

Reentrant localization induced by short-range hopping in the fractal Rosenzweig-Porter model

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Typically, metallic systems localized under strong disorder exhibit a transition to delocalization as the kinetic terms increase. In this Letter, we reveal the opposite effect—increasing kinetic terms leads to an unexpected reduction of mobility, enhancing localization of the system, and even leading to reentrant delocalization transitions. Specifically, we add a nearest-neighbor hopping with amplitude κ to the Rosenzweig-Porter (RP) model with fractal on-site disorder and surprisingly see that, as κ grows, the system initially tends to localization from the fractal phase, but then reenters the ergodic phase. We build an analytical framework to explain this re-entrant behavior, supported by exact diagonalization results. The interplay between the spatially local κ term, insensitive to fractal disorder, and the energy-local RP coupling, sensitive to fine-level spacing structures, drives the observed reentrant behavior. This mechanism offers a distinct pathway to reentrant localization phenomena in many-body quantum systems.

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Introduction: Ergodicity in quantum many-body systems [1,2] and its breakdown by a disordered potential [3–5], usually called the many-body localization (MBL) transition, has attracted a lot of attention for the past two decades. This stems both from fundamental challenges in determining whether the MBL phase persists in the thermodynamic limit [6–12], and from the potential applications of this phase in quantum information processing and machine learning [13,14].

According to common knowledge, disordered quantum systems typically exhibit localization behavior, such as in Anderson transitions [15] and many-body localizations [3,4], which occur monotonically with increasing disorder strength (or equivalently, decreasing the amplitude of the corresponding kinetic term). This behavior is believed to be universal both in many-body disordered systems [5] and in the corresponding random-matrix (RM) ensembles utilized to understand them with Anderson [16] and ergodicity breaking [17] phase transitions. The latter models, such as the so-called Rosenzweig-Porter random-matrix ensemble [17–24], show not only Anderson localized and ergodic (metallic) phases, but also an intermediate nonergodic extended phase (relevant for the description of the Hilbert-space structure of the MBL states [25–27]), in which the eigenstates span over an extensive number but measure zero of all Hilbert-space configurations characterized by fractal dimensions $0 \leq D_2 \leq 1$ [see Eq. (2)] [16]. In this Letter, using RM ensembles, we analytically reveal a pathway that challenges the conventional

view of a monotonic transition from localization to ergodicity. Starting with a system characterized by a fractal distribution of on-site disorder [28,29], we show that adding a short-range kinetic term with coupling strength κ initially steers the system towards localization. Only beyond a critical strength of κ , this trend reverses, and further increasing κ drives the system to an ergodic phase (see Fig. 1). Our analytical understanding provides a different perspective on reentrant delocalization transitions in the disordered systems, robust to perturbations [30]. We then verify our predictions with exact numerical simulations.

Model: The standard Rosenzweig-Porter model [17,31] is a random-matrix model where both the diagonal and off-diagonal elements of the $L \times L$ Hermitian matrix $H_{mn} = H_{nm}^*$ are taken from a random normal distribution such that

$$H_{mn} = h_n \delta_{mn} + R_{mn} \cdot L^{-\gamma/2}. \quad (1)$$

Here, $\overline{h_n} = \overline{R_{mn}} = 0$ and $\overline{h_n^2} = \overline{R_{mn}^2} = 1$ and γ is the tuning parameter which rescales the off-diagonal elements and controls the phase diagram. The ergodic ($\gamma \leq 1$) and localized ($\gamma \geq 2$) phases are separated by a nonergodic extended fractal phase ($1 \leq \gamma \leq 2$), which is squeezed between the ergodic transition (ET) at $\gamma = 1$ and the Anderson localization transition (AT) at $\gamma = 2$. The eigenstate nature in different phases can be characterized by the fractal dimension D_2 , calculated from the eigenfunctions $|\psi\rangle$, as [16]

$$L^{-D_2} = \text{IPR} = \sum_{i=1}^L |\langle i|\psi\rangle|^4, \quad (2)$$

where $|i\rangle$ denotes the computational basis states and IPR denotes the inverse participation ratio. When we compute D_2 numerically, we fit the IPR for different system sizes L and extrapolate to obtain the thermodynamic limit of D_2 . Corrections $O(1/\log L)$ to D_2 are known to be present for

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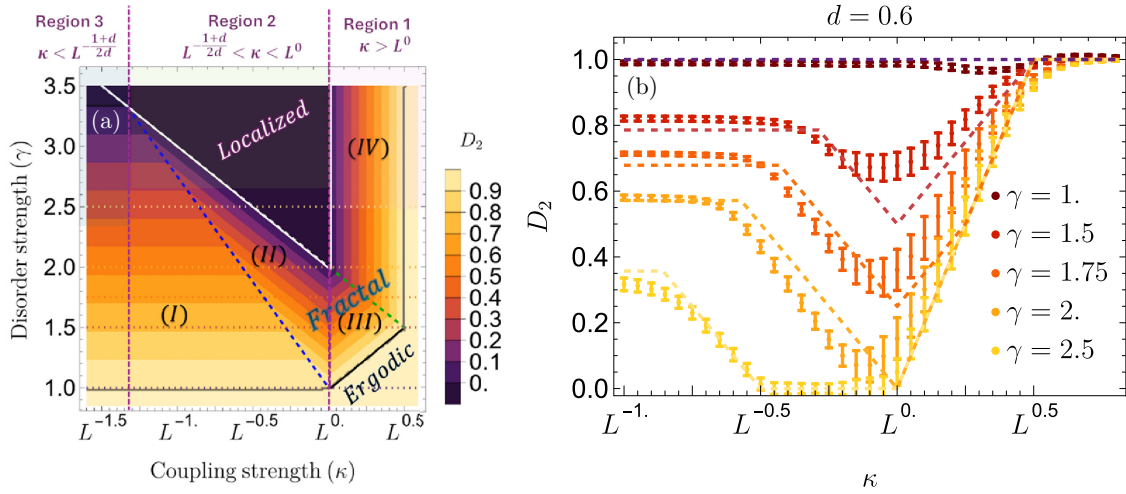


FIG. 1. (a) Localization phase diagram in terms of fractal dimension D_2 for $d = 0.6$ for different nearest-neighbor couplings κ and effective disorder strengths (γ), obtained from exact analytical computation. The three regions described in the text are demarcated by vertical dashed lines. Additionally, the localized and fractal phases are separated by the white line and the fractal and ergodic phases are separated by the black line. Inside the fractal phase the regions with different behaviors are separated by dashed lines and marked with roman numbers. The horizontal dotted lines denote the values of γ chosen for (b). (b) D_2 with varying κ for specific γ 's. The numerical data shown via points are obtained by fitting IPR for different system sizes 2^n where n runs from 7 to 14 and averaged over 10^2 – 10^5 realizations (the smaller system sizes have a larger number of realizations). The dashed lines denote the analytical results.

the RP models [32], which were also seen in our case and were taken into account during fitting. The ergodic states are characterized by $D_2 = 1$, and the extended (multi)fractal states are characterized by $0 < D_2 < 1$ and for the localized state $D_2 = 0$. Recently, it has been shown [29] that the fractal distribution of diagonal elements h_n 's tunes the phase diagram of the Rosenzweig-Porter (RP) model by extending the nonergodic fractal phase. Simply put, in this scenario of fractal diagonal disorder one considers the level spacing $s_i = h_{n_{i+1}} - h_{n_i}$, where $h_{n_i} < h_{n_{i+1}}$ are sorted diagonal elements h_n , to be independent and identically distributed (i.i.d.) numbers, distributed as a Pareto distribution $P(s) = \frac{d \cdot \delta_{\text{typ}}^d}{s^{d+1}} \theta(s - \delta_{\text{typ}})$, where $\delta_{\text{typ}} \sim L^{-1/d}$ is the typical level spacing. The elements h_n are randomly shuffled with respect to the sorted sequence $h_{n_i} < h_{n_{i+1}}$ to realize random disorder. Consequently, the number of elements h_n in the interval L^{-b} around a certain energy E scales as $N\{|E - h_n| \in [L^{-b-db}, L^{-b}]\} \sim L^{1-f(b)} db$ [33], where $f(x) = d \cdot x$, $df(x)/dx \leq 1$ for Hermitian matrices, and $f(0) = 0$ and d can be regarded as the fractal dimension of the disorder. In this Letter, we add an extra nearest-neighbor hopping term to modify Eq. (1) to

$$H_{mn} = h_n \delta_{mn} + \kappa (\delta_{m,n+1} + \text{H.c.}) + R_{mn} L^{-\gamma/2}, \quad (3)$$

where h_n is distributed as the aforementioned fractal disorder [34], and the amplitude of the extra nearest-neighbor hopping term scales with the system size, $\kappa \sim L^k$, with positive or negative k . In what follows, we shall focus on the properties of the bulk spectral states and discuss the nonmonotonic behavior of their fractal dimension by tuning this extra term k .

Phase diagram: By increasing the nearest-neighbor hopping strength κ from zero, a strikingly counterintuitive phenomenon occurs. For small but significant coupling, the fractal dimension D_2 initially decreases, driving the system towards localization. Only beyond a certain critical hopping

amplitude κ does the standard increasing behavior of D_2 (indicating more delocalization) dominate. This reveals a rich reentrant phase diagram as shown in Fig. 1(a), where we plot the fractal dimension (D_2) versus varying coupling strength (κ) and disorder strength (which is controlled by γ) (see Supplemental Material [34] for results for other values of d). From a fully analytic study described later, we compute the fractal dimension D_2 of the different regions of the diagram as follows:

Region 1: For $k > 0$, $\kappa \gg 1$ there are no effects of the fractal diagonal disorder,

$$D_2(\gamma) = \begin{cases} 1, & \gamma < \gamma_{\text{ET}}^{(1)}, \\ 2 - \gamma + k, & \gamma_{\text{ET}}^{(1)} < \gamma < \gamma_P, \\ 2k, & \gamma_P < \gamma, \end{cases} \quad (4)$$

because of the spatial locality of the dominant hopping term κ . Here, there is no localized phase, only the ergodic, $\gamma < \gamma_{\text{ET}}^{(1)} = 1 + k$, and fractal phases. We distinguish the two fractal phases as RP-like [fractal III in Fig. 1(a)] for the middle regime, and blocklike (fractal IV), $\gamma > \gamma_P = 2 - k$, for the final regime in Fig. 1(a). This is because fractal IV has energy-level spacings distributed according to Poisson statistics in spite of it not being localized, distinct from the other regime. The middle regime disappears at $k = 1/2$ and the system becomes ergodic for all $k > 1/2$ at any γ .

Region 2: For intermediate $-(1+d)/(2d) < k < 0$, there is competition between the hopping terms and the fractal disorder resulting in four distinct regimes of γ ,

$$D_2(\gamma) = \begin{cases} 1, & \gamma < \gamma_{\text{ET}}^{(2)}, \\ \frac{2-d \cdot \gamma}{2-d}, & \gamma_{\text{ET}}^{(2)} < \gamma < \gamma_{\text{FT}}, \\ 2 - \gamma + |2k| \frac{2-2d}{1+d}, & \gamma_{\text{FT}} < \gamma < \gamma_{\text{AT}}^{(2)}, \\ 0, & \gamma_{\text{AT}}^{(2)} < \gamma, \end{cases} \quad (5)$$

ergodic, $\gamma < \gamma_{\text{ET}}^{(2)} = 1$, and localized, $\gamma > \gamma_{\text{AT}}^{(2)} = 2 + \frac{2-d}{1+d}|2k|$, ones which are complimented by two fractal phases—the fractal-disorder dominated [29], $\gamma < \gamma_{\text{FT}} = 1 + \frac{2-d}{1+d}|2k|$ [fractal I in Fig. 1(a)], and the standard RP-like [17], $\gamma > \gamma_{\text{FT}}$ denoted by fractal II in Fig. 1(a) to distinguish their origins.

Region 3: For small $k < -(1+d)/(2d)$, there are no effects of the short-range hopping and the results coincide with the ones of Ref. [29] with the enlarged fractal-disorder-dominated phase, also denoted by fractal I,

$$D_2(\gamma) = \begin{cases} 1, & \gamma < \gamma_{\text{ET}}^{(3)}, \\ \frac{2-d\gamma}{2-d}, & \gamma_{\text{ET}}^{(3)} < \gamma < \gamma_{\text{AT}}^{(3)}, \\ 0, & \gamma_{\text{AT}}^{(3)} < \gamma, \end{cases} \quad (6)$$

with $\gamma_{\text{ET}}^{(3)} = 1$ and $\gamma_{\text{AT}}^{(3)} = 2/d$. The reentrant behavior is even more evident in Fig. 1(b) by considering fixed γ cuts from Fig. 1(a), marked by the dotted lines on the plot.

The numerical data, represented by points, align well with the analytical prediction of the phase diagram outlined above, which is plotted alongside the data with dashed lines. As is clearly evident for $\gamma > 1$, D_2 shows a reduction with an increasing value of κ first, before reverting back to the standard behavior towards delocalization.

In what follows, we provide a detailed analytical explanation for the different regions of the phase diagram. In summary, the key insight lies in identifying how the nearest-neighbor coupling hybridizes the fractal RP eigenstates, thereby modifying the spectral distribution. In certain parameter regimes, due to the hybridization, the effective on-site disorder distribution is transformed from correlated fractal to uncorrelated Gaussian, which triggers localization. We shall treat the cases $k > 0$ and $k < 0$ separately, as they give rise to distinct phenomena.

Analytical explanation: Let us first recall the main results for $\kappa = 0$, i.e., $k \rightarrow -\infty$, the fractal RP model limit. The eigenfunction of any RP model has a Lorentzian structure [20,21,35] (including the fractal disorder case [29])

$$|\psi_m(n)|^2 = \frac{A}{(h_n - E_m)^2 + \Gamma_d^2}, \quad (7)$$

where A is the normalization coefficient. We also have $\Gamma_d \sim L^{-a}$ with the parameter a being the solution of the self-consistency equation

$$1 + 2a - f(a) = \gamma. \quad (8)$$

Similarly, the wave-function fractal dimension is given by the formula

$$D_2 = 1 - f(a).$$

Clearly, Γ_d is the width of the eigenfunction miniband, within which all the eigenstates are fully hybridized, and is found to be [29]

$$\Gamma_d \sim L^{-\frac{\gamma-1}{2-d}}. \quad (9)$$

For small enough short-range hopping κ , the fractal phase is dominated by the local in-energy RP broadening Γ_d [17], highly sensitive to the fractal on-site disorder [29]. At large $\kappa \gtrsim 1$, the standard picture of a one-dimensional (1D)

Anderson transition prevails [36], removing all the fine (fractal) details of the diagonal-energy distribution via strong short-range resonances. In the intermediate region the competition between Γ_d and κ comes into play and gradually destroys the fractal structure of the diagonal disorder: The interplay between two delocalizing mechanisms reduces the fractal dimension and can even localize the system. Let us now consider the situation in details for the two different limits,

Strong nearest-neighbor hopping $k > 0 \Rightarrow \kappa \gg 1$ (*region I*): When $\kappa \gg 1$, the nearest-neighbor term dominates the Hamiltonian and strongly hybridizes the nearby sites, thus making the diagonal elements reshuffled with no fractal structure and effectively creating a block diagonal structure in the Hamiltonian, before the effects of RP coupling kick in. The block sizes scale as $\xi \sim \kappa^2 = L^{2k} \gg 1$ (see Refs. [36,37]), where ξ is the localization length in an effective 1D Anderson model with disorder strength κ^{-1} [36,38]. Within each block, wave functions become fully hybridized, rendering the fractal disorder distribution in diagonal elements h_n insignificant. The bandwidth of the $\xi \times \xi$ block is equal to the total bandwidth and can be estimated as $E_{\text{BW}}^2 = \frac{1}{L} \sum_{m,n=1}^L \langle |H_{mn}|^2 \rangle = \frac{1}{\xi} \sum_{m,n=1}^{\xi} \langle |H_{mn}|^2 \rangle = 2\kappa^2 + 1$. The corresponding mean level spacing δ_ξ in each of such blocks reads as $\delta_\xi \simeq \frac{E_{\text{BW}}}{\xi}$, as the number of levels in each block is given by the block size ξ .

Using $\kappa \sim \xi^{1/2} \gg 1$, we have $E_{\text{BW}} \simeq \xi^{1/2}$. Therefore the local level spacing in each block, $\delta_\xi \sim \xi^{-1/2}$, is parametrically larger than the global one, $\delta_L = E_{\text{BW}}/L \simeq \xi^{1/2}/L$, as soon as $\xi \ll L$. For $\xi \gtrsim L$ there is only one block, therefore $\delta_\xi = \delta_L$ and all the wave functions are ergodic, $D_2 = 1$. This happens when $k > 1/2$ [36]. Therefore in what follows, we focus on the case $0 \leq k < 1/2$.

For such k , the effect of fractal distribution is suppressed due to the strong hybridization in each block (see also the consideration of region 2 below with $b_* < 0$ for the current case of $k > 0$), and hence the RP all-to-all coupling will hybridize all the energy levels of the blocks in the interval Γ_1 , similar to the case of nonfractal disorder $d = 1$ [17], where Γ_1 is the width of the Lorentzian eigenfunction with magnitude, $\Gamma_1 \sim L^{1-\gamma}$, which replaces Γ_d in Eq. (7), according to Fermi's golden rule. In order to find the wave-function fractal dimension, we should now compare Γ_1 to the block-local δ_ξ and global δ_L .

For $\Gamma_1 < \delta_L \ll \delta_\xi$ we have no additional hybridization given by Γ_1 and the fractal support set L^{D_2} is given by $D_2 = \frac{\ln \xi}{\ln L} = 2k$ similar to a generalized 1D Anderson model [36]. This corresponds to the limit $\gamma > 2 - k$ and constitutes the fractal IV behavior with Poisson level statistics. For $\delta_L < \Gamma_1 < \delta_\xi$, in the energy interval Γ_1 there is at most $O(1)$ levels from each block. Therefore the number of blocks, hybridized by Γ_1 , will be equal to the number of hybridized levels Γ_1/δ_L . As a result the fractal support sets $\sim \xi$ of all Γ_1/δ_L hybridized wave functions, combined together, will determine the fractal dimension as (cf. Ref. [37])

$$L^{D_2} = \xi \cdot \frac{\Gamma_1}{\delta_L} \Leftrightarrow D_2 = 2 - \gamma + k, \quad (10)$$

where we used the fact that for $\xi \gg 1$, $\delta_\xi = \xi^{-1/2} \sim L^{-k}$, and that the support set of each eigenstate in each block is ξ . This behavior constitutes the fractal III RP-like region. For

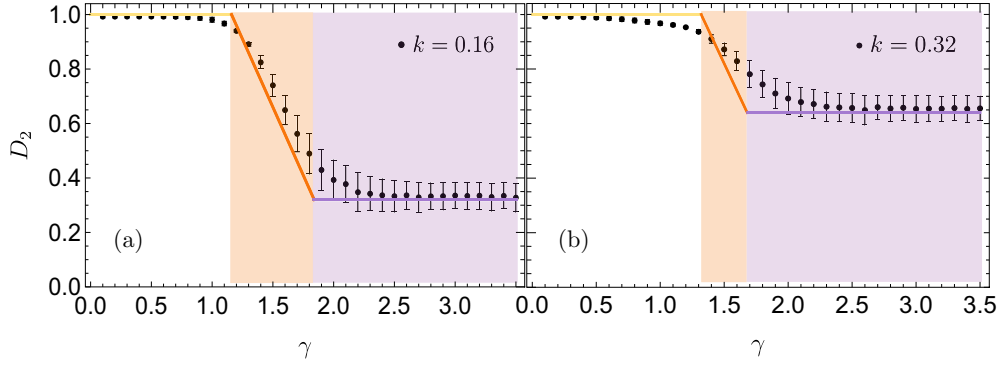


FIG. 2. Comparison of numerically computed variation fractal dimension D_2 with γ (points with error bars) to analytically computed Eq. (4) (solid lines) for $k = 0.16$ and $k = 0.32$. The orange (lilac) line and shaded regions denote the fractal III (IV) phase, and the yellow line stands for the ergodic phase.

$\Gamma_1 > \delta_\xi$, i.e., $\gamma < 1 + k$, the fractal dimension (10) saturates at $D_2 = 1$ and the wave functions are ergodic. Therefore, for $k > 0$, we reproduce Eq. (4).

There are several items to note here. First, since $k > 0$, in the fractal regime, if present, the fractal dimension of the eigenfunctions is bigger than in the usual RP case [17], $D_2 > 2 - \gamma$. Second, the ergodic transition now shifts to $\gamma_{\text{ET}}^{(1)} = 1 + k$ and there is no localization transition $\gamma_{\text{AT}}^{(1)}$ for any γ , unlike the standard RP models. What we have instead is a change of level statistics to Poisson statistics at $\gamma_P = 2 - k$, while the wave functions are extended with a γ -independent fractal dimension of $2k$ within each of the nearly independent blocks of size ξ [36,39]. This is a hallmark of the extensively varying local hopping term. At $k = 1/2$ the ergodic γ_{ET} , and Poisson γ_P , transitions merge simultaneously with the change of the statistics at $\gamma > \gamma_P$ from Poisson to Wigner-Dyson and thus only the ergodic phase survives. The above predictions for the fractal dimensions, Eq. (10), are verified with exact numerics in Fig. 2, where the solid lines represent Eq. (4).

Weak nearest-neighbor hopping, $k \leq 0 \Rightarrow \kappa \lesssim 1$ (regions 2 and 3): In this region, we first need to recompute the localization length ξ and the hybridized wave functions. This step is crucial, as our primary goal is to accurately compute how hybridization due to the additional term impacts the fractal-distributed disorder h_n . However, since κ is small, we can employ perturbation theory [38].

If the wave function $\psi_m(n)$ reaches its maximum at $n = m$, then, applying perturbation theory for $\kappa < 1$, we obtain $\psi_m(m \pm 1) = \frac{\kappa}{h_m - h_{m \pm 1}} \psi_m(m) \simeq \psi_m(m) e^{-1/\xi}$, from which we can extract $\xi^{-1} = -\ln \kappa + \langle \ln |h_m| \rangle = -\ln(2\epsilon\kappa)$ by approximating h_n 's via a box distribution with $|h_m| < 1/2$ [38]. The corresponding energy shift is given by the second-order perturbation theory

$$\epsilon_m = h_m + \sum_{\eta=\pm 1} \frac{\kappa^2}{h_m - h_{m+\eta}}, \quad (11)$$

which typically shifts the diagonal disorder h_m by

$$\exp(\langle \ln |\epsilon_m - h_m| \rangle) \simeq \kappa^2. \quad (12)$$

This typical energy scale defines the lower-energy bound below which all the fractal diagonal elements are reshuffled and thus they lose their fractal nature. Beyond the typical

shift, the probability to have $|\epsilon_m - h_m| \sim L^{-b} \gg \kappa^2$, i.e., with $b < -2k$, is given by the probability p_b to have small $|h_m - h_{m \pm 1}| \sim L^{b+2k} \ll 1$ [40]. Note that, as before, here we consider the randomly reshuffled diagonal elements h_n , but not the monotonic sequence $h_{n_i} < h_{n_{i+1}} = h_{n_i} + s_i$, given by i.i.d. $s_i > 0$. Therefore the marginal distribution $P(h_m - h_{m \pm 1})$ of $h_m - h_{m \pm 1}$ is regular close to the origin. For such distributions, regular at the origin, p_b is proportional to the small interval, $|h_m - h_{m \pm 1}|$, i.e., $p_b \sim L^{b+2k} \ll 1$. Thus, among L differences $|h_m - h_{m-1}|$ in the entire sample there are $M_b \sim L \cdot p_b \sim L^{b+2k+1}$ levels shifted by $\delta h \equiv |\epsilon_m - h_m| \sim L^{-b}$. Note that as we have only L level differences δh , the typical maximal energy hybridization interval δh_{max} reaches only at $M_b \sim L^0$ levels, and scales as $\delta h_{\text{max}} \sim L^{2k+1}$. Thus, δh_{max} will play the role of the crucial energy scale in this regime.

Region 2: In the case when $\delta h_{\text{max}} \gg \delta_{\text{typ}}$, which we denote by region 2, the hybridization from the short-range hopping already affects the fractal diagonal disorder ($\delta h_{\text{max}} \gg \Gamma_d$) at least at some γ values, but the corresponding block RP picture is not applicable as the block size $\xi_\kappa \simeq \{\ln[1/(2\epsilon\kappa)]\}^{-1} \ll 1$ is small [38].

As $\xi \ll 1$, the above effect of short-range hopping is significant only when the hybridization in energy is larger than the typical level spacing $\delta_{\text{typ}} \sim L^{-1/d}$. This leads to the following condition,

$$\delta h_{\text{max}} \gg \delta_{\text{typ}} \Leftrightarrow -1 - 1/d < 2k < 0. \quad (13)$$

Now let us focus on the effect of the nearest-neighbor hopping on the fractal diagonal disorder. For this, we will compare the fractal structure of the latter with the hybridization δh , shifting this levels. Indeed, the number of fractal levels in the interval L^{-b} are given by $L^{1-f(b)}$, while the number of levels shifted by the same energy $\delta h \sim L^{-b}$ is $M_b \sim L^{1+b+2k}$. This means that as soon as $L^{1-f(b)} \ll M_b$, corresponding to $b > b_*$, all the levels in the interval L^{-b} will be hybridized by at least $\delta h \sim L^{-b}$ and, thus losing their fractal distribution, they will be redistributed homogeneously on that interval L^{-b} . Otherwise, if $L^{1-f(b)} \gg M_b$, there will just be a measure zero of such hybridization events with a negligible impact on counting.

For small $b < b_*$ (large enough intervals L^{-b}), we have $L^{1-f(b)} > M_b$. Since $f(b) = d \cdot b$ at such $b < b_*$, the solution

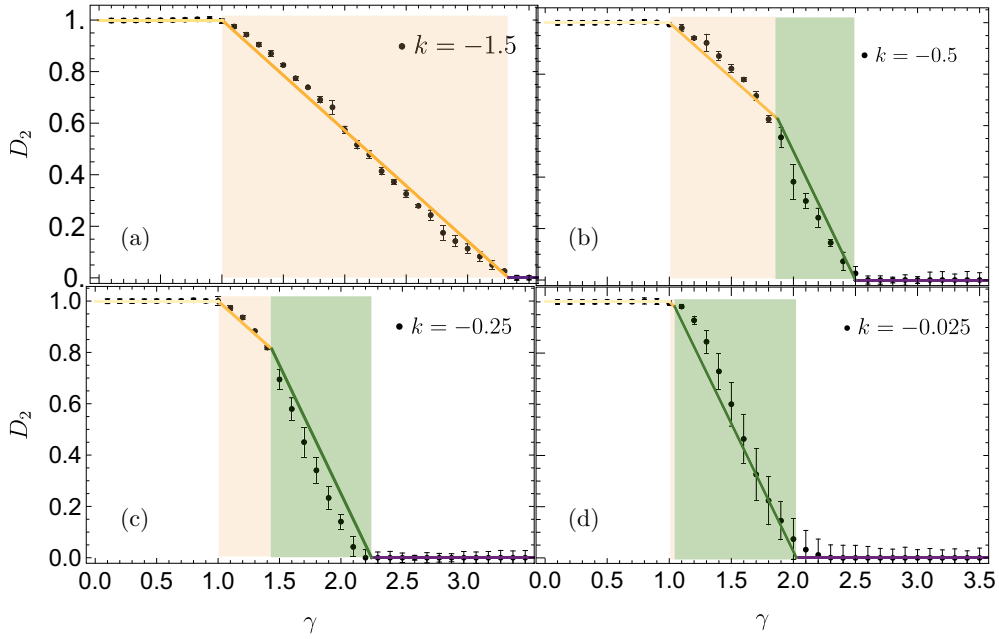


FIG. 3. (a) D_2 vs γ for $k = -1.5$ which is in region 3. The solid line is Eq. (6). (b)–(d) Three values of k showing the behavior in region 2. The solid lines are obtained from Eq. (5), where yellow, dark yellow, dark green, and purple denote ergodic, fractal I, fractal II, and localized phases, respectively. Fractal I and II regions are also shaded by yellow and green colors, respectively.

of the equation $L^{1-f(b_*)} = M_{b_*}$ is given by

$$1 - |2k| + b_* = 1 - d \cdot b_* \implies b_* = \frac{|2k|}{1+d}, \quad (14)$$

which describes the intervals L^{-b_*} , smaller than which everything in the interval is fully hybridized, i.e., $df(b)/db = 1$. Consequently, the corresponding $f(b)$, changed by κ elements, can be read from the continuity equation at $b = b_*$ as

$$f(b) = \begin{cases} d \cdot b, & b < b_*, \\ b - (1-d)b_*, & b_* < b < b_{**}, \end{cases} \quad (15)$$

where $b_{**} = 1 + (1-d)b_*$ is determined by $f(b_{**}) = 1$. This concludes the calculation of lower-energy scale L^{-b_*} below which (at $b > b_*$) all the fractal diagonal potential levels are fully hybridized. All the later self-consistency calculations of Eq. (8) use this modified $f(a)$ and do not need any information about M_b .

This means that as soon as the solution of the self-consistency Eq. (8) corresponds to $f(a < b_*) = d \cdot a$, i.e., at $\gamma < \gamma_{\text{FT}} = 1 + (2-d)b_*$, the fractal I solution [29] is valid, given by the second line in Eq. (5) using Γ_d from (9). In the opposite case of $\gamma > \gamma_{\text{FT}}$, substituting $f(b)$ from Eq. (15) to Eq. (8) gives

$$\Gamma_\kappa = L^{b_{**}-\gamma}, \quad D_2(\gamma > \gamma_{\text{FT}}) = 2b_{**} - \gamma. \quad (16)$$

It works until the Anderson transition at $\gamma_{\text{AT}}^{(2)} = 2b_{**} = 2 + \frac{2-2d}{1+d}|2k|$ describes the fractal II region, matching all the other limiting cases, and corresponds to the third line in Eq. (5).

To corroborate the analytical results we plot numerical data for D_2 vs γ for $k = -0.5, -0.25, -0.025$ in Figs. 3(b)–(3d) with the analytical predictions shown in solid lines. Here, dark

yellow and dark green solid lines have been used to denote fractal I and II behavior, respectively. The numerical data matches very well with our analytical model.

Region 3: This is the scenario where the entire phase diagram remains unaffected by short-range hopping and is exactly predicted by the analysis in Ref. [29]. From Eq. (15), one can see that this occurs when $b_{**} \leq b_*$, effectively eliminating any trace of the fractal II region from the phase diagram. This yields the condition

$$1 + \frac{1-d}{1+d}|2k| \leq \frac{|2k|}{1+d} \Leftrightarrow |2k| \geq 1 + \frac{1}{d}. \quad (17)$$

This is also fully consistent with the condition $\delta h_{\text{max}} \ll \delta_{\text{typ}}$, which is the reverse of the inequality provided in Eq. (13). In this case, the maximal energy hybridization interval δh_{max} of the generalized model is smaller than Γ_κ for all values of $\gamma < \gamma_{\text{AT}}$. Note that when $\gamma > \gamma_{\text{AT}}$, no effects of hybridization are present whatsoever and it does not matter if $\Gamma_\kappa > \delta h_{\text{max}}$ in this regime as δ_{typ} is larger than both.

Although for $\Gamma_\kappa \gg \delta_{\text{typ}}$ the above condition may initially appear to be stronger than necessary, it is important to ensure that this inequality holds throughout the fractal regime—including regions near localization where $\Gamma_\kappa \sim \delta_{\text{typ}}$.

This describes Eq. (6), which is given by the results of Ref. [29]. In Fig. 3(a) we numerically compute the phase diagram of fractal dimension D_2 for $d = 0.6$ and $k = -1.5$, i.e., $|k| > 1/2 + 1/(2d)$, which shows excellent agreement with our theoretical prediction.

Conclusion: In this Letter, utilizing the Rosenzweig-Porter model with fractal disorder, we show that a short-range kinetic term may not always drive the system towards delocalization. In fact, we analytically demonstrate that increasing the

amplitude of this term results in an unexpected nonmonotonic behavior of the localization phase diagram and leads to reentrant delocalization. This Letter elevates the tunability of the fractal phase in the RP model, as introduced in Ref. [29], to a different level. It shows that by applying local perturbations, a system with fractal diagonal disorder can be driven to localized, fractal, or ergodic states. The underlying mechanism is based on the interplay between the local-in-space tight-binding hopping term κ and the local-in-energy Fermi's golden rule broadening Γ_d in the Rosenzweig-Porter model. In the intermediate regime, these competing delocalization mechanisms reduce the fractal dimension and can even induce localization. In quantum many-body systems, locality emerges in energy, real, and Hilbert space. Extending this

mechanism from random matrix models to such systems could enable competition between localization in different spaces, potentially inducing reentrant ergodic transitions in a many-body setting.

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Data availability: The data are available from the authors upon reasonable request.

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