

Reply to “Comment on ‘Scaling and universality at noisy quench dynamical quantum phase transitions’”

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(Received 11 December 2025; accepted 26 August 2026; published 5 October 2026)

The Comment by Sirker [arXiv:2511.16509] raises an important issue concerning dynamical quantum phase transitions (DQPTs) in noisy and mixed-state dynamics, namely that the extension of the Loschmidt echo from pure to mixed states is not unique and different extensions preserve different physical properties. The Comment examines a noise-averaged mixed-state fidelity and shows that, DQPTs cannot occur for any nonzero noise when the return rate is defined through the Uhlmann-Bures fidelity of the noise-averaged density matrix. This conclusion is valid for the mixed-state fidelity observable discussed in the Comment and is consistent with prior studies [Jafari, Langari, Eggert, and Johannesson, *Phys. Rev. B* **109**, L180303 (2024); Jafari, Akbari, Biderang, and Sirker, arXiv:2504.03005]. Our article [Ansari, Jafari, Akbari, and Abdi, *Phys. Rev. B* **112**, 054304 (2025)] investigated a different operationally defined quantity: the logarithm of the Loschmidt echo obtained by first determining the noise-averaged excitation probabilities generated during the noisy ramp and then performing a coherent postramp evolution of a pure state constructed from these noise-averaged transition probabilities. As emphasized explicitly in our original publication, this observable is defined through an operational assumption and is not the same quantity as the mixed-state fidelity. The nonanalyticities reported in [Ansari, Jafari, Akbari, and Abdi, *Phys. Rev. B* **112**, 054304 (2025)], therefore, concern this two-stage operational protocol and should not be identified with zeros of the Uhlmann-Bures fidelity. There is, therefore, no direct contradiction between the theorem established for the Uhlmann-Bures return rate and the conclusions obtained for the different operational protocol studied in [Ansari, Jafari, Akbari, and Abdi, *Phys. Rev. B* **112**, 054304 (2025)].

DOI: 10.1103/j7bd-hl5g

Agreement with the Comment’s formalism. We first emphasize the point on which there is no disagreement. The Comment provides a clear classification of several noise-averaging procedures for Loschmidt-type observables. This taxonomy is fully consistent with earlier analyses, including Refs. [1,2]. The fidelity definition studied in the Comment corresponds to one specific member of this taxonomy, namely the return rate constructed from the Uhlmann-Bures fidelity between noise-averaged mixed states. Within that definition the Comment correctly proves that dynamical quantum phase transitions cannot occur for any finite noise.

The central issue is therefore not the validity of this theorem, but its domain of applicability. Indeed, extending the Loschmidt echo from pure to mixed states is not unique: different constructions preserve different physical properties and therefore probe different aspects of nonequilibrium dynamics. Our article [3] did not employ this mixed-state

fidelity. Instead, it introduced explicitly an operational assumption: the postramp state is represented by a pure state whose populations match the noise-averaged excitation probabilities obtained from the master equation. This operational choice was explicitly stated in the original publication and corresponds to a different Loschmidt-type protocol from the Uhlmann-Bures construction considered in the Comment. Since the observable considered in our work is not equivalent to the mixed-state fidelity used in the Comment, the no-dynamical quantum phase transitions (DQPT) conclusions proven there do not apply to the operational protocol analyzed in our article [3].

Standard approaches to noise averaging. For clarity we summarize the two noise-averaging procedures that are most commonly used in the study of DQPTs and that also appear in the taxonomy of the Comment.

(1) *Averaging over single noise realizations.* The system is initialized in the ground state at t_i , $|g_k(t_i)\rangle$, and the noisy ramp is applied up to $t_f \rightarrow 0^-$. For each individual realization of the noise the Loschmidt echo is

$$L_k^{(S)}(t) = |\langle g_k(t_f) | g_k(t) \rangle|^2. \quad (1)$$

Averaging is then performed over realizations at fixed time, producing $\langle L_k(t) \rangle$. As shown in Ref. [1], DQPTs occur in

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each realization but at different critical times, so the averaging procedure removes the nonanalyticities and leads to no DQPT.

(2) *Averaging over the noisy ramp only.* Here the averaging is performed over the noise that acts during the interval t_i to t_f by solving the exact noise master equation, which yields a mixed state $\rho_k(t_f)$ [2,4,5]. The natural generalization of the Loschmidt echo within a fidelity-based mixed-state formulation is then the Uhlmann-Bures fidelity. As demonstrated in Refs. [1,2], this fidelity-based return rate has no zeros for any finite noise intensity. Hence DQPTs are absent within this definition.

These two procedures correspond to two branches of the classification discussed in the Comment. They are mathematically well-defined and physically useful, but they do not exhaust all possible operational definitions of Loschmidt-type observables. The observable employed in Ref. [3] does not belong to either of these fidelity-based categories, since it is based on a different operational assumption about the post-ramp state.

What our results in Ref. [3] actually studied. In our original article noise acts only during the ramp. Solving the exact noise master equation yields a mixed state $\rho_k(t_f)$ at the end of the ramp.

Our work therefore introduced an explicit operational assumption motivated by coherent postramp dynamics: rather than evaluating the Loschmidt return rate directly from the mixed state, we considered a population-preserving pure state whose excitation probability matches that obtained from the noise-averaged master equation.

The state used at $t = 0$ was

$$|\psi_k(0)\rangle = \sqrt{1-p_k}|g_k^f\rangle + \sqrt{p_k}|e_k^f\rangle, \quad (2)$$

where $|g_k^f\rangle$ and $|e_k^f\rangle$ are the postramp eigenstates and p_k is the noise-averaged excitation probability obtained from the master equation. This construction was not introduced as the optimal pure-state approximation to $\rho_k(t_f)$ in the Uhlmann-Bures fidelity. Rather, it defines an operational protocol in which the noise-averaged populations obtained during the ramp are used as input for a subsequent coherent evolution. The protocol is experimentally motivated because mode-resolved excitation probabilities are directly measurable in engineered two-level systems and quantum simulators [6–9].

Dynamical free energy. Within this operational protocol the return rate is determined by evolving the population-matched pure state under the noiseless postramp Hamiltonian H_f . The resulting dynamical free energy takes the form

$$g(t) = -\frac{1}{2\pi} \int_0^\pi dk \ln [1 - 4p_k(1-p_k) \sin^2(\varepsilon_k^f t)]. \quad (3)$$

Equation (3) is precisely the return rate analyzed in Ref. [3]. Unlike the Uhlmann-Bures return rate, it is constructed from the operational protocol defined above. Related interferometric and two-band simulation schemes provide experimentally relevant ways to access Loschmidt-type amplitudes in coherent few-level settings [7–9]. For a pure two-level state with the same populations, an arbitrary relative phase $e^{i\phi_k}$ between the two energy eigenstates drops out of the Loschmidt echo. Thus, our construction retains the population imbalance p_k generated during the noisy ramp and the coherent postramp

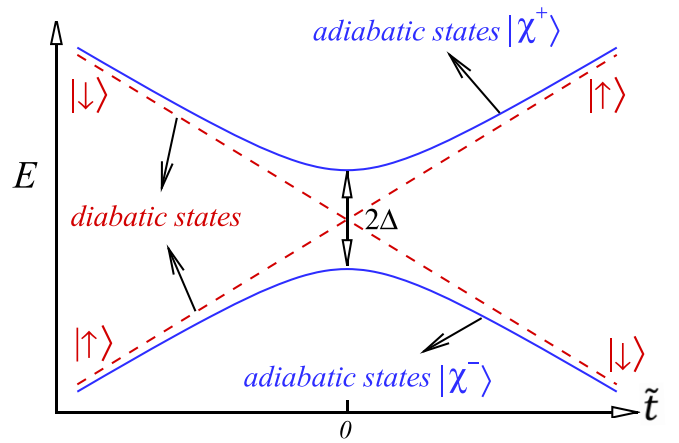


FIG. 1. Adiabatic (solid) and diabatic (dashed) energies for the standard Landau-Zener problem.

interference underlying the condition $p_{k^*} = 1/2$, while it is not intended to preserve the full noise-averaged mixed state. Although the pure state $|\psi_k(0)\rangle$ is not identical to the mixed state $\rho_k(t_f)$, the excitation probability p_k encodes information about both diagonal and off-diagonal components of $\rho_k(t_f)$ once it is transformed into the adiabatic basis. This point is important because the Comment argues that a protocol based on excitation probabilities is blind to coherence information. We show below, through explicit Landau-Zener analysis, that p_k is generally a function of both diagonal and off-diagonal elements of the density matrix in the diabatic basis. This supports the internal consistency of the observable evaluated in our paper and explains why nonanalytic behavior can emerge within this operational framework.

Landau-Zener analysis. The revised Comment argues that replacing the mixed state by a pure state specified through p_k neglects coherence information and that determining the excitation probability in this way is valid only for a pure state. To address these points, we analyze the generic Landau-Zener problem. The purpose of this analysis is not the basis transformation itself, but to show explicitly, in the simplest setting, that p_k is well defined and can be extracted from the noise-averaged density matrix also when the state is mixed, and that it generally depends on both diagonal and off-diagonal elements of the density matrix in the diabatic basis.

As illustrated in Fig. 1, we consider the standard time-dependent two-level Hamiltonian

$$H(t) = \begin{pmatrix} h(t) & \Delta \\ \Delta & -h(t) \end{pmatrix}, \quad (4)$$

where $h(t)$ is a slowly varying control field and Δ is the minimum gap. The instantaneous eigenstates of $H(t)$ form the adiabatic basis. They are given by

$$|\chi^-(t)\rangle = \alpha(t)|\uparrow\rangle + \beta(t)|\downarrow\rangle = \begin{pmatrix} \alpha(t) \\ \beta(t) \end{pmatrix}, \quad (5)$$

$$|\chi^+(t)\rangle = -\beta(t)|\uparrow\rangle + \alpha(t)|\downarrow\rangle = \begin{pmatrix} -\beta(t) \\ \alpha(t) \end{pmatrix}, \quad (6)$$

with corresponding instantaneous energies

$$\varepsilon^\pm(t) = \pm \varepsilon(t), \quad \varepsilon(t) = \sqrt{h^2(t) + \Delta^2}. \quad (7)$$

Here the coefficients $\alpha(t) = (h(t) - \varepsilon(t))/N(t)$, and $\beta(t) = \Delta/N(t)$, encode the rotation between the diabatic and adiabatic bases, and their explicit forms depend only on $h(t)$, Δ , and the normalization convention $N(t) = \sqrt{2\varepsilon(t)(\varepsilon(t) - h(t))}$. This parametrization makes it possible to express $\rho_k(t_f)$ in either basis and to show how the measured excitation probability

$$p_k = \langle \chi^+(t_f) | \rho_k(t_f) | \chi^+(t_f) \rangle \quad (8)$$

combines contributions from both the diagonal and off-diagonal matrix elements of $\rho_k(t_f)$ in the diabatic basis. This quantity is simply the expectation value of the excited-state projector, $p_k = \text{Tr}[\rho_k(t_f) | \chi^+(t_f) \rangle \langle \chi^+(t_f) |]$, and is therefore well defined for an arbitrary density matrix, whether pure or mixed.

The unitary transformation under which the Hamiltonian is diagonal at fixed t' is given as

$$H^{(d)}(t') = U^\dagger(t') H(t') U(t') = \begin{pmatrix} -\varepsilon(t') & 0 \\ 0 & \varepsilon(t') \end{pmatrix}, \quad (9)$$

$$\rho^{(d)}(t') = U^\dagger(t') \rho(t') U(t') = \begin{pmatrix} \alpha(t') & \beta(t') \\ -\beta(t') & \alpha(t') \end{pmatrix} \begin{pmatrix} \rho_{11}(t') & \rho_{12}(t') \\ \rho_{21}(t') & \rho_{22}(t') \end{pmatrix} \begin{pmatrix} \alpha(t') & -\beta(t') \\ \beta(t') & \alpha(t') \end{pmatrix}, \quad (12)$$

which gives

$$\rho^{(d)}(t') = \begin{pmatrix} \alpha^2 \rho_{11} + \alpha \beta (\rho_{12} + \rho_{21}) + \beta^2 \rho_{22} & \alpha^2 \rho_{12} + \alpha \beta (\rho_{22} - \rho_{11}) - \beta^2 \rho_{21} \\ \alpha^2 \rho_{21} + \alpha \beta (\rho_{22} - \rho_{11}) - \beta^2 \rho_{12} & \alpha^2 \rho_{22} - \alpha \beta (\rho_{12} + \rho_{21}) + \beta^2 \rho_{11} \end{pmatrix}, \quad (13)$$

where $\rho_{11}^{(d)}(t')$ and $\rho_{22}^{(d)}(t')$ are the probabilities of occupying the lower and upper adiabatic levels at t' , respectively. Equivalently,

$$\rho^{(d)}(t') = \begin{pmatrix} \langle \chi^-(t') | \rho(t') | \chi^-(t') \rangle & \langle \chi^-(t') | \rho(t') | \chi^+(t') \rangle \\ \langle \chi^+(t') | \rho(t') | \chi^-(t') \rangle & \langle \chi^+(t') | \rho(t') | \chi^+(t') \rangle \end{pmatrix}, \quad (14)$$

which yields

$$\begin{aligned} \rho_{11}^{(d)}(t') &= \langle \chi^-(t') | \rho(t') | \chi^-(t') \rangle \\ &= \alpha^2 \rho_{11} + \alpha \beta (\rho_{12} + \rho_{21}) + \beta^2 \rho_{22}, \\ \rho_{12}^{(d)}(t') &= \langle \chi^-(t') | \rho(t') | \chi^+(t') \rangle \\ &= \alpha^2 \rho_{12} + \alpha \beta (\rho_{22} - \rho_{11}) - \beta^2 \rho_{21}, \\ \rho_{21}^{(d)}(t') &= \langle \chi^+(t') | \rho(t') | \chi^-(t') \rangle \\ &= \alpha^2 \rho_{21} + \alpha \beta (\rho_{22} - \rho_{11}) - \beta^2 \rho_{12}, \\ \rho_{22}^{(d)}(t') &= \langle \chi^+(t') | \rho(t') | \chi^+(t') \rangle \\ &= \alpha^2 \rho_{22} - \alpha \beta (\rho_{12} + \rho_{21}) + \beta^2 \rho_{11}. \end{aligned} \quad (15)$$

Equations (13) and (15) show explicitly that the adiabatic excitation probability

$$\begin{aligned} p_k &= \rho_{22}^{(d)}(t_f) \\ &= \alpha^2 \rho_{22}(t_f) - \alpha \beta [\rho_{12}(t_f) + \rho_{21}(t_f)] + \beta^2 \rho_{11}(t_f) \end{aligned} \quad (16)$$

with

$$U(t') = \begin{pmatrix} \alpha(t') & -\beta(t') \\ \beta(t') & \alpha(t') \end{pmatrix}; \quad U^\dagger(t') = \begin{pmatrix} \alpha(t') & \beta(t') \\ -\beta(t') & \alpha(t') \end{pmatrix}. \quad (10)$$

If the system is initially prepared in the ground state $|\chi^-(t_i)\rangle$ at time t_i , then to obtain the transition probability to the upper level $|\chi^+(t')\rangle$ at time t' , one must solve the von Neumann equation or the noise averaged master equation.

$$\dot{\rho}_k(t) = -i[H_k^{(0)}(t), \rho_k(t)] - \frac{\xi^2}{2} [H_1, [H_1, \rho_k(t)]]. \quad (11)$$

Here $H_k^{(0)}(t)$ denotes the noiseless Hamiltonian, H_1 is the operator coupled to the noise, and ξ denotes the noise strength. Since the Hamiltonian is given in the diabatic basis ($|\uparrow\rangle, |\downarrow\rangle$), the density matrix $\rho(t)$ obtained from Eq. (11) is expressed in that basis. Therefore, the diagonal elements of $\rho(t)$ are diabatic populations, not adiabatic excitation probabilities. To obtain the excitation probability at a fixed time t' in the adiabatic basis, the density matrix should be transformed to the adiabatic basis $|\chi^\pm(t')\rangle$ using the unitary transformation, i.e.,

contains the off-diagonal elements of the density matrix in the diabatic basis, except in special limits where the basis rotation is trivial. These expressions show explicitly that the excitation probability entering our protocol generally depends on both diagonal and off-diagonal elements of the density matrix in the diabatic basis. Consequently, the operational protocol should not, in general, be interpreted as retaining only classical population information. In the presence of noise the time evolution of the density matrix is given by the exact noise master equation which has been used in Refs. [10–12]. For more details about the difference between transition probability in adiabatic and diabatic state please see Eqs. (4) and Eq. (7) in Ref. [13] and also Refs. [6,10,14,15].

Position of our observable within the taxonomy. In our protocol noise acts only during the ramp, while the postramp evolution is fully coherent under the same Hamiltonian H_f for all realizations. The only quantity carried over from the noisy stage is the excitation probability, Eq. (8), which is obtained from the noise-averaged master equation. Inserting this population into Eq. (3) determines the return rate. A non-analyticity arises whenever there exists a momentum k^* with $p_{k^*} = 1/2$, which is the condition highlighted in our paper. This condition is a property of the operational return rate in Eq. (3); it is not claimed to be a zero of the Uhlmann-Bures fidelity.

The operational assumption underlying this construction was explicitly stated in Ref. [3]. Therefore, the no-DQPT

theorem established for the Uhlmann-Bures fidelity does not apply to the observable investigated in our work.

Operational interpretation. Our protocol has a direct experimental interpretation consisting of two sequential steps:

(1) Apply the noisy ramp, solve the master equation, and measure the mode-resolved excitation probabilities p_k at $t = 0$.

(2) Prepare for each k a pure state with noise-averaged populations $(1 - p_k, p_k)$ and let it evolve coherently under the fixed Hamiltonian H_f .

The return rate in our paper is precisely the interferometric signal generated by this second, fully coherent stage.

As noted in the revised Comment, this protocol may be viewed as an interferometric protocol rather than as a measurement of the Uhlmann-Bures fidelity.

Conclusion. The Comment's results [16], which correctly demonstrate the absence of DQPTs when using the

mixed-state fidelity, are in agreement with previous studies [1,2]. Our claim in Ref. [3], however, concerns a different and experimentally motivated observable based on population-preserving pure states, a distinction that was clearly stated in the original publication. Within that operationally realizable protocol the return rate can exhibit nonanalyticities.

Moreover, although not required for the arguments presented here, the physical picture underlying the dynamical phase structure reported in Ref. [3] is further supported by an independent theoretical approach developed in our subsequent work [17], which reaches the same qualitative conclusions using a different formulation and connected to the DQPTs using the reduced fidelity [18,19]. Therefore, while the Comment establishes an important no-go theorem for the Uhlmann-Bures mixed-state return rate, it does not invalidate the conclusions obtained for the operational two-stage protocol studied in Ref. [3].

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