

Exact polaron-polaron interactions between heavy impurities in a quantum Hall fluidJia Wang¹, Xia-Ji Liu, and Hui Hu²*Centre for Quantum Technology Theory, Swinburne University of Technology, Melbourne 3122, Australia*

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We present an exact solution for effective polaron-polaron interactions between heavy impurities, mediated by a sea of noninteracting light fermions in the quantum Hall regime with highly degenerate Landau levels. For weak attraction between impurities and fermions, where only the manifold of lowest Landau levels is relevant, we obtain an analytical expression of mediated polaron-polaron interactions. Remarkably, polaron interactions are exactly zero when fermions in the lowest Landau levels outnumber heavy impurities. For strong attraction, different manifolds of higher Landau levels come into play and we derive a set of equations that can be used to numerically solve the mediated polaron interaction potential. We find that the potential vanishes when the distance R between impurities is larger than the magnetic length, but strongly diverges at short range following a Coulomb form $-1/R$. Our exact results of polaron-polaron interactions might be examined in cold-atom setups, where a system of Fermi polarons in the quantum Hall regime is realized with a synthetic gauge field or under fast rotation. Our predictions could also be useful to understand the effective interaction between exciton-polarons in electron-doped semiconductors under strong magnetic field.

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Introduction. The exchange of particles leading to effective interactions is a profound conceptual advance in physics. Fundamentally, forces mediated by gauge bosons explain all interactions between elementary particles [1–3]. In superconductors, Cooper pairing is induced by exchanging phonons between electrons [4]. The nature of effective interactions depends on the dispersion relation of exchanged particles. For instance, relativistic massive bosons induce Yukawa potentials [5,6], and nonrelativistic fermions with parabolic dispersion mediate Ruderman-Kittel-Kasuya-Yosida (RKKY) interactions [7–9]. Recent developments in engineering quantum many-body systems make the investigation of these mediated interactions experimentally accessible [10–14], in the previously untouched territory of strong coupling. Specific interest is given to the situations where the number of exchange particles is large, such as in a filled Fermi sea, so the whole system can be well described as a polaron problem [15], and the mediated interactions can be regarded as polaron-polaron interactions. In controllable cold-atom experiments, great efforts have been taken to measure fermion-mediated polaron interactions by immersing a few impurity atoms into a Fermi sea [11,13,14]. In two-dimensional materials such as WS_2 , interaction effects between Fermi polarons have also been observed [12]. However, accurate theoretical descriptions of polaron-polaron interactions are notoriously difficult to establish [13,14], particularly in the strongly correlated regime beyond the perturbative RKKY paradigm.

In this Letter, we aim to explore an intriguing question: what happens to polaron interactions if the energy spectrum of exchange particles is highly degenerate? Energy degeneracy, where distinct quantum states share the same energy, is ubiquitous in quantum mechanics. The presence of degenerate energy levels typically indicates symmetry in the corresponding Hamiltonian, with higher degrees of symmetry leading to greater degeneracy. Notable examples include highly excited states in a spherical harmonic trap and Rydberg electronic states in a Coulomb potential [16]. A highly excited Rydberg electron can mediate an exotic trilobite potential, binding another ground state atom into an ultra-long-range molecule [17,18].

We focus on a prime example of Landau levels (LLs), formed by fermions like electrons in two dimensions (2D) under a perpendicular magnetic field, which exhibit macroscopic degeneracy. These LLs are central to understanding the integer and fractional quantum Hall effects, noted for the robustness of Hall conductivity quantization against impurities [19,20]. Impurities in LLs have attracted interest for their role in quantum Hall robustness, with short-range impurities causing significant level broadening [21–24]. However, the interplay between impurity properties and polaron physics, particularly the emergence of polaron-mediated interactions induced by scattering of highly degenerate particles like those in LLs, remains largely unexplored, aside from the pioneering work by Efimkin and MacDonald [25].

We analyze a model with infinitely heavy impurities, allowing for exact solutions. Such solvable cases are rare in many-body physics [26], akin to the Ising model [27], 1D systems [28], the Kitaev model [29,30], the Dicke model [31], and the heavy polaron problem [32–34]. These exact solutions provide benchmarks for approximate methods and insights into complex systems. Our analytical and exact

results for polaron-polaron interactions in LLs, though limited to the heavy polaron limit, can provide crucial insights of fermion-mediated interactions in strongly correlated systems and serve as benchmarks for testing theoretical approaches, which sometimes yield conflicting predictions [13,14].

Our model is also relevant in both cold-atom setups and 2D semiconductor experiments, where the mass ratio of impurities to fermions in the Fermi sea is selectable, so the heavy impurity limit can be approximately realized. In cold-atom setups, the quantum Hall regime is within reach either with a strong synthetic gauge field [35,36] or under fast rotation [37–39]. Experimentally, exciton-polariton polarons in GaAs quantum wells were already studied via polariton spectroscopy in the integer and fractional quantum Hall regimes [40].

Our system consists of a mixture of heavy impurities and light fermions, where the light fermions are governed by a Hamiltonian with highly degenerate single-particle energy levels, such as LLs, and with negligible intraspecies interactions. The short-range impurity-background interaction, characterized by a lengthscale r_0 , can be well modeled by contact interactions for a dilute background where $r_0 \ll k_F^{-1}$, with k_F being the Fermi momentum [25,33,41,42]. In a Born-Oppenheimer approximation or in the rigorous infinitely heavy limit achievable by confinement in a deep optical trap or an optical tweezer, the heavy impurities are localized. The single-particle Hamiltonian for the light fermions with the presence of S impurities located at positions $\mathbf{r}_s$ is thus given by $H = T + V \equiv T + \sum_{s=1}^S g_s \delta(\mathbf{r} - \mathbf{r}_s)$, where T is a noninteracting Hamiltonian describing certain highly degenerate manifolds. Here, $\mathbf{r}$ is the position operator for the light fermions, and g_s is the coupling strength. We focus on attractive coupling $g_s < 0$ for simplicity and leave the discussions of the repulsive coupling cases to the Supplemental Material (SM) [43]. In the following, we always discuss the general situation of highly degenerate levels first and then consider LLs as a concrete example.

Weak impurity-fermion coupling. We start by investigating the simplest case of a single weakly interacting impurity, where the interaction term reduces to $g_1 \delta(\mathbf{r} - \mathbf{r}_1)$, and g_1 is weak enough so that only the lowest degenerate manifold with D -fold degeneracy is relevant. Expanding the Hamiltonian with the degenerate eigenfunctions leads to a matrix form with elements $H_{nn'} = T_{nn'} + V_{nn'}$, where $T_{nn'} = \varepsilon \delta_{nn'}$ and $V_{nn'} = g_1 \phi_n^*(\mathbf{r}_1) \phi_{n'}(\mathbf{r}_1)$. Here $\phi_n^*(\vec{r})$ are the eigenstates with degenerate energy ε . To solve the Hamiltonian, we only need to diagonalize V , as T is proportional to an identity matrix. A crucial observation is that V is a rank-1 matrix and can be expressed as $V = g_1 \mathcal{N}_1 \vec{v}_1 \vec{v}_1^\dagger$, where $\vec{v}_1^\dagger = [\phi_1(\mathbf{r}_1), \phi_2(\mathbf{r}_1), \dots, \phi_D(\mathbf{r}_1)]/\sqrt{\mathcal{N}_1}$ is a row vector, and $\mathcal{N}_1 = \sum_{n=1}^D |\phi_n(\mathbf{r}_1)|^2$ can be regarded as a normalization constant. The diagonalization can then be obtained analytically as $V \vec{v}_1 = g_1 \mathcal{N}_1 \vec{v}_1$, making $\vec{v}_1$ an eigenvector with eigenvalue $g_1 \mathcal{N}_1$. The corresponding real-space wave function is given by $\psi_1(\mathbf{r}) = \sum_{n=1}^D \phi_n^*(\mathbf{r}_1) \phi_n(\mathbf{r})/\sqrt{\mathcal{N}_1}$. There exist $D-1$ normalized vectors orthogonal to $\vec{v}_1$ in a D -dimension Hilbert space, denoted as $\vec{u}_n$, $n = 2, 3, \dots, D$ with $V \vec{u}_n = 0 \vec{u}_n$. These vectors, $\{\vec{v}_1, \vec{u}_2, \dots, \vec{u}_D\}$, form a complete set of all the eigenvectors. This method for diagonalizing rank-1 matrices has

been used to obtain trilobite potentials between a Rydberg atom and a ground-state atom induced by high angular momentum Rydberg electrons [17,18]. With known eigenstates and eigenenergies for T and H , the many-body problem for light fermions can be solved exactly. At zero temperature, noninteracting fermions occupy all the lowest levels, and the many-body ground-state energy is the summation of energies of the filled levels. The inclusion of the interacting impurity shifts only one energy level from the manifold by $g_1 \mathcal{N}_1$, corresponding to the spatial wave function $\psi_1(\mathbf{r})$. The polaron energy, defined as the total energy difference due to the presence of an impurity, is $E_p = g_1 \mathcal{N}_1$.

For multiple impurities with $g_s < 0$, the V matrix can be written as $V = \sum_{s=1}^S V_s \equiv \sum_{s=1}^S g_s \mathcal{N}_s \vec{v}_s \vec{v}_s^\dagger$, where $\vec{v}_s^\dagger = [\phi_1(\mathbf{r}_s), \phi_2(\mathbf{r}_s), \dots, \phi_D(\mathbf{r}_s)]/\sqrt{\mathcal{N}_s}$ and $\mathcal{N}_s = \sum_{n=1}^D |\phi_n(\mathbf{r}_s)|^2$. This is a rank- S matrix if $S < D$, with S nonzero eigenvalues $\lambda_s < 0$ in ascending order. One can define orthogonal vectors $\vec{u}_n$ for $n = S+1, S+2, \dots, D$, orthogonal to all $\vec{v}_s$, $s = 1, 2, \dots, S$. These vectors satisfy $V \vec{u}_n = 0 \vec{u}_n$ and the corresponding eigenenergies remain unchanged in the manifold despite the presence of impurities. The shifted eigenstates can be solved in the Hilbert space spanned by the nonorthogonal $\vec{v}_n$ as a generalized eigenvalue problem $\tilde{V} \vec{c}_s = \lambda_s \tilde{S} \vec{c}_s$, where $\tilde{S}$ and $\tilde{V}$ are $S \times S$ matrices with elements $\tilde{S}_{ij} = \vec{v}_i^\dagger \vec{v}_j$ and $\tilde{V}_{ij} = \vec{v}_i^\dagger V \vec{v}_j = \sum_{s=1}^S g_s \mathcal{N}_s \tilde{S}_{is} \tilde{S}_{sj}$, respectively. The S eigenvalues λ_s correspond to the S eigenvectors $\vec{w}_s = \sum_{s'=1}^S \vec{v}_{s'} c_{s's}$, where $c_{s's}$ is the s' th element of the column vector $\vec{c}_s$.

At zero temperature, all shifted energy levels are occupied if the number of fermions in the degenerate manifold, N , exceeds the number of impurities. The total energy change is given by $E_S = \sum_{s=1}^S \lambda_s = \text{Tr}[V]$. Since $\text{Tr}[V] = \sum_{s=1}^S \text{Tr}[V_s] = \sum_{s=1}^S E_p^{(s)}$, where $E_p^{(s)} = g_s \mathcal{N}_s$ represents the single-polaron energy of the s th impurity, we thus find that the total interaction energy is zero, implying that the polaron-polaron interactions are *exactly* zero, provided $N \geq S$. Importantly, this result holds even when the degenerate manifold is not fully filled, highlighting a nontrivial cancellation that stems from the intrinsic structure of the highly degenerate levels.

As a solid example, let us consider weakly interacting impurities immersed in a 2D gas of noninteracting light fermions subjected to a perpendicular uniform effective magnetic field in the x - y plane with area $L_x L_y$. Using the Landau gauge $\mathbf{A} = Bx\hat{y}$, the Landau Hamiltonian of the light fermions is

$$T = \frac{(\mathbf{p} - q\mathbf{A})^2}{2m} = \frac{p^2}{2m} - \frac{qBp_y}{m}x + \frac{q^2 B^2}{2m}x^2, \quad (1)$$

which results in highly degenerate Landau levels $E_{k_y, n} = \hbar\omega_c(n + 1/2)$, recognizing the Hamiltonian as a harmonic oscillator centered at $x = k_y \ell_B^2$. Here the cyclotron frequency $\omega_c = |qB|/m$ and the magnetic length $\ell_B = \sqrt{\hbar/|qB|}$ are defined by the effective magnetic field strength B , effective charge q , and mass m of the light fermions. The momentum in the y -direction $p_y \equiv \hbar k_y$ is a good quantum number.

Focusing on the case where only the lowest manifold is filled, and assuming all impurity interactions are identical, $g_s = g$, the vector $\vec{v}_s$ for the impurity at $\mathbf{r}_s = (x_s, y_s)$ is then

a column vector indexed by k_y as

$$(\vec{v}_s)_{k_y} = \frac{e^{ik_y y_s}}{\sqrt[4]{\pi \mathcal{N}_s \ell_B^2 L_y^2}} \exp \left[-\frac{(x_s - k_y \ell_B^2)^2}{2\ell_B^2} \right]. \quad (2)$$

The normalization constant is $\mathcal{N}_s = \mathcal{N} \equiv 1/(2\pi \ell_B^2)$. The polaron energy for a single impurity is therefore given by

$$E_p = \frac{g}{2\pi \ell_B^2}. \quad (3)$$

To investigate the induced interaction between two impurities, we place them at $(0,0)$ and $(0,R)$, respectively. The overlap can be analytically obtained as $\vec{v}_1^\dagger \vec{v}_2 = \exp(-R^2/4\ell_B^2)$, which gives the two nonzero eigenvalues of V :

$$\lambda_{\pm}(R) = \frac{g}{2\pi \ell_B^2} \left[1 \pm \exp \left(-\frac{R^2}{4\ell_B^2} \right) \right], \quad (4)$$

corresponding to the symmetric and antisymmetric solutions $\vec{w}_{\pm} = (\vec{v}_1 \pm \vec{v}_2)/\sqrt{2} \pm 2 \exp(-R^2/4\ell_B^2)$. It is convenient to define the contribution from each state as $U_{\pm}(R) \equiv \lambda_{\pm}(R) - \lambda_{\pm}(\infty)$, whose analytical expression reads

$$U_{\pm}(R) = \pm \frac{g}{2\pi \ell_B^2} \exp \left(-\frac{R^2}{4\ell_B^2} \right). \quad (5)$$

It implies that the Born-Oppenheimer potential between two heavy impurities is an attractive Gaussian potential if there is only one light fermion to mediate interactions. It is also obvious that for two or more light fermions, the total induced interaction is exactly zero as expected.

Arbitrary impurity-fermion coupling. For arbitrarily strong coupling between impurities and fermions, we must go beyond the lowest manifold approximation. We assume that there are M manifolds coupled by the impurity-fermion coupling, each with degeneracy $D_i > S$ and degenerate energy ε_i , where $i = 1, 2, \dots, M$. The dimension of the total Hamiltonian with the M manifolds is $G = \sum_{i=1}^M D_i$. For each manifold, we define $\vec{v}_s^{(i)\dagger} = [\phi_1^{(i)}(\mathbf{r}_s), \phi_2^{(i)}(\mathbf{r}_s), \dots, \phi_{D_i}^{(i)}(\mathbf{r}_s)]/\sqrt{\mathcal{N}_s^{(i)}}$, where the superscript (i) denotes the i th manifold. The corresponding orthogonal vectors are denoted as $\vec{u}_n^{(i)}$, where $n = S, S+1, \dots, D_i$. Defining $S \times M$ vectors $\mathcal{V}_{s,i}^\dagger = [0_{D_1}, 0_{D_2}, \dots, \vec{v}_s^{(i)\dagger}, \dots, 0_{D_M}]$ and similarly for $(G-S \times M)$ orthogonal vectors $\mathcal{U}_{n,i}$ in the full Hilbert space, the contact interaction matrix $V = \sum_s g_s \delta(\mathbf{r} - \mathbf{r}_s)$ can be expanded as a block matrix with block elements as a $D_i \times D_j$ matrix given by $V_{ij} = \sum_s g_s \mathcal{V}_{s,i} \mathcal{V}_{s,j}^\dagger$. We find $V \mathcal{U}_{n,i} = 0$, indicating that $\mathcal{U}_{n,i}$ are eigenvectors of the Hamiltonian with the same eigenvalues as the noninteracting Hamiltonian T . Thus, only S states shift in each manifold, and these shifted eigenstates can be solved as a generalized eigenvalue problem in the Hilbert space spanned by the $S \times M$ unorthogonal vectors, which is much smaller than the full Hilbert space of dimension G . While this problem generally requires large-scale numerical solutions, interestingly we are able to derive a set of much simplified equations to exactly solve the case of heavy and strongly interacting impurities immersed in a Fermi sea of light fermions in LLs [see Eq. (10) below]. In this case, the coupling constant g must be regularized by the 2D scattering length a_{2D} , which naturally couples infinitely many manifolds.

To derive Eq. (10), let us first examine the single impurity case using the symmetric gauge $\mathbf{A} = (-By\hat{x} + Bx\hat{y})/2$ and cylindrical coordinates centered at the impurity. Due to cylindrical symmetry, the angular quantum number m_ℓ is a good quantum number, and the noninteracting Hamiltonian is

$$\hat{T} = -\frac{\hbar^2}{2m} \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{m_\ell^2}{2mr^2} + \frac{m}{8} \omega_c^2 r^2 - \frac{\hbar \omega_c}{2} m_\ell, \quad (6)$$

equivalent to a 2D isotropic harmonic trap problem with an energy shift of $-m_\ell \hbar \omega_c/2$. The eigenenergies are $E = \hbar \omega_c [n_r + 1/2 + (|m_\ell| - m_\ell)/2]$, with $n_r = 0, 1, 2, \dots$ and $m_\ell = 0, 1, 2, \dots$. The degeneracy pattern matches that of the Landau gauge, with each manifold having one s -wave ($m_\ell = 0$). In the presence of an impurity at the origin with contact interaction, only the s -wave state is affected, agreeing with our analytical conclusion that only one state shifts away from each manifold.

Since $m_\ell = 0$, the s -wave problem is equivalent to the case in harmonic traps studied in Refs. [44,45]. Solving the s -wave problem with the Bethe-Peierls boundary condition, the radial Schrödinger equation yields solutions that vanish at infinity: $\Phi(\mathbf{r}; v) = e^{-\xi/2} U(v, 1, \xi)$, where $v = -E/\hbar \omega_c + 1/2$, $\xi = r^2/(2\ell_B^2)$, and $U(\alpha, \beta, \xi)$ is the Tricomi confluent hypergeometric function [45]. Near the origin,

$$e^{-\xi/2} U(v, 1, \xi) \rightarrow \frac{\log(\sqrt{2}\ell_B) - \psi[v]/2 - \gamma - \log[r]}{\Gamma[v]/2}, \quad (7)$$

where γ is the Euler-Mascheroni constant, $\psi[n, z]$ is the n th derivative of digamma function $\psi[z]$ at z , and $\Gamma[z]$ denotes the gamma function. Applying the Bethe-Peierls boundary condition by comparing with the free-space scattering wave solution

$$J_0(k\rho) - \tan(\delta)Y_0(k\rho) \rightarrow \frac{\log(2a_{2D}) - \gamma - \log(r)}{\log(ka_{2D})} \quad (8)$$

yields the transcendental equation

$$\psi[v] = C^{-1} \equiv \log \left(\frac{\ell_B^2}{2a_{2D}^2} \right), \quad (9)$$

matching Ref. [44], with results shown as the black solid curves of Fig. 1.

Not only can the polaron energy be exactly determined, but the entire polaron spectrum can also be determined analytically using the functional determinant approach [34,46–50]. This spectrum is experimentally accessible via radiofrequency spectroscopy, where impurities transition between noninteracting and interacting hyperfine states [51]. In the linear response regime, it corresponds to the spectral function $A(\omega)$, which is proportional to the imaginary part of the retarded impurity Green function. At zero temperature, with only the lowest LLs populated, the spectral function is given by $A(\omega) = \text{Re} \int_0^\infty S(t) e^{i\omega t} dt / \pi$, where $S(t) = \sum_v e^{-i\omega_c t} / (v^2 \psi[1, v])$ is the many-body overlap function (details are in the SM [43]). The resulting $A(\omega)$, shown in Fig. 2, reveals multiple polaron branches at positive frequencies (highlighted in the inset, logarithmic scale), all with infinitesimal widths. This unique behavior, unlike conventional repulsive polarons, likely arises from the suppression

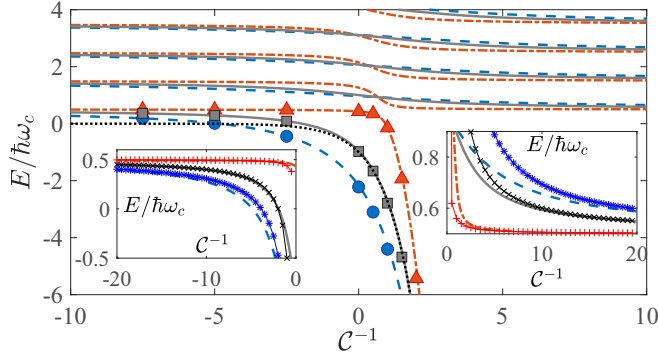


FIG. 1. Shifted single-particle energy levels due to impurities as a function of the regularized coupling constant $C^{-1} \equiv \log[\ell_B^2/(2a_{2D}^2)]$. Black solid curves show results for a single impurity, while blue dashed and red dash-dotted curves correspond to E_+ and E_- [defined below Eq. (10)] for two impurities separated by $R = 0.5\ell_B$. The black dotted curve represents the two-body binding energy $-\hbar^2/2ma_{2D}^2$. Black squares, blue circles, and red triangles represent numerical results using 2D van der Waals potentials (see text). Insets compare with analytical expressions from Eqs. (3) and (4) in the weak-coupling limit, depicted as thin curves with symbols.

of particle-hole excitations due to energy gaps between LL manifolds.

For multiple impurities, inspired by the singularity regularization approach [44], we express the total wave function as a superposition of single impurity states: $\Psi(\mathbf{r}; E) \sim \sum_s c_s \Phi_s(\mathbf{r} - \mathbf{r}_s; \nu)$, where $\Phi_s(\mathbf{r} - \mathbf{r}_s; \nu) = e^{i\varphi_s} e^{-\xi_s/2} U(\nu, 1, \xi_s)$ with $\xi_s = (\mathbf{r} - \mathbf{r}_s)^2/(2\ell_B^2)$, and $\varphi_s = |\mathbf{r}_s \times \mathbf{r}|/(2\ell_B^2)$ accounts for the harmonic states around each impurity (see the SM for details [43]). $\Psi(\mathbf{r}; E)$ is an eigenstate with eigenenergy $\nu = -E/\hbar\omega_c + 1/2$ except at impurity positions $\mathbf{r}_s$. Imposing the Bethe-Peierls boundary conditions yields S transcendental equations for the coefficients c_s ,

$$(\psi[\nu] - C_s^{-1})c_s = \Gamma[\nu] \sum_{s' \neq s} \Phi(\mathbf{r}_s - \mathbf{r}_{s'}; \nu)c_{s'}, \quad (10)$$

whose solutions determine the eigenenergies. Figure 1 shows results for two identical impurities separated by $R = 0.5\ell_B$,

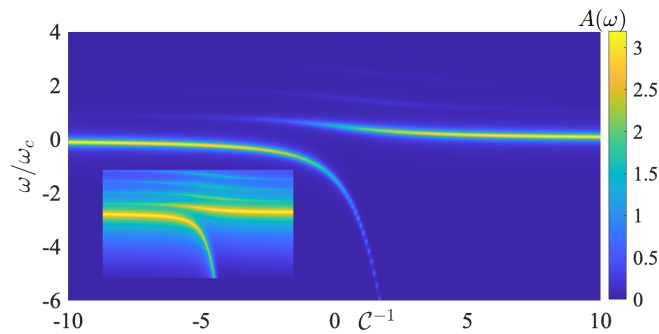


FIG. 2. The spectral function $A(\omega)$ for a single infinitely heavy impurity immersed in a Fermi sea with fully populated lowest LLs at zero temperature, as a function of frequency ω and C^{-1} . The inset shows $A(\omega)$ in a logarithmic scale. All peaks have infinitesimally small widths, with an artificial spectral broadening of $0.1\omega_c$ added for visibility.

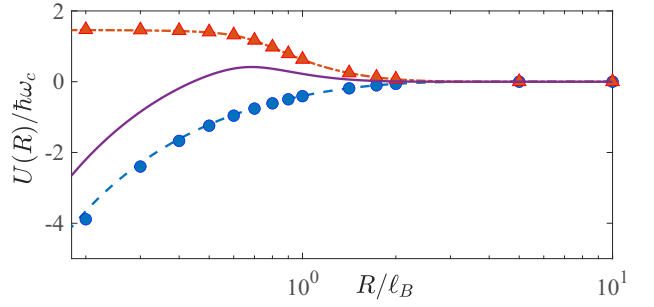


FIG. 3. Potential between two heavy impurities mediated by light fermions in the strong-coupling limit ($C^{-1} \rightarrow 0$). Blue dashed and red dash-dotted curves represent symmetric and antisymmetric state potentials, respectively. The purple solid curve shows the polaron interaction $U_+(R) + U_-(R)$ for $N \geq 2$ fermions. Blue circles and red triangles denote numerical results, as in Fig. 1.

revealing two energy shifts in each manifold, consistent with our previous analysis. The lowest eigenenergies, $E_{\pm}$, correspond to the symmetric and antisymmetric wave functions $\Psi_{\pm}(\mathbf{r}) \sim \Phi(\mathbf{r} + \mathbf{R}/2) \pm \Phi(\mathbf{r} - \mathbf{R}/2)$. The insets in Fig. 1 show excellent comparison with analytical expressions Eqs. (3) and (4) in the weak-coupling limit (see the SM for details [43]).

Next, we analyze the distance dependence of the mediated interaction between two impurities. Figure 3 shows potentials in the strong-coupling limit ($C^{-1} \rightarrow 0$). The symmetric state potential U_+ asymptotically behaves as $-\exp(-\tilde{c}R^2/\ell_B^2)$ with fitted coefficient $\tilde{c} \approx 0.275$ for $R \gg \ell_B$, and as $-\tilde{d}\ell_B/R$ ($\tilde{d} \approx 0.8$) for $R \ll \ell_B$. For a single light fermion, $U_+(R)$ corresponds to the Born-Oppenheimer potential. The antisymmetric state potential $U_-(R)$ has a repulsive tail $\exp(-\tilde{c}R^2/\ell_B^2)$ at large distances and saturates at short range. For $N \geq 2$ fermions, the induced polaron interaction $U_+(R) + U_-(R)$ vanishes for $R \gg \ell_B$, becomes strongly attractive as $1/R$ at short-range, and features a barrier at $R \approx 0.685\ell_B$.

To verify that our analytical results are robust against the short-range details of realistic interactions, we finally conduct direct, large-scale numerical calculations using 2D van der Waals potentials with a lengthscale $l_{\text{vdW}} \ll \ell_B$ (see the SM for details and definitions [43]). The numerical results, shown in Figs. 1 and 3 as symbols, align perfectly with our analytical results, except for slight deviations at very small $R \sim l_{\text{vdW}}$, as expected.

Conclusions. We derived analytical expressions for polaron energies and induced interactions between heavy impurities mediated by light fermions in the weak-coupling regime and exactly solved the polaron-polaron interactions numerically for arbitrary coupling. At strong coupling, the short-range attractive potential suggests that impurities within ℓ_B will bind into molecules, while those farther apart do not interact. These exact results are directly testable in fermionic quantum Hall systems, either in cold-atom experiments [36] or 2D condensed-matter setups [40]. Our analysis of polaron-polaron interactions with highly degenerate levels is broadly applicable, including scenarios with bosonic mediators. Since bosons typically occupy the lowest energy state at zero

temperature, our results could inform the engineering of Landau level occupations in experiments with rotating Bose-Einstein condensates [35,37–39] with pointlike localized potentials created by lasers.

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