

Anharmonic collective oscillations and gap generation from thermal fluctuations in isotropic classical spin systems

Anna Fancelli ^{1,2,*} Matías G. Gonzalez ^{1,2,3} Subhankar Khatua ^{4,5,6} Bella Lake ^{1,7} Michel J. P. Gingras ^{4,8}
Jeffrey G. Rau ⁵ and Johannes Reuther ^{1,2,†}

¹*Helmholtz-Zentrum Berlin für Materialien und Energie, Hahn-Meitner-Platz 1, 14109 Berlin, Germany*

²*Dahlem Center for Complex Quantum Systems and Fachbereich Physik, Freie Universität Berlin, Arnimallee 14, 14195 Berlin, Germany*

³*Institute of Physics, University of Bonn, Nussallee 12, 53115 Bonn, Germany*

⁴*Department of Physics and Astronomy, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada*

⁵*Department of Physics, University of Windsor, 401 Sunset Avenue, Windsor, Ontario N9B 3P4, Canada*

⁶*Institute for Theoretical Solid State Physics, IFW Dresden and Würzburg-Dresden Cluster of Excellence ctd.qmat, Helmholtzstr. 20, 01069 Dresden, Germany*

⁷*Institut für Festkörperforschung, Technische Universität Berlin, 10623 Berlin, Germany*

⁸*Waterloo Institute for Nanotechnology, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada*

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Spin waves are the fundamental excitations in magnetically ordered spin systems and are ubiquitously observed in magnetic materials. However, the standard understanding of spin waves as collective spin oscillations in an effective harmonic potential does not consider the possibility of soft modes, such as those due to an effective *quartic* potential. In this work, we show that such quartic potentials arise under very general conditions in a broad class of isotropic spin systems without a fine-tuning of the interaction parameters. Considering models with spin-spiral ground states in two and three spatial dimensions, we numerically demonstrate that quartic amplitude spin oscillations produce a fluctuation-induced spin-wave gap which *grows* with temperature according to a characteristic power-law. In conjunction with a phenomenological theory, the present work provides a general theoretical framework for describing soft spin modes. This framework generalizes earlier treatments of soft-spin dynamics and gap generation in order-by-disorder systems arising from accidental classical ground-state degeneracies, and highlights the important role of finite-size effects relevant to numerical studies. Our predictions of a temperature-dependent gap in spiral spin systems could be tested in inelastic neutron scattering experiments, providing direct spectroscopic evidence for thermal effects arising from soft spin modes in magnetic materials.

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Introduction. The notion of a quasiparticle is widely used to describe collective excitations in many-body systems [1]. This term refers to a common situation in which, similarly to fundamental particles in high-energy physics, a collective excitation in a solid-state system can be described by characteristic properties such as its effective mass or, more generally, its dispersion relation. Well-known examples of quasiparticles include phonons, which arise from collective lattice vibrations, and magnons, which result from collective spin precessions [2]. In both cases, the quasiparticles can be classically understood as oscillations in an effective harmonic potential $V(x) = \alpha x^2$ with a frequency ω , and with the quantum mechanical excitation energy E given by $E = \hbar\omega$.

In this Letter, we address the fundamental question of how the concept of a quasiparticle changes when the effective potential is softer than quadratic and how such anharmonicity can be exposed in experiments. For concreteness, we specifically consider the case of *quartic* potentials where $V(x) \propto x^4$ [3]. Such potentials and the associated collective excitations are of relevance in quite distinct physical systems, including trapped Bose-Einstein condensates [4,5], chaotic systems [6,7], soft phonons [8,9], and, in high-energy physics, in the context of the Higgs self-coupling [10–12]. Here, we investigate quartic potentials in classical Heisenberg spin systems, where they emerge under fairly general conditions and without the necessity of a situation with fine-tuned model parameters to eliminate the harmonic αx^2 component. Our starting point is a Heisenberg spin Hamiltonian that has several degenerate symmetry-related planar spin-spiral ground states. We show that in a system exhibiting a planar spin-spiral long-range ordered state at one of several symmetry-related ground-state ordering wave vectors, an oscillation of the spin component perpendicular to the spiral ordering plane, taken at the wave vector of a different ground-state spiral, has a quartic dependence of the excitation energy on the amplitude. We henceforth refer to such an oscillation as a *quartic* oscillation.

*Contact author: anna.fancelli.work@gmail.com

†Contact author: reuther@zedat.fu-berlin.de

Since a system with a purely quartic potential has a vanishing harmonic component ($\alpha = 0$), one might naively expect it to display gapless excitations. While this is true in the zero-temperature limit, our numerical simulations reveal that thermal fluctuations dynamically generate a spin-wave gap Δ ; a so-called pseudo-Goldstone mode. This implies a remarkable situation in which the elementary properties of a quasiparticle, such as its effective mass, are no longer determined by the system's microscopic parameters, but instead become state-dependent, allowing them to be modified by external parameters such as the temperature T . Our numerical simulations of finite-size spin systems in two and three spatial dimensions indicate a characteristic scaling of this fluctuation-induced spin-wave gap of the form $\Delta(T) \sim T^\mu$ where $\mu = 1/4$ ($\mu \approx 1/2$) at low (intermediate) temperatures. We explain these results using a phenomenological theory that takes into account the quartic oscillations and their coupling to other quadratic modes that inevitably exist in the system. Specifically, this theory reveals an intriguing finite-size effect resulting from the small number [$O(1)$] of quartic modes that couple to $O(N)$ quadratic modes (where N is the total number of spins). Consequently, although the low-temperature gap scaling with $\mu = 1/4$ is a direct consequence of the quartic oscillation, the low-temperature regime in which it occurs vanishes in the thermodynamic limit, leaving only a thermally generated gap with $\mu = 1/2$.

This detailed analysis of the temperature-dependent scaling of a spin-wave gap associated with quartic spin oscillations uncovers a general and robust mechanism for the emergence of pseudo-Goldstone spin-wave excitations and their gapping through fluctuation effects. Whereas previous theoretical work tied fluctuation-induced gap generation to thermal or quantum order-by-disorder effects acting on exact accidental ground-state zero modes [13–16], our results demonstrate that such modes and their dynamically generated gaps do not require fine-tuned degeneracies. Because exact zero modes are rare in real materials, our results significantly extend the physical relevance of pseudo-Goldstone physics and how these become gapped thanks to fluctuations. We show that quartic potentials are sufficient to produce these effects, demonstrating that pseudo-Goldstone modes can arise in a substantially broader range of systems.

On the experimental front, spin-wave gaps in the dynamical response have already been detected in neutron scattering experiments on magnetic materials, such as the Heisenberg pyrochlore helimagnet ZnCr_2Se_4 [17,18] and the half-Heusler compound GdPtBi [19]. Here, we provide the theoretical basis for these observations by relating them to quartic oscillations using a classical description, in part motivated by the “large spin” $S = 3/2$ and $S = 7/2$ values of Cr and Gd in ZnCr_2Se_4 and GdPtBi , respectively, to predict the ensuing temperature dependence of the spin-wave gap.

Spin systems and quartic perturbations. To set the stage for our study of anharmonic spin oscillations in isotropic spin systems, we first consider a minimal two-dimensional (2D) square lattice model that supports soft modes in a quartic potential. Further below, we extend this analysis to a three-dimensional (3D) cubic lattice to confirm that our results apply to a wider range of systems and the more common 3D situation. In the square lattice case, we consider a Heisen-

berg Hamiltonian with exchange couplings extending up to third-nearest neighbors, with ferromagnetic $J_1 < 0$ and anti-ferromagnetic $J_2, J_3 > 0$ couplings:

$$H = J_1 \sum_{\langle i,j \rangle_1} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{\langle i,j \rangle_2} \mathbf{S}_i \cdot \mathbf{S}_j + J_3 \sum_{\langle i,j \rangle_3} \mathbf{S}_i \cdot \mathbf{S}_j. \quad (1)$$

Here, $\langle \dots \rangle_n$ indicates the sum over the n th-nearest neighbors with the spins treated classically as three-dimensional vectors of unitary norm, $|\mathbf{S}_i| = 1$. The ground-state spin configuration for Eq. (1), $\mathbf{S}_i^{\text{GS}}$, is a coplanar spiral [20],

$$\mathbf{S}_i^{\text{GS}} = \begin{pmatrix} \cos(\mathbf{Q} \cdot \mathbf{r}_i + \phi) \\ \sin(\mathbf{Q} \cdot \mathbf{r}_i + \phi) \\ 0 \end{pmatrix}, \quad (2)$$

where $\mathbf{Q} = (Q_x, Q_y)$ is the spiral wave vector, $\mathbf{r}_i$ is the position of site i , and $\phi \in [0, 2\pi)$ is a phase. Without loss of generality, we assume the spiral to be in the x - y plane and also set $\phi = 0$. The wave vector $\mathbf{Q}$ is then determined by substituting the spiral spin configuration into the Hamiltonian and identifying the $\mathbf{Q}$ that minimizes the system's energy. We fix $J_1 = -1$, $J_2 = 0.476393205$, and $J_3 = \frac{1}{2}J_2 - \frac{1}{5}$ such that the ground state is a one-dimensional spiral with two possible commensurate wave vectors $\mathbf{Q} = (0, \pm \frac{2\pi}{5})$ and $(\pm \frac{2\pi}{5}, 0)$, related by a $\pi/2$ lattice rotation [see Fig. 1(a)]. While this simplifies numerical calculations tremendously by enabling the usage of periodic boundary conditions, the commensurability of the spiral is irrelevant to the physical properties we study herein. Specifically, our predictions apply to the entire yellow region in Fig. 1(a), and generally to any isotropic Heisenberg system with symmetry-related spin-spiral ground states, such that no fine-tuning is needed.

The continuous $O(3)$ symmetry of the Hamiltonian cannot be broken at finite temperatures in two dimensions, excluding a transition into a long-range magnetically ordered phase [21]. However, the system undergoes a finite-temperature Ising transition associated with the discrete $\mathbb{Z}_2$ symmetry between the two (energetically degenerate) spiral wave vector directions [22]. Specifically, the one-dimensional spiral phase breaks the square lattice's fourfold rotational symmetry down to a twofold symmetry, with the spiral oriented along either of the Cartesian directions [see Fig. 1(b)]. The selected ordering wave vector, $\mathbf{Q}_B$, is responsible for the Bragg peaks of the ground-state order, and we refer to the wave vector that is not selected as $\mathbf{Q}_{aB}$, where “aB” stands for anti-Bragg.

This system has two key properties that are essential in harboring the physics of anharmonic collective excitations that we aim to explore. First, the system supports spin oscillations in an effective quartic potential which correspond to spin fluctuations in the direction *perpendicular* to the planar spiral ground state in Eq. (2) and characterized by the wave vector $\mathbf{Q}_{aB}$. Specifically, we consider $\mathbf{S}_i^{\text{quartic}}$ as a perturbed state about the long-range ordered ground state

$$\mathbf{S}_i^{\text{quartic}} = \frac{1}{N_i} \left[\mathbf{S}_i^{\text{GS}} + \delta \begin{pmatrix} 0 \\ 0 \\ \cos(\mathbf{Q}_{aB} \cdot \mathbf{r}_i + \phi') \end{pmatrix} \right], \quad (3)$$

see Fig. 1(b) for an illustration. Here, δ is the perturbation amplitude, N_i is the site (i)-dependent normalization to ensure $|\mathbf{S}_i^{\text{quartic}}| = 1$, and $\phi' \in [0, 2\pi)$. We show in the Supplemental

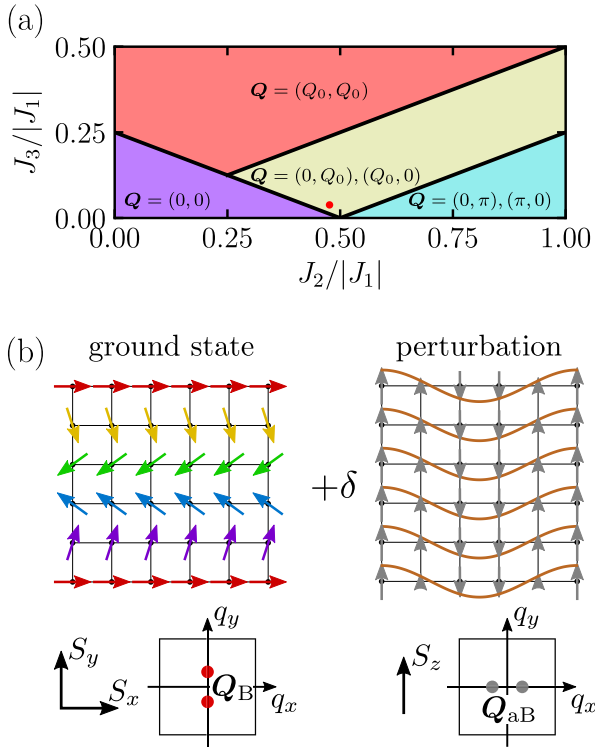


FIG. 1. (a) Zero-temperature phase diagram for the J_1 - J_2 - J_3 Heisenberg model on the square lattice, with ferromagnetic $J_1 < 0$. Four different ground states are realized: ferromagnetic order with $\mathbf{Q} = (0, 0)$ (purple region), stripe order with $\mathbf{Q} = (0, \pi), (\pi, 0)$ (light blue region), one dimensional spiral with $\mathbf{Q} = (0, Q_0), (Q_0, 0)$ (yellow region), two dimensional spiral with $\mathbf{Q} = (Q_0, Q_0)$ (red region). The red point corresponds to the specific values of J_2 and J_3 used in this work. (b) Illustration of the ground-state spiral configuration in the x - y plane corresponding to a $\mathbf{Q}_B$ wave vector (colored arrows), plus the quartic perturbation along the z axis with a $\mathbf{Q}_{AB}$ wave vector. These two wave vectors are symmetry-related and are chosen as $\mathbf{Q}_B = (0, \pm \frac{2\pi}{5})$ and $\mathbf{Q}_{AB} = (\pm \frac{2\pi}{5}, 0)$ in the illustrated state. This is a pictorial representation of Eq. (3) in the main text.

Material (SM) [23] that such a perturbed noncoplanar state always has a vanishing δ^2 term in the energy, regardless of the range of the interactions, as long as the Hamiltonian conserves the C_4 , reflection, and translation symmetries of the square lattice. For our particular set of interactions, the energy per site is $e(\delta) = e_{\text{GS}} + 0.039\delta^4 + O(\delta^6)$ [23]. Second, aside from three zero modes arising from perturbations with the wave vector $\mathbf{Q}_B$ —which correspond to global spin rotations that preserve the spiral order and would represent Goldstone modes at $\mathbf{Q}_B$ were the system magnetically long-range ordered at $T > 0$ —the system does not exhibit any additional modes softer than the quartic ones. In particular, the system has no accidental ground-state degeneracies [24] leading to zero modes, such as occurs in the context of order-by-disorder phenomena [15]. Therefore, the system considered isolates the effects of quartic modes, which have heretofore attracted limited attention.

To further establish the general scope and quantitative validity of our findings, we extend our analysis to a 3D classical Heisenberg model on a cubic lattice. As in the 2D case, we consider a model with general spiral ground states. This is

achieved by using a Hamiltonian with the following interaction parameters: $J_1 = -1$, $J_2 = 1/(4 + 1.8 \cos \gamma)$, and $J_4 = 0.45/(4 + 1.8 \cos \gamma)$, where J_n is the coupling between n th-nearest neighbors [17]. In this case, the ground state is also given by a coplanar one-dimensional spiral given by Eq. (2), but with ordering wave vectors $\mathbf{Q} = (0, 0, \pm\gamma)$, $(0, \pm\gamma, 0)$, or $(\pm\gamma, 0, 0)$. We fix $\gamma = 2\pi/5$, yielding a five-site periodic spiral again. Below a critical temperature T_c , one of the three spiral configurations is selected. The perpendicular spin fluctuations at the two remaining wave vectors correspond to quartic oscillations.

There are two main differences between this 3D model and the 2D model discussed above. First, quartic oscillations can now involve contributions from *two* wave vectors, corresponding to the nonselected spirals in the ordered state. Second, in 3D, the $\text{SO}(3)$ spin rotation symmetry is explicitly broken in the ordered phase below T_c , giving rise to true magnetic long-range order (we find that both types of symmetry breaking, lattice rotation and spin rotation, occur at the same critical temperature T_c).

Numerical simulations. To investigate the dynamic signatures of quartic modes, we perform classical Monte Carlo (cMC) and spin molecular dynamics (sMD) [25] simulations on the models introduced above. Using equilibrated states generated from cMC as initial configurations for the sMD simulations, we compute the spin structure factor, $\mathcal{S}(\mathbf{Q}, \omega)$, as a function of temperature [23]. Specifically, our focus is on the determination of the excitation gap Δ at the wave vector $\mathbf{q} = \mathbf{Q}_{AB}$ of the quartic perturbation in Eq. (3), which can be extracted by fitting the spectra with a Lorentzian function [23].

In the 2D case, the cMC results show that at $T_c = 0.071(3)$ the system breaks the fourfold lattice rotation symmetry and develops a one-dimensional spiral along the $\hat{x}$ or $\hat{y}$ direction. Below T_c , $\mathcal{S}(\mathbf{Q}_{AB}, \omega)$ displays a peak at a finite frequency that decreases with temperature [see Fig. 2(a)]. Extracting the value of the gap for each temperature results in Fig. 2(b), where we show the gap Δ as a function of temperature in a log-log plot for several square systems with linear sizes $L = 20, 40, 60$, and 80 . This gap can be attributed to an anharmonic effect because linear spin-wave theory would predict a gapless mode at this point as it treats quartic oscillations as zero modes [17].

In the 3D case, we use cubic systems with $N = L^3$ spins to compute $\mathcal{S}(\mathbf{Q}_{AB}, \omega)$. The cMC calculations show that at $T_c \simeq 0.17(1)$, the system orders in a one-dimensional spiral along $\hat{x}$, $\hat{y}$, or $\hat{z}$. As expected, the dynamical structure factor at $\mathbf{q} = \mathbf{Q}_{AB_1}$ and $\mathbf{q} = \mathbf{Q}_{AB_2}$ displays a peak at the same frequency [23]. The results are shown in Fig. 2(c) for $L = 10$ and 15 . The similarities between the results for the 2D and 3D models show that the gap behavior is a characteristic property of a system that possesses quartic oscillations.

Overall, our results can be summarized by three key observations, which we will explain in detail below

(1) At low temperatures, the gap is well-described by the scaling $\Delta \sim T^{1/4}$ for both the 2D and 3D models, as can be seen in Figs. 2(b) and 2(c), respectively.

(2) At higher temperatures, the gap increases *faster* than $\Delta \sim T^{1/4}$. Remarkably, and maybe counter-intuitively, the largest gap is found right below the phase transition, although

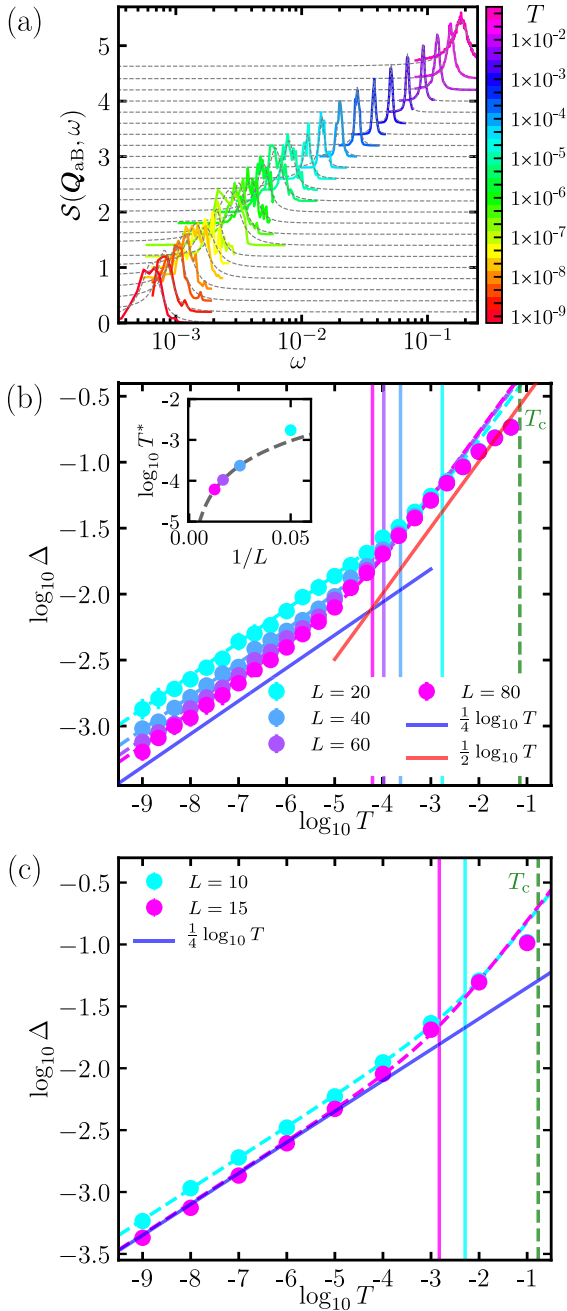


FIG. 2. (a) Normalized dynamical structure factor $S(q, \omega)$ at $q = Q_{\text{AB}}$ for different temperatures, obtained using sMD simulations for $L = 80$ in the 2D case. The gray-dashed lines show Lorentzian function fits. The position of the maximum indicates the gap Δ . (b) Logarithm of the gap Δ as a function of the logarithm of the temperature in the 2D case. To guide the eye, we plot two reference lines with slopes of $1/4$ (blue line) and $1/2$ (red line). The colored points and the dashed lines correspond to the data (with error bars defined in the Supplemental Material [23]) and fitting function [Eq. (6)], respectively. The vertical lines correspond to $\log_{10} T^*$ with $T^* = 6\lambda/\alpha^2$, where the fitted function changes slope for the different sizes. The inset shows the finite-size scaling analysis for this quantity. (c) Same information as in panel (b), but for the 3D model.

the order nearly vanishes in this regime. This indicates that the gap at Q_{AB} is a robust feature of the ordered phase.

(3) Finite-size effects are more prominent at low T , where $\Delta \sim T^{1/4}$, as seen in Figs. 2(b) and 2(c).

The $T^{1/4}$ behavior. To explain the origin of the exponent $1/4$ observed in Fig. 2 at low temperatures, we model the system's dynamics by a simple classical oscillation in a quartic potential and in thermal equilibrium, described by the Hamiltonian

$$H_4 = \frac{p^2}{2m} + \lambda x^4, \quad (4)$$

where the first term is an effective kinetic term. Then, by a straightforward dimensional analysis, one finds that the frequency of the oscillation scales as $\omega \sim \lambda^{1/4} E^{1/4}$ for an excitation of energy E . Associating the frequency with the gap, $\Delta \sim \omega$, and taking $E \sim T$, we obtain $\Delta \sim T^{1/4}$, explaining our numerical observations (see the SM for an alternative numerical approach to the $T^{1/4}$ gap scaling [23]). The effective Hamiltonian H_4 accounts only for the *energetic* contribution of the quartic mode, and is, therefore, expected to be accurate only at very low temperatures.

Entropic effects. At moderate finite temperatures, entropic effects must be considered. These contributions can be effectively modeled by including an additional $\sim T x^2$ term in Eq. (4), which arises from integrating out other spin-wave modes that interact with the quartic mode. This leads to a minimal modified effective Hamiltonian

$$H_{\text{eff}} = H_4 + \alpha T x^2 = \frac{p^2}{2m} + \lambda x^4 + \alpha T x^2. \quad (5)$$

The additional term is negligible at very low temperatures, but it becomes comparable to, and ultimately dominates over the quartic term as temperature increases. To determine the temperature dependence of the frequency of the oscillator in Eq. (5), we perform a mean-field decoupling of the x^4 term [23], yielding

$$\Delta = \sqrt{\frac{\alpha T}{m}} \left(1 + \sqrt{1 + \frac{2T^*}{T}} \right)^{1/2}, \quad (6)$$

where $T^* \equiv 6\lambda/\alpha^2$. This result has two interesting limits: i) for $T \gg T^*$, where the entropic contribution dominates over the quartic term, $\Delta \sim \sqrt{T}$, and ii) for $T \ll T^*$, where the quartic term remains dominant, $\Delta \sim T^{1/4}$. Thus, the temperature T^* sets a rough crossover energy scale below which the quartic mode gap scales as $T^{1/4}$ and above which it follows $\sqrt{T}$. This result not only explains our observation that the gap increases more rapidly at higher temperatures, but also provides a phenomenological equation that can be fitted to our numerical simulations.

In Figs. 2(b) and 2(c), we fit our numerical results for the 2D and 3D models to the prediction of the phenomenological theory of Eq. (6) [23]. For both models, we find a good agreement between the simulations and the phenomenological theory, especially at low temperatures. The region where the phenomenological model predicts a $\sim \sqrt{T}$ behavior is harder

to identify, but is still identifiable in our numerical data. Specifically, the red line in Fig. 2(b) corresponds to a plain $\sim\sqrt{T}$ function demonstrating that in a small intermediate temperature regime, our results indeed follow this behavior approximately. The vertical lines in Figs. 2(b) and 2(c) show the characteristic crossover temperature T^* obtained from our fits.

Finally, for both the 2D and 3D models, significant deviations between the simulations and the phenomenological model occur near the critical temperature T_c . This is expected, since the order parameter fluctuations become strong and the fundamental underlying assumption of a state selection with wave vector $\mathbf{Q}_B$ ceases to be fulfilled.

Finite-size effects. As stated previously, finite-size effects become important at low temperatures, which is also evident from the L -dependent crossover temperatures T^* highlighted in Figs. 2(b) and 2(c). To explain this, we formally derive here the phenomenological model of Eq. (5), presented above based on heuristic arguments. The first step consists of writing the spins in terms of canonically conjugate variables $\{x_i, p_j\} = \delta_{ij}$,

$$\mathbf{S}_i = \sqrt{1 - \frac{1}{4}(x_i^2 + p_i^2)} x_i \hat{\mathbf{x}}_i + \sqrt{1 - \frac{1}{4}(x_i^2 + p_i^2)} p_i \hat{\mathbf{y}}_i + \left[1 - \frac{1}{2}(x_i^2 + p_i^2)\right] \hat{\mathbf{z}}_i, \quad (7)$$

where $\{\hat{\mathbf{x}}_i, \hat{\mathbf{y}}_i, \hat{\mathbf{z}}_i\}$ is any right-handed *local* orthogonal frame. This is a faithful representation of spins since $\{x_i, p_j\} = \delta_{ij}$ ensures the canonical Poisson bracket relation for the components of $\mathbf{S}_i$. Inserting Eq. (7) into the spin Hamiltonians above, with the local frame aligned with the ground-state spiral ordering at wave vector $\mathbf{Q}_B$, and Taylor expanding in x_i and p_i , allows us to isolate the quartic mode and derive the phenomenological model (see details in the SM [23]). The key finding is that λ in Eq. (5) depends on the system size as $\lambda \sim 1/N$. This result can be traced back to a prefactor $1/\sqrt{N}$ in the Fourier transform, and implies that the crossover temperature behaves as $T^* \sim 1/N$, which is confirmed by our numerical results [inset of Fig. 2(b)]. This also affects the low-temperature regime, where the gap behaves as $\Delta \sim (T/N)^{1/4}$.

This result not only explains our observations in the numerical calculations, but also provides important nontrivial information about the approach to the thermodynamic limit $N \rightarrow \infty$. On one hand, the gap closes in the low-temperature regime, $\Delta \rightarrow 0$ for $N \rightarrow \infty$. At the same time, the crossover temperature T^* vanishes for $N \rightarrow \infty$. This implies that only the $\Delta \sim \sqrt{T}$ regime arising from entropic effects survives for large enough systems. Interestingly, this is the same result that was previously found for order-by-thermal-disorder cases with (type-I) pseudo-Goldstone mode with dispersion $\omega \sim |q|$ [15].

Although the quartic term in the energy is suppressed by a factor of $1/N$ for individual magnon excitations, it can re-emerge in other settings. For example, in the presence of true or accidental degeneracies, one can form wide domain walls connecting states within the degenerate manifold at vanishing (or nearly vanishing) cost [26]. In our case, however, domain walls between the two symmetry-related incommensurate orders cost $O(J)$ because of the quartic potential. Relatedly,

interactions among parametrically pumped magnons in ferromagnets can overcome the same $1/N$ suppression when the mode population is large, leading to a range of nonlinear dynamical phenomena [27].

Conclusion. In this Letter, we demonstrated that quartic potentials which generically arise in isotropic spiral spin systems lead to a distinctive spin-wave gap of the form $\Delta \sim \sqrt{T}$ in the thermodynamic limit. Interestingly, this gap scaling does not result directly from spin oscillations in the quartic potential, but rather from entropic effects that consider the interaction between the quartic mode and the other (macroscopic number of) harmonic modes. The absence of an explicit signature of quartic oscillation in the thermodynamic limit (which generates $\Delta \sim T^{1/4}$ for finite N) reflects a subtle finite-size effect, as demonstrated analytically and numerically.

In real materials, spin-isotropic Heisenberg interactions are often supplemented by single-ion anisotropy, symmetric anisotropic exchange, or Dzyaloshinskii–Moriya interactions. Such terms generally allow a quadratic contribution to the energy of the anti-Bragg collective coordinate and would, therefore, produce a conventional anisotropy-induced gap in addition to the thermally generated gap discussed here. Nevertheless, in weakly anisotropic spiral magnets, the effective potential may be viewed schematically as $V_{\text{eff}}(x, T) \sim K_{\text{aniso}} x^2 + \lambda x^4 + \alpha T x^2$. When K_{aniso} is sufficiently small, the thermally generated stiffness may dominate over an intermediate temperature window below the critical temperature, so that the mechanism discussed in this work may remain experimentally relevant. This situation may be particularly favorable in systems with weak spin-orbit-induced anisotropies.

The anti-Bragg mode studied here may be viewed as a many-body equilibrium analog of a thermally dressed nonlinear oscillator. Similarly to Duffing-type oscillator problems [28], anharmonicity makes the characteristic frequency amplitude dependent, so thermal fluctuations generate a finite dynamical frequency scale. In contrast to the standard driven and damped Duffing problem, the effective harmonic term in the present case is not a bare stiffness or a drive-induced effect, but an entropic stiffness generated by integrating out the remaining spin-wave modes.

The firm understanding of the role of quartic oscillations developed here for classical spin systems paves the way for future investigations of similar quantum effects at zero and nonzero temperature [16]. By being controlled by the inverse spin length $1/S$, such effects are expected to conspire with thermal fluctuations to again generate dynamical spin-wave gaps Δ . The ultimate goal of such studies will be to investigate the impact of finite spin lengths $1/S > 0$ on the temperature dependence $\Delta(T)$ in isotropic spiral spin systems and, in a parallel pursuit, seek to expose such behavior in real candidate materials such as ZnCr_2Se_4 [17,18] and GdPtBi [19].

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Data availability. The data that support the findings of this article are openly available [40].

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