

Observation of a topological phase transition in random coaxial cable structures with chiral symmetry

D. M. Whittaker *Physics and Astronomy, School of Mathematical and Physical Sciences, [University of Sheffield](#), Sheffield S3 7RH, United Kingdom*

Maxine M. McCarthy

*Physics and Astronomy, School of Mathematical and Physical Sciences, [University of Sheffield](#), Sheffield S3 7RH, United Kingdom
and Max Planck Institute for the Science of Light, Staudstraße 2, 91058 Erlangen, Germany*

Qingqing Duan

*Wenzhou Institute and Wenzhou Key Laboratory of Biophysics, [University of Chinese Academy of Sciences](#),
Wenzhou, Zhejiang 325001, China*

(Received 17 June 2024; revised 26 June 2026; accepted 12 August 2026; published 16 September 2026)

We report an experimental study of the disordered Su-Schrieffer-Heeger (SSH) model, implemented in a system of coaxial cables, whose radio frequency properties map onto the SSH Hamiltonian. By measuring multiple chains with random hopping terms, we demonstrate the presence of a topologically protected state, with frequency variation of less than 0.2% over the ensemble, evidence of near-perfect chiral symmetry. When the ends of the chains are connected to form loops, we observe a topological phase transition, characterized by the closure of the band gap and the appearance of states that are delocalized, despite the strong disorder.

DOI: [10.1103/hnqs-xh69](https://doi.org/10.1103/hnqs-xh69)

I. INTRODUCTION

The Su-Schrieffer-Heeger (SSH) model [1], originally a description of the electronic states in polyacetylene, is one of the simplest systems of topological physics. It consists of a chain of sites, representing carbon atoms, connected by hopping terms that alternate in strength, corresponding to the bonds of the dimerized molecule. In this periodic form, it has a band gap that closes when the two hopping strengths are the same. The gap closure separates two topological phases, determined by the relative magnitudes of the hopping amplitudes. With appropriate termination, SSH chains can support localized boundary states that are said to be topologically protected, because their energy is independent of disorder in the hopping amplitudes. These nontrivial topological properties are a consequence of the chiral, or sublattice, symmetry of the SSH model [2,3]; the sites can be divided into two sublattices, such that there are only hopping terms connecting the two types.

The topology of the SSH model is robust in the presence of disorder, provided that the chiral symmetry is not broken. A chain with a random sequence of hopping amplitudes can still be assigned to one of two topological phases. We can thus talk about a topological phase transition in an ensemble of random SSH loops, driven by varying the parameters in the probability

distribution from which the hopping amplitudes are drawn [4]. The physics of the transition in the random chains is much richer than in their periodic counterpart, a consequence of the larger parameter space. In the periodic case, there are only two parameters, the weak and strong hopping amplitudes, and the phase boundary corresponds to the trivial uniform chain where these are the same. For a random chain of length N , the parameter space is N dimensional, one for each hopping amplitude, and the structures on the phase boundary are disordered.

The theory of such topologically marginal random chains, with off-diagonal disorder, has a long history, dating back to work by Dyson [5–7] and continuing through modern scaling treatments of the Anderson transition [8–12]. In contrast to the more usual case of on-site disorder, even in one dimension delocalized states are expected to be found, occurring at zero energy. Measurements of localization properties can thus provide a signature of a topological phase transition. In an infinite chain, the transition is also predicted to be accompanied by a singularity in the density of states.

There have been numerous experimental studies of implementations of the SSH model using electromagnetic waves, in photonic and microwave structures [13–17], acoustic lattices [18], and discrete electronics [19–21]. It is generally hard to control all the couplings in these systems so as to maintain chiral symmetry with sufficient accuracy to observe the effects we discuss, particularly while introducing controlled disorder. As a result, most demonstrations have focused on periodic structures, without addressing topologically significant questions about symmetry preserving disorder.

However, delocalization at a topological phase boundary has been observed [22] for a momentum-space SSH structure in cold-atom systems [23].

II. COAXIAL CABLE NETWORKS

Coaxial cable networks are a very simple electromagnetic system that can be used to investigate disorder [24] and topological [25–28] physics. We have shown [29] that cable structures can be fabricated with radio frequency properties that map very accurately onto the SSH Hamiltonian. The hopping amplitudes are determined by the impedances of the corresponding cables, so it is easy to make a random ensemble of chains with full chiral symmetry. In this work, we use cable structures to experimentally investigate the properties of random SSH chains. By making impedance and transmission measurements, we demonstrate very precise topological protection of a state, and show the delocalization and closing of the gap at the phase transition.

The derivation of the matrix description of a coaxial cable network is given in Appendix A. We consider a network consisting of a set of sites, labeled n , connected by sections of coaxial cable, all of which have the same transmission time, τ , the length divided by the transmission speed. The cable connecting sites n and n' has electrical impedance $Z_{nn'}$. The network has radio frequency resonances that are determined by a matrix eigenvalue equation $Hv = \varepsilon v$, where the dimensionless “energy,” ε , is related to the frequency, ω , by $\varepsilon = \cos \omega\tau$. The components of the vector v are the voltages at the sites, scaled such that the actual voltage is $V_n = \sigma_n v_n$. Here $\sigma_n = (\sum_{n'} Z_{nn'}^{-1})^{-1/2}$, with the sum taken over the sites n' directly connected to n . The matrix elements of the “Hamiltonian” are then the hopping amplitudes

$$H_{nn'} = \sigma_n Z_{nn'}^{-1} \sigma_{n'}. \quad (1)$$

This one-to-one mapping from cables connecting sites to hopping amplitudes means it is possible to create a network corresponding to any finite real matrix Hamiltonian, though in practice this is limited by the availability of cables with arbitrary impedances.

We measure the radio frequency properties of the cable structures using a vector network analyzer (VNA). Two sorts of measurement are useful. A single port reflection measurement of the S_{11} parameter gives us the impedance of the structure measured at a given site. In Appendix A, Eq. (A27), we show that the real part of this is proportional to the local density of states, broadened only by losses in the cables. This enables us to accurately determine the frequencies of the resonances of the structure. Two port transmission measurements (S_{21}) provide information about the spatial extent of the states, allowing us to detect the delocalization that occurs around the phase transition.

III. SPECTRUM AND TOPOLOGICAL PROTECTION

A Hamiltonian has chiral symmetry if the sites can be divided into two sublattices, which we call “black” and “white” (Fig. 1), such that there is only hopping between sites on different sublattices. There can be no intrasublattice terms, including on-site energies. If this is the case, ordering the basis

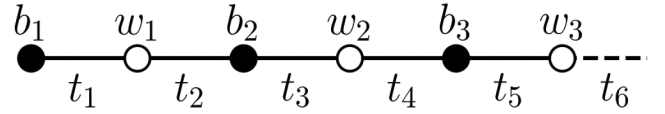


FIG. 1. Sublattice coloring of an SSH chain, with black and white sites labeled b_n and w_n , as in Eq. (2). The hopping amplitudes, t_n , follow Eq. (3).

such that all the black sites precede the white sites gives a block antidiagonal form:

$$H \begin{pmatrix} b \\ w \end{pmatrix} = \begin{pmatrix} 0 & Q \\ Q^\dagger & 0 \end{pmatrix} \begin{pmatrix} b \\ w \end{pmatrix} = \varepsilon \begin{pmatrix} b \\ w \end{pmatrix}, \quad (2)$$

where b and w are vectors of the amplitudes on the black and white sites. From this we obtain $(Q^\dagger Q)w = \varepsilon^2 w$, so the eigenvalues must either be zero, or occur in symmetric pairs with opposite signs. It immediately follows that for a chain with an odd number of sites there must be at least one zero-energy state. Since this conclusion does not depend on the values of the hopping amplitudes that form the matrix elements of Q , the zero-energy state is topologically protected against disorder. More generally, for a chiral network with n_b black sites and n_w white sites, there are at least $|n_b - n_w|$ protected states.

Figure 2 shows the local density of states measurement for a number of structures consisting of sequences of 16 cables connected end to end [30]. The individual cables are randomly selected from two impedances: 50 and 93 Ω . The structures thus map onto finite-length SSH chains with randomized hopping terms. More details of the cables are given in Appendix B. Figure 2(a) shows spectra for unlooped chains, with the measurement on a site at the end of the structure. The 16 cables correspond to 17 lattice sites, so we see, as expected, a topologically protected $\varepsilon = 0$ state, at a frequency of approximately 114 MHz. The topological protection is very good: the inset shows combined results for this state in 41 random structures. The standard deviation of the resonance frequency is approximately 0.22 MHz, which we believe is due to small errors in the lengths of the cables.

In an infinite periodic SSH structure, a topological phase boundary occurs only for the trivial situation where the inter- and intracell hopping terms are identical. However, in disordered chains, the possibilities for marginal structures are more interesting. A phase transition is signaled by the presence of a pair of degenerate states at zero energy, equivalent to the gap closure in a periodic structure. When $n_b = n_w$, this corresponds to the condition that the determinant $|Q| = 0$ [31]. For our unlooped structures, $|Q|$ is a simple product of hopping amplitudes, the first term in Eq. (3), which cannot be made zero without cutting the chain, so there is, trivially, just one topological phase. However, by joining the ends of the chains to form loops, we can observe a transition between the two phases of the SSH model.

To have chiral symmetry, the loops must consist of an even number of sites, and thus be made from an even number of cables. In a loop with N cables, labeling hopping amplitudes rather than the sites, $t_1, t_2, \dots, t_N$ (Fig. 1), we obtain

$$|Q| = t_1 t_3 \cdots t_{N-1} - (-1)^{(N/2)} t_2 t_4 \cdots t_N. \quad (3)$$

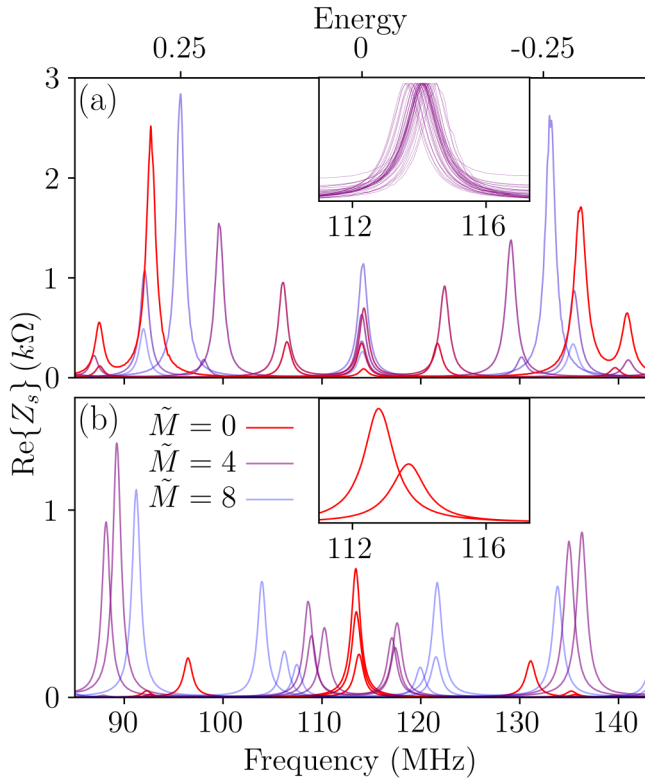


FIG. 2. (a) Measured impedance spectra (local density of states) for a selection of unlooped length $N = 16$ random SSH chains. The colors are an aid to distinguishing curves. The spectra show the expected symmetry about the chiral frequency (~ 114 MHz), which corresponds to zero energy in the SSH Hamiltonian. The topological protection of the state at $\varepsilon = 0$ is apparent. The inset shows, expanded and normalized, the protected state in an ensemble of 41 such chains. The standard deviation of the peak frequency is 0.22 MHz, demonstrating the minimal chiral symmetry breaking in our cable structures. (b) Impedance spectra for looped length 16 random chains with various reduced lengths, Eq. (5): $\tilde{M} = 0$ (red), $\tilde{M} = 4$ (purple), and $\tilde{M} = 8$ (blue). As $\tilde{M}$ decreases, the gap closes, leading to a doubly degenerate state at the chiral frequency for the topologically marginal $\tilde{M} = 0$ structures. The inset shows spectra for $\tilde{M} = 0$ around $\varepsilon = 0$ measured on adjacent sites, revealing the two states, one localized on each sublattice. Compared to (a), there is more chiral symmetry breaking, and a slight lowering of the chiral frequency, due to the extra length of the T-connector inserted in the loop to make the measurement.

However, since the t_n contain the scaling factors, σ_n , as well as the cable impedances Z_n , Eq. (1), it is convenient to look at the quotient of the two terms, where these cancel, and define a quantity

$$M = 2 \ln \left(\frac{t_2 t_4 \cdots t_N}{t_1 t_3 \cdots t_{N-1}} \right) = 2 \ln \left(\frac{Z_1 Z_3 \cdots Z_{N-1}}{Z_2 Z_4 \cdots Z_N} \right). \quad (4)$$

For our structures, where the impedances are taken from a binary distribution, Z_a or Z_b , there are typically cancellations in the ratio of the impedances, and we can write

$$M = \tilde{M} \ln(Z_a/Z_b), \quad (5)$$

where $\tilde{M}$ is an even integer, which we call the “reduced length” of the structure. For a loop where the length N is

an even multiple of 2 ($N = 4, 8 \dots$) we get $|Q| = 0$ when M is zero. The sign of M is thus a topological invariant: the impedances, and corresponding hopping amplitudes, are real, so if they were changed continuously, it would not be possible to flip the sign of M without passing through a marginal structure with $|Q| = 0$. For odd multiples of 2 ($N = 2, 6 \dots$) the two terms in the expansion of $|Q|$ have the same sign, so, though M can be zero, to make a topologically marginal structure would require a negative hopping amplitude in the loop. Note that the sign of M , and thus the attribution of phases, is arbitrary, because it is a property of the looped structure and depends on the choices of the starting point and direction of numbering. However, if this is done consistently, the difference in sign distinguishes the two phases.

If, instead of making a loop, the chain is infinitely repeated to form a periodic structure, we find that the topological classification from the sign of the reduced length always agrees with the generalized Berry phase [32] and winding number invariants obtained from Brillouin zone based calculations. These methods also predict a phase transition when $M = 0$ in a chain with an odd number of pairs of cables. In a periodic structure this is correct, because there are gap closures somewhere in the Brillouin zone for both even and odd numbers of pairs. The two cases differ because, for an even number of pairs, the closure is at wave number $k = 0$ where the state is the same at the end of each period, corresponding to the boundary condition for a loop. For an odd number of pairs, the closure is at $k = \pi$, so the loop boundary condition is not satisfied. However, as we show below, the delocalization associated with the phase transition can be seen for both even and odd numbers of pairs. The $M = 0$ condition also corresponds to the phase boundary found in Ref. [4] and observed experimentally in Ref. [22]; the unusual reentrant shape of the boundary in these works is due to the particular choice of rectangular probability distribution from which the hopping amplitudes are drawn.

In Fig. 2(b), we plot the local density of states for some random looped chains with length $N = 16$ and different values of the reduced length $\tilde{M}$. As expected, there is always a gap around $\varepsilon = 0$, except in the marginal case $\tilde{M} = 0$, where the degenerate pair of zero-energy states is found. From this pair, it is always possible to make states that are localized entirely on separate sublattices. In the inset, this is demonstrated experimentally by comparing spectra from two adjacent sites, one on each sublattice. The peaks correspond to two distinct states, as can be seen by the small separation in energy.

IV. TRANSMISSION AND LOCALIZATION

In order to explore the localization of the zero-energy states, we make use of transmission measurements on the unlooped chains. These are most simply described using a transfer matrix treatment, which relates the currents and voltages entering and leaving the structure. At zero energy, the transfer matrix, Appendix A, Eq. (A6), for a single cable is

$$\begin{pmatrix} V_{\text{out}} \\ I_{\text{out}} \end{pmatrix} = \mathcal{M}_n \begin{pmatrix} V_{\text{in}} \\ I_{\text{in}} \end{pmatrix} = \begin{pmatrix} 0 & iZ_n \\ i/Z_n & 0 \end{pmatrix} \begin{pmatrix} V_{\text{in}} \\ I_{\text{in}} \end{pmatrix}. \quad (6)$$

The matrix representing a sequence of N cables is then just the product of the $\mathcal{M}_n$ for each cable, $\mathcal{M} = \mathcal{M}_N \mathcal{M}_{N-1} \cdots \mathcal{M}_1$. The nonzero elements of $\mathcal{M}$ are the same ratios of impedance

products as appear in M , Eq. (4), so we write, for even N ,

$$\mathcal{M} = (-1)^{(N/2)} \begin{pmatrix} e^{-M/2} & 0 \\ 0 & e^{M/2} \end{pmatrix}. \quad (7)$$

The measured transmission amplitude, S_{21} , at $\varepsilon = 0$ is then

$$\begin{aligned} S_{21} &= (-1)^{N/2} \operatorname{sech}\left(\frac{M}{2}\right) \\ &= (-1)^{N/2} \operatorname{sech}\left(\frac{\tilde{M}}{2} \ln\left(\frac{Z_a}{Z_b}\right)\right). \end{aligned} \quad (8)$$

For our binary distribution, this is the same as the transmission for a periodic chain, $Z_a Z_b Z_a Z_b \dots$, in which the number of cables is equal to the reduced length $\tilde{M}$ (for negative $\tilde{M}$ the sequence starts with Z_b). This follows because, at $\varepsilon = 0$, the transfer matrix for an adjacent pair of cables with the same impedance is just minus the unit matrix, so in calculating S_{21} we can iteratively remove such pairs from the structure until it is reduced to a periodic chain.

Equation (8) shows that the transmission at $\varepsilon = 0$ depends only on the value of M , and has a value of unity in chains with $M = 0$, which are topologically marginal when joined to form a loop or repeated periodically. The topologically protected states in the marginal cables are delocalized [6,7], in that they have the same amplitude at either end. Away from $M = 0$, the state is localized, with a larger amplitude at one or the other end, depending on the sign of M , and thus the topological phase. The simple treatment leading to Eq. (8) does not account for the small resistive losses occurring in the cables, though these are easily included numerically. The losses always reduce the transmission, but they also cause some spread of the $\varepsilon = 0$ values for a given M .

Experimental transmission results for our length 16 chains are shown in Fig. 3, where we plot $|S_{21}|$ as a function of frequency for different values of the reduced length $\tilde{M}$. The spectra consist of peaks that correspond to the states found in the S_{11} measurements of Fig. 2, but with much greater broadening, a result of losses due to the finite (50 Ω) input and output impedances of the VNA. As predicted, the value at $\varepsilon = 0$ is fairly similar for all structures with the same $\tilde{M}$. In the inset, the average of $|S_{21}|$ at $\varepsilon = 0$ is plotted as a function of $\tilde{M}$, along with the hyperbolic secant dependence predicted in Eq. (8), scaled by a constant factor to account for losses in the cables. With this scaling, the agreement is good, and both the absolute values and the spread are consistent with numerics using the losses deduced from the measured broadening of the peaks in Fig. 2.

In addition to the delocalization that we have demonstrated, Refs. [6–12] predict a singular peak, the Dyson singularity, in the density of states, $\rho(\varepsilon)$, around $\varepsilon = 0$ for topologically marginal random structures of infinite length. This has a functional form $\rho(\varepsilon) \sim |\varepsilon \ln \varepsilon|^{-1}$. However, in finite structures this singularity is replaced by a broader peak, which narrows as N increases. Numerical simulations suggest that, with our choice of impedances, it is not possible to see such a feature in the short structures we investigate here. However, it should be feasible in an ensemble with the lengths increased to 50–100 cables.

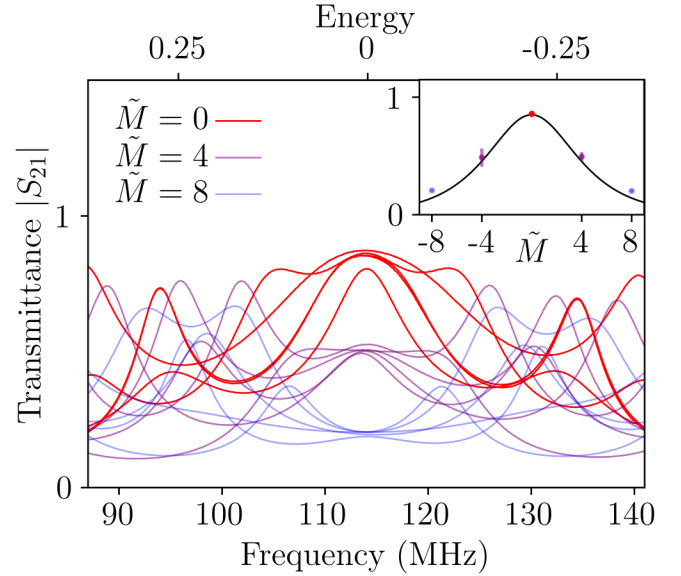


FIG. 3. Measured transmission spectra, $|S_{21}|$, for a selection of random unlooped chains with length $N = 16$. The curves are colored according to the reduced length of the structures, Eq. (5): $\tilde{M} = 0$ (red), $\tilde{M} = 4$ (purple), and $\tilde{M} = 8$ (blue), as in Fig. 2. The transmission at zero energy (top scale) is, neglecting losses, predicted to depend only on $\tilde{M}$. The inset shows the dependence of this transmission on reduced length (points), averaging over seven to nine structures for each $\tilde{M}$. The error bars show the standard deviation of the measured values. The solid line is the behavior predicted in Eq. (8), with a constant scaling to account for losses.

V. DISCUSSION AND CONCLUSIONS

We have shown that when a looped chiral structure is split, the transmission through the corresponding chain has unit value, in the absence of losses, if the original loop was topologically marginal. Such perfect transmission is thus an experimental signature of a topological phase boundary. The result generalizes, with some caveats, to more complicated networks with chiral symmetry. If we start from a balanced structure, having equal numbers of sites on each sublattice, and break a loop by unplugging a cable, we split a site, creating an imbalance, and thus a topologically protected state through which transmission can occur. The transfer matrix, which determines the transmission between the two sides of the break, will always be diagonal at zero energy, like Eq. (7), of the form

$$\mathcal{M} = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda^{-1} \end{pmatrix}. \quad (9)$$

As we have seen, this leads to perfect transmission when $\lambda = 1$ [33]. However, this is also the condition for the unsplit structure to be topologically marginal; then the voltages and currents on either side of the break are identical, which is the boundary condition that must be satisfied to obtain a zero-energy state when they are joined [34].

The connection between topological phase boundaries and perfect transmission is not, however, universal. In more complicated networks, there are cases where a structure is marginal but it can be split in such a way that the transmission

between the ends is less than 1, sometimes zero. Though a full discussion is beyond the scope of the present paper, this occurs when, in the split structure, either the topologically protected state has zero amplitude on the input or output site, or there is more than one zero-energy state on the same sublattice.

To conclude, we have carried out an experimental study of the topological properties of a coaxial cable system that maps onto the SSH model. The accuracy of this mapping is demonstrated by the small variation, less than 0.2%, in the frequencies of the topologically protected state in an ensemble of random structures. By varying the parameters in the random distribution, we have shown that looped structures can be taken through a topological phase transition, characterized by the closure of the gap and the appearance of a delocalized state at zero energy. Coaxial cable structures provide an excellent platform for such topological physics experiments. Structures with a high degree of chiral symmetry can be obtained, and disorder introduced in a controlled way, without breaking the symmetry. The simple chains considered here can readily be extended to networks representing more complicated Hamiltonians, where similar signatures of phase transitions are predicted to be observable.

ACKNOWLEDGMENTS

D.M.W. would like to thank Art Gower for many helpful lockdown discussions on this topic. M.M.M. wishes to thank the EPSRC for support from a studentship (Project No. 2482664). The work of Q.D. is supported by the National Natural Science Foundation of China under Grant No. 12090052.

DATA AVAILABILITY

The experimental data used to support the findings of this work are publicly available [30].

APPENDIX A: TIGHT-BINDING MODEL OF A CABLE NETWORK

In this Appendix, we derive the transfer matrix for a single coaxial cable and use it to obtain a generalized eigenvalue equation relating the voltages at the nodes of a network. By scaling these voltages, this is turned into a standard eigenvalue problem. We then show how to describe the experimental setup by adding driving and loss terms to represent the ports of the vector network analyzer. Finally, we demonstrate the relationship between the impedance of the network measured at a site and the local density of states.

Our system consists of a series of sections of transmission line of length d_n , with transmission speed c_n and impedance Z_n . Wave propagation in such a structure is determined by the telegraph equations. In each section, the voltage $V(x)$ is determined by a Helmholtz equation

$$\frac{d^2 V}{dx^2} + \left(\frac{\omega}{c_n}\right)^2 V = 0, \quad (\text{A1})$$

where ω is the frequency. At the boundaries between sections both V and the current,

$$I = -i \frac{c_n}{\omega Z_n} \frac{dV}{dx}, \quad (\text{A2})$$

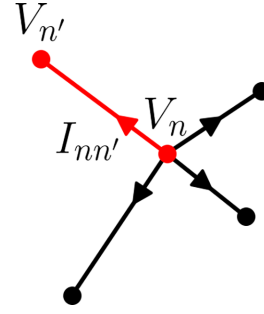


FIG. 4. Site n of the network and its neighbors. $I_{nn'}$ is the current flowing out of site n towards n' .

are continuous.

For a piecewise continuous system, we conventionally solve this equation using transfer matrices. In a given section, the solution can be written

$$V(x) = V(0) \cos(\omega x/c_n) + iZ_n I(0) \sin(\omega x/c_n), \quad (\text{A3})$$

$$I(x) = iZ_n^{-1} V(0) \sin(\omega x/c_n) + I(0) \cos(\omega x/c_n). \quad (\text{A4})$$

Thus, at the end of the section $x = d_n$,

$$\begin{pmatrix} V(d_n) \\ I(d_n) \end{pmatrix} = \mathcal{M}_n \begin{pmatrix} V(0) \\ I(0) \end{pmatrix}, \quad (\text{A5})$$

where the transfer matrix

$$\mathcal{M}_n = \begin{pmatrix} \cos(\omega d_n/c_n) & iZ_n \sin(\omega d_n/c_n) \\ i/Z_n \sin(\omega d_n/c_n) & \cos(\omega d_n/c_n) \end{pmatrix}. \quad (\text{A6})$$

We next demonstrate that a transmission line network is equivalent to a tight-binding system when each section has the same transit time, $\tau = d_n/c_n$. To show this, let us add to the notation a little so that V_n is the voltage at the n th junction (site), and $I_{nn'}$ is the current flowing out of that junction to connected site n' (see Fig. 4). $Z_{nn'}$ is the impedance of the section between sites n and n' . Then the transfer matrix gives

$$V_{n'} = V_n \cos \omega \tau + iZ_{nn'} I_{nn'} \sin \omega \tau. \quad (\text{A7})$$

We can use this to find $I_{nn'}$ in terms of V_n and $V_{n'}$, then apply Kirchhoff's junction rule, $\sum_{n'} I_{nn'} = 0$, to get

$$\sum_{n'} Z_{nn'}^{-1} (V_{n'} - V_n \cos \omega \tau) = 0. \quad (\text{A8})$$

Here, the sum over n' includes the sites that are directly connected to n . Identifying $\varepsilon = \cos \omega \tau$, this becomes

$$\sum_{n'} Z_{nn'}^{-1} V_{n'} = \varepsilon \left(\sum_{n'} Z_{nn'}^{-1} \right) V_n. \quad (\text{A9})$$

This is a generalized eigenvalue problem, but we can turn it into the standard form by defining scaled voltages depending on the impedances of the cables connected to site n :

$$v_n = \left(\sum_{n'} Z_{nn'}^{-1} \right)^{1/2} V_n = \sigma_n^{-1} V_n. \quad (\text{A10})$$

Then we have the tight-binding system

$$\sum_{n'} H_{nn'} v_{n'} = \varepsilon v_n, \quad (\text{A11})$$

with

$$H_{nn'} = \sigma_n Z_{nn'}^{-1} \sigma_{n'}. \quad (\text{A12})$$

Note that the “energy,” $\varepsilon = \cos \omega \tau$, is not the frequency, so zero energy corresponds to finite frequency; indeed the spectrum repeats periodically in frequency.

We now consider an experiment where we connect an input to site α , which we model as an ideal voltage source with amplitude V_{in} in series with an impedance Z_{in} . We measure an output voltage at site β , using a detector that is modeled as an ideal voltmeter in parallel with an impedance Z_{out} . The measured output voltage is simply the site voltage $V_{\text{out}} = V_\beta$.

We start with the detector. From the circuit point of view, this is simply an impedance Z_{out} , which causes an additional current out of the site β , with magnitude $I_{\text{out}} = V_\beta / Z_{\text{out}}$. The expression from Kirchhoff’s rule for site β is then modified to

$$\sum_{n'} Z_{\beta n'}^{-1} V_{n'} + i Z_{\text{out}}^{-1} \sqrt{1 - \varepsilon^2} V_\beta = \varepsilon \sum_{n'} Z_{\beta n'}^{-1} V_\beta, \quad (\text{A13})$$

since $\sin \omega \tau = \sqrt{1 - \varepsilon^2}$. Now scaling the voltages as in Eq. (A10), the tight-binding equation for site β becomes

$$\sum_{n'} H_{\beta n'} v_{n'} + i \Gamma_{\text{out}} \sqrt{1 - \varepsilon^2} v_\beta = \varepsilon v_\beta, \quad (\text{A14})$$

where $\Gamma_{\text{out}} = \sigma_\beta^2 Z_{\text{out}}^{-1}$; the finite impedance leads to an on-site imaginary energy for site β .

Doing a similar thing for the source, on site α , we get a slightly more complicated result. If the source voltage is V_{in} , and it has an impedance Z_{in} , there is an extra current flowing into site α , I_{in} , determined by

$$V_\alpha = V_{\text{in}} - I_{\text{in}} Z_{\text{in}}. \quad (\text{A15})$$

subtracting this current from the Kirchhoff expression gives

$$\sum_{n'} Z_{\alpha n'}^{-1} V_{n'} + i Z_{\text{in}}^{-1} \sqrt{1 - \varepsilon^2} (V_\alpha - V_{\text{in}}) = \varepsilon \sum_{n'} Z_{\alpha n'}^{-1} V_\alpha. \quad (\text{A16})$$

Rearranging, and scaling, we get

$$\sum_{n'} H_{\alpha n'} v_{n'} + i \Gamma_{\text{in}} \sqrt{1 - \varepsilon^2} v_\alpha = \varepsilon v_\alpha + i \sqrt{1 - \varepsilon^2} v_{\text{in}}, \quad (\text{A17})$$

where $\Gamma_{\text{in}} = \sigma_\alpha^2 Z_{\text{in}}^{-1}$ and $v_{\text{in}} = \sigma_\alpha Z_{\text{in}}^{-1} V_{\text{in}}$. The input adds an on-site imaginary energy and a driving term at site α .

We now have a matrix system that takes the form

$$(H + i\Gamma)v = \varepsilon v + i\mathcal{V}, \quad (\text{A18})$$

where the Hamiltonian H is the same as in Eq. (A11), and Γ is diagonal with the only entries the loss terms $\Gamma_{\text{in}} \sqrt{1 - \varepsilon^2}$ and $\Gamma_{\text{out}} \sqrt{1 - \varepsilon^2}$ on sites α and β . The driving term $\mathcal{V}$ has the single entry $\sqrt{1 - \varepsilon^2} v_{\text{in}}$ on site α .

We can diagonalize H to find its eigenvalues ε_k , and eigenvectors $u^{(k)}$. Then $H = UDU^\dagger$ where U is the unitary with matrix elements $U_{ij} = u_i^{(j)}$ and D is a diagonal matrix with $D_{ii} = \varepsilon_i$. This can be used to invert $H - \varepsilon \mathbb{1}$ to get the Green’s function

$$g = (H - \varepsilon \mathbb{1})^{-1} = U(D - \varepsilon \mathbb{1})^{-1} U^\dagger, \quad (\text{A19})$$

so that

$$g_{ij} = \sum_k \frac{u_i^{(k)} u_j^{(k)}}{\varepsilon_k - \varepsilon}. \quad (\text{A20})$$

Note that the eigenvectors are real, so the complex conjugate $u_j^{(k)*}$ is not required. With this, we can relate the output and input voltages for the case where the input and output impedances are infinite, so the loss terms Γ_{in} and Γ_{out} are zero:

$$v_{\text{out}} = v_\beta = i g_{\beta\alpha} \sqrt{1 - \varepsilon^2} v_{\text{in}}. \quad (\text{A21})$$

However, we really want to find $G = (H + i\Gamma - \varepsilon \mathbb{1})^{-1}$, to deal with the case where there are finite losses. Since Γ has only two nonzero entries, this can be calculated using the Woodbury matrix identity to obtain

$$G_{\beta\alpha} = \frac{g_{\beta\alpha}}{1 + \Gamma_{\text{in}} \Gamma_{\text{out}} (1 - \varepsilon^2) (g_{\beta\alpha} g_{\alpha\beta} - g_{\alpha\alpha} g_{\beta\beta}) + i \sqrt{1 - \varepsilon^2} (\Gamma_{\text{in}} g_{\alpha\alpha} + \Gamma_{\text{out}} g_{\beta\beta})}. \quad (\text{A22})$$

Then

$$v_{\text{out}} = i G_{\beta\alpha} \sqrt{1 - \varepsilon^2} v_{\text{in}}, \quad (\text{A23})$$

or, in terms of the unscaled physical quantities,

$$V_{\text{out}} = i \sigma_\beta \sigma_\alpha G_{\beta\alpha} Z_{\text{in}}^{-1} \sqrt{1 - \varepsilon^2} V_{\text{in}}. \quad (\text{A24})$$

For a single port measurement, connecting only to site α and measuring $V_{\text{out}} = V_\alpha$ to obtain the complex reflectance, we put $\Gamma_{\text{out}} = 0$ and get

$$\begin{aligned} \frac{V_{\text{out}}}{V_{\text{in}}} &= i \sigma_\alpha^2 Z_{\text{in}}^{-1} \frac{\sqrt{1 - \varepsilon^2} g_{\alpha\alpha}}{1 + i \Gamma_{\text{in}} \sqrt{1 - \varepsilon^2} g_{\alpha\alpha}} \\ &= \frac{i \sigma_\alpha^2 \sqrt{1 - \varepsilon^2} g_{\alpha\alpha}}{Z_{\text{in}} + i \sigma_\alpha^2 \sqrt{1 - \varepsilon^2} g_{\alpha\alpha}}. \end{aligned} \quad (\text{A25})$$

However, we can also think of the circuit as simply a potential divider, with the input V_{in} connected across the input impedance Z_{in} in series with an effective impedance Z_α representing the network. In these terms, we get

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{Z_\alpha}{Z_{\text{in}} + Z_\alpha}. \quad (\text{A26})$$

Comparing to Eq. (A25), the network impedance is thus

$$Z_\alpha = i \sigma_\alpha^2 \sqrt{1 - \varepsilon^2} g_{\alpha\alpha}. \quad (\text{A27})$$

Hence

$$\frac{\text{Re}\{Z_\alpha\}}{\pi \sigma_\alpha^2 \sqrt{1 - \varepsilon^2}} = -\frac{1}{\pi} \text{Im}\{g_{\alpha\alpha}\} = \sum_k (u_\alpha^{(k)})^2 \delta(\varepsilon - \varepsilon_k), \quad (\text{A28})$$

which is the unbroadened local density of states on site α . In practice, the delta function peaks are broadened by the small resistive losses in the cables.

When there is a single nondegenerate zero-energy state, as for the even length open structures considered here, the expression for $G_{\beta\alpha}$ at zero energy simplifies. Around zero energy, $g_{\beta\alpha}$ can be replaced by the divergent term in Eq. (A20), $-u_{\beta}^{(0)}u_{\alpha}^{(0)}/\varepsilon$. Then, $g_{\beta\alpha}g_{\alpha\beta} - g_{\alpha\alpha}g_{\beta\beta} = 0$, so using the expressions for Γ_{in} and Γ_{out} from above, and putting $Z_{\text{in}} = Z_{\text{out}} = Z_0$,

$$G_{\beta\alpha}(\varepsilon = 0) = \frac{-iu_{\beta}^{(0)}u_{\alpha}^{(0)}Z_0}{(\sigma_{\alpha}u_{\alpha}^{(0)})^2 + (\sigma_{\beta}u_{\beta}^{(0)})^2}. \quad (\text{A29})$$

This gives the transmission amplitude

$$\begin{aligned} S_{21} &= 2 \frac{V_{\text{out}}}{V_{\text{in}}} = 2 \left(\frac{\sigma_{\alpha} u_{\alpha}^{(0)}}{\sigma_{\beta} u_{\beta}^{(0)}} + \frac{\sigma_{\beta} u_{\beta}^{(0)}}{\sigma_{\alpha} u_{\alpha}^{(0)}} \right)^{-1} \\ &= 2 \left(\frac{V_{\alpha}^{(0)}}{V_{\beta}^{(0)}} + \frac{V_{\beta}^{(0)}}{V_{\alpha}^{(0)}} \right)^{-1}, \end{aligned} \quad (\text{A30})$$

where $V_{\alpha}^{(0)}$, $V_{\beta}^{(0)}$ are the physical voltages corresponding to the scaled quantities $u_{\alpha}^{(0)}$ and $u_{\beta}^{(0)}$.

For the zero-energy state, Eq. (A9) simplifies to

$$Z_{n-1}^{-1} V_{n-1}^{(0)} + Z_n^{-1} V_{n+1}^{(0)} = 0. \quad (\text{A31})$$

This can be solved iteratively to get, for an even number of cables, N ,

$$\frac{V_1^{(0)}}{V_{N+1}^{(0)}} = (-1)^{N/2} \left(\frac{Z_1 Z_3 \cdots Z_{N-1}}{Z_2 Z_4 \cdots Z_N} \right). \quad (\text{A32})$$

Using the definition of M in Eq. (4), the ratio of the products of odd and even impedances is $e^{M/2}$, so Eq. (A30) recovers the expression for S_{21} obtained from the transfer matrix treatment, Eq. (8).

APPENDIX B: EXPERIMENTAL DETAILS

We make our experimental structures using two types of coaxial cable, RG58 and RG62, with impedances of, respectively, 50 and 93 Ω . In order to obtain the mapping onto the tight-binding Hamiltonian, and thus chiral symmetry, it is essential that the transmission time, τ , in each section of the

cable is the same. For our choice of zero energy at ~ 114 MHz, this corresponds to nominal cable lengths of approximately 41 and 55 cm for the RG58 and RG62 cables, as they have different propagation speeds. To obtain the accurate chiral symmetry in our results, it was necessary to consider the contribution of the SMA (SubMiniature version A) connectors used to join the cables, which all have 50 Ω impedance. To account for these, the RG58 cables were shortened and the RG62 cables lengthened, such that, in a structure where they alternate, the transmission times in the 50 and 93 Ω sections were the same. However, double sections of the same cable type are then the wrong length, and the RG62 doubles contain a pair of 50 Ω connectors in the middle. We avoided these problems by making double-length cables of each type. It is clearly possible also to make triple and greater lengths, but instead, we restricted our random sequences to those containing no more than pair repeats.

Radio frequency spectra were obtained using a vector network analyser (NanoVNA V2 Plus4). Our results use two types of measurements. We find the impedance, and thus the local density of states, using a single port measurement of the S_{11} parameter. The structure impedance is then found using

$$Z_{\alpha} = \frac{1 + S_{11}}{1 - S_{11}} Z_{\text{in}}, \quad (\text{B1})$$

where Z_{in} is the output impedance of the VNA. The value of the transmission amplitude, S_{21} , is obtained directly from a two-port measurement between the ends of the cable.

The adjustments to the cable lengths to account for the connectors, as described above, moves the effective junctions, and hence the sites in the tight-binding model, to the points where the RG62 cables enter their SMA connectors. In the impedance measurements on the terminated chains, we accounted for this by calculating the correction to the impedance due to the transmission through the SMA connector between the physical junction with the VNA and the effective site position. At the opposite end to the measurement, the length of the terminating cable also needed to be modified, so that the final site corresponds to the end of the cable. For an RG62 termination, we simply removed the connector at the end, while in the RG58 case a slightly longer cable accounted for the pair of SMA connectors required to obtain the correct position.

-
- [1] W. P. Su, J. R. Schrieffer, and A. J. Heeger, Solitons in polyacetylene, *Phys. Rev. Lett.* **42**, 1698 (1979).
 - [2] A. Kitaev, Periodic table for topological insulators and superconductors, *AIP Conf. Proc.* **1134**, 22 (2009).
 - [3] A. P. Schnyder, S. Ryu, A. Furusaki, and A. W. W. Ludwig, Classification of topological insulators and superconductors, *AIP Conf. Proc.* **1134**, 10 (2009).
 - [4] I. Mondragon-Shem, T. L. Hughes, J. Song, and E. Prodan, Topological criticality in the chiral-symmetric AIII class at strong disorder, *Phys. Rev. Lett.* **113**, 046802 (2014).
 - [5] F. J. Dyson, The dynamics of a disordered linear chain, *Phys. Rev.* **92**, 1331 (1953).
 - [6] G. Theodorou and M. H. Cohen, Extended states in a one-dimensional system with off-diagonal disorder, *Phys. Rev. B* **13**, 4597 (1976).
 - [7] T. P. Eggarter and R. Riedinger, Singular behavior of tight-binding chains with off-diagonal disorder, *Phys. Rev. B* **18**, 569 (1978).
 - [8] R. H. McKenzie, Exact results for quantum phase transitions in random XY spin chains, *Phys. Rev. Lett.* **77**, 4804 (1996).

- [9] L. Balents and M. P. A. Fisher, Delocalization transition via supersymmetry in one dimension, *Phys. Rev. B* **56**, 12970 (1997).
- [10] P. W. Brouwer, C. Mudry, and A. Furusaki, Density of states in coupled chains with off-diagonal disorder, *Phys. Rev. Lett.* **84**, 2913 (2000).
- [11] M. Titov, P. W. Brouwer, A. Furusaki, and C. Mudry, Fokker-Planck equations and density of states in disordered quantum wires, *Phys. Rev. B* **63**, 235318 (2001).
- [12] F. Evers and A. D. Mirlin, Anderson transitions, *Rev. Mod. Phys.* **80**, 1355 (2008).
- [13] N. Malkova, I. Hromada, X. Wang, G. Bryant, and Z. Chen, Observation of optical Shockley-like surface states in photonic superlattices, *Opt. Lett.* **34**, 1633 (2009).
- [14] T. Ozawa, H. M. Price, A. Amo, N. Goldman, M. Hafezi, L. Lu, M. C. Rechtsman, D. Schuster, J. Simon, O. Zilberberg, and I. Carusotto, Topological photonics, *Rev. Mod. Phys.* **91**, 015006 (2019).
- [15] C. Poli, M. Bellec, U. Kuhl, F. Mortessagne, and H. Schomerus, Selective enhancement of topologically induced interface states in a dielectric resonator chain, *Nat. Commun.* **6**, 6710 (2015).
- [16] P. St-Jean, V. Goblot, E. Galopin, A. Lemaître, T. Ozawa, L. Le Gratiet, I. Sagnes, J. Bloch, and A. Amo, Lasing in topological edge states of a one-dimensional lattice, *Nat. Photon.* **11**, 651 (2017).
- [17] F. Bleckmann, Z. Cherpakova, S. Linden, and A. Alberti, Spectral imaging of topological edge states in plasmonic waveguide arrays, *Phys. Rev. B* **96**, 045417 (2017).
- [18] A. Coutant, A. Sivadon, L. Zheng, V. Achilleos, O. Richoux, G. Theocharis, and V. Pagneux, Acoustic Su-Schrieffer-Heeger lattice: Direct mapping of acoustic waveguides to the Su-Schrieffer-Heeger model, *Phys. Rev. B* **103**, 224309 (2021).
- [19] W. Tan, Y. Sun, H. Chen, and S.-Q. Shen, Photonic simulation of topological excitations in metamaterials, *Sci. Rep.* **4**, 3842 (2014).
- [20] J. Ningyuan, C. Owens, A. Sommer, D. Schuster, and J. Simon, Time- and site-resolved dynamics in a topological circuit, *Phys. Rev. X* **5**, 021031 (2015).
- [21] C. H. Lee, S. Imhof, C. Berger, F. Bayer, J. Brehm, L. W. Molenkamp, T. Kiessling, and R. Thomale, Topoelectrical circuits, *Commun. Phys.* **1**, 39 (2018).
- [22] E. J. Meier, F. A. An, A. Dauphin, M. Maffei, P. Massignan, T. L. Hughes, and B. Gadway, Observation of the topological Anderson insulator in disordered atomic wires, *Science* **362**, 929 (2018).
- [23] M. Atala, M. Aidelsburger, J. T. Barreiro, D. Abanin, T. Kitagawa, E. Demler, and T. Bloch, Direct measurement of the Zak phase in topological Bloch bands, *Nat. Phys.* **9**, 795 (2013).
- [24] Z. Q. Zhang, C. C. Wong, K. K. Fung, Y. L. Ho, W. L. Chan, S. C. Kan, T. L. Chan, and N. Cheung, Observation of localized electromagnetic waves in three-dimensional networks of waveguides, *Phys. Rev. Lett.* **81**, 5540 (1998).
- [25] T. Jiang, M. Xiao, W.-J. Chen, L. Yang, Y. Fang, W. Y. Tam, and C. T. Chan, Experimental demonstration of angular momentum-dependent topological transport using a transmission line network, *Nat. Commun.* **10**, 434 (2019).
- [26] Q. Guo, T. Jiang, R.-Y. Zhang, L. Zhang, Z.-Q. Zhang, B. Yang, S. Zhang, and C. T. Chan, Experimental observation of non-Abelian topological charges and edge states, *Nature (London)* **594**, 195 (2021).
- [27] C. Oliver, D. Nabari, H. M. Price, L. Ricci, and I. Carusotto, Photonic lattices of coaxial cables: Flat bands and artificial magnetic fields, [arXiv:2310.18325](https://arxiv.org/abs/2310.18325).
- [28] M. M. McCarthy and D. M. Whittaker, Classifying the topology of finite chiral structures using complete matchings, *Phys. Rev. B* **114**, 024210 (2026).
- [29] D. M. Whittaker and R. Ellis, Topological protection in disordered photonic multilayers and transmission lines, [arXiv:2102.03641](https://arxiv.org/abs/2102.03641).
- [30] All the experimental data shown in the figures is available at, D. M. Whittaker, M. M. McCarthy, and Q. Duan, Experimental data for “Observation of a topological phase transition in random coaxial cable structures with chiral symmetry”, The University of Sheffield, Dataset, <https://doi.org/10.15131/shef.data.29226071>.
- [31] In more complicated networks, it is also possible to have pairs of topologically protected states when $n_b = n_w$, in which case $|Q|$ is always zero, However, this does not occur in the simple structures considered here.
- [32] N. Marzari and D. Vanderbilt, Maximally localized generalized Wannier functions for composite energy bands, *Phys. Rev. B* **56**, 12847 (1997).
- [33] There is also unit transmission when $\lambda = -1$, which corresponds to a phase transition when the network is periodically repeated, but not for the finite structure.
- [34] I. C. Fulga, F. Hassler, and A. R. Akhmerov, Scattering theory of topological insulators and superconductors, *Phys. Rev. B* **85**, 165409 (2012).