

Temporal interface in wire media: Generation of multiple waves

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In this work, we consider an instantaneous transition of a uniaxial dielectric formed by an array of short conducting wires (split-wire medium) into a usual wire medium (WM) of continuous infinitely long wires. Due to strong spatial dispersion in the WM, the known theory by Morgecnthaler of temporal discontinuities in continuous media cannot be applied, and we solve this problem in the time domain using temporal boundary conditions that we specially derive. We find that a wave with transverse magnetic polarization propagating in the split WM (before transition) transforms into four waves: a pair of transverse magnetic waves and a pair of transverse electromagnetic waves. The power flow splits so that one of the new propagation directions is along the wires. The analytical model is validated by full-wave simulations.

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I. INTRODUCTION

Time-varying materials have recently gained significant interest due to their potential for advanced wave control. Modulating properties of metamaterials over time has opened up new possibilities for achieving magnetless nonreciprocity, wave amplification, frequency conversion, pulse shaping, etc. One key type of time modulation involves temporal discontinuities, when the electromagnetic properties of a medium change abruptly at specific moments, practically within one cycle of the wave traveling through the medium. Such transitions can be treated as instantaneous temporal interfaces. For lossless, dispersion-free media they were first studied in 1958 by F. Morgecnthaler [1]. His theory showed that a monochromatic plane wave at a temporal interface transforms into two waves at a new frequency: one called the time-refracted wave that propagates in the same direction as the original wave, and another one called the time-reflected wave that propagates in the opposite direction. That theory is based on the fact that in space-uniform media the wave number of these new waves remains the same across the temporal interface.

Unfortunately, Morgecnthaler's theory does not address the cases when the medium before or after the temporal interface has temporal or spatial dispersion. This limitation is particularly relevant for materials like wire media (WM), which exhibit resonant spatial dispersion at all frequencies [2]. In Ref. [3], the transition of a WM into a uniaxial dielectric formed by thin and short metal rods was analytically considered. It was found that a wave with transverse electric and magnetic (TEM) polarization converts into a pair of waves

with transverse magnetic (TM) polarization at a higher frequency. In this paper, we develop an analytical model of the inverse transition: a uniaxial dielectric of metal rods is transformed into a WM. That is, a medium with negligible spatial dispersion is converted into a medium with extremely strong spatial dispersion. We present an exact solution by solving an initial value problem in the time domain. The theoretical results are validated by numerical simulations.

II. USUAL WIRE MEDIUM AND SPLIT-WIRE MEDIUM

A usual WM, whose properties and prospective applications were reviewed in Refs. [4,5], is a square array of parallel thin conducting wires, whose period a is much smaller than the free-space wavelength ($a \ll \lambda_0$). The period a of the wire array is much larger than the wire radius r_0 . A wire-medium sample is schematically depicted in Fig. 1. In 1960s Brown [6] and Rotman [7] showed that for a plane wave whose wave vector $\mathbf{k}$ is orthogonal to the wires and the electric field has a component parallel to them, this lattice acts as a plasma, whose effective relative permittivity in the lossless approximation is

$$\varepsilon = 1 - \left(\frac{k_p}{k_0} \right)^2, \quad (1)$$

where $k_0 = \omega_0/c = \omega_0 \sqrt{\varepsilon_0 \mu_0}$ is the free-space wave number and $k_p = \omega_p/c = \omega_p \sqrt{\varepsilon_0 \mu_0}$ is called the WM plasma wave number, which depends on the geometrical parameters of the array of wires in accordance with the formula [8,9]

$$k_p^2 = \frac{2\pi}{a^2} \frac{1}{\log \frac{a}{2\pi r_0} + 0.5275}, \quad (2)$$

where r_0 is the radius of the wire cross section.

Two decades ago in Ref. [2], it was shown that for the general case of propagation, when the wave vector $\mathbf{k}$ may be not orthogonal to the medium axis y , the permittivity tensor of the WM strongly depends on the axial (y) component of the wave vector $\mathbf{k}$, which means that there is resonant spatial

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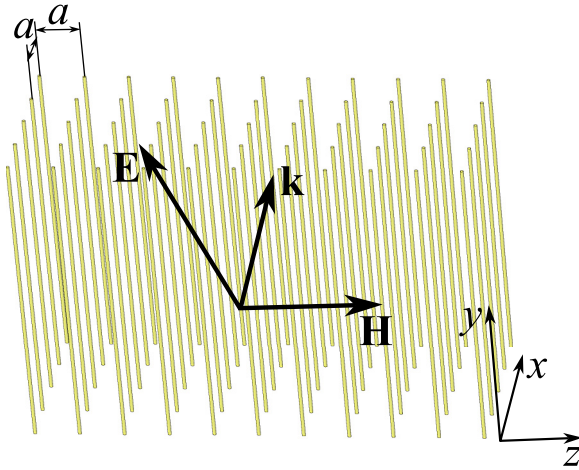


FIG. 1. A schematic view of a wire medium.

dispersion:

$$\tilde{\varepsilon} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \varepsilon_{yy} = 1 - \frac{k_p^2}{k_0^2 - k_y^2}. \quad (3)$$

This model assumes that the wires are infinitely long and the period a at the operation frequency does not exceed $(0.25-0.3)\lambda$ (see, e.g., in Refs. [10–15]). However, in fact (see, e.g., in Ref. [10]), arrays of finite wires of equivalent length can be considered as slabs of WM whose properties inside the slab correspond to the WM of infinitely long wires and are described by formula (3). In this case special boundary conditions are introduced at the ends of the wires (slab interfaces). Moreover, arrays of wires that are finite in the plane xz can be also considered as samples of WM [11,12].

Now let us consider another artificial medium that we call a split-wire medium (SWM). It is an orthorhombic lattice of thin metal rods of radius r_0 whose length l , period a in the xz plane, and period $d = l + g$ along the axis y are all small compared to λ_0 , and g ($r_0 \ll g \ll d, a$) is the gap between the ends of adjacent rods. The lattice is schematically depicted in Fig. 2. As it is explained in Ref. [3], this medium is a uniaxial

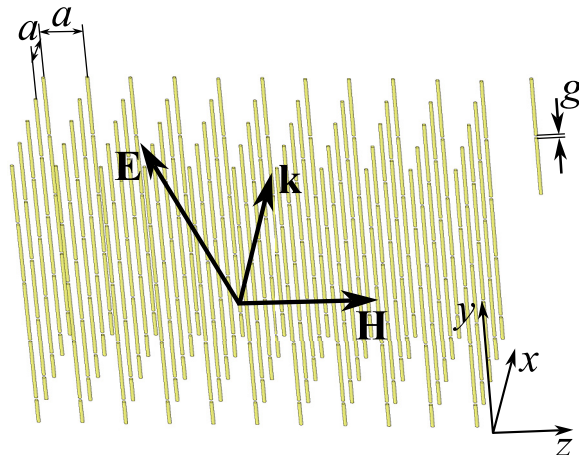


FIG. 2. A schematic view of a split wire medium. On the inset a piece of split wire is shown.

artificial dielectric: both temporal and spatial dispersion are negligibly small in it. The permittivity tensor takes the simple form (see, e.g., in Ref. [16])

$$\tilde{\varepsilon} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \varepsilon_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (4)$$

where ε_1 , in accordance with Refs. [9,17], is equal to

$$\varepsilon_1 \approx 1 + \frac{\alpha}{V} \frac{1}{1 - aC}, \quad \alpha = \frac{4l^3}{3} \log \frac{2l}{r_0},$$

$$C = \frac{1.2\pi}{d^3} - \frac{8\pi}{d^3} K_0 \left(\frac{2\pi a}{d} \right). \quad (5)$$

Here, $V = da^2$ is the unit cell volume, α is the normalized rod polarizability, K_0 is the zero-order McDonald function, and C is called the interaction constant of the lattice. Here, we neglect the small polarizability of the wires in the xz plane.

Let a plane wave at the frequency $\omega_1 < \omega_p$ with the electric field $\mathbf{E} = (E_x, E_y, 0)$, magnetic field $\mathbf{H} = (0, 0, H_z)$, and wave vector $\mathbf{k} = (k_x, k_y, 0)$ propagate in the space filled with a SWM. Using Maxwell's equations and formula (4), the field components can be easily expressed in terms of the amplitude H_0 and the axial permittivity ε_1 . Also, the wave frequency can be easily related with the wave number. The result reads

$$H_z = H_0 \cos(\mathbf{k} \cdot \mathbf{r} - \omega_1 t), \quad \omega_1 = c \sqrt{\frac{k_x^2}{\varepsilon_1} + k_y^2}, \quad (6)$$

$$E_x = -H_0 \frac{k_y c}{\omega_1} \sqrt{\frac{\mu_0}{\varepsilon_0}} \cos(\mathbf{k} \cdot \mathbf{r} - \omega_1 t),$$

$$E_y = H_0 \frac{k_x c}{\omega_1 \varepsilon_1} \sqrt{\frac{\mu_0}{\varepsilon_0}} \cos(\mathbf{k} \cdot \mathbf{r} - \omega_1 t). \quad (7)$$

In the next section, we will consider a temporal interface when the whole space filled with such SWM instantaneously turns into a conventional WM. The known analytical methods are not suitable for the solution of this time-boundary problem. None of the known approaches can be applied because none of them takes time-varying spatial dispersion into account. In this situation, we must solve the transition problem in the time domain as the initial value problem, and the key point of our model is derivation of the needed additional temporal boundary conditions.

III. TRANSITION BETWEEN SPLIT-WIRE MEDIUM AND WIRE MEDIUM

Because the temporal interface conditions depend on a particular mechanism of switching the material parameters [18], we first need to define the switching method. The needed transition may be implemented by using a gas discharge, more specifically, a nanoplasma discharge [19]. A nanoplasma discharge is a discharge in a very small gap between electrodes of a size smaller than a few micrometers. For nanoplasma, the effect of the field emission electron dominates over the Townsend discharge mechanism. This leads to a very fast evolution of plasma. In this transition, the total inductance per unit length of the split WM increases because the inductance of newly created conductive segments (plasma discharges)

is added to the effective inductance of the split WM—that granted by the initial wire segments. The temporal interface with an increasing effective medium inductance was analyzed in recent work [18] (for natural media with Drude frequency dispersion). If this transition is passive (that is, there is no effective delta-function-like source acting at the temporal interface), it implies a continuous volumetric current density $\mathbf{J}$. Its continuity in time means that the vector $\mathbf{B}$ (and, consequently, H_z) is continuous. In the following, it will be shown that the continuity of $\mathbf{J}$ also implies the continuity of the axial electric field (and even its first spatial derivative). It is easy to see that the continuity of the mean current implies the continuity of the axial polarization $\mathbf{P}$ of the medium. Together with the continuity of E_y , it means that the vector $\mathbf{D}$ is also continuous. Thus, for this temporal interface Morgenthaler's boundary conditions (temporal continuity of $\mathbf{D}$, $\mathbf{B}$, k_x and wave number k_y) [1] are satisfied. However, these conditions are insufficient because spatial dispersion in the wire medium results in excitation of more than two waves. We should extend the set of time boundary conditions.

A. Governing equations

In both considered media, a uniaxial dielectric called SWM and the wire medium, electric currents flow only along one axis y . The transversal polarization is negligibly small because the wires are very thin. The mean (averaged over the period $a \times a$) bulk current density denoted as $\mathbf{J} = (0, J, 0)$ is the time derivative of the averaged bulk polarization density $\mathbf{P} = (0, P, 0)$. Maxwell's equations for the macroscopic electromagnetic field and polarization read

$$\begin{aligned} \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} &= -\mu_0 \frac{\partial H_z}{\partial t} \\ \frac{\partial H_z}{\partial x} &= -\varepsilon_0 \frac{\partial E_y}{\partial t} - \frac{\partial P}{\partial t} \\ \frac{\partial H_z}{\partial y} &= \varepsilon_0 \frac{\partial E_x}{\partial t}. \end{aligned} \quad (8)$$

First, we apply the operator $\partial/\partial t$ to the first equation and substitute it into the third equation of the system (8) (see Supplemental Material [20] for details). Next, we apply $\partial/\partial x$ to the equation obtained at the previous step and substitute the second equation from the system into the result. Since the field does not contain constant terms, we may remove one time derivative and obtain

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} - \frac{1}{c^2} \frac{\partial^2 E_y}{\partial t^2} = \mu_0 \frac{\partial^2 P}{\partial t^2} - \frac{1}{\varepsilon_0} \frac{\partial^2 P}{\partial y^2}. \quad (9)$$

This is our first governing equation that is valid for both WM and SWM. In addition to macroscopic Maxwell equations, it only uses the fact that the polarization of the medium is electric, and its vector $\mathbf{P}$ is directed along y .

Now, let us look at the formula for the axial component of the displacement vector $\mathbf{D}$. In the limit case $t \rightarrow \infty$ there are no transition processes, and we may write the usual equations for complex amplitudes of time-harmonic quantities, denoting them by the symbol $\hat{}$:

$$\hat{D}_y = \varepsilon_0 \hat{E}_y + \hat{P} \equiv \varepsilon_0 \varepsilon_{yy} \hat{E}_y. \quad (10)$$

This gives $\hat{P} = (\varepsilon_{yy} - 1) \hat{E}_y$. Since this relation is written in the Fourier domain, we may use the formula (3). It evidently results in the following equation:

$$(k_0^2 - k_y^2) \hat{P} = -\varepsilon_0 k_p^2 \hat{E}_y. \quad (11)$$

Returning to the space-time domain, we should consider k_0^2 as the Fourier counterpart of the differential operator $-\partial^2/c^2 \partial t^2$, and k_y^2 as that of the operator $-\partial^2/\partial y^2$. Then the equations for E_y and P form the following system:

$$\begin{aligned} \frac{\partial^2 P}{\partial y^2} - \frac{1}{c^2} \frac{\partial^2 P}{\partial t^2} &= -\varepsilon_0 k_p^2 E_y, \\ \Delta E_y - \frac{1}{c^2} \frac{\partial^2 E_y}{\partial t^2} &= k_p^2 E_y, \end{aligned} \quad (12)$$

where the symbol $\Delta \equiv \partial^2/\partial x^2 + \partial^2/\partial y^2$. This system of equations describes the polarization properties of WM after the temporal discontinuity $t = 0$. Since after the jump these properties do not change, the equations (12) are valid starting from the moment $t = 0$.

B. Initial-value problem and solution

We assume that the transition time is much smaller than $2\pi/\omega_1$ and consider it as a temporal interface that occurs at the moment $t = 0$. To find the electromagnetic field after the transition of the medium at $t = +0$ we pose conditions that formulate an initial-value problem for the system of governing equations (12):

$$\begin{aligned} P|_{t=0} &= \varepsilon_0(\varepsilon_1 - 1)E_0 \cos(\mathbf{k} \cdot \mathbf{r}), \\ \frac{\partial P}{\partial t} \Big|_{t=0} &= \varepsilon_0(\varepsilon_1 - 1)\omega_1 E_0 \sin(\mathbf{k} \cdot \mathbf{r}), \\ E_y|_{t=0} &= E_0 \cos(\mathbf{k} \cdot \mathbf{r}), \\ \frac{\partial E_y}{\partial t} \Big|_{t=0} &= \omega_1 E_0 \sin(\mathbf{k} \cdot \mathbf{r}). \end{aligned} \quad (13)$$

The key assumption here is the temporal continuity of the macroscopic current density $J = \partial P/\partial t$. It is the only condition that we postulate. The continuity of other electromagnetic values follows from it. The temporal continuity of P follows from the fact that the time derivative $\partial P/\partial t$ is continuous at $t = 0$. Strictly speaking, the continuity of the derivative does not guarantee the continuity of the function. In physics, however, jumps of functions whose derivatives are continuous usually do not arise. Also, in our particular case of plasma discharge, plasma is created via field emission from the electrodes and does not instantaneously fill the whole gap volume. This implies that E_y is also continuous in time if $\partial E_y/\partial t$ is continuous.

Let us show that the continuity of $\partial E_y/\partial t$ follows from the continuity of the current density. Consider the second equation in (8). In this equation, the left-hand side is preserved at $t = 0$ because the whole spatial distribution of the magnetic field is preserved. The second term on the right-hand side is J , which is preserved by assumption. Therefore, the axial electric field derivative is also preserved at $t = 0$. So, the initial value problem (13) makes sense if we assume the continuity of the macroscopic current density across the temporal interface.

Note that two of our four conditions (13)—continuity of the mean medium polarization and its time derivative—are equivalent to Morgenthaler's conditions (continuity of the displacement vector and magnetic field vector). To find four amplitudes (those of the time-refracted and time-reflected waves at two new frequencies), we added the conditions of continuity of the electric field and its time derivative. Four temporal continuity conditions (13) are both necessary and sufficient for our purely temporal interface scenario.

The electromagnetic field that satisfies the governing equations (12) with the initial conditions is found in the Supplemental Material [20]. It has the following components:

$$E_x(\mathbf{r}, t) = \frac{-k_y}{k_x \varepsilon_0} \left\{ \left[\frac{\varepsilon_0 E_0}{2} \left(1 + \frac{\omega_1}{\omega_2} \right) + A^+ \right] \cos(\mathbf{k} \cdot \mathbf{r} - \omega_2 t) + \left[\frac{\varepsilon_0 E_0}{2} \left(1 - \frac{\omega_1}{\omega_2} \right) + A^- \right] \cos(\mathbf{k} \cdot \mathbf{r} + \omega_2 t) + B^+ \cos(\mathbf{k} \cdot \mathbf{r} - \Omega_2 t) + B^- \cos(\mathbf{k} \cdot \mathbf{r} + \Omega_2 t) \right\}, \quad (14)$$

$$E_y(\mathbf{r}, t) = \frac{E_0}{2} \left(1 + \frac{\omega_1}{\omega_2} \right) \cos(\mathbf{k} \cdot \mathbf{r} - \omega_2 t) + \frac{E_0}{2} \left(1 - \frac{\omega_1}{\omega_2} \right) \cos(\mathbf{k} \cdot \mathbf{r} + \omega_2 t), \quad (15)$$

$$H_z(\mathbf{r}, t) = \frac{\omega_2}{k_x} \left\{ \left[\frac{\varepsilon_0 E_0}{2} \left(1 + \frac{\omega_1}{\omega_2} \right) + A^+ \right] \cos(\mathbf{k} \cdot \mathbf{r} - \omega_2 t) - \left[\frac{\varepsilon_0 E_0}{2} \left(1 - \frac{\omega_1}{\omega_2} \right) + A^- \right] \cos(\mathbf{k} \cdot \mathbf{r} + \omega_2 t) \right\} + \frac{\Omega_2}{k_x} [B^+ \cos(\mathbf{k} \cdot \mathbf{r} - \Omega_2 t) - B^- \cos(\mathbf{k} \cdot \mathbf{r} + \Omega_2 t)], \quad (16)$$

$$P(\mathbf{r}, t) = A^+ \cos(\mathbf{k} \cdot \mathbf{r} - \omega_2 t) + A^- \cos(\mathbf{k} \cdot \mathbf{r} + \omega_2 t) + B^+ \cos(\mathbf{k} \cdot \mathbf{r} - \Omega_2 t) + B^- \cos(\mathbf{k} \cdot \mathbf{r} + \Omega_2 t), \quad (17)$$

where

$$A^\pm = \frac{E_0}{2} \frac{\varepsilon_0 k_p^2}{k_x^2 + k_p^2} \left(1 \pm \frac{\omega_1}{\omega_2} \right), \quad \omega_2 = c \sqrt{k_x^2 + k_y^2 + k_p^2}, \quad \Omega_2 = k_y c, \quad B^\pm = -\frac{E_0}{4} \frac{\varepsilon_0 k_p^2}{k_x^2 + k_p^2} \left[\left(1 + \frac{\omega_1}{\omega_2} \right) \left(1 \pm \frac{\omega_2}{\Omega_2} \right) + \left(1 - \frac{\omega_1}{\omega_2} \right) \left(1 \mp \frac{\omega_2}{\Omega_2} \right) \right] + \frac{\varepsilon_0 (\varepsilon_1 - 1) E_0}{2} \left(1 \pm \frac{\omega_1}{\Omega_2} \right). \quad (18)$$

Since the initial frequency ω_1 is related to the wave-vector components as $\omega_1 = c \sqrt{k_x^2/\varepsilon_1 + k_y^2}$ (see above) we can easily express the new frequencies ω_2 and Ω_2 through ω_1 . These expressions are discussed below.

C. Analysis of the formulas

At first, let us check that formulas (15) and (17) with substitutions of (18) satisfy the initial conditions (13). To see it, one should take the time derivative of (15) and (17) and set $t = 0$ in the expressions for P , E_y , $\partial P/\partial t$, and $\partial E_y/\partial t$. Next, if we set $t = 0$ in (16), we obtain $H_z = \varepsilon_0 \varepsilon_1 \omega_1 E_0 \cos(\mathbf{k} \cdot \mathbf{r})/k_x$. It is the magnetic field of the initial wave at the moment $t = 0$. Our check is over. Second, let us substitute $t = 0$ in (14). It gives $E_x(\mathbf{r}, 0) = -k_y \varepsilon_1 E_0 \cos(\mathbf{k} \cdot \mathbf{r})/k_x$. This is nothing but the transverse electric field of the initial TM wave at the moment $t = 0$. It is not surprising that this field component is also preserved because the transverse susceptibility of the medium does not change (see above). We see that all electromagnetic field vectors ($\mathbf{E}$, $\mathbf{H}$, $\mathbf{B}$, $\mathbf{D}$) as well as the wave vector $\mathbf{k}$ and even the time derivative of the axial electric field E_y are preserved at the transition.

The solution shows that after the temporal interface, a frequency conversion occurs so that the initial frequency ω_1 is converted into two new frequencies ω_2 and Ω_2 . These two frequencies correspond to two different types of waves. One new frequency is above the plasma frequency and corresponds to two TM waves (time-refracted and time-reflected) whose energy propagates obliquely to the axis y . We call the dominant TM wave *time-refracted* because after the temporal interface, the same tilt ψ of the propagation axis corresponds to two TM-polarized plane waves with the new frequency ω_2 . This new frequency can be expressed via ω_1 and ψ (the angle between the wave vector $\mathbf{k}$ and the axis x) as

$$\omega_2 = \sqrt{\omega_1^2 \varepsilon_1 \frac{1 + \tan^2 \psi}{1 + \varepsilon_1 \tan^2 \psi} + c^2 k_p^2}. \quad (19)$$

Since $\varepsilon_1 > 0$, this frequency is above the plasma frequency ω_p of the WM after the temporal interface. Waves at such frequencies can propagate in a WM in any direction. Both waves with the frequency ω_2 carry energy along the same line as the initial wave and have the same TM polarization. One of these two waves is the time-refracted wave, and its propagation direction coincides with the direction of $\mathbf{k}$ of the initial wave. The second of these two waves is the time-reflected wave, and its propagation direction is opposite to that of the initial wave. For both these waves, the phase and group velocities can be expressed via the wave-vector components in the following way:

$$v_{ph}^{(TM)} = \frac{\omega_2}{k} = c \frac{\sqrt{k_x^2 + k_y^2 + k_p^2}}{\sqrt{k_x^2 + k_y^2}}, \quad (20)$$

$$\mathbf{v}_g^{(TM)} = \pm \nabla_k \omega_2 = \pm \frac{c}{\sqrt{k_x^2 + k_y^2 + k_p^2}} (k_x, k_y, 0), \quad (21)$$

where the plus sign corresponds to the time-refracted wave and the minus sign to the time-reflected wave. This pair of TM waves has the electric field vectors $(E_x^{(1\pm)}, E_y^{(1\pm)}, 0)$ and the magnetic field vector with components $(0, 0, H_z^{(1\pm)})$. These waves correspond to terms with the space-time dependence $\cos(\mathbf{k} \cdot \mathbf{r} \mp \omega_2 t)$ in the formulas above.

The second new frequency, Ω_2 , is below the plasma frequency of the WM because ω_1 is assumed to be below it:

$$\Omega_2 = \omega_1 \sqrt{\varepsilon_1 \frac{\tan^2 \psi}{1 + \varepsilon_1 \tan^2 \psi}} < \omega_1. \quad (22)$$

In the case where $\omega_1 > \omega_p$, our lattice of thin metal rods is not a uniaxial dielectric with dispersion-free permittivity ε_1 , but a photonic crystal, a medium with strong spatial dispersion. In this case, our model is not valid. We assume that $\omega_1 < \omega_p$, and the new frequency Ω_2 corresponds to a pair of nonuniform TEM waves. For this type of waves, the time-reflected wave is also present; it vanishes only if the initial wave is propagated along y . The time-refracted and time-reflected TEM waves have the electric field with the components $(E_x^{(2\pm)}, 0, 0)$ and the magnetic field $(0, 0, H_z^{(2\pm)})$. These waves correspond to terms with the space-time dependence $\cos(\mathbf{k} \cdot \mathbf{r} \mp \Omega_2 t)$ in the formulas above. The direction of the phase velocity of the TEM waves coincides with the direction of the vector $\mathbf{k}$, and its value is equal to

$$v_{ph}^{(TEM)} = \frac{\Omega_2}{k} = c \frac{k_y}{\sqrt{k_x^2 + k_y^2}}. \quad (23)$$

The expression for the group velocity reads as

$$\mathbf{v}_g^{(TEM)} = \pm \nabla_k \Omega_2 = (0, \pm c, 0). \quad (24)$$

So, the energy of both TEM waves propagates along the y axis with the speed c . The velocity of this energy transfer does not depend on the direction of the initial wave vector $\mathbf{k}$. These waves result from strong far-field interactions in the array of continuous wires, i.e., from the spatial dispersion of the WM.

The total electromagnetic field at any point in the WM has components $E_y^{(1+)} + E_y^{(1-)} = E_y$, $E_x^{(1+)} + E_x^{(1-)} + E_x^{(2+)} + E_x^{(2-)} = E_x$, $H_z^{(1+)} + H_z^{(1-)} + H_z^{(2+)} + H_z^{(2-)} = H_z$, as follows from the formulas deduced above.

As a check of our results, one can see that the formulas obtained for the amplitudes and frequencies satisfy the initial conditions (13). They also satisfy the continuity of the magnetic field vector because the result for H_z is equivalent to formula (7). As another check, the final formulas give reasonable results for two basic incidence cases when $k_y = 0$ and $k_x = 0$. In the first case, the amplitudes of the time-reflected and time-refracted TEM waves are equivalent, as must be due to the problem symmetry. In the second case, the medium does not feel the temporal discontinuity. The field remains a uniform TEM wave because in this case $\Omega_2 = \omega_1$. Indeed, it means that $B^- = 0$, and the amplitudes of both time-refracted and time-reflected TM waves vanish.

IV. NUMERICAL SIMULATIONS

To confirm the predicted behavior of the electromagnetic field under the temporal discontinuity, we have carried out numerical simulations using commercial simulation software comsol multiphysics 6.2. For simplicity, we consider a particular case of uniaxial media, when the initial medium is free space. The wire medium arising after the transition has the

wires parallel to the y axis, and the field does not depend on the z coordinate (as in the theory above). The system of equations (12) was solved in a rectangular two-dimensional (2D) region (box) with effective Dirichlet boundaries. To avoid the impact of these boundaries on the wave transformation, we needed a localized solution of the wave equation with constant coefficients. A suitable localized solution $u(\tilde{\mathbf{r}}, t)$ of the homogeneous wave equation $\partial^2 u / \partial t^2 - c^2 \Delta u = 0$, $\tilde{\mathbf{r}} = (\tilde{x}, \tilde{y})$ in two dimensions was found in Refs. [21,22] in the form

$$u(\tilde{\mathbf{r}}, t) = \text{Re} \left[\frac{A \exp(-ps)}{\sqrt{\tilde{x} + ct - j\eta}} \right], \quad s = \sqrt{1 - \frac{j\theta}{\gamma}},$$

$$\theta = \tilde{x} - ct + \frac{\tilde{y}^2}{\tilde{x} + ct - j\eta}, \quad \tilde{\mathbf{r}} = (\tilde{x}, \tilde{y}), \quad (25)$$

where η , p , and γ are real positive parameters that determine the pulse frequency and spatial spectra, and A is the amplitude whose arbitrary dimensionality allows us to treat $u(\tilde{\mathbf{r}}, t)$ either as E_y or P . The solution (25) is real and has the form of the Gaussian envelope, filled with oscillations and propagating along the $\tilde{x}$ axis with the speed c being practically undistorted.

Letting this wave package and its time derivative at $t = 0$ be an initial data set for E_y and $\partial E_y / \partial t$, which can be found explicitly from Eq. (25), we avoid the parasitic impact of the lateral boundaries of the simulation box. For the polarization, we assume $P(t = 0) = 0$. This simplification practically does not change the numerical results compared to the case of the split wire medium having the same geometric parameters as in our previous paper [3] (in this case $\varepsilon_1 \approx 1.03$ —close to that of free space), and drastically shortens the simulation time.

Solving Eqs. (12) with $E_y(t = 0)$ given by (25), we simulate the evolution of the initial TM wave after the temporal interface. The results of this simulation must numerically differ from our analytical model, in which the fields are not spatially localized and the initial wave is monochromatic. However, since the frequency spectrum of our pulse decays away from the central frequency in a Gaussian manner, and this decay is adjustable via parameters η , p , and γ , as well as the localization of the wave package, we may reasonably shrink the frequency and special spectra of the initial wave package by choosing these parameters properly. It allows us to compare the simulation results with the theory of temporal scattering of plane waves.

Some results of the simulations are presented in Fig. 3. The parameters of the initial wave package are chosen as follows: $\eta = 2$, $\gamma = 0.2$, $p = 20$. This set of parameters defines the period of oscillations $\lambda_1 = 0.125$ m, which corresponds to the central frequency $\omega_1 = 1.507 \times 10^{10}$ rad/s and the wave number $k_1 = 2\pi / \lambda_1$. The propagation axis of the wave package $\tilde{x}$ is tilted by 30° to the axis x . The plasma wave number is set to $k_p = 5k_1$. In Fig. 3(a) the initial distribution for E_y is shown. The electric field $E_y(\mathbf{r}, t)|_{t=0}$ is set equal to $u(\mathbf{r}, 0)$ from (25), and the time derivative $\partial E_y / \partial t|_{t=0}$ is set equal to $\partial u / \partial t|_{t=0}$. In Figs. 3(b) and 3(c), the simulated axial electric field and polarization for the time $t = 6.5$ ns after the temporal interface are presented. In the plot of E_y , the distribution for two TM waves—time-reflected wave and time-refracted

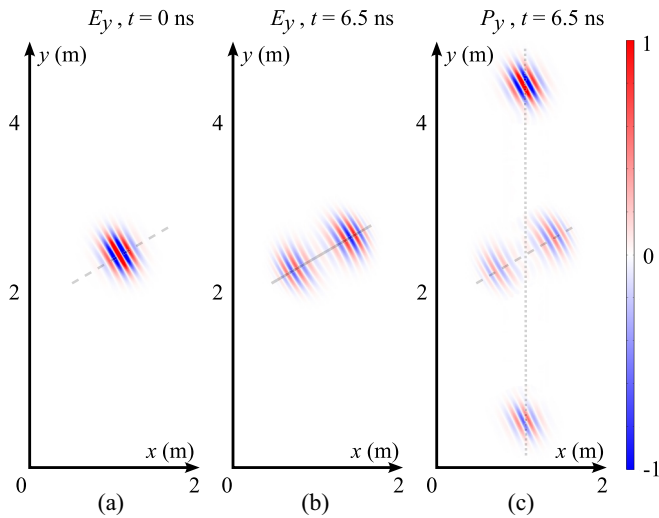


FIG. 3. Simulation results: (a) electric field $E_y(\mathbf{r})$ at the moment of time $t = 0$; (b) simulated electric field $E_y(\mathbf{r})$ at time $t = 6.5$ ns; (c) simulated polarization $P_y(\mathbf{r})$ at time $t = 6.5$ ns. All values are normalized to the maxima of their absolute values. Gray dashed lines at each subplot indicate the initial direction of propagation of the wave packet. Gray dotted line at subplot (c) indicates the direction of propagation of the TEM mode, created after medium switching.

wave—is seen. In the plot of P_y , all four waves—two TM waves and two TEM waves—are seen because TEM waves are also manifested via medium polarization. For TEM waves, the group velocity vector coincides with the direction y and does not depend on the initial wave-package direction. The amplitudes of the individual wave packages are predicted by (15), (17), and (18). Other field components can be found from formulas (14) and (16).

Let us discuss the agreement of the simulations with our analytical model. From formulas (15), (17), and (18) for the chosen set of parameters, the ratio of the amplitude of the TM wave time refracted to that of the time-reflected wave over time is equal to 1.49. For TEM waves, the similar ratio is equal to 3.0. From our simulations, we obtain, respectively, 1.51 (disagreement 2%) and 2.74 (disagreement 8.5%). This level of disagreement, in our opinion, is acceptable, taking in account the difference of the simulated pulse from the analytically modeled wave. The ratios of the amplitudes of the resulting TM and TEM waves to the initial wave amplitude were also calculated analytically and numerically. The disagreements for these ratios is below 9% as well. The theoretical and simulated values of ω_2 and Ω_2 agree even better than the theoretical and simulated amplitudes. For the TEM waves, the new frequency Ω_2 is equal to 7.26×10^9 rad/s from the simulations, while its analytically predicted value is 7.53×10^9 rad/s (the disagreement is equal 3.7%). For our TM waves, the simulated frequency ω_2 is equal to 7.746×10^{10} rad/s, whereas the analytically predicted result for ω_2 is equal to 7.683×10^{10} rad/s. Here the disagreement is smaller than 1%. In our Supplemental Material, [20] there is a reference to the video file that convincingly shows that the space-time evolution of both E_y and P in the four re-

sulting pulses fits our theory not only qualitatively but also quantitatively.

V. CONCLUSIONS

In this paper, we have studied a temporal interface between a split wire medium (a uniaxial dielectric) and a usual wire medium which possesses strong spatial dispersion. The main scientific novelty of our work is the finding of generation of four waves at two new frequencies as a result of the temporal transition. In this case, the classical Morgenthaler temporal boundary conditions are not sufficient to solve the problem. The analytical results have been obtained in the time domain based on temporal boundary conditions that we have introduced for the axial electric field, medium polarization, and their time derivatives. Within the frameworks of the WM homogenization model governed by Eq. (3), we have obtained exact analytical results describing two TM-polarized waves and two TEM waves after the temporal interface. These pairs of time-refracted and time-reflected waves propagate along two different directions.

Indeed, an assumption of an instantaneous jump in the effective medium permittivity due to an ignition of the nanoplasma discharge is an idealization. Realistic transition time can be estimated from experiments by Nikoo *et al.* [19] and bin Roslan *et al.* [23] that report nanoplasma formation times of the order of 5 ps. We consider the switching time, which is below 25% of an oscillation cycle, as approximately instantaneous. This condition is satisfied for the frequencies below 50 GHz (for the 5-ps switching time). Together with the homogenization constraint $a < 0.3\lambda$ it determines the allowed geometries of our WM. At microwave frequencies with a shorter wavelength, and, moreover, at millimeter wave frequencies, these times are not negligibly small compared to the oscillation cycle. Such finite-rise transition will produce a sinc-shaped frequency envelope, i.e., finite-band spectra of waves at the predicted TM and TEM branches. In some cases, a significant transition time may even result in partial overlapping of these modes. This issue is beyond the scope of the present paper and needs to be considered separately.

This phenomenon can have applications, for example, in sensors. The considered transition transforms the field distributions over the split wire medium sample into a package of traveling waves that propagate along the axis of a WM endoscope and carry information about the field distribution before the switch. Moreover, the results of the present work combined with those of our previous paper [3] offer the necessary information for studies of periodic modulations of wire media with strong spatial dispersion.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not

technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of

this research project. The data are available from the authors upon reasonable request.

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