, 2$ are possible and we will discuss examples with both $d_\Gamma = 1$ and $d_\Gamma = 2$ in this work.

As explained above, our goal is to compare transport anisotropies in the superconducting and normal state. As such, we will consider normal-state Hamiltonians, $\mathcal{H}_0 = \sum_{\mathbf{k},v=\pm} c_{\mathbf{k},v}^\dagger E_{\mathbf{k},v} c_{\mathbf{k},v}$, where C_{3z} rotational symmetry is broken. The associated dispersion can take either of the following forms:

$$E_{\mathbf{k},v} = \xi_{\mathbf{k},v} + \Phi \cdot \mathbf{g}(\mathbf{k}) \quad \text{or} \quad (2a)$$

$$E_{\mathbf{k},v} = \xi_{\mathbf{k},v} + \delta\epsilon_{\mathbf{k},v}. \quad (2b)$$

Here, $\xi_{\mathbf{k},v} = \epsilon_{\mathbf{v},\mathbf{k}}$, such that $\xi_{\mathbf{k},v} = \xi_{-\mathbf{k},-v}$, is the single-particle dispersion for valley v , which obeys $\epsilon_{C_{3z},\mathbf{k}} = \epsilon_{\mathbf{k}}$. The first form (2a) describes symmetry breaking as a consequence of moiré nematic order or effectively the presence of strain [82,86–89,104], which we will parameterize by $\Phi = \Phi(\cos \theta, \sin \theta)$; meanwhile, Eq. (2b) models the consequence of a vestigial order $\delta\epsilon_{\mathbf{v},\mathbf{k}}$ that modifies the bare dispersion. These are the two principal sources of breaking the symmetry that we will discuss in this work. To preserve TRS (and, if present, C_{2z}

symmetry) in the normal state, we choose the form factor $\mathbf{g}(\mathbf{k})$ to be even in momentum $\mathbf{k}$ and to transform as a vector under C_{3z} (specifically IR E of the point group D_3 or E_2 of D_6). In the limit of small $\mathbf{k}$, its leading behavior is given by $\mathbf{g}(\mathbf{k}) \sim (k_y^2 - k_x^2, 2k_x k_y)$ and we take the Brillouin-zone-periodic extension with the fewest nodal lines.

In order to probe such C_{3z} symmetry breaking in experiments, one typically measures the resistivity/conductivity tensor, which is sensitive to such anisotropies. Within a Boltzmann transport approach, the conductivity tensor in the relaxation-time approximation is given as (see e.g., [105] where a similar notation is used)

$$\sigma_{\alpha\beta} = \frac{2e^2\tau}{\mathcal{V}} \sum_{\nu=\pm} \langle (\mathbf{v}_{\mathbf{k},\nu})_{\alpha} (\mathbf{v}_{\mathbf{k},\nu})_{\beta} \rangle_{\nu}, \quad (3a)$$

where $(\alpha, \beta) \in \{x, y\}$ and all the nonuniversal properties are encoded in the generalized average

$$\langle \dots \rangle_{\nu} \equiv \frac{4}{TN} \sum_{\mathbf{k}} \frac{\tau_{\mathbf{k},\nu}/\tau}{\cosh^2\left(\frac{E_{\mathbf{k},\nu}-\mu}{2T}\right)} \dots \quad (3b)$$

and in the velocities $\mathbf{v}_{\mathbf{k},\nu} = \nabla_{\mathbf{k}} E_{\mathbf{k},\nu}$. Here, N is the number of moiré unit cells, $\mathcal{V}$ is the area of the moiré unit cell, and $\tau_{\mathbf{k},\nu}^{-1}$ is the relaxation-time scattering rate for momentum $\mathbf{k}$ in valley ν . The average scattering time is defined as $\tau = N^{-1} \sum_{\mathbf{k}} \tau_{\mathbf{k},\nu}$, which is the same in both valleys, because of TRS. Considering a current applied along a direction that makes an angle Ω with the x axis, and assuming a momentum-independent scattering time $\tau_{\mathbf{k},\nu} = \text{const.}$, one can now extract the angular dependence of longitudinal resistivity as

$$\rho_L(\Omega) = \frac{1}{\sigma_0} - \frac{1}{\sigma_0^2} (\lambda \cos 2\Omega + \delta \sin 2\Omega), \quad (4)$$

where $\lambda = (\sigma_{xx} - \sigma_{yy})/2$, $\sigma_0 = (\sigma_{xx} + \sigma_{yy})/2$, and $\delta = \sigma_{xy}$.

Equipped with normal state transport, we now focus on the superconducting phase. Assuming that the SC arises from such a normal state, we take an attractive interaction ($g_c > 0$) and perform a mean-field decoupling to obtain $H_s = \mathcal{H}_0 + \mathcal{H}_c + \sum_{\mathbf{q},j} |\eta_j(\mathbf{q})|^2 / g_c$. Integrating out the electrons, and expanding up to quadratic order in the free energy, we get

$$\mathcal{F} \sim \sum_{\mathbf{q},j=1}^{d_{\Gamma}} a_j(\mathbf{q}) |\eta_j(\mathbf{q})|^2 + \mathcal{O}(\eta^4). \quad (5)$$

It holds $a_j(\mathbf{q}) = \frac{1}{g_c} - \Gamma^j(\mathbf{q})$ with

$$\Gamma^j(\mathbf{q}) = \frac{1}{2N} \sum_{\mathbf{k}} \frac{\tanh \frac{\mathcal{E}_{\mathbf{k},\mathbf{q},+}^{\Phi}}{2T} + \tanh \frac{\mathcal{E}_{\mathbf{k},\mathbf{q},-}^{\Phi}}{2T}}{\mathcal{E}_{\mathbf{k},\mathbf{q},+}^{\Phi} + \mathcal{E}_{\mathbf{k},\mathbf{q},-}^{\Phi}} |f_j(\mathbf{k})|^2, \quad (6)$$

where N is the number of unit cells and $\mathcal{E}_{\mathbf{k},\mathbf{q},\nu}^{\Phi} = E_{\mathbf{k}+\nu\mathbf{q},+}^{\Phi}$. Restricting the analysis to single- $\mathbf{q}$ states, $\eta(\mathbf{q}) \propto \delta_{\mathbf{q},\mathbf{q}_0}$, we can then obtain the current $\mathbf{J}(\mathbf{q}) = 2e\partial_{\mathbf{q}}\mathcal{F}$ [106]. The critical current $J_c(\Omega)$ along a certain direction Ω is then given by the maximum of $|\mathbf{J}(\mathbf{q})|$ with $\mathbf{J}(\mathbf{q})$ oriented along Ω . To compare the relative anisotropies between the superconducting critical current and normal state resistivity, we define the quantity

$$\zeta_{xy} := \frac{\text{sgn}[\bar{J}_c(\Omega=0) - \bar{J}_c(\Omega=\pi/2)]}{\text{sgn}[\bar{\rho}_L(\Omega=0) - \bar{\rho}_L(\Omega=\pi/2)]}. \quad (7)$$

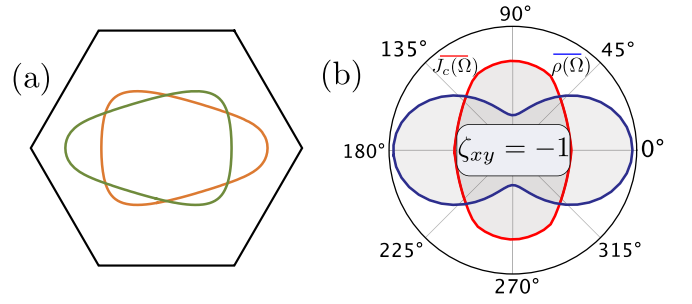


FIG. 1. (a) Fermi surfaces (FSs) for the two-valley model. The presence of nematic order along x ($\theta = 0$) breaks the C_{3z} symmetry in the normal state. Panel (b) shows the resistivity (in blue) and critical current (in red) for $\Phi = 0.2$. The rest of the parameters are chosen to be $T = 0.2$, $\mu = -0.68$, $\phi = -0.5\pi$, $g_c^{-1} = 0.5\Gamma(0)$.

We here defined $\bar{J}_c(\Omega) := \max_{n=0,1,2} J_c(\Omega + n2\pi/3)$ and $\bar{\rho}_L(\Omega) := \max_{n=0,1,2} \rho_L(\Omega + n2\pi/3)$ to account for all three, C_{3z} -related, orientations of the nematicity. By construction, $\zeta_{xy} \in \{-1, 1\}$, indicating whether the anisotropy of the critical current is aligned ($\zeta_{xy} = 1$) or antialigned ($\zeta_{xy} = -1$) with that of the resistivity. While equivalent for many of our model calculations, we note that we use ζ_{xy} instead of stating whether the maxima of J_c and ρ_L are aligned or not; the reason is that in some cases, these quantities have their global maxima close to, but not exactly at, the axes $\Omega = 0$ and $\pi/2$ [where there are technically only local minima, pinned by reflection symmetries, see e.g., Fig. 3(a) below]. In those cases, ζ_{xy} is a more useful quantity to distinguish the behavior.

B. Model calculations

Having established our general formalism, we next discuss the results of model-specific calculations of $\rho_L(\Omega)$ and $J_c(\Omega)$. In all cases, we describe the C_{3z} -symmetric part of the normal-state dispersion using a minimal model with nearest-neighbor dispersion on a triangular lattice in the presence of a finite flux ϕ , which induces trigonal warping of the band dispersion,

$$\epsilon_{\mathbf{k}} = - \sum_{j=1}^3 t \cos(\mathbf{a}_j \cdot \mathbf{k} - \phi/3). \quad (8)$$

Here, $\mathbf{a}_j$ are three C_{3z} -related primitive vectors. In the following, we measure all energy scales in units of t , thus, setting $t \equiv 1$.

1. k -independent SC

We start with a simple, k -independent superconducting order parameter, which thus transforms under the trivial IR (with $d_{\Gamma} = 1$), and the form factors are $f_j(\mathbf{k}) = 1$. As SC is isotropic, the rotational symmetry breaking must come from strain or nematic order in the normal state, described by the second term in Eq. (2a). We see in Fig. 1 that a strain/nematic field Φ along the x direction leads to maximum resistivity in x as well, while the maximum critical current is rotated by 90° along y , leading to $\zeta_{xy} = -1$. The same conclusion was reached by the theoretical analysis of [93], where, for a quadratic band structure, a Drude-like conductivity was

shown to align with the direction of the highest critical current, contrary to the experimental observations.

2. Nematic SC

To analyze nematic SC, we need to consider one of the two-dimensional IRs of the point group. For instance, for D_6 (or C_6 with displacement field), relevant to twisted bi- and trilayer graphene, these are E_1, E_2 . The E_2 pairing is entirely band-off diagonal in presence of spin polarization [54,103], with many subtleties that are not of direct relevance here. We shall thus focus only on the E_1 representation with intervalley pairing. The two possible sources of symmetry breaking for the nematic SC in a normal state can be classified into two broad categories.

Arising from C_{3z} broken normal state. Here, the threefold rotational symmetry is already significantly broken in the normal electron liquid—either by a nematic transition or by a strain field—and this symmetry reduction is not the result of the SC. From this normal state, a superconducting instability emerges with order parameter transforming under E_1 of the (undistorted) point group with C_{3z} . Importantly, which order-parameter direction $\eta = (\eta_1, \eta_2)^T$ will be favored in the superconductor is determined by the normal-state anisotropy, linking normal-state and superconducting anisotropies.

To study this, we investigate the superconducting part of the free-energy expansion, up to quadratic order given by Eq. (5) with $d_\Gamma = 2$. Assuming a strain field with $\Phi = (\Phi_V, 0)^T$ in the normal state, we first determine η . The easiest way to do this is to compute the quadratic coefficients $a_1(\mathbf{q} = 0)$ and $a_2(\mathbf{q} = 0)$ and see which one is smaller. The expressions for the corresponding particle–particle bubbles are then given as Eq. (6). Although the exact point symmetries of the systems we consider here do not constrain $\mathbf{f}(\mathbf{k})$ to be even or odd in $\mathbf{k}$, we still discuss the even and odd cases separately. This can be thought of as limiting cases to probe the dependence of ζ_{xy} on $\mathbf{f}(\mathbf{k})$. We further note that twisted WSe₂ was found to be close to an emergent intravalley inversion symmetry with respect to the pairing instabilities, which can explain why the numerically obtained superconducting order parameters are indeed very close to being even or odd under $\mathbf{k} \rightarrow -\mathbf{k}$ [107]. Let us first consider the case in which $\mathbf{f}(\mathbf{k})$ is odd, $\mathbf{f}(\mathbf{k}) = -\mathbf{f}(-\mathbf{k})$. We calculate $a_j(\mathbf{q} = 0)$ and find that $a_1(\mathbf{q} = 0) < a_2(\mathbf{q} = 0)$; as such, pairing with $\eta = (\eta_1, 0)^T$ and thus a superconducting order parameter $\Delta_{\mathbf{k}} \propto k_x$ is stabilized. Using the formalism described in Sec. II A, we next compute the angular dependence of resistivity $\rho(\Omega)$ and critical current $J_c(\Omega)$ (see Fig. 2). The strain/nematic order parameter with $\Phi = (\Phi_V, 0)^T$, assumed to select the superconducting component as described above, implies $\rho_{xx} > \rho_{yy}$ and therefore the maximum of the resistivity aligns along x , irrespective of the value of Φ_V . Interestingly, for small values of Φ_V , we see that the maximum of the critical current is rotated [Fig. 2b(i)] compared to Fig. 1 and is now larger along x than y , leading to $\zeta_{xy} = 1$. This is consistent with the observation of the experiment [93]. Increasing Φ further results in the case where the maximum critical current is along y , which is perpendicular to the direction of maximum $\rho(\Omega)$, reverting to the naive expectation [Fig. 2b(ii)]. Intuitively, this is because at some point, the increase in the nematic order is so significant

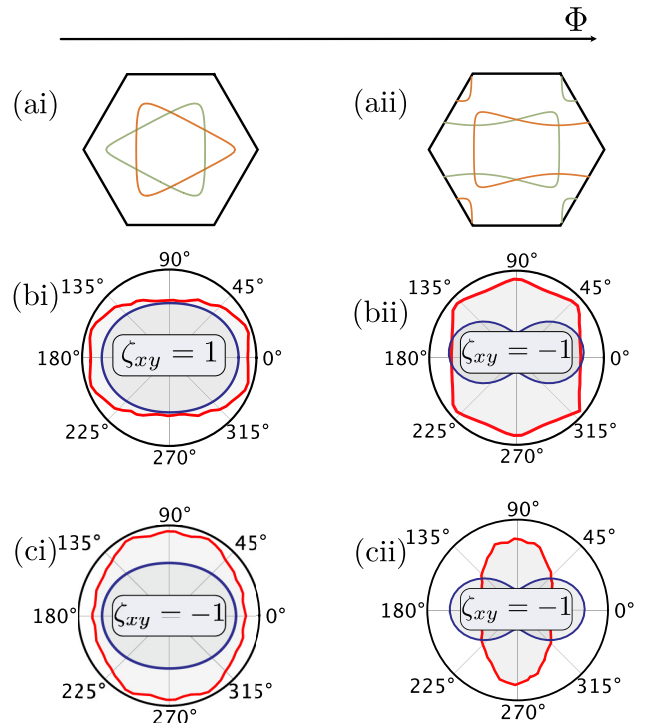


FIG. 2. Nematic SC arising from C_{3z} breaking in the normal state. (i) Weak nematic/strain order ($\Phi_V = 0.03$); (ii) strong nematic/strain order ($\Phi_V = 0.3$). (a) Fermi surfaces (FSs) for the two-valley model. The presence of nematic order along x ($\theta = 0$) breaks the C_{3z} symmetry in the normal state. The critical currents (in red) and resistivity (in blue) are shown for (b) odd and (c) even form factors. Explicit parameters are same as Fig. 1 except $\phi = -0.7\pi$.

that the anisotropy coming from the superconducting order parameter is subdominant and the critical current asymmetry is the same as that of an isotropic superconducting order parameter, discussed in the previous subsection [cf. Fig. 1(b)].

Performing the computation for the case in which $\mathbf{f}(\mathbf{k}) \sim (2k_x k_y, k_y^2 - k_x^2)$ is even, we see that for Φ along x , again the first component is stabilized, i.e., $\Delta_{\mathbf{k}} \sim k_x k_y$. However, unlike the odd- $\mathbf{k}$ case, here the maximum critical current of the anisotropic superconductor for small normal-state anisotropy Φ_V is oriented already along the y direction [see Fig. 2c(i)]. As such, the directions of maximal resistivity and critical current are orthogonal here, which also does not change upon varying Φ_V .

Arising from vestigial nematic order. Next, we shall explore the scenario where the source of C_{3z} symmetry breaking is a remnant nematic order persisting after the melting of a nematic superconducting order parameter—which itself arose from a C_{3z} -symmetric normal state at higher temperatures. As such, the reduced rotation symmetry in the superconducting and normal state are both rooted in superconducting correlations and, hence, again intertwined, although in a different way than in the previous scenario.

To proceed, we adopt the action formalism, promoting the operators $c_{k,v}$ to Grassmann fields. As before, we focus on a superconducting order parameter, belonging to the E_1 representation, which explicitly breaks C_{3z} rotational symmetry. The resulting action in Matsubara notation is given by

$S = S_0 + S_{\text{int}}$, where $S_0 = \sum_v \int_k c_{k,v}^\dagger (\mathcal{G}_0^{-1})_{k,v} c_{k,v}$ and

$$S_{\text{int}} = \int_{k,q} [f(\mathbf{k}) \cdot \boldsymbol{\eta}(q) c_{k+q,-}^\dagger c_{-k+q,+}^\dagger + \text{H.c.}] + \int_q \boldsymbol{\eta}^\dagger(q) \chi^{-1}(q) \boldsymbol{\eta}(q) + \int_x V(\boldsymbol{\eta}^\dagger(x), \boldsymbol{\eta}(x)). \quad (9)$$

Here, $(\mathcal{G}_0^{-1})_{k,v} = (-i\omega_n + \xi_{k,v})$ is the bare Green's function, with $k = (i\omega_n, \mathbf{k})$ comprising of 2D momentum and fermionic Matsubara frequencies; $\chi^{-1}(q)$ is the susceptibility taken to be of the form $\chi(q) = 1/(m^2 + \mathbf{q}^2 + \Omega_n^2)$ where $q = (\mathbf{q}, i\Omega_n)$ comprises of the bosonic momenta and frequencies, and $V(\boldsymbol{\eta}^\dagger, \boldsymbol{\eta})$ encodes the local bosonic interactions. We want $f(C_{3z}\mathbf{k}) = C_{3z}f(\mathbf{k})$ to describe pairing in the E_1 representation. Consequently, $V(\boldsymbol{\eta}^\dagger, \boldsymbol{\eta})$ has to be invariant under $\boldsymbol{\eta} \rightarrow C_{3z}\boldsymbol{\eta}$. Together with time-reversal symmetry, acting as $\boldsymbol{\eta} \rightarrow \boldsymbol{\eta}^*$, the only allowed (independent) terms are

$$V(\boldsymbol{\eta}^\dagger, \boldsymbol{\eta}) = u(\boldsymbol{\eta}^\dagger \cdot \boldsymbol{\eta})^2 + v(\boldsymbol{\eta}^\dagger \sigma_y \boldsymbol{\eta})^2 + \mathcal{O}(\eta^6), \quad (10)$$

in agreement with [52], where σ_j 's are the Pauli matrices. Negative v favors a chiral pairing state, $\boldsymbol{\eta} = e^{i\varphi}(1, i)^T$ while $v > 0$ favors nematic pairing, $\boldsymbol{\eta} = (\cos \phi, \sin \phi)^T$. Note that the direction ϕ is only determined by higher-order terms [52], which we neglect here. We focus on $v > 0$ from now on as we are interested in nematic pairing. The additional inclusion of the intravalley in-plane rotation symmetry C_{2y} , acting as $c_{k,v} \rightarrow c_{(-k_x, k_y), v}$ on our fermions in Eq. (9), further requires that the basis functions obey $f_1(\mathbf{k}) = -f_1(C_{2y}\mathbf{k})$ and $f_2(\mathbf{k}) = f_2(C_{2y}\mathbf{k})$; meanwhile, the potential should then also be invariant under $\boldsymbol{\eta} \rightarrow \sigma_z \boldsymbol{\eta}$, which, however, does not further constrain Eq. (10) up to quartic order—it only matters at higher orders.

To describe vestigial orders, we perform a Hubbard-Stratonovich decoupling [18] of the quartic terms in V such that Eq. (9) effectively becomes

$$S_{\text{int}} \rightarrow \int_{k,q} [f(\mathbf{k}) \cdot \boldsymbol{\eta}(q) c_{k+q,-}^\dagger c_{-k+q,+}^\dagger + \text{H.c.}] + \int_q \boldsymbol{\eta}^\dagger(q) \chi^{-1}(q) \boldsymbol{\eta}(q) + \int_x \frac{1}{4v} \tilde{\Phi}^2 + \int_x \frac{1}{4\tilde{u}} \Psi^2 + \int_x \boldsymbol{\eta}^\dagger (i\Psi \sigma_0 + \tilde{\Phi}_x \sigma_z + \tilde{\Phi}_y \sigma_x) \boldsymbol{\eta}, \quad (11)$$

where $\tilde{u} = u + v$. We now discuss vestigial order in the large- N limit of [18], where the saddle-point equations of the fields Ψ and $\tilde{\Phi}_{x,y}$ become exact. As the saddle-point value $-i\Psi^0$ of Ψ just leads to an isotropic mass renormalization of the bosons $\boldsymbol{\eta}$, we can neglect it here. Instead, we will focus on the saddle-point value $\tilde{\Phi}^0 = (\tilde{\Phi}_x^0, \tilde{\Phi}_y^0)$ of $\tilde{\Phi}$; the two components $\tilde{\Phi}_x^0$ and $\tilde{\Phi}_y^0$ correspond to the composite order parameters

$$|\eta_1|^2 - |\eta_2|^2 \quad \text{and} \quad 2\text{Re}(\eta_1^* \eta_2), \quad (12)$$

respectively; these two components are gauge invariant, even under time-reversal, and transform under E_2 . They hence describe the vestigial C_{3z} symmetry breaking and represent the analog of Φ studied above. Symmetry dictates that if $\tilde{\Phi}^0 \neq 0$, there will be three degenerate solutions of the saddle-point equations, $\tilde{\Phi}^0 = \tilde{\Phi}_0 \hat{n}$, $\tilde{\Phi}_0 C_{3z} \hat{n}$, $\tilde{\Phi}_0 C_{3z}^2 \hat{n}$ and the system will

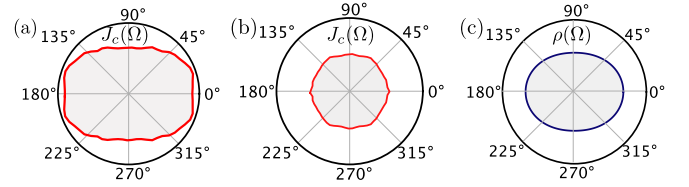


FIG. 3. Critical current of a nematic superconductor with (a) odd form factor, (b) even form factor. Panel (c) depicts the angular dependence of resistivity arising from a vestigial nematic order. For both the even and odd cases, the angular dependence $\rho(\Omega)$ looks qualitatively identical. Parameters are the same as Fig. 1.

spontaneously choose one. Without C_{2y} , $\hat{n}$ will point along a generic direction, while C_{2y} will fix $\hat{n}$ to be either $(1, 0)^T$ or $(0, 1)^T$.

For concreteness, we here assume that $\tilde{\Phi}_y \rightarrow 0$ and $\tilde{\Phi}_x \rightarrow \tilde{\Phi}_x^0$ upon its condensation. This leads to distinct renormalized masses $m_1^2 = m^2 + \tilde{\Phi}_x^0$ and $m_2^2 = m^2 - \tilde{\Phi}_x^0$ for the two components of $\boldsymbol{\eta}$ and the associated action reduces to

$$S_{\text{int}} \rightarrow \int_{k,q} [f(\mathbf{k}) \cdot \boldsymbol{\eta}(q) c_{k+q,-}^\dagger c_{-k+q,+}^\dagger + \text{H.c.}] + \int_q \boldsymbol{\eta}^\dagger(q) \begin{pmatrix} m_1^2 + \mathbf{q}^2 + \Omega_n^2 & 0 \\ 0 & m_2^2 + \mathbf{q}^2 + \Omega_n^2 \end{pmatrix} \boldsymbol{\eta}(q), \quad (13)$$

where we suppressed an irrelevant constant term.

Mean-field theory. We now integrate over the $\boldsymbol{\eta}(q)$ fields and perform a mean-field decoupling of the resultant effective electron-electron interaction (see the Supplemental Material [108]). Assuming that $\langle c_{k,+}^\dagger c_{k,-} \rangle = 0$, i.e., that there is no intervalley coherent (IVC) order or spontaneous TRS breaking and focusing only on the static $i\Omega_n = 0$ contributions, the resulting mean-field Hamiltonian describing a fluctuation-driven vestigial nematic order then reads

$$H_{\text{MF}}^{\text{eff}} = \sum_{k,v} \left(\epsilon_{k,v} - \sum_{j,q} f_j^2(\mathbf{k}) \frac{N_{k-2q}}{m_j^2 + \mathbf{q}^2} \right) c_{k,v}^\dagger c_{k,v}, \quad (14)$$

where $N_k = \langle c_{-k,-}^\dagger c_{-k,-} \rangle = \langle c_{-k,+}^\dagger c_{-k,+} \rangle$. Equation (14) captures the contribution $\delta \epsilon_{k,v}$ in Eq. (2b). To demonstrate the features of this vestigial nematic phase explicitly, we now turn to explicit model calculations. For completeness and just as before, we consider cases with both odd and even form factors.

Assuming that the nematic state that is spontaneously selected is $\boldsymbol{\eta} = (1, 0)^T$, which means that we consider superconducting order parameters of the form $\Delta_k \propto k_x$ (odd parity) or $\Delta_k \propto k_x k_y$ for even parity. In either case, the resulting superconductor leads to the breaking of C_{3z} symmetry, and consequently gives rise to an anisotropic critical current profile. Specifically, the critical current along the x direction [see Fig. 3(a) for odd and 3(b) for even parity] exceeds that of y , leading to transport anisotropy characterized by $\text{sgn}(J_c(\Omega = 0) - J_c(\Omega = \pi/2)) = 1$ for both cases, albeit with a weaker anisotropy in Fig. 3(b).

When the superconducting order melts, the broken rotational symmetry associated with $\boldsymbol{\eta} = (1, 0)^T$ remains encoded in $\tilde{\Phi}_x^0 < 0$ and $\tilde{\Phi}_y^0 = 0$; this follows by noting that only

the first component in Eq. (12) is nonzero and setting $\eta = (1, 0)^T$ in Eq. (11) favors negative $\tilde{\Phi}_x$. Consequently, we have $m_1^2 < m_2^2$ in Eq. (13), which breaks the C_{3z} symmetry and defines a vestigial nematic phase. This defines $\delta\epsilon_{k,v}$, which enters Eq. (3) and allows us to obtain the result $\rho_{xx} > \rho_{yy}$, irrespective of the parity of the underlying form factor [see also as seen in Fig. 3(c)]. Consequently, the direction of the maximum of the normal-state resistivity from the nematic vestigial order parameter is aligned with the direction of the maximum critical current in the superconducting phase, leading to $\zeta_{xy} = 1$. This behavior is different from the previous case in which the C_{3z} breaking in the normal state was unrelated to superconductivity and we found $\zeta_{xy} = -1$, except for odd-parity pairing and weak Φ , see Fig. 2.

C. Nematic paraconductivity via Aslamazov-Larkin

Instead of working with fermions within mean-field theory, we can work in an effective theory of dynamic bosonic order parameter fields η , with dynamics endowed by integrating out fermions, and account for coupling to vestigial fields $\tilde{\Phi}_{x/y}$, Ψ . Within this bosonic framework, one computes resistivity via the Aslamazov-Larkin diagram [29]. This provides a complementary approach to mean-field theory, and we confirm here that the two approaches give consistent results—the relation $\rho_{xx} > \rho_{yy}$ for $\tilde{\Phi}_x^0 < 0$, $\tilde{\Phi}_y^0 = 0$, and $\Psi^0 = 0$.

Bosonic susceptibility. Integrating out fermions, we find from the couplings in Eq. (9) the correction to the bosonic action given by

$$\delta S_{\text{eff}}[\eta] = \int_q \eta_i^*(q) \delta \chi_{ij}^{-1}(q) \eta_j(q),$$

$$\delta \chi^{-1}(q) = \delta \chi_0^{-1}(q) \sigma_0 + d_z(q_x^2 - q_y^2) \sigma_z + 2d_x q_x q_y \sigma_x. \quad (15)$$

Most important to the present analysis are the generated terms with prefactors d_z , d_x , which are not proportional to the identity matrix. By direct integration, taking $\epsilon_{\tau k}$ of Eq. (8) and sweeping across all fluxes ϕ and at a fixed temperature $T = 0.2t$, we find that $d_z < 0$, $d_x > 0$.

The $\delta \chi_0^{-1}(q)$ is absorbed into the diagonal component of the susceptibility. Including the off-diagonal components, we arrive at the bare susceptibility,

$$\chi_0^{-1} = (\Omega_n^2 + q^2 + m^2) \sigma_0 + d_z(q_x^2 - q_y^2) \sigma_z + 2d_x q_x q_y \sigma_x, \quad (16)$$

while the vestigial order-parameter correction to the susceptibility is given by

$$\chi_\Phi^{-1}(q) = i\Psi^0 \sigma_0 + \tilde{\Phi}_x^0 \sigma_z + \tilde{\Phi}_y^0 \sigma_x, \quad (17)$$

which immediately follows from Eq. (11), upon taking the saddle-point values Ψ , $\tilde{\Phi}_{x,y} \rightarrow \Psi^0$, $\tilde{\Phi}_{x,y}^0$. Armed with this, we now define the *full susceptibility* as

$$\chi^{-1}(q) = \chi_0^{-1}(q) + \chi_\Phi^{-1}(q). \quad (18)$$

Paraconductivity via Aslamazov-Larkin. The current vertex is obtained as $J_\mu = \partial_{q_\mu} \chi^{-1}(q)$, i.e.,

$$J_x = 2q_x \sigma_0 + 2d_z q_x \sigma_z + 2d_x q_y \sigma_x,$$

$$J_y = 2q_y \sigma_0 + 2d_z q_y \sigma_z + 2d_x q_x \sigma_x. \quad (19)$$

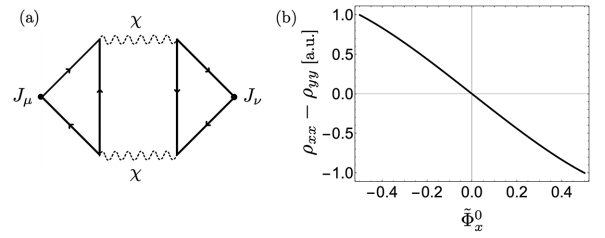


FIG. 4. (a) Depiction of the Aslamazov-Larkin diagram. (b) Resistivity anisotropy $\rho_{xx} - \rho_{yy}$ vs vestigial order $\tilde{\Phi}_x^0$, as computed via the Aslamazov-Larkin diagram.

Meanwhile the conductivity tensor, as computed via the Aslamazov-Larkin diagram, is $\sigma_{\mu\nu} \propto (e^2/\hbar) \int_q \text{Tr}[J_\mu(q) \chi(q) \chi(q) J_\nu(q)]$ [109,110]. Treating $d_{x/z}$ and $\tilde{\Phi}_{x,y}^0$ as expansion parameters, we obtain $\sigma_{xx} - \sigma_{yy} \propto -d_z \tilde{\Phi}_x^0$ (and $\sigma_{xy} \propto -d_x \tilde{\Phi}_y^0$). The anisotropy in the resistivity immediately follows, $\rho_{xx} - \rho_{yy} \propto d_z \tilde{\Phi}_x^0$, see also Fig. 4(b) for the result obtained by direct computation of the diagram without expansion. Finally, since $d_z < 0$, then $\text{sign}[\rho_{xx} - \rho_{yy}] = -\text{sign}(\tilde{\Phi}_x^0)$, which is consistent with our mean-field treatment of the vestigial phase in the previous subsection.

III. INTRAVALLEY PAIRING

With observations of pairing in rhombohedral tetralayer graphene and twisted MoTe₂ [67,100] emerging from a valley-imbalanced normal state as our inspiration, we next examine an analogous scenario in a model with only one active valley degree of freedom and, hence, broken time-reversal symmetry in the normal state. As at least in some of those superconducting phases also spin seems to be polarized, we further start from spinless electrons with creation operators $c_k^\dagger$. Denoting the single-valley dispersion by ϵ_k , the Hamiltonian in this case reads as

$$\mathcal{H} = \sum_k c_k^\dagger c_k \epsilon_k + \sum_{k,q} [\Delta_q f(k) c_{k+q}^\dagger c_{-k+q}^\dagger + \text{H.c.}], \quad (20)$$

where $f(k)$ encodes the nature of the pairing state. Since it must obey $f(k) = -f(-k)$ as a result of Fermi-Dirac statistics, we choose a chiral superconducting order parameter with $f(k) \sim k_x \pm ik_y$ (for small k ; appropriately extended to the full Brillouin zone), which will lead to a full gap and preserve C_{3z} symmetry in gauge-invariant observables. In fact, this type of chiral state is expected to be favored, at least within mean-field theory, in line with many theoretical works [68–81], with the sign $\pm$ determined by the chirality of the normal state [70,77]. For concreteness, we will focus on $f(k) \sim k_x + ik_y$ in the following.

Importantly, as a result of the broken time-reversal symmetry associated with valley imbalance, the maximum of the particle-particle $\Gamma(q)$ might not be at $q = 0$ [111], leading to finite-momentum pairing already in equilibrium; this is the regime we will focus on here. In the presence of C_{3z} in the normal state, there will be three degenerate maxima at positions $q_j = (C_{3z})^{j-1} q_1$, $j = 1, 2, 3$. To define a free-energy expansion, we allow for a superposition of all three momenta in Eq. (20) and write $\Delta_q = \sum_{j=1}^3 \eta_j (\delta q) \delta_{q,q_j+\delta q}$, where δq is

a small momentum deviation from the minima, which will be used to parametrize fluctuations around the equilibrium configuration and to impose a finite supercurrent later. Up to quartic order in η_j , the free energy reads as

$$\mathcal{F}_{3q} \sim \sum_{\delta q, j=1}^3 a_{q_j+\delta q} |\eta_j(\delta q)|^2 + \int_x V_{3q}(\eta_j^*(\mathbf{x}), \eta_j(\mathbf{x})), \quad (21)$$

where we wrote the quartic terms, with

$$V_{3q}(\eta_j^*, \eta_j) = u \left(\sum_{j=1}^3 |\eta_j|^2 \right)^2 + v \sum_{j \neq j'=1}^3 |\eta_j|^2 |\eta_{j'}|^2, \quad (22)$$

in real space—for future reference and to keep the notation more compact. Here, $\eta_j(\mathbf{x})$ is the Fourier transform of $\eta_j(\delta \mathbf{q})$. Depending on the sign of v , we either obtain a “ $1\mathbf{q}$ state” or a “ $3\mathbf{q}$ state”. In the case of the former, only one of $\eta_{1,2,3}$ is nonzero such that translational symmetry is preserved in observables, while C_{3z} is broken. In the $3\mathbf{q}$ state, C_{3z} is preserved while translational invariance is broken [78].

It was recently observed in Ref. [112] that at the onset of valley polarization in rhombohedral graphene also a nematic transition occurs. Upon entering the superconducting phase from this nematic phase, another experiment [95] provides evidence for “striped” superconductivity. The plethora of intertwined phases with different sorts of symmetry breaking (translational, time-reversal) in such superconductors naturally motivates the question regarding the emergence of vestigial phases, and how these phases might influence the directional dependence on transport properties. This is the question we turn to next. To this end, we first devise a theoretical framework for the corresponding vestigial orders.

A. Vestigial phases of superconducting order

To develop a theory of vestigial phases of this form of finite-momentum superconductivity, we start from the associated action description,

$$\begin{aligned} S_{3q} = S_0 &+ \sum_{j=1}^3 \int_{k, \delta q} \eta_j(\delta q) f(\mathbf{k}) c_{k+q_j+\delta q}^\dagger c_{-k+q_j+\delta q}^\dagger \\ &+ \sum_{j=1}^3 \int_{\delta q} |\eta_j(\delta q)|^2 (m^2 + \delta q^2 + \Omega_n^2) + S_V, \end{aligned} \quad (23)$$

where $S_0 = \int_k c_k^\dagger (-i\omega_n + \epsilon_k) c_k$ and

$$S_V = \int_x \left[u(\text{tr}[\rho])^2 + v \sum_{j \neq j'=1}^3 \rho_{j,j'} \rho_{j',j} \right]. \quad (24)$$

Here, we introduced $\rho_{j,j'} = \eta_j^* \eta_{j'}$ as possible composite order parameters. As before, we decouple the quartic terms using Hubbard-Stratonovich fields $\psi_{j,j'}$ such that effectively

$$\begin{aligned} S_V \rightarrow \int_x &\left[\sum_{j \neq j'} [\rho_{j,j'} \psi_{j',j} + \text{H.c.}] + i \sum_j \phi_j \rho_{j,j} \right] \\ &+ \int_x \sum_j \frac{1}{4u} \phi_j^2 - \int_x \sum_{j \neq j'} \frac{1}{2\tilde{v}} \psi_{j,j'} \psi_{j',j}, \end{aligned} \quad (25)$$

where $\tilde{v} = u + v$. We then treat $\psi_{j,j'}$, ϕ_j on the saddle-point level, which becomes exact in the limit where the number of components of η is infinitely large (large- N limit). For instance, for the $1\mathbf{q}$ state, one would only have one of the components of ϕ_j nonzero, i.e., $i\phi_j = \psi_0 \delta_{j,1}$, without loss of generality. But other configurations are also possible. Driven by the above-mentioned experimental claim of striped superconductivity [95] and the fact that this order leads to particularly interesting symmetry breaking, we here focus on a $2\mathbf{q}$ state (and defer the discussion of the $1\mathbf{q}$ state to the Supplemental Material [108]). On the level of the saddle-point fields, the associated vestigial phase is characterized by $\psi_{2,3} = \psi_{3,2}^* = \psi_0 \neq 0$ (picking 2 and 3 without loss of generality) with all other components zero. Integrating out the η fields, yields an effective $2\mathbf{q}$ action (see the Supplemental Material [108]) $S_{2q}^{\text{eff}} = S_0 + S_1 + S_2$ with

$$S_1 = - \sum_j \int_{\delta q} \frac{1}{[\tilde{\chi}_j(\delta q)]^{-1}} D_j^\dagger(\delta q) D_j(\delta q), \quad (26a)$$

$$S_2 = \int_{\delta q} \frac{1}{[M_{j=2}(\delta q)]^{-1}} (\psi_0 D_2^\dagger(\delta q) D_3(\delta q) + \text{H.c.}). \quad (26b)$$

Here, $D_j(\delta q) = \sum_k f(\mathbf{k}) c_{-k+q_j+\delta q} c_{k+q_j+\delta q}$ is a fermionic bilinear and the renormalized propagators are defined by $[M_j(\delta q)]^{-1} = [\chi(\delta q)]^{-2} - |\psi_0|^2 (\delta_{j,2} + \delta_{j,3})$ and $[\tilde{\chi}_j(\delta q)]^{-1} = [M_j(\delta q)]^{-1} [\chi(\delta q)]$. As expected, the $j = 1$ component remains unchanged, whereas the propagators for $j = 2, 3$ are renormalized, owing to the presence of the ψ_0 field. Notably, as a consequence of the off-diagonal nature of ψ_0 , the term S_2 explicitly breaks translational symmetry through a nonzero momentum transfer vector $2\mathbf{Q}$ where $\mathbf{Q} = \mathbf{q}_3 - \mathbf{q}_2$. As a result, two distinct symmetries are broken: C_{3z} rotational symmetry associated with nematic order, and translational symmetry along the direction of $\mathbf{Q}$. As such, we obtain a unidirectional and, thus, nematic vestigial charge-density wave phase emerging out of the superconductor.

Mean-field theory. To be able to compute the associated transport anisotropies, we subsequently go back to a Hamiltonian description and perform a mean-field decoupling, assuming no additional symmetries are broken. As detailed in the Supplemental Material [108], we finally arrive at an effective Hamiltonian of the form

$$\mathcal{H}_{\text{MF}} = \sum_k [\epsilon_k - \tilde{\alpha}(\mathbf{k})] c_k^\dagger c_k + \beta \sum_k c_{k+2\mathbf{Q}}^\dagger c_k. \quad (27)$$

Here, $\tilde{\alpha}(\mathbf{k}) = \sum_j \Lambda_j |f(\mathbf{k} - \mathbf{q}_j)|^2 n_F(\epsilon_k)$, where $\Lambda_j = \Lambda(\delta_{j,2} + \delta_{j,3})$ and β are mean-field parameters. The first correction $\propto \tilde{\alpha}(\mathbf{k})$ in Eq. (27) corresponds to a translational symmetry-obeying yet in general nematic contribution. The second correction ($\propto \beta$) is a translational symmetry-breaking term, as indicated by the coupling of momenta related by $2\mathbf{Q}$.

In Fig. 5, we illustrate the resultant FS of the associated vestigial orders. For ϵ_k , we here use a continuum model with a quadratic dispersion, deformed by a trigonal warping term γ , i.e., $\epsilon_k = \mathbf{k}^2 [1 + \gamma \cos(3\theta_k)] - \mu$. With both $\tilde{\alpha}(\mathbf{k})$, $\beta = 0$, the FS retains its expected C_{3z} with trigonal warping [shown in orange in Fig. 5(a)], which leads to an isotropic resistivity. When $\tilde{\alpha}(\mathbf{k})$ is now turned on—on account of the vestigial phase of the parent $2\mathbf{q}$ superconductor with $\psi_{2,3} \neq 0$, the parameters

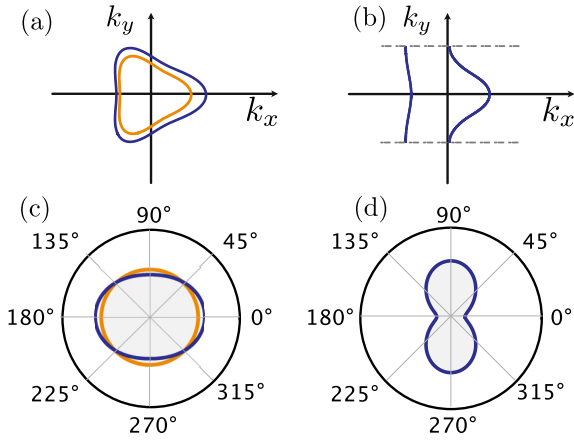


FIG. 5. Normal-state vestigial phases and their resistivities. In (a) and (c), we show the Fermi surfaces and resistivities for the cases with $\beta = 0$, and in (b) and (d) with $\tilde{\alpha}(\mathbf{k}) = 0$, $\beta \neq 0$. In (a) and (c), the orange FS and resistivity curve indicates the case without nematicity, whereas the dark blue curves are with $\Lambda_1 = 0$, $\Lambda_2 = \Lambda_3 = \Lambda \neq 0$. The Brillouin-zone strip is indicated by the dashed lines in (b).

satisfy $\Lambda_1 = 0$, $\Lambda_2 = \Lambda_3 \neq 0$. This leads to breaking of the threefold rotational symmetry, resulting in an anisotropic FS as can be seen in dark blue in Fig. 5(a). The associated elongation along x induces a resistivity maximum along x , as can be seen in Fig. 5(c).

To isolate the impact of the term $\propto \beta$ in Eq. (27) and compare with the previous case, we next consider $\tilde{\alpha}(\mathbf{k}) = 0$ but $\beta \neq 0$. This leads to charge-density modulations, where the band copies separated by the wave vector $2\mathbf{Q}$ hybridize. With our $\mathbf{Q}$ along the y direction, the Brillouin zone is finite along k_y but still infinite along k_x . This can be seen in Fig. 5(b) where we find open FSs resulting from bands with much smaller velocities along y than along x . This is also why the current flow is much more impeded along y compared to x , leading to a resistivity maximum in that direction. This is in sharp contrast to the case in which $\tilde{\alpha}(\mathbf{k}) \neq 0$, $\beta = 0$ where the associated orientation of resistivity is orthogonal to this case. Consequently, the competition between the translational-symmetry breaking (β) and nematic contribution (α) in Eq. (27) determines the sign of $\rho_L(\Omega = 0) - \rho_L(\Omega = \pi/2)$.

B. Critical current of $2\mathbf{q}$ state

Having established the normal-state transport anisotropy, we next turn to the parent superconductor where not only the composite order parameter $\rho_{2,3} = \eta_2^* \eta_3$ is condensed, but also η_2 and η_3 individually. With only two of the three components $\eta_{1,2,3}$ nonzero, rotation and translation symmetries are, of course, also broken in the superconductor and we have a “striped superconductor”. In order to compute its critical currents, we adopt a minimal setting, where the two stabilized components η_2 and η_3 are assumed to continue to have equal values, $\eta_2(\delta\mathbf{q}) = \eta_3(\delta\mathbf{q}) =: \eta_{2q}(\delta\mathbf{q})$, even when we detune their momentum from their respective equilibrium values

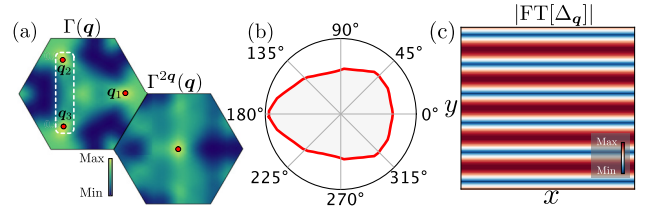


FIG. 6. $2\mathbf{q}$ superconductor. Panel (a) shows $\Gamma(\mathbf{q})$ and $\Gamma^{2q}(\mathbf{q})$ where the red dots indicate the maxima of the particle–particle bubbles. Panel (b) shows the angular dependence of critical current, which is seen to break C_{3z} symmetry as expected. Here, $\phi = -0.8\pi$, the rest of the parameters remain the same as Fig. 1. Panel (c) shows real-space modulation of the Fourier transform (FT) of $|\Delta_q|$, illustrating stripe order.

($\delta\mathbf{q} \neq 0$). Then, the free energy can be effectively written as

$$\mathcal{F}_{2q} = \sum_{\delta\mathbf{q}} [a_{q_2+\delta\mathbf{q}} + a_{q_3+\delta\mathbf{q}}] |\eta_{2q}(\delta\mathbf{q})|^2 + \mathcal{O}(\Delta^4). \quad (28)$$

Our procedure to construct the particle–particle bubble for the $2\mathbf{q}$ state is as follows. First, we compute the particle–particle bubble (see the Supplemental Material [108] for expression) to determine $a_{\mathbf{q}}$ in Eq. (28) and the three $\mathbf{q}_j$ associated with superconducting instability. Then, to model the $2\mathbf{q}$ state, we restrict ourselves to retain $\mathbf{q}_2$ and its C_{3z} -rotated partner $\mathbf{q}_3$, highlighted with white dashed lines in Fig. 6(a), upper panel. The resulting effective particle–particle bubble of the $2\mathbf{q}$ state from Eq. (28) is then given by

$$\Gamma^{2q}(\delta\mathbf{q}) = \Gamma(\mathbf{q}_2 + \delta\mathbf{q}) + \Gamma(\mathbf{q}_3 + \delta\mathbf{q}). \quad (29)$$

In Fig. 6(a), we show $\Gamma(\mathbf{q})$ and the corresponding $\Gamma^{2q}(\mathbf{q})$, obtained from the above procedure, in the upper and lower panel, respectively. By design, $\Gamma^{2q}(\mathbf{q})$ now has a maximum at $\mathbf{q} = 0$ and explicitly breaks C_{3z} symmetry. As before, the current is given by $\mathbf{J}^{2q}(\mathbf{q}) = 2e\Gamma^{2q}\nabla_{\mathbf{q}}\Gamma^{2q}$ from which we extract the critical currents. As expected, since both time-reversal and inversion symmetry are broken, the critical currents in opposite directions are unequal, leading to a nonzero diode effect [67,69,100,111,113,114]. However, this effect is not the focus of our work, since we are primarily interested in (sources of) rotational symmetry breaking, and will not pursue the diode effect further.

The resulting angular dependence of the critical current is shown in Fig. 6(b). The rotational symmetry is clearly broken, with the extrema of the critical current pointing in the direction $\pm\hat{\mathbf{q}}_1$, i.e., the momentum component that remains uncondensed. This behavior is consistent with the real-space modulation of the superconductor [see Fig. 6(c)], which exhibits stripe order oriented in the y direction.

Comparing with the angular dependency of the resistivity of the nematic order $\zeta_{xy} = 1$ while for the charge-density wave order $\zeta_{xy} = -1$. This is consistent with the observations in Ref. [95], where it was also noted that in the SC displaying stripe order, the maximum of critical current is anticorrelated with the direction of maximum resistivity in the metallic state above the superconducting transition temperature.

TABLE I. Summary of our results, where $\parallel$ ($\perp$) indicates that the direction of maximum critical current of the superconductor is parallel (perpendicular) to the direction of maximum resistance in the normal state, or, more precisely, $\zeta_{xy} = 1$ ($\zeta_{xy} = -1$) with ζ_{xy} as defined in Eq. (7). The arrow in the second line signals how the behavior changes upon increasing the strength of the anisotropy in the normal state. The normal state has time-reversal symmetry in all cases, except for the last two lines where we take it to be valley polarized leading to finite center-of-mass pairing $\mathbf{q} \neq 0$.

Superconductor	Normal state	Orientation
Isotropic	strain/other nematic	$\perp$
Nematic, odd	strain/other nematic	$\parallel \rightarrow \perp$
Nematic, even	strain/other nematic	$\perp$
Nematic, odd/even	vestigial nematic	$\parallel$
$2\mathbf{q}$ PDW	vestigial CDW dominant	$\perp$
	vestigial nematic dominant	$\parallel$

IV. CONCLUSIONS

In this work, we have studied the relation between the angular dependence of the normal-state resistivity $\rho(\Omega)$ and of the critical current $J_c(\Omega)$ for different microscopic origins of rotational symmetry breaking, which we summarize in Table I. Our study is primarily motivated by two-dimensional systems, such as rhombohedral graphene or twisted van der Waals materials, where these angular dependencies can be accessed via sunbeam Hall bar geometries [95,115] and rotational symmetry breaking and superconductivity are ubiquitous in the phase diagram. This is why we use models with two valley degrees of freedom and a hexagonal lattice symmetry, although the formalism can be straightforwardly applied to other scenarios as well.

We started with the scenario in which the only source of rotational symmetry breaking comes from the normal state, while the superconducting order parameter is isotropic. In that case, the maxima of the critical current and of the normal-state resistivity occur along perpendicular directions. Since these maxima need not be perfectly aligned with high-symmetry directions of the lattice, we here use $\zeta_{xy} = -1$ [see Eq. (7) for formal definition] to compactly describe the situation of misaligned anisotropies along the two perpendicular high-symmetry directions $\Omega = 0$ and $\pi/2$. We then contrasted this behavior with the scenario where also the superconducting order itself is nematic, i.e., transforms nontrivially under rotations, taking into account that the normal-state anisotropies determine the specific superconducting order parameter configuration. Here, the result depends on the parity of the superconducting order parameter $\Delta(\mathbf{k})$. If $\Delta(\mathbf{k}) = \Delta(-\mathbf{k})$ (“nematic, even” in Table I), we obtain the same behavior as in the first scenario with an isotropic pairing state. However, an odd-parity order parameter, $\Delta(\mathbf{k}) = -\Delta(-\mathbf{k})$, yields the opposite behavior, $\zeta_{xy} = 1$, if the strain/nematic order in the normal state is sufficiently weak. If the latter is large enough to dominate the anisotropy inside the superconductor, we

are then effectively again back to the first scenario and find $\zeta_{xy} = -1$.

The third distinct type of scenario arises when the superconductor is the *only* source of broken rotational symmetry, including in the resistive state via the emergence of a vestigial nematic state. We show that this leads to the opposite behavior, $\zeta_{xy} = 1$, i.e., the maximal critical current and normal-state resistivity are along the same direction, irrespective of the parity of the order parameter.

Finally, we also considered pairing within a single valley, i.e., when time-reversal symmetry is already broken in the normal state, which naturally leads to pairing at finite center-of-mass momentum $\mathbf{q} \neq 0$. This, in turn, allows for interesting forms of nonsuperconducting vestigial orders. Motivated by the experimental results in Ref. [95], we focus on a translational-symmetry-breaking unidirectional pair-density wave ($2\mathbf{q}$ PDW in Table I), which also leads to a charge-density modulated vestigial phase in the normal state. We show that both $\zeta_{xy} = -1$ and $\zeta_{xy} = 1$ are possible, depending on whether the charge-density modulations or additional translational-symmetry-preserving terms associated with the superconducting vestiges dominate.

Taken together, our findings show that these relative transport anisotropies in the superconductor and related normal state can provide crucial insights. For instance, the maximum critical current and normal-state resistance being aligned, as seen in Ref. [93], indicates that the superconducting order parameter is either odd parity with its direction determined by a normal-state anisotropy that is unrelated to the superconductor or the dominant source of rotational symmetry breaking comes from the superconductor alone, including in the normal state because of nematic paraconductivity or vestigial nematic order. If the relative anisotropy ζ_{xy} is found to change sign, e.g., as a function of displacement field or filling, only the former of the two scenarios is naturally consistent. Furthermore, our findings for the valley polarized case show that the observations of Ref. [95], where $\zeta_{xy} = -1$, imply that the charge-density-modulated part of the vestigial order dominates.

ACKNOWLEDGMENTS

S.B. and M.S.S. acknowledge funding by the European Union (Grant No. ERC-2021-STG, Project No. 101040651—SuperCorr). Views and opinions expressed are however those of the authors only and do not necessarily reflect those of the European Union or the European Research Council Executive Agency. Neither the European Union nor the granting authority can be held responsible for them. M.S.S. thanks J. Li for bringing the question of correlations between normal-state and superconducting transport anisotropies to our attention.

DATA AVAILABILITY

The data sets generated in this study are available in the Zenodo database [116].

- [1] M. Sigrist and K. Ueda, Phenomenological theory of unconventional superconductivity, *Rev. Mod. Phys.* **63**, 239 (1991).
- [2] V. Oganesyan, S. A. Kivelson, and E. Fradkin, Quantum theory of a nematic Fermi fluid, *Phys. Rev. B* **64**, 195109 (2001).
- [3] E. Fradkin, S. A. Kivelson, M. J. Lawler, J. P. Eisenstein, and A. P. Mackenzie, Nematic Fermi fluids in condensed matter physics, *Annu. Rev. Condens. Matter Phys.* **1**, 153 (2010).
- [4] I. Khavkine, C.-H. Chung, V. Oganesyan, and H.-Y. Kee, Formation of an electronic nematic phase in interacting fermion systems, *Phys. Rev. B* **70**, 155110 (2004).
- [5] H. Yamase, V. Oganesyan, and W. Metzner, Mean-field theory for symmetry-breaking Fermi surface deformations on a square lattice, *Phys. Rev. B* **72**, 035114 (2005).
- [6] H. Yamase and W. Metzner, Competition of Fermi surface symmetry breaking and superconductivity, *Phys. Rev. B* **75**, 155117 (2007).
- [7] P. Fulde and R. A. Ferrell, Superconductivity in a strong spin-exchange field, *Phys. Rev.* **135**, A550 (1964).
- [8] A. I. Larkin and Y. N. Ovchinnikov, Nonuniform state of superconductors, *Zh. Eksp. Teor. Fiz.* **47**, 1136 (1964) [*Sov. Phys. JETP* **20**, 762 (1965)].
- [9] E. Berg, E. Fradkin, and S. A. Kivelson, Theory of the striped superconductor, *Phys. Rev. B* **79**, 064515 (2009).
- [10] E. Berg, E. Fradkin, and S. A. Kivelson, Charge-4e superconductivity from pair-density-wave order in certain high-temperature superconductors, *Nat. Phys.* **5**, 830 (2009).
- [11] D. F. Agterberg, J. S. Davis, S. D. Edkins, E. Fradkin, D. J. Van Harlingen, S. A. Kivelson, P. A. Lee, L. Radzihovsky, J. M. Tranquada, and Y. Wang, The physics of pair-density waves: Cuprate superconductors and beyond, *Annu. Rev. Condens. Matter Phys.* **11**, 231 (2020).
- [12] S. A. Kivelson, E. Fradkin, and V. J. Emery, Electronic liquid-crystal phases of a doped Mott insulator, *Nature (London)* **393**, 550 (1998).
- [13] R. M. Fernandes, P. P. Orth, and J. Schmalian, Intertwined vestigial order in quantum materials: Nematicity and beyond, *Annu. Rev. Condens. Matter Phys.* **10**, 133 (2019).
- [14] E. V. Herland, E. Babaev, and A. Sudbø, Phase transitions in a three dimensional $U(1) \times U(1)$ lattice London superconductor: Metallic superfluid and charge-4e superconducting states, *Phys. Rev. B* **82**, 134511 (2010).
- [15] V. Grinenko, D. Weston, F. Caglieris, C. Wuttke, C. Hess, T. Gottschall, I. Maccari, D. Gorbunov, S. Zherlitsyn, J. Wosnitza, *et al.*, State with spontaneously broken time-reversal symmetry above the superconducting phase transition, *Nat. Phys.* **17**, 1254 (2021).
- [16] J. Ge, P. Wang, Y. Xing, Q. Yin, H. Lei, Z. Wang, and J. Wang, Discovery of charge-4e and charge-6e superconductivity in kagome superconductor CsV_3Sb_5 , *Phys. Rev. X* **14**, 021025 (2024).
- [17] R. M. Fernandes and L. Fu, Charge-4e superconductivity from multicomponent nematic pairing: Application to twisted bilayer graphene, *Phys. Rev. Lett.* **127**, 047001 (2021).
- [18] R. M. Fernandes, A. V. Chubukov, J. Knolle, I. Eremin, and J. Schmalian, Preemptive nematic order, pseudogap, and orbital order in the iron pnictides, *Phys. Rev. B* **85**, 024534 (2012).
- [19] S.-K. Jian, Y. Huang, and H. Yao, Charge-4e superconductivity from nematic superconductors in two and three dimensions, *Phys. Rev. Lett.* **127**, 227001 (2021).
- [20] M. Zeng, L.-H. Hu, H.-Y. Hu, Y.-Z. You, and C. Wu, High-order time-reversal symmetry breaking normal state, [arXiv:2102.06158](https://arxiv.org/abs/2102.06158).
- [21] F.-F. Song and G.-M. Zhang, Phase coherence of pairs of Cooper pairs as quasi-long-range order of half-vortex pairs in a two-dimensional bilayer system, *Phys. Rev. Lett.* **128**, 195301 (2022).
- [22] I. Maccari, J. Carlström, and E. Babaev, Prediction of time-reversal-symmetry breaking fermionic quadrupling condensate in twisted bilayer graphene, *Phys. Rev. B* **107**, 064501 (2023).
- [23] S. B. Chung and S. K. Kim, Berezinskii-Kosterlitz-Thouless transition transport in spin-triplet superconductor, *SciPost Phys.* **5**, 003 (2022).
- [24] Y.-F. Jiang, Z.-X. Li, S. A. Kivelson, and H. Yao, Charge-4e superconductors: A Majorana quantum Monte Carlo study, *Phys. Rev. B* **95**, 241103(R) (2017).
- [25] P. Li, K. Jiang, and J. Hu, Charge 4e superconductor: A wavefunction approach, *Sci. Bull.* **69**, 2328 (2024).
- [26] N. V. Gnezdilov and Y. Wang, Solvable model for a charge-4e superconductor, *Phys. Rev. B* **106**, 094508 (2022).
- [27] J. Garaud and E. Babaev, Effective model and magnetic properties of the resistive electron quadrupling state, *Phys. Rev. Lett.* **129**, 087602 (2022).
- [28] Z. Pan, C. Lu, F. Yang, and C. Wu, Frustrated superconductivity and sextetting order, *Sci. China-Phys. Mech. Astron.* **67**, 287412 (2024).
- [29] M. Hecker and J. Schmalian, Vestigial nematic order and superconductivity in the doped topological insulator $\text{Cu}_x\text{Bi}_2\text{Se}_3$, *npj Quantum Mater.* **3**, 26 (2018).
- [30] Y. Yu, Nondegenerate surface pair density wave in the kagome superconductor CSV_3Sb_5 : Application to vestigial orders, *Phys. Rev. B* **108**, 054517 (2023).
- [31] S. Zhou and Z. Wang, Chern Fermi pocket, topological pair density wave, and charge-4e and charge-6e superconductivity in kagomé superconductors, *Nat. Commun.* **13**, 7288 (2022).
- [32] P. P. Poduval and M. S. Scheurer, Vestigial singlet pairing in a fluctuating magnetic triplet superconductor and its implications for graphene superlattices, *Nat. Commun.* **15**, 1713 (2024).
- [33] Y.-M. Wu and Y. Wang, *d*-wave charge-4e superconductivity from fluctuating pair density waves, *npj Quantum Mater.* **9**, 66 (2024).
- [34] Y.-B. Liu, J. Zhou, and F. Yang, Nematic superconductivity and its critical vestigial phases in the quasicrystal, *Phys. Rev. Lett.* **133**, 136002 (2024).
- [35] P. T. How and S. K. Yip, Broken time reversal symmetry vestigial state for a two-component superconductor in two spatial dimensions, *Phys. Rev. B* **110**, 054519 (2024).
- [36] J. Ingham, R. Thomale, and H. D. Scammell, Vestigial order from an excitonic mother state in kagome superconductors AV_3Sb_5 , [arXiv:2503.02929v1](https://arxiv.org/abs/2503.02929v1).
- [37] Y. Verghis, D. Sedov, J. Weßling, P. P. Poduval, and M. S. Scheurer, Vestigial pairing from fluctuating magnetism and triplet superconductivity, [arXiv:2510.02474](https://arxiv.org/abs/2510.02474).
- [38] M. Hecker and R. M. Fernandes, Local condensation of charge-4e superconductivity at a nematic domain wall, *Phys. Rev. B* **109**, 134514 (2024).
- [39] M. Hecker, R. Willa, J. Schmalian, and R. M. Fernandes, Cascade of vestigial orders in two-component superconduct-

- tors: Nematic, ferromagnetic, s -wave charge- $4e$, and d -wave charge- $4e$ states, *Phys. Rev. B* **107**, 224503 (2023).
- [40] Y.-X. Dai and Q.-F. Sun, Andreev reflection in normal metal/charge- $4e$ superconductor junctions, *Phys. Rev. B* **109**, 144504 (2024).
- [41] G. E. Volovik, Fermionic quartet and vestigial gravity, *JETP Lett.* **119**, 330 (2024).
- [42] E. Huecker and Y. Wang, Vestigial d -wave charge- $4e$ superconductivity from bidirectional pair density waves, *Phys. Rev. B* **113**, 045121 (2026).
- [43] Y. Cao, V. Fatemi, S. Fang, K. Watanabe, T. Taniguchi, E. Kaxiras, and P. Jarillo-Herrero, Unconventional superconductivity in magic-angle graphene superlattices, *Nature (London)* **556**, 43 (2018).
- [44] M. Yankowitz, S. Chen, H. Polshyn, Y. Zhang, K. Watanabe, T. Taniguchi, D. Graf, A. F. Young, and C. R. Dean, Tuning superconductivity in twisted bilayer graphene, *Science* **363**, 1059 (2019).
- [45] J. M. Park, Y. Cao, K. Watanabe, T. Taniguchi, and P. Jarillo-Herrero, Tunable strongly coupled superconductivity in magic-angle twisted trilayer graphene, *Nature (London)* **590**, 249 (2021).
- [46] J. M. Park, Y. Cao, L.-Q. Xia, S. Sun, K. Watanabe, T. Taniguchi, and P. Jarillo-Herrero, Robust superconductivity in magic-angle multilayer graphene family, *Nat. Mater.* **21**, 877 (2022).
- [47] M. Oh, K. P. Nuckolls, D. Wong, R. L. Lee, X. Liu, K. Watanabe, T. Taniguchi, and A. Yazdani, Evidence for unconventional superconductivity in twisted bilayer graphene, *Nature (London)* **600**, 240 (2021).
- [48] E. Lake, A. S. Patri, and T. Senthil, Pairing symmetry of twisted bilayer graphene: A phenomenological synthesis, *Phys. Rev. B* **106**, 104506 (2022).
- [49] Y. Waschitz, A. Stern, and Y. Oreg, Momentum-resolved spectroscopy of superconductivity with the quantum twisting microscope, *Phys. Rev. Lett.* **136**, 156501 (2026).
- [50] T. J. Peltonen, R. Ojajarvi, and T. T. Heikkilä, Mean-field theory for superconductivity in twisted bilayer graphene, *Phys. Rev. B* **98**, 220504(R) (2018).
- [51] D. M. Kennes, J. Lischner, and C. Karrasch, Strong correlations and $d + id$ superconductivity in twisted bilayer graphene, *Phys. Rev. B* **98**, 241407(R) (2018).
- [52] M. S. Scheurer and R. Samajdar, Pairing in graphene-based moiré superlattices, *Phys. Rev. Res.* **2**, 033062 (2020).
- [53] H. Isobe, N. F. Q. Yuan, and L. Fu, Unconventional superconductivity and density waves in twisted bilayer graphene, *Phys. Rev. X* **8**, 041041 (2018).
- [54] M. Christos, S. Sachdev, and M. S. Scheurer, Nodal band-off-diagonal superconductivity in twisted graphene superlattices, *Nat. Commun.* **14**, 7134 (2023).
- [55] F. Wu, Topological chiral superconductivity with spontaneous vortices and supercurrent in twisted bilayer graphene, *Phys. Rev. B* **99**, 195114 (2019).
- [56] F. Wu, A. H. MacDonald, and I. Martin, Theory of phonon-mediated superconductivity in twisted bilayer graphene, *Phys. Rev. Lett.* **121**, 257001 (2018).
- [57] T. Huang, L. Zhang, and T. Ma, Antiferromagnetically ordered Mott insulator and $d + id$ superconductivity in twisted bilayer graphene: A quantum Monte Carlo study, *Sci. Bull.* **64**, 310 (2019).
- [58] V. Kozii, H. Isobe, J. W. F. Venderbos, and L. Fu, Nematic superconductivity stabilized by density wave fluctuations: Possible application to twisted bilayer graphene, *Phys. Rev. B* **99**, 144507 (2019).
- [59] T. Cea and F. Guinea, Coulomb interaction, phonons, and superconductivity in twisted bilayer graphene, *Proc. Natl. Acad. Sci. USA* **118**, e2107874118 (2021).
- [60] G. Shavit, E. Berg, A. Stern, and Y. Oreg, Theory of correlated insulators and superconductivity in twisted bilayer graphene, *Phys. Rev. Lett.* **127**, 247703 (2021).
- [61] Y. Wang, J. Kang, and R. M. Fernandes, Topological and nematic superconductivity mediated by ferro-SU(4) fluctuations in twisted bilayer graphene, *Phys. Rev. B* **103**, 024506 (2021).
- [62] C.-X. Liu and B. A. Bernevig, Nodal nematic superconductivity in multiple flat-band systems, *Phys. Rev. B* **111**, L020502 (2025).
- [63] Y.-J. Wang, G.-D. Zhou, S.-Y. Peng, B. Lian, and Z.-D. Song, Molecular pairing in twisted bilayer graphene superconductivity, *Phys. Rev. Lett.* **133**, 146001 (2024).
- [64] J. Yu, M. Xie, F. Wu, and S. Das Sarma, Euler-obstructed nematic nodal superconductivity in twisted bilayer graphene, *Phys. Rev. B* **107**, L201106 (2023).
- [65] G. Shavit, Nematic enhancement of superconductivity in multilayer graphene via quantum geometry, *Phys. Rev. B* **113**, 094502 (2026).
- [66] H. Zhou, T. Xie, T. Taniguchi, K. Watanabe, and A. F. Young, Superconductivity in rhombohedral trilayer graphene, *Nature (London)* **598**, 434 (2021).
- [67] T. Han, Z. Lu, Z. Hadjri, L. Shi, Z. Wu, W. Xu, Y. Yao, A. A. Cotten, O. S. Sedeh, H. Weldeyesus, *et al.*, Signatures of chiral superconductivity in rhombohedral graphene, *Nature (London)* **643**, 654 (2025).
- [68] Y.-Z. Chou, J. Zhu, and S. D. Sarma, Intravalley spin-polarized superconductivity in rhombohedral tetralayer graphene, *Phys. Rev. B* **111**, 174523 (2025).
- [69] C. Yoon, T. Xu, Y. Barlas, and F. Zhang, Quarter-metal superconductivity in rhombohedral graphene, *Phys. Rev. Lett.* **136**, 026603 (2026).
- [70] J. May-Mann, T. Helbig, and T. Devakul, How pairing mechanism dictates topology in valley-polarized superconductors with Berry curvature, *arXiv:2503.05697*.
- [71] S. A. Murshed and B. Roy, Nodal pair density waves from a quarter-metal in crystalline graphene multilayers, *Phys. Rev. B* **112**, 085121 (2025).
- [72] M. Geier, M. Davydova, and L. Fu, Chiral and topological superconductivity in isospin polarized multilayer graphene, *arXiv:2409.13829*.
- [73] J. D. Sau and S. Wang, Theory of anomalous Hall effect from screened vortex charge in a phase disordered superconductor, *arXiv:2411.08969*.
- [74] H. Yang and Y.-H. Zhang, Topological incommensurate Fulde-Ferrell-Larkin-Ovchinnikov superconductor and Bogoliubov Fermi surface in rhombohedral tetra-layer graphene, *arXiv:2411.02503*.
- [75] Q. Qin and C. Wu, Chiral finite-momentum superconductivity in the tetralayer graphene, *Chin. Phys. Lett.* **43**, 030708 (2026).

- [76] A. Jahin and S.-Z. Lin, Enhanced Kohn-Luttinger superconductivity in geometric bands, *Phys. Rev. B* **113**, 014504 (2026).
- [77] M. Christos, P. M. Bonetti, and M. S. Scheurer, Finite-momentum pairing and superlattice superconductivity in valley-imbalanced rhombohedral graphene, *arXiv:2503.15471*.
- [78] D. Sedov and M. S. Scheurer, Probing superconductivity with tunneling spectroscopy in rhombohedral graphene, *Phys. Rev. B* **113**, L140503 (2026).
- [79] A. Gil and E. Berg, Charge and pair density waves in a spin and valley-polarized system at a Van-Hove singularity, *arXiv:2504.19321*.
- [80] G. Parra-Martínez, A. Jimeno-Pozo, V. T. Phong, H. Sainz-Cruz, D. Kaplan, P. Emanuel, Y. Oreg, P. A. Pantaleón, J. A. Silva-Guillén, and F. Guinea, Band renormalization, quarter metals, and chiral superconductivity in rhombohedral tetralayer graphene, *Phys. Rev. Lett.* **135**, 136503 (2025).
- [81] F. Gaggioli, D. Guerci, and L. Fu, Spontaneous vortex-antivortex lattice and Majorana fermions in rhombohedral graphene, *Phys. Rev. Lett.* **135**, 116001 (2025).
- [82] Y. Cao, D. Rodan-Legrain, J. M. Park, N. F. Yuan, K. Watanabe, T. Taniguchi, R. M. Fernandes, L. Fu, and P. Jarillo-Herrero, Nematicity and competing orders in superconducting magic-angle graphene, *Science* **372**, 264 (2021).
- [83] Z. Bi, N. F. Q. Yuan, and L. Fu, Designing flat bands by strain, *Phys. Rev. B* **100**, 035448 (2019).
- [84] T. Cea, N. R. Walet, and F. Guinea, Electronic band structure and pinning of Fermi energy to Van Hove singularities in twisted bilayer graphene: A self-consistent approach, *Phys. Rev. B* **100**, 205113 (2019).
- [85] A. Uri, S. Grover, Y. Cao, J. A. Crosse, K. Bagani, D. Rodan-Legrain, Y. Myasoedov, K. Watanabe, T. Taniguchi, P. Moon, *et al.*, Mapping the twist-angle disorder and Landau levels in magic-angle graphene, *Nature (London)* **581**, 47 (2020).
- [86] A. Kerelsky, L. J. McGilly, D. M. Kennes, L. Xian, M. Yankowitz, S. Chen, K. Watanabe, T. Taniguchi, J. Hone, C. Dean, *et al.*, Maximized electron interactions at the magic angle in twisted bilayer graphene, *Nature (London)* **572**, 95 (2019).
- [87] Y. Choi, J. Kemmer, Y. Peng, A. Thomson, H. Arora, R. Polski, Y. Zhang, H. Ren, J. Alicea, G. Refael, *et al.*, Electronic correlations in twisted bilayer graphene near the magic angle, *Nat. Phys.* **15**, 1174 (2019).
- [88] P. Stepanov, I. Das, X. Lu, A. Fahimniya, K. Watanabe, T. Taniguchi, F. H. Koppens, J. Lischner, L. Levitov, and D. K. Efetov, Untying the insulating and superconducting orders in magic-angle graphene, *Nature (London)* **583**, 375 (2020).
- [89] Y. Jiang, X. Lai, K. Watanabe, T. Taniguchi, K. Haule, J. Mao, and E. Y. Andrei, Charge order and broken rotational symmetry in magic-angle twisted bilayer graphene, *Nature (London)* **573**, 91 (2019).
- [90] Y. Su and S.-Z. Lin, Pairing symmetry and spontaneous vortex-antivortex lattice in superconducting twisted-bilayer graphene: Bogoliubov-de Gennes approach, *Phys. Rev. B* **98**, 195101 (2018).
- [91] D. V. Chichinadze, L. Classen, and A. V. Chubukov, Nematic superconductivity in twisted bilayer graphene, *Phys. Rev. B* **101**, 224513 (2020).
- [92] T. Yu, D. M. Kennes, A. Rubio, and M. A. Sentef, Nematicity arising from a chiral superconducting ground state in magic-angle twisted bilayer graphene under in-plane magnetic fields, *Phys. Rev. Lett.* **127**, 127001 (2021).
- [93] N. J. Zhang, P. A. Nosov, O. E. Sommer, Y. Wang, K. Watanabe, T. Taniguchi, E. Khalaf, and J. I. A. Li, Angular interplay of nematicity, superconductivity, and strange metallicity in a moiré flat band, *arXiv:2503.15767*.
- [94] S. A. Murshed and B. Roy, Charge-density waves and stripes in quarter metals of graphene heterostructures, *arXiv:2510.20816*.
- [95] P. Qin, H.-T. Wu, R. Q. Nguyen, E. Morissette, N. J. Zhang, K. Watanabe, T. Taniguchi, and J. I. A. Li, Extreme anisotropy in the metallic and superconducting phases of rhombohedral hexalayer graphene, *arXiv:2504.05129*.
- [96] R. Q. Nguyen, H.-T. Wu, E. Morissette, N. J. Zhang, P. Qin, K. Watanabe, T. Taniguchi, A. W. Hui, D. E. Feldman, and J. I. A. Li, A hierarchy of superconductivity and topological charge density wave states in rhombohedral graphene, *arXiv:2507.22026*.
- [97] Y. Xiang, Q. Li, Y. Li, W. Xie, H. Yang, Z. Wang, Y. Yao, and H.-H. Wen, Twofold symmetry of *c*-axis resistivity in topological kagome superconductor CsV₃Sb₅ with in-plane rotating magnetic field, *Nat. Commun.* **12**, 6727 (2021).
- [98] Y. Guo, J. Pack, J. Swann, L. Holtzman, M. Cothrine, K. Watanabe, T. Taniguchi, D. G. Mandrus, K. Barmak, J. Hone, A. J. Millis, A. Pasupathy, and C. R. Dean, Superconductivity in 5.0° twisted bilayer WSe₂, *Nature (London)* **637**, 839 (2025).
- [99] Y. Xia, Z. Han, K. Watanabe, T. Taniguchi, J. Shan, and K. F. Mak, Superconductivity in twisted bilayer WSe₂, *Nature (London)* **637**, 833 (2025).
- [100] F. Xu, Z. Sun, J. Li, C. Zheng, C. Xu, J. Gao, T. Jia, K. Watanabe, T. Taniguchi, B. Tong, L. Lu, J. Jia, Z. Shi, S. Jiang, Y. Zhang, Y. Zhang, S. Lei, X. Liu, and T. Li, Signatures of unconventional superconductivity near reentrant and fractional quantum anomalous Hall insulators, *arXiv:2504.06972*.
- [101] Y. Cao, J. M. Park, K. Watanabe, T. Taniguchi, and P. Jarillo-Herrero, Pauli-limit violation and re-entrant superconductivity in moiré graphene, *Nature (London)* **595**, 526 (2021).
- [102] E. Morissette, J.-X. Lin, D. Sun, L. Zhang, S. Liu, D. Rhodes, K. Watanabe, T. Taniguchi, J. Hone, J. Pollanen, *et al.*, Dirac revivals drive a resonance response in twisted bilayer graphene, *Nat. Phys.* **19**, 1156 (2023).
- [103] B. Putzer and M. S. Scheurer, Eliashberg theory and superfluid stiffness of band-off-diagonal pairing in twisted graphene, *Phys. Rev. B* **111**, 144513 (2025).
- [104] X. Ma, Z. Liu, J. Cai, K. Watanabe, T. Taniguchi, X. Xu, J.-H. Chu, and M. Yankowitz, Giant elastoresistance in magic-angle twisted bilayer graphene, *arXiv:2505.10506*.
- [105] R. Samajdar, M. S. Scheurer, S. Turkel, C. Rubio-Verdú, A. N. Pasupathy, J. W. F. Venderbos, and R. M. Fernandes, Electric-field-tunable electronic nematic order in twisted double-bilayer graphene, *2D Mater.* **8**, 034005 (2021).
- [106] A. Daido, Y. Ikeda, and Y. Yanase, Intrinsic superconducting diode effect, *Phys. Rev. Lett.* **128**, 037001 (2022).

- [107] M. Christos, P. M. Bonetti, and M. S. Scheurer, Approximate symmetries, insulators, and superconductivity in the continuum-model description of twisted WSe₂, *Phys. Rev. Lett.* **135**, 046503 (2025).
- [108] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/kjyl-rxg7> for more details on the theory of nematic vestigial order for intervalley pairing, critical currents in $1q$ state, and vestigial orders of the $2q$ superconductor.
- [109] L. G. Aslamazov and A. I. Larkin, The influence of fluctuation pairing of electrons on the conductivity of normal metal, *Phys. Lett. A* **26**, 238 (1968).
- [110] L. G. Aslamazov and A. I. Larkin, Effect of fluctuations on the properties of a superconductor above the critical temperature, *Sov. Phys. - Solid State (Engl. Transl.)* **10**, 875 (1968).
- [111] H. D. Scammell, J. I. A. Li, and M. S. Scheurer, Theory of zero-field superconducting diode effect in twisted trilayer graphene, *2D Mater.* **9**, 025027 (2022).
- [112] E. Morissette, P. Qin, K. Watanabe, T. Taniguchi, and J. I. A. Li, Evidence of momentum space condensation in rhombohedral hexalayer graphene, [arXiv:2503.09954](https://arxiv.org/abs/2503.09954).
- [113] Y. Chen, M. S. Scheurer, and C. Schrade, Intrinsic superconducting diode effect and nonreciprocal superconductivity in rhombohedral graphene multilayers, *Phys. Rev. B* **112**, L060505 (2025).
- [114] S. Banerjee and M. S. Scheurer, Enhanced superconducting diode effect due to coexisting phases, *Phys. Rev. Lett.* **132**, 046003 (2024).
- [115] D. V. Chichinadze, N. J. Zhang, J.-X. Lin, E. Morissette, X. Wang, K. Watanabe, T. Taniguchi, O. Vafek, and J. I. A. Li, Observation of giant nonlinear Hall conductivity in Bernal bilayer graphene, [arXiv:2411.11156](https://arxiv.org/abs/2411.11156).
- [116] S. Banerjee, H. D. Scammell, and M. S. Scheurer, Dataset associated with correlations between superconducting and resistive anisotropies, Zenodo (2026), <https://zenodo.org/records/20070547>.