

Erratum: Transport theory and spin-transfer physics in d -wave altermagnets **[Phys. Rev. B **111**, 064422 (2025)]**

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The magnetic torques (19) and (21) have been calculated under the assumption that the itinerant spin density follows the localized spin background adiabatically, so that they represent the *adiabatic* contributions to the spin torques $\partial_t \mathbf{m}$ and $\partial_t \mathbf{n}$, respectively. Other terms not captured by our Hamiltonian approach originate in the spin-mistracking of the itinerant carriers and therefore constitute the *nonadiabatic* contributions to the magnetic torques. In this regard, the phenomenological constants η_{FL} and η_{DL} must be replaced by η_{ad} and η_{nad} for the sake of notational accuracy.

The spin torques acting on the Néel order $\mathbf{n}$ and the macroscopic spin density $\mathbf{m}$ must be invariant under the combination of sublattice exchange ($A \leftrightarrow B$) and the $C_{4(001)}$ rotation in the real space. In this regard, the Néel order and $j_x \partial_y + j_y \partial_x$ are odd under this symmetry combination, whereas the total magnetization is even. Therefore, the nonadiabatic term in Eq. (19) is wrong. A correct phenomenological analysis yields the expressions $\boldsymbol{\tau}_m = \frac{1}{s} \eta_{\text{nad}} \mathbf{m} \times (j_x^e \partial_y + j_y^e \partial_x) \mathbf{n}$ and $\boldsymbol{\tau}_n = \frac{1}{s} \eta_{\text{nad}} \mathbf{n} \times (j_x^e \partial_y + j_y^e \partial_x) \mathbf{n}$ for the leading altermagnetic nonadiabatic current-induced contributions to the magnetic torques. Therefore, the altermagnetic spin-transfer torques acting on the order parameters of a d -wave altermagnet can be cast, phenomenologically, as

$$\boldsymbol{\tau}_m = \eta_{\text{ad}}^m (j_x^e \partial_y + j_y^e \partial_x) \mathbf{n} + \frac{1}{s} \eta_{\text{nad}} \mathbf{m} \times (j_x^e \partial_y + j_y^e \partial_x) \mathbf{n}, \quad (1)$$

$$\boldsymbol{\tau}_n = \eta_{\text{ad}}^n (\vec{j}^e \cdot \vec{\nabla}) \mathbf{n} + \frac{1}{s} \eta_{\text{nad}} \mathbf{n} \times (j_x^e \partial_y + j_y^e \partial_x) \mathbf{n}, \quad (2)$$

in agreement with Ref. [44]. Equations (1) and (2) yield the identities $[\underline{L}^{(\text{mq})}]_{ji} = \vartheta \sigma_{x|i\beta} [\eta_{\text{ad}}^m \partial_\beta n^j + (\eta_{\text{nad}}/s) \epsilon_{jlk} m^l \partial_\beta n^k]$ and $[\underline{L}^{(\text{nq})}]_{ji} = \vartheta [\eta_{\text{ad}}^n \partial_i n^j + (\eta_{\text{nad}}/s) \sigma_{x|i\beta} \epsilon_{jlk} n^l \partial_\beta n^k]$, respectively, for the off-diagonal blocks of the Onsager matrix

$$\begin{pmatrix} \partial_t \mathbf{m} \\ \partial_t \mathbf{n} \\ \vec{j}^e \end{pmatrix} = \begin{pmatrix} \cdot \star \cdot & \cdot \star \cdot & \underline{L}^{(\text{mq})} \\ \cdot \star \cdot & \cdot \star \cdot & \underline{L}^{(\text{nq})} \\ \underline{L}^{(\text{qm})} & \underline{L}^{(\text{qn})} & \vartheta \end{pmatrix} \begin{pmatrix} \mathbf{f}_m \\ \mathbf{f}_n \\ \vec{E} \end{pmatrix}, \quad (3)$$

relating the thermodynamic fluxes and forces between the spin (including both the total magnetization and the Néel order) and charge sectors. Again, from the Onsager reciprocity principle and the expressions $\mathbf{f}_m \simeq -\frac{1}{s} \mathbf{n} \times \partial_t \mathbf{n}$ and $\mathbf{f}_n \simeq \frac{\alpha}{s} \partial_t \mathbf{n} - \frac{1}{s^2} \mathbf{n} \times \partial_t \mathbf{m}$ for the thermodynamic forces [see Eqs. (22)], we obtain the correct expressions

$$\begin{aligned} j_x^{\text{e,pump}} &= \frac{\vartheta}{s^2} [\eta_{\text{ad}}^m - \alpha \eta_{\text{nad}}] (\mathbf{n} \times \partial_y \mathbf{n}) \bullet \partial_t \mathbf{n} + \frac{\vartheta \alpha \eta_{\text{ad}}^n}{s} \partial_x \mathbf{n} \bullet \partial_t \mathbf{n} + \frac{\vartheta \eta_{\text{ad}}^n}{s^2} (\mathbf{n} \times \partial_x \mathbf{n}) \bullet \partial_t \mathbf{m} + \frac{\vartheta \eta_{\text{nad}}}{s} \partial_y \mathbf{n} \bullet \partial_t \mathbf{m}, \\ j_y^{\text{e,pump}} &= \frac{\vartheta}{s^2} [\eta_{\text{ad}}^m - \alpha \eta_{\text{nad}}] (\mathbf{n} \times \partial_x \mathbf{n}) \bullet \partial_t \mathbf{n} + \frac{\vartheta \alpha \eta_{\text{ad}}^n}{s} \partial_y \mathbf{n} \bullet \partial_t \mathbf{n} + \frac{\vartheta \eta_{\text{ad}}^n}{s^2} (\mathbf{n} \times \partial_y \mathbf{n}) \bullet \partial_t \mathbf{m} + \frac{\vartheta \eta_{\text{nad}}}{s} \partial_x \mathbf{n} \bullet \partial_t \mathbf{m}, \end{aligned} \quad (4)$$

for the components of the spin-pumping charge current $\vec{j}^{\text{e,pump}} = \underline{L}^{(\text{qm})} \mathbf{f}_m + \underline{L}^{(\text{qn})} \mathbf{f}_n$. Note that, in systems with low damping, those terms parametrized by the damping constant α contribute at the subleading order.

The collective-variable approach for altermagnetic textures used in the main text, with account of the correct expressions for the spin-transfer torques (1) and (2), yields again the Thiele equation (28), but with the caveat that the expression for the spin-transfer force differs from that in the main text, $F_{\text{ST},i} = \int d\vec{r} [-\frac{\eta_{\text{ad}}^m}{\chi} \mathbf{n} \bullet [\partial_t \mathbf{n} \times (j_x^e \partial_y + j_y^e \partial_x) \mathbf{n}] + \frac{\eta_{\text{nad}}}{s} \mathbf{n} \bullet [\partial_t \mathbf{n} \times (\frac{dj_x^e}{dt} \partial_y + \frac{dj_y^e}{dt} \partial_x) \mathbf{n}] - \eta_{\text{ad}}^n \partial_i \mathbf{n} \bullet (\frac{dj_x^e}{dt} \partial_x + \frac{dj_y^e}{dt} \partial_y) \mathbf{n}]$. We note in passing that the nonadiabatic part of $\boldsymbol{\tau}_m$ does not contribute to the spin-transfer force as a result of the orthogonality condition $\mathbf{n} \bullet \mathbf{m} \equiv 0$. The explicit expressions for the components of the spin-transfer force read

$$\begin{aligned} F_{\text{ST},x} &= 4\pi s^2 \left[-\frac{s \eta_{\text{ad}}^m}{\chi} j_x^e + \eta_{\text{nad}} \frac{dj_x^e}{dt} \right] \mathcal{Q} - \eta_{\text{ad}}^n \left[\mathcal{M}_{xx} \frac{dj_x^e}{dt} + \mathcal{M}_{xy} \frac{dj_y^e}{dt} \right], \\ F_{\text{ST},y} &= 4\pi s^2 \left[\frac{s \eta_{\text{ad}}^m}{\chi} j_y^e - \eta_{\text{nad}} \frac{dj_y^e}{dt} \right] \mathcal{Q} - \eta_{\text{ad}}^n \left[\mathcal{M}_{yx} \frac{dj_x^e}{dt} + \mathcal{M}_{yy} \frac{dj_y^e}{dt} \right]. \end{aligned} \quad (5)$$

The first term on the right-hand side of the above equations is proportional to the skyrmion charge of the magnet, as discussed in the main text, and therefore vanishes for (one-dimensional) domain walls. We thus conclude that, in the case of steady charge currents, the altermagnetic contributions to the spin-transfer torques cannot drive the motion of domain walls, contrary to what we claimed in the Discussion section and Fig. 2 of the main text.

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