

**Erratum: Gilbert damping in metallic ferromagnets from Schwinger-Keldysh field theory:
Intrinsically nonlocal, nonuniform, and made anisotropic by spin-orbit coupling
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A mistake was made in our paper when presenting general expressions in Eqs. (15a) and (15b) for the nonlocal damping tensor $\lambda_{nn'}^{\alpha\beta}$, which we offer as the input for possible first-principles calculations via the usage of the retarded and advanced Green's functions (GFs) $\hat{G}^{R/A}(\varepsilon)$ as input in Eqs. (15a) and (15b). Originally, we stated that the symmetric and antisymmetric components of $\lambda_{nn'}^{\alpha\beta}$ are given by

$$\lambda_{nn'}^{(\alpha\beta)} = -\frac{J_{sd}^2}{2\pi} \int d\varepsilon \frac{\partial f}{\partial \varepsilon} \text{Tr}_{\text{spin}}[\sigma^\alpha A_{nn'} \sigma^\beta A_{n'n}], \quad (1a)$$

$$\begin{aligned} \lambda_{nn'}^{[\alpha\beta]} = & -\frac{2J_{sd}^2}{\pi} \int d\varepsilon \frac{\partial f}{\partial \varepsilon} \text{Tr}_{\text{spin}}[\sigma^\alpha \text{Re} \hat{G}_{nn'}^R \sigma^\beta A_{n'n} - \sigma^\alpha A_{nn'} \sigma^\beta \text{Re} \hat{G}_{n'n}^R] \\ & + \frac{J_{sd}^2}{2\pi} \int d\varepsilon (1 - 2f) \text{Tr}_{\text{spin}} \left[\sigma^\alpha \text{Re} \hat{G}_{nn'}^R \sigma^\beta \frac{\partial A_{n'n}}{\partial \varepsilon} - \sigma^\alpha \frac{\partial A_{nn'}}{\partial \varepsilon} \sigma^\beta \text{Re} \hat{G}_{n'n}^R \right], \end{aligned} \quad (1b)$$

where $(\alpha\beta)$ or $[\alpha\beta]$ indicate that the encompassed indices have been symmetrized or antisymmetrized, respectively, and $A(\varepsilon) = i[\hat{G}^R(\varepsilon) - \hat{G}^A(\varepsilon)]$ is the spectral function. However, Eq. (1a) already contains contributions that are both symmetric and antisymmetric in the Cartesian indices α, β . Therefore, the complete nonlocal damping tensor is given *solely* by the expression

$$\lambda_{nn'}^{\alpha\beta} = -\frac{J_{sd}^2}{2\pi} \int d\varepsilon \frac{\partial f}{\partial \varepsilon} \text{Tr}_{\text{spin}}[\sigma^\alpha A_{nn'} \sigma^\beta A_{n'n}], \quad (2)$$

which was already displayed as the correct Eq. (15a) of our paper. It can be verified that Eq. (2) satisfies the expected symmetry $\lambda_{nn'}^{\alpha\beta} = \lambda_{n'n}^{\beta\alpha}$. We note that this correction does not affect the results presented in our paper, but should be taken into account in first-principles calculations of $\lambda_{nn'}^{\alpha\beta}$, as outlined in Sec. III F of our paper.

It is also worth comparing our correct Eq. (2) for nonlocal Gilbert damping with other formulas in the literature. Recent studies driven by increased interest to obtain the nonlocal damping tensor from first principles for bulk ferromagnets [1,2], their heterostructures [3,4] and low-dimensional systems [5] have employed standard formulas (such as, torque-torque correlation or breathing Fermi surface) for Gilbert damping as scalar parameter, but plausibly generalized [6] to make it dependent on sites of the system and its whole magnetic configuration. For example, Eq. (4) in Refs. [1,2] has similar structure as our Eq. (2) but instead of the Pauli matrices σ^α it uses site-resolved torque operators ($\sigma^\alpha \mapsto \hat{T}_i^\alpha$) within the trace. We re-emphasize that our expression in Eq. (2) is rigorously (field-theoretically) derived and confirmed by comparing it with exact numerics in our paper. Furthermore, it is identical to Eq. (13) of Ref. [7] that also employs field-theoretic derivation by starting from *sd* model of exchange interaction of flowing and localized spins. For anticipated usage of Eq. (2) [or, equivalently, Eq. (13) of Ref. [7]] in first-principles calculation, besides computing GFs with input Hamiltonian from density functional theory (DFT), one also needs to extract J_{sd} parameter. In principle, this can be done by analyzing exchange-correlation magnetic field [8] of noncollinear DFT.

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