

Quantum contact processes on a topological lattice

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Contact processes play an important role in classical nonequilibrium dynamics, describing the spreading of diseases, the dynamics of earthquakes and forest fires, and the distribution of information through the internet. Here we show that their quantum counterpart, where the spreading occurs through coherent couplings, displays even richer dynamics and offers new means of control. A quantum contact process on a topologically nontrivial one-dimensional lattice can be confined to a protected subspace corresponding to either a single site or a fully excited lattice. Furthermore, excitation spreading can be controlled to occur in quantized steps and on demand when employing topological pumps. We show that the many-body dynamics of excited domains can be mapped to an effective single-particle model, which also determines the topological properties. Throughout this work, we consider a specific type of contact process corresponding to coherent Rydberg facilitation in a tweezer array of trapped atoms in a one-dimensional lattice.

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I. INTRODUCTION

Spreading phenomena such as epidemics or the flow of information are described by classical contact processes. One of their characteristic features is the competition between excitation spreading and dissipation. This results in nonequilibrium phase transitions between an absorbing state of vanishingly small activity and an active phase of permanent excitation spreading, generically belonging to the directed percolation (DP) universality class [1]. Apart from facilitating such a phase transition, there is, however, little control over the spreading process.

In its quantum counterpart, the quantum contact process (QCP) [2], the spreading dynamics can be much richer due to interference phenomena associated with *coherent* transport. As a consequence, the QCP can display dynamic behavior, which does not exist in classical spreading processes, such as Bloch oscillations [3]. Moreover, quantum fluctuations become important and may modify critical phenomena and the universal properties of the absorbing-state phase transition [2,4–6].

In this paper, we show that, in the quantum case, spreading processes can be controlled by topology. They can be confined to topologically protected subspaces or can proceed in quantized steps controlled by topological invariants. For this, we consider a specific model of the QCP in one dimension. Here, each site can either be in an excited state, $|\bullet\rangle$, or in its ground state, $|\circ\rangle$. Specifically, we focus on the QXP model, where excitations can only be created if exactly *one* neighboring site is already excited, i.e., $|\bullet\circ\rangle \leftrightarrow |\bullet\bullet\rangle$, while they are

suppressed if either *none* or *two* neighbors are excited, i.e., $|\circ\circ\rangle \not\leftrightarrow |\circ\bullet\rangle$ and $|\bullet\bullet\rangle \not\leftrightarrow |\bullet\bullet\bullet\rangle$ (cf. Fig. 1).

The kinetic constraint on the many-body dynamics makes the QXP model the quantum counterpart to classical epidemic models, including the concrete implementation of an initial seed (“patient zero”).

It is a variant of the PXP model [7–9], a paradigmatic quantum system with kinetic constraints, where excitation is only possible in the absence of any excited (nearest or next-nearest, etc.) neighbor, as can be realized, e.g., by Rydberg blockade [10–16].

The dynamics of the QXP contact process on a semi-infinite chain is governed by the Hamiltonian

$$H_{\text{qxp}} = \sum_{j=1} \lambda_j (\hat{Q}_{j-1} \hat{X}_j \hat{P}_{j+1} + \hat{P}_{j-1} \hat{X}_j \hat{Q}_{j+1}) + \delta_j \hat{Q}_j, \quad (1)$$

where $\hat{Q}_j = |\bullet\rangle_{jj}\langle\bullet|$, $\hat{P}_j = |\circ\rangle_{jj}\langle\circ|$, $\hat{X}_j = |\circ\rangle_{jj}\langle\bullet| + |\bullet\rangle_{jj}\langle\circ|$, and we allow for a site-dependent coherent excitation rate, λ_j , and an on-site energy, δ_j . It has been shown that adding spontaneous decay at rate γ from the excited to the ground state, and assuming homogeneous excitation rates, also this (quantum) model shows an absorbing-state phase transition at some critical value of λ/γ depending on the spatial dimension [2,4]. Although in dimensions $d = 2$ and higher the transition is believed to be of second order and of DP universality, the character of the transition in $d = 1$ remains not fully understood, but is likely not of the DP universality class [2,4–6]. The critical behavior of quantum contact processes is however not the subject of the present paper. Instead we focus on the coherent control of the spreading process exploiting topology.

We first numerically simulate the coherent contact process starting from an initial seed excitation at site $j = 1$ for the case of alternating excitation rates $\lambda_{j=v,w}$ [see Fig. 1(a)]. A classical spreading process, whose dynamics is fully incoherent and is described by rate equations, as detailed in Sec. 1

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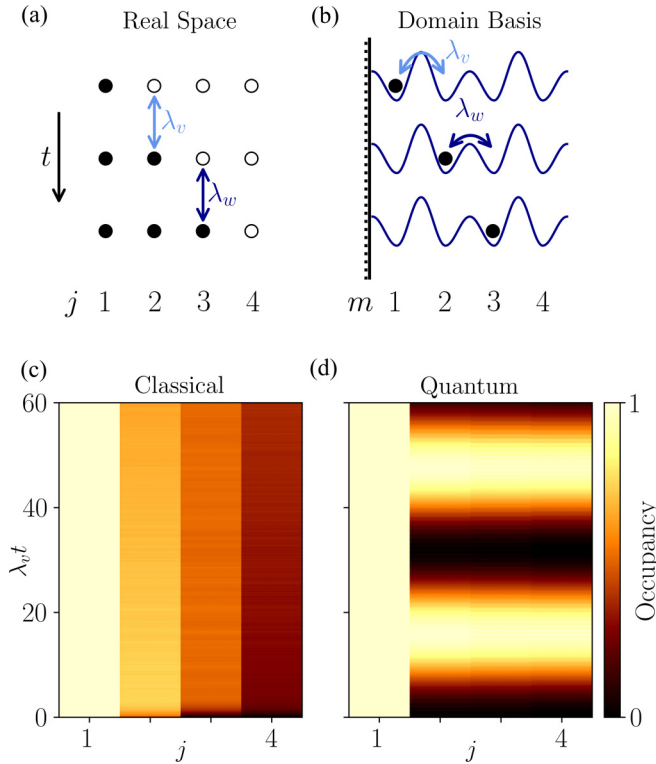


FIG. 1. QXP quantum contact process on a one-dimensional lattice. (a) In real space, $\lambda_v = 1$ and $\lambda_w = 10$ denote alternating coherent excitation rates (Rabi frequencies) on even and odd sites. (b) In the basis of domains of length m domain space, the rates from panel (a) correspond to alternating hopping rates. Time- and space-resolved dynamics for the classical (c) and quantum (d) QXP contact process for four sites with open boundary conditions and with one initial seed excitation. The color code represents the excitation probability.

of the Appendix, would simply lead to a diffusive spreading as shown in Fig. 1(c). The quantum case is dramatically different. For instance, if the excitation rate at odd sites λ_v is much smaller than that at even sites λ_w , and the lattice is finite with an even number of sites, one observes an oscillation of the excitation between only one site—the initial seed site—and the entire lattice. This is evident in Fig. 1(d). As we show, these dynamics occur as the network on which the spreading process takes place corresponds to the Su-Schrieffer-Heeger model [17], which possesses topologically protected edge states. Second, we show that one is able to control the spreading of excitations by means of a topological Thouless pump [18,19], where on-site energies and excitation rates are periodically modulated in time.

II. QUANTUM CONTACT PROCESS IN A ONE-DIMENSIONAL LATTICE USING RYDBERG FACILITATION

A. QXP model and Rydberg facilitation

The QXP model, studied in our work, can be realized through coherent Rydberg facilitation in a finite chain of neutral atoms trapped in optical tweezers with an initially excited edge. In Rydberg facilitation a laser field couples the atoms

between their ground state, $|g\rangle$, and a high-lying Rydberg state, $|r\rangle$, off resonantly. In the Rydberg state, the atoms interact via a strong van der Waals potential, i.e., $V(r) \sim r^{-6}$, which falls off very rapidly with distance. Therefore, it can be ignored beyond nearest-neighbor distances. The resulting many-body Hamiltonian is given by

$$H_{\text{ryd}} = \sum_{j=1}^N (\lambda_j \hat{\sigma}_j^x + \Delta_j \hat{n}_j + V_{\text{NN}} \hat{n}_j \hat{n}_{j+1}), \quad (2)$$

with $\hat{\sigma}_j^x = |g\rangle_{jj}\langle r| + |r\rangle_{jj}\langle g|$, the projector onto the Rydberg state of the j th atom $\hat{n}_j = |r\rangle_{jj}\langle r|$, laser detuning $\Delta_j = \Delta_0 + \delta_j$, Rabi frequency $\lambda_j \propto \lambda_0$, and interaction potential V_{NN} between neighboring Rydberg atoms.

By choosing laser parameters such that $|\Delta_j| \gg \lambda_0$ for all j , the driving becomes far off-resonant and individual excitations are suppressed. Furthermore, the lattice spacing a_0 is chosen to meet the *facilitation constraint*,

$$V(a_0) + \Delta_0 = V_{\text{NN}} + \Delta_0 = 0, \quad (3)$$

with $\delta_j \ll \Delta_0$ remaining as the site-specific detuning. Under this condition, to good approximation, atoms are only resonantly driven in the presence of exactly *one* neighboring Rydberg atom, as required by the QXP model (1).

To simulate the dynamics of Eq. (2), we employ time-evolving block decimation (TEBD) [20,21]. For the presented results we use parameters $\Delta_0/\lambda_0 = -500$, with $V_{\text{NN}} + \Delta_0 = 0$ throughout this work. We note that for resonant driving, i.e., $\Delta_j = 0$, the interaction between Rydberg atoms results in a full suppression of excitations surrounding a single Rydberg atom, i.e., Rydberg blockade. This is the situation required for the realization of the PXP model [11–14], which is hence a special case of the more general model we study here.

B. Domain model

Starting from Hamiltonian (1), we realize that the many-body dynamics are kinetically constrained, such that excitations can only be created in the presence of one excited neighbor (cf. Fig. 1). We consider a finite one-dimensional lattice with the initial (seed) state having a single excitation at the edge, i.e., $|\psi_{m=1}\rangle \equiv |\bullet \circ \circ \dots \circ\rangle$, and an even number of sites N . Due to the QXP property, the many-body dynamics reduces to the dynamics of domains of contiguous excited sites. Domains can grow and shrink at the edge, but they can neither split nor coalesce—if initially multiple domains exist. Remarkably, for the case of a single initial seed excitation, corresponding to a domain size of $m = 1$, the mapping of Eq. (2) onto the dynamics of the domain size allows one to describe the physics in terms of a simple effective single-particle model [22]:

$$H_{\text{dom}} = \sum_{m=2}^N \lambda_m (\hat{c}_m^\dagger \hat{c}_{m-1} + \text{H.c.}) + \sum_{m=1}^N \eta_m \hat{c}_m^\dagger \hat{c}_m. \quad (4)$$

Here, the creation and annihilation operators $\hat{c}_m^{(\dagger)}$ represent raising and lowering operators of the domain size m . Within Eq. (4), they take the role of hard-core bosons, which can tunnel with hopping rates λ_m between sites. These rates relate

to the site-dependent excitation rate as $\lambda_{j=m}$ (cf. Fig. 1), and

$$\eta_m = \sum_{j=1}^m \Delta_j + \sum_{j=2}^m V_{\text{NN}} = \Delta_0 + \sum_{j=1}^m \delta_j. \quad (5)$$

The constant term Δ_0 is only an energy-offset in the effective model and can therefore be neglected. Furthermore, this mapping to the domain space is unique if and only if the first atom is not driven, i.e., $\lambda_1 = 0$. For obvious reasons, the first atom is then also not detuned, i.e., $\delta_1 = 0$. With this additional restriction, the effective potential term becomes

$$\eta_m = \sum_{j=2}^m \delta_j. \quad (6)$$

One can now choose different effective domain models and lead back to the corresponding site-dependent detunings and Rabi frequencies. A simple example for a nontrivial coherent spreading dynamics would be the realization of a Wannier-Stark ladder [23], where the δ_j values are nonzero but site independent. This gives rise to a linear gradient in the effective on-site potentials η_m of the domain model. There, Bloch oscillations can be observed [3]. Further, one could realize time-independent disorder by allowing for positive and negative random values of δ_j . In such a system, Anderson localization [24] could be realized. In this paper we consider the mapping of a quantum contact process to two topological models, namely, the Su-Schrieffer-Heeger model [17] as well as a Thouless pump [18,19].

III. TOPOLOGICAL CONTROL OF EXCITATION SPREADING

A. Su-Schrieffer-Heeger lattice

Let us first consider a lattice with an even number of sites and allowing for alternating flip-rates λ_m , with

$$\lambda_m = \begin{cases} \lambda_v = \lambda_0 = 1, & \text{even } m, \\ \lambda_w, & \text{odd } m, \end{cases} \quad (7)$$

where $\lambda_0 = 1$ is setting the energy scale. The detuning is site independent and chosen as $\Delta_j = \Delta_0 = -V_{\text{NN}}$. Therefore, the potential term in the effective Hamiltonian (4) vanishes. This corresponds to the Su-Schrieffer-Heeger (SSH) model [17], a simple model system featuring topological properties.

The SSH model with open boundary conditions is topologically nontrivial if $\lambda_w > \lambda_v$. For an even number of lattice sites N , it possesses two exponentially localized edge states with the localization length $\xi = (\log |\lambda_w| - \log |\lambda_v|)^{-1}$ for large N .

In Fig. 2, we show the time-resolved dynamics of the excitation spreading of the QXP process in the finite SSH lattice starting from the single excited site $j = 1$, for both the topological trivial case and the nontrivial case. The simulations are performed for the many-body Rydberg system, Eq. (2). Both, the topologically trivial case $\lambda_w < \lambda_v$ (left column) and the nontrivial one (right column) are considered in the strong localization limit $\xi < 1$.

The dynamics exhibit a striking difference for the two cases. In the topological trivial case, where no edge states exist, the excitation simply spreads coherently with rates λ_v and λ_w until it encounters the boundary, followed by a

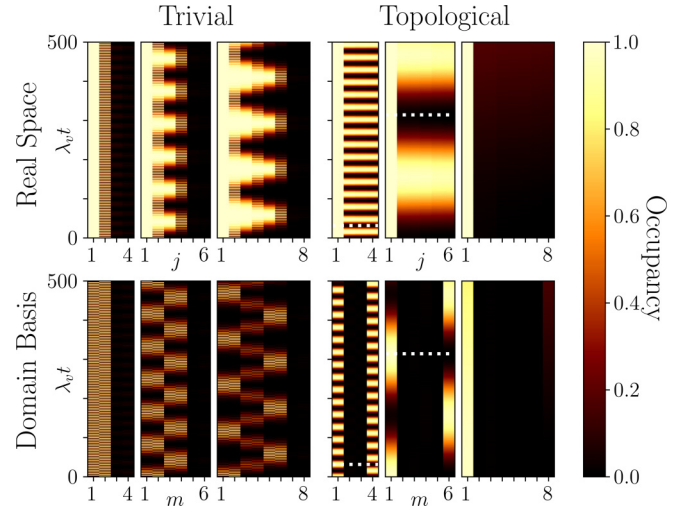


FIG. 2. Time-resolved dynamics. The top row shows the real space and the bottom row shows the domain space for the trivial $\lambda_w = 1/10 < \lambda_v = 1$ (left column) and topological $\lambda_w = 10 > \lambda_v = 1$ (right column) cases. The dotted lines correspond to the analytically expected period $T_{\text{hyb}} \propto (E_+ - E_-)^{-1}$, where $E_{\pm}$ are the corresponding energies of the two perfectly localized edge states.

coherent defacilitation cascade propagating to the left. When the initial site $j = 1$ is reached, the process repeats. This can be understood in the domain picture, where it corresponds to the oscillation of a wave function of a single particle initially prepared in a superposition of bulk states. In the topologically nontrivial case, the dynamics are completely different. There is an oscillation between a state where only site $j = 1$ is excited and a state where all sites are excited. This can be understood as follows: In the nontrivial configuration, midgap edge states exist which are energetically degenerate in an infinite chain. In a finite chain of system size N , they hybridize with a small energy splitting, which decreases exponentially with the system size

$$E_{\pm} = \pm |n_1| e^{-\frac{(N-1)}{\xi}} \lambda_v, \quad (8)$$

where n_1 is the population of the edge site [25]. As the left and right edge states are coherent superpositions of the hybridized states, this leads to edge-state oscillations. The period of this oscillation, set by the energy splitting, rapidly increases with system size N as the two edge states hybridize for finite N . For $\lambda_w \gg \lambda_v$, an initial excitation at $j = 1$ approximately prepares the system in an edge state. The second edge mode corresponds to all sites being excited $|\bullet \dots \bullet\rangle$, hence $m = N$. As a result of the hybridization, the system then oscillates between the edge states with period $T_{\text{hyb}} = \frac{2\pi}{E_+ - E_-}$, shown as dotted white lines in Fig. 2, assuming $n_1 = 1$.

The domain states $|\psi_{m=1}\rangle$ (only one excited site) and $|\psi_{m=N}\rangle$ (the whole lattice is excited) correspond to the edge states only for very different hopping rates λ_w and λ_v . As shown in Fig. 3, many more domain states are involved for smaller ratios, and only if λ_w/λ_v is increased, the edge-state oscillations in the domain basis become much clearer and the oscillation period converges to the expected value for perfect localization.

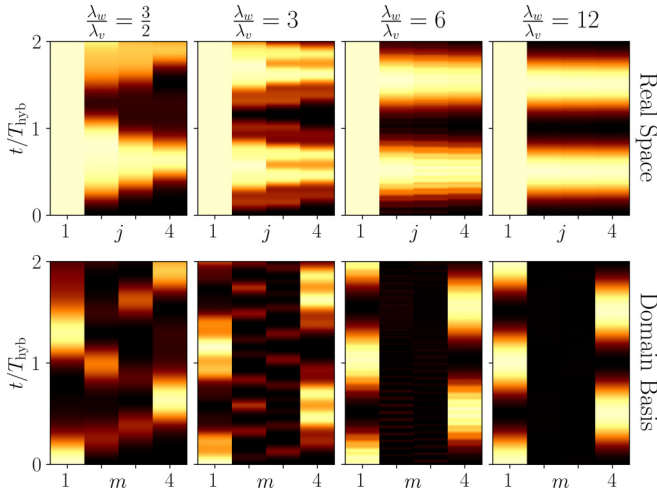


FIG. 3. Hybridization of edge states. The states $\Psi_L = |\bullet \circ \circ \dots \circ\rangle$ and $\Psi_R = |\bullet \bullet \bullet \dots \bullet\rangle$ correspond to topological edge states in the limit $\lambda_w/\lambda_v \rightarrow \infty$. As the ratio λ_w/λ_v increases, oscillations between Ψ_L and Ψ_R become clearer and the frequency approaches the analytical expectation T_{hyp} (see main text). The color code is the same as that in Fig. 2.

B. Coherent control of excitation spreading via Thouless pumping

The mapping onto the SSH model reveals that the system can be manipulated and controlled using techniques borrowed from topological systems. To demonstrate this, we now turn to enacting topological control over the system dynamics via a Thouless pump [18]. This allows us to precisely control the domain size and with that the spread of excitations. To this end, we add a time-dependent on-site potential, $\eta_m(t)$, in the domain basis and allow for time-dependent hopping rates $\lambda_m(t)$,

$$H_{\text{dom}} = \sum_{m=2}^N \lambda_m(t) (\hat{c}_m^\dagger \hat{c}_{m-1} + \text{H.c.}) + \sum_{m=2}^N \eta_m(t) \hat{c}_m^\dagger \hat{c}_m. \quad (9)$$

The values of $\lambda_m(t)$ and $\eta_m(t)$ are chosen to be periodic in m with lattice period q such that H_{dom} represents the topological (time-dependent) Aubry-André-Harper (AAH) model [26,27] with q bands (for $N \rightarrow \infty$). Time-periodic modulation of the parameters leads to a quantized transport of the center of mass of a single-particle wave function, governed by a topological invariant (Thouless pump) [19]. Due to the finite widths of the single-particle bands, the spatial wave function will, however, disperse. This spread, which for the contact process would result in blurring the local excitation probability, can be suppressed by following the high-order tunneling suppression protocol from Ref. [28]. Specifically, we here consider a model with a unit-cell size $q = 3$ and the parameters

$$\lambda_m(t) = \lambda_0 \sin\left(\omega t + \frac{4\pi}{3}(m+1)\right), \quad (10a)$$

$$\eta_m(t) = \eta_0 \cos\left(\omega t + \frac{4\pi}{3}(m+1)\right), \quad (10b)$$

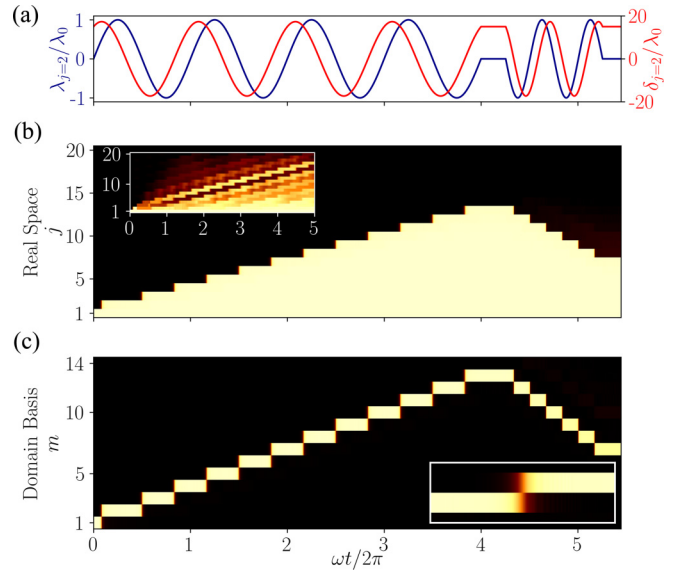


FIG. 4. Quantum control of domain size by manipulating the AAH topological pump. In panel (a), the corresponding parameters $\lambda_2(t)$ and $\delta_2(t)$ are shown. Panels (b) and (c) show the dynamics in the real space and the domain basis, respectively. The inset in panel (b) shows three periods of a time evolution, with small detuning $\Delta_0/\lambda_0 = -22$, where the mapping to the single-particle domain model fails. The inset in panel (c) shows the rapid transition between the domain sizes. To ensure the adiabaticity of the time evolution, the frequency is chosen as $\omega = 0.02\lambda_0$.

with $\lambda_0 = 1$ again setting the energy scale. Most importantly, the dispersion suppression requires a strong on-site potential, which we realize by choosing $\eta_0 = -10\lambda_0$.

As written in the introduction of the effective model, the hopping amplitude λ_m , which couples the sites $m-1$ and m in the effective model, is equivalent to the Rabi frequency $\lambda_{j=m}$ of the m th atom in the full Rydberg model (2). On the other hand, the on-site potentials η_m are the sum of the detunings and interaction potentials of the atoms that are inside the excited domain $\eta_m = \sum_{j=2}^m \delta_j$. The following choice of δ_j fulfills this condition:

$$\delta_j = \eta_{m=j} - \eta_{m=j-1} = -\sqrt{3}\eta_0 \sin\left(\omega t + \frac{4\pi}{3}j + \frac{2\pi}{3}\right). \quad (11)$$

The limits of this mapping are investigated in Sec. 2 of the Appendix.

In Fig. 4 we show the real-space time evolution of the QXP chain, obtained by TEBD simulations of the Rydberg system. The initial seed starting at site index $j = 1$ facilitates its neighbor controlled by the underlying Thouless dynamics. To check whether the facilitated domain is not a superposition of different sizes, the lower plot shows a projection of the many-body state onto the domain space, with the projection operator $\hat{P}_m = |m\rangle\langle m|$, demonstrating perfect matching with Fock states even after several pump cycles. Here, a pumping process is shown, in which the domain is first grown with a constant rate. It is then kept at the same size and subsequently reduced again, before finally being kept at the same size for a short period by modifying the frequency ω through the system

parameters as displayed in Fig. 4(a). We emphasize that the topological control of the spreading dynamics in the QXP model requires that the many-body dynamics, Eq. (2), can be faithfully mapped to the single-particle domain model. If this is not the case [see the inset of Fig. 4(b)], the control of the spreading process is lost.

Our results show that, by employing coherent manipulation and topological properties of the network on which coherent spreading occurs, the domain size of excitations can be completely controlled by the underlying physics of the topological pump. There are only two limiting factors: (i) Because of nonadiabatic effects, when the lattice is switched to become static, a minimum time in the static scenario is necessary before turning on again the pump in either direction. (ii) The speed of the pumping process is limited due to adiabaticity requirements of the underlying Thouless pump. The latter is determined by the band gap of the effective single-particle AAH model, which is on the order of λ_0 , and thus we require for the frequency of the Thouless pump $\omega \ll \lambda_0$.

C. Experimental realization in Rydberg tweezer arrays

The experimental realization of Rydberg facilitation in a one-dimensional chain with site-dependent Rabi-frequencies and detunings according to H_{ryd} in Eq. (2) is challenging as it requires individual addressing of atoms trapped, e.g., in a tweezer array. For the topological control discussed in the previous subsections, such an individual addressing is, however, not necessary. The hopping rates λ_m in the domain model have a spatial periodicity of 2 or 3, which requires one to implement laser driving of the atoms into the Rydberg state with Rabi frequencies that have the same spatial periodicity. The simplest realization is the use of tweezer arrays with two, respectively three, different species of atoms, or atoms prepared in different internal states, and two or three different laser fields applied globally. Such a configuration has already been realized in experiments, see Ref. [29]. Similar arguments can be made for the potentials η_m , required in the realization of the Thouless pump with dispersion suppression, Eq. (10). As can be seen from Eq. (11), the pump merely requires laser detunings with a spatial periodicity of 3. The only additional restriction is that the first atom has to be shielded from the laser field in order to fulfill the condition that the first atom is undriven.

IV. SUMMARY AND CONCLUSION

We have shown that the dynamics of quantum contact processes on regular networks can be much richer than its classical counterpart. In particular, if the network is topologically nontrivial, the spreading can be controlled and this control is robust due to topological protection. We here have demonstrated such a topological control for a particular quantum contact process, the QXP model (which is an idealization of coherent Rydberg facilitation) for a single initial seed excitation on a one-dimensional lattice and the quantum counterpart to classical epidemic models. The constrained many-body dynamics of this model can be mapped to an effective single-particle lattice model where the size of the contiguous domain of excited Rydberg atoms

corresponds to the position of a particle in the lattice. For a finite one-dimensional network with alternating coherent excitation rates, which, in the domain picture, corresponds to the topological Su-Shrieffer-Heeger (SSH) lattice, we have shown that coherent excitation spreading can be confined to a topologically protected subspace. Here oscillations occur between states where either one edge site or all sites are excited, if the SSH lattice is in the nontrivial phase and the excitation rates are sufficiently different. Since edge states are topologically protected by generalized symmetries, this confinement of the spreading dynamics is robust against small perturbations which do not close the energy gap of the SSH model or destroy its symmetries. Extending the SSH model to an Aubry-André-Harper model with time-periodic hopping amplitudes implements a Thouless pump with controlled and quantized growth of the excitation domain in the quantum contact process. Our work demonstrated that, in contrast to classical processes, the dynamics of quantum contact processes can be significantly affected by topological properties of the underlying networks, offering entirely new avenues of control of spreading processes.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX

1. Rate equation approach for the classical contact process

The classical contact process in a one-dimensional lattice, corresponding to the constrained QXP dynamics, is described by rate equations for the excitation probability p_j of the j th site:

$$\frac{d}{dt}p_j = -(\Gamma_f + \gamma)p_j + \Gamma_f(1 - p_j) \quad (\text{A1})$$

where γ is the rate of spontaneous decay and Γ_f is the facilitation rate that is constrained by the excitation probability of the neighboring sites:

$$\Gamma_f = \Gamma_f^0 \cdot [p_{j-1}(1 - p_{j+1}) + (1 - p_{j-1})p_{j+1}]. \quad (\text{A2})$$

For the Rydberg facilitation considered here, it holds $\Gamma_f^0 = 2\Omega^2/\gamma_\perp$, where $\gamma_\perp$ is the dephasing rate of the ground-Rydberg transition.

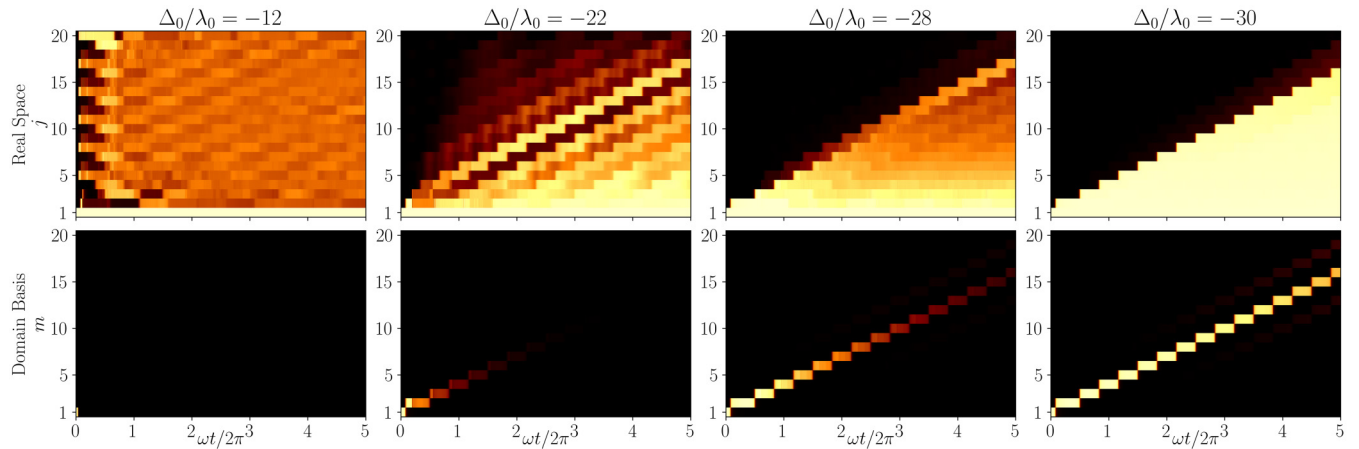


FIG. 5. Accuracy of the single-particle QXP model. For smaller detunings Δ_0/λ_0 , the QXP model of the facilitation breaks down. Top row: Real space Rydberg excitation probability. Bottom row: Population in domain picture. The color code is the same as that in Fig. 2 of the main text.

2. Comparison of QXP and Rydberg dynamics

In the main text, the many-body dynamics of coherent Rydberg facilitation was mapped to an effective single-particle Hamiltonian including the derivation of the parameters. Here, we give some details on the validity limits of this mapping.

With the choice of detunings in Eq. (11), the maximum amplitude of the variation is $\delta_{\max} = \sqrt{3}\eta_0 \approx 17\lambda_0$. To ensure the facilitation regime for all times, the offset Δ_0 has to be large compared to $\delta_{\max}$. To underline this, the results of simulations for different values of Δ_0 are shown in Fig. 5. As can be seen, values below the critical amplitude are dominated by individual excitations. The periodicity of the excitations in

space are simply a consequence of the spatial periodicity of the detunings. As soon as Δ_0 becomes of the order of $\delta_{\max}$, the dynamics changes towards the one expected by the QXP model.

Since individual excitations and the facilitation process do not exclude each other, there is no sudden transition to the facilitation regime, when increasing Δ_0 . Individual excitations have an amplitude $\propto \frac{\lambda_0^2}{\lambda_0^2 + \Delta_j^2}$, which vanishes for large detunings Δ_j .

Throughout this paper, we ensured the facilitation regime by setting $\Delta_0 = 500\lambda_0$, exceeding the critical value $\delta_{\max}$ by more than 1 order of magnitude.

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