

# Crystal structure effects on vortex dynamics in superconducting $\text{MgB}_2$ thin films

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The current-driven resistive transition is central to superconducting single-photon detectors, transition-edge sensors, and fluxonic devices. Depending on sample uniformity, dimensions, and heat removal, it can be driven by phase-slip events, flux-flow instabilities (FFI), and normal-domain formation. Here we investigate the influence of two types of microstructural defects on vortex dynamics in  $\text{MgB}_2$  films: columnar growth in textured films and buffer-layer roughness in single-crystal films. The current-voltage ( $I$ - $V$ ) curves measured at  $T \approx 0.25T_c$  for both films exhibit multiple steps. Time-dependent Ginzburg-Landau simulations reproduce the major features of the experimental  $I$ - $V$  curves and suggest that the resistive transitions for both films are mediated by the formation and growth of normal domains rather than FFI. The single-crystal film with buffer-layer roughness exhibits superconductivity breakdown at higher currents and pinning activation energies approximately twice those of the textured film, along with more pronounced multistep features in the  $I$ - $V$  curves. These features are primarily attributed to stronger pinning induced by lateral variations of the superconducting order parameter along the MgO buffer layer. Our results show that both the film microstructure and the film-buffer interface are critical for the resistive transition, offering insights for superconducting devices requiring controlled dissipation at high transport currents.

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## I. INTRODUCTION

The current-induced transition to the high-resistive or normal state is central to superconducting single-photon detectors (SSPDs) [1,2], transition-edge sensors [3], and various Josephson [4] and Abrikosov [5] fluxonic devices. Within the Ginzburg-Landau framework, the maximal dissipationless current in superconductors is set by the depairing current density  $j_{\text{dep}}$  [6]. In practice, however, superconductivity is typically destroyed at a critical current density  $j_c$  [7] well below this theoretical limit [8]. The primary mechanisms driving the transition to a high-resistive state include the increase of the phase difference across weak links under overcritical currents [9], formation and growth of normal (N) domains [10], flux-flow instabilities (FFI) [11], slips of the phase of the superconducting order parameter [12], and their combinations [13–16]. Which of the mechanisms dominates depends on the

sample uniformity and dimensions relative to the coherence length  $\xi$  and magnetic penetration depth  $\lambda$  [17], and the rates of electron energy relaxation and heat removal [18]. In the case of FFIs, the current-voltage ( $I$ - $V$ ) curve of a superconductor exhibits an abrupt jump [19] while for N domains [10] and phase slips [20] multiple voltage transitions are usually observed [12,21].

In the FFI framework, the voltage  $V^*$  measured just before the resistance jump defines the maximal vortex velocity,  $v^* = V^*/(BL)$ , where  $B$  is the magnetic flux density and  $L$  the voltage-lead separation. Near  $T_c$ , FFI is electronic [19], whereas at lower temperatures, electron-electron scattering dominates and FFI becomes thermal [22]. A larger  $v^*$  implies a shorter energy-relaxation time  $\tau_e$ , a desirable property for SSPDs [8]. In uniform systems, the highest  $v^*$  of a few tens of kilometers per second were experimentally observed in Nb-C [5] and MoSi [23] strips with small electron diffusion coefficient  $D \simeq 0.5 \text{ cm}^2/\text{s}$  and fast electron energy relaxation ( $\tau_e \simeq 20\text{--}30 \text{ ps}$ ) and in Pb bridges [14] with large  $D \simeq 30 \text{ cm}^2/\text{s}$  and short inelastic electron-phonon relaxation time ( $\tau_{\text{ep}} \simeq 20 \text{ ps}$ ) [24].

However, in many superconductors relevant for SSPDs, the  $v^*$  and  $\tau_e$  values inferred from FFI often conflict with the fast relaxation observed in photon-counting experiments [23]. This discrepancy arises because FFI may nucleate locally

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rather than across the entire sample [15]. In inhomogeneous systems, regions with slow and fast vortices can coexist [13,25], leading to the formation of N domains where  $v^*$  is first reached and rendering the extracted  $v^*$  nonquantitative.

Once the current density exceeds the threshold [26,27]

$$j_{\text{eq}} = \left[ \frac{2h}{R_{\square} d^2} (T_c - T) \right]^{1/2}, \quad (1)$$

which determines the equilibrium of the nonisothermal normal/superconductor (N/S) boundary, domains will expand until the entire sample transitions to the normal state [15]. In Eq. (1),  $h$  is the heat-removal coefficient,  $R_{\square}$  the film sheet resistance in the normal state, and  $d$  the film thickness. By contrast, if the transport current density  $j < j_{\text{eq}}$ , then a non-stationary N domain can appear in the film: It initially grows, reaches a maximal size, and then shrinks until it vanishes [27]. In the presence of N domains, the relation  $v^* = V^*/(BL)$  no longer provides a quantitative measure of the instability velocity. At the same time, a crossover from local to global FFI can be facilitated by vortex ordering through a reduction of intrinsic random disorder [25], vortex guiding effects [28], and coupling to spin waves [29]. High-quality sample edges are also crucial [30], as they promote FFI nucleation at multiple points along the sample border [23], thereby mitigating local overheating [15].

Here we investigate the impact of two defect types—interfacial and volumetric crystal defects—on vortex pinning, dynamics, and resistive states in single-crystal  $\text{MgB}_2$  films with rough  $\text{MgO}$  buffer layers and in textured  $\text{MgB}_2$  films grown on atomically flat amorphous  $\text{MgBAlO}_x$  buffers.  $\text{MgB}_2$  is a two-band superconductor [31–33], with  $T_c$  exceeding 20 K in films just a few tens of nanometers thick and reaching 39 K in bulk samples. Recently,  $\text{MgB}_2$  films have enabled SSPDs up to 20 K [2], high-kinetic-inductance devices [34], and spin injection at  $\text{MgB}_2$ /ferromagnet interfaces [35]. Yet, vortex dynamics and resistive states in  $\text{MgB}_2$  films at high transport currents remain largely unexplored. Here we bridge this gap by measuring  $I$ - $V$  curves of single-crystal and textured  $\text{MgB}_2$  films at  $T \approx 0.25 T_c$ , both exhibiting multistep characteristics at low magnetic fields. Based on a comparison of our experimental data with time-dependent Ginzburg-Landau simulations, we rule out an FFI scenario and attribute the resistive behavior to the nucleation and growth of N domains. We then examine how the two types of structural defects influence pinning activation energy, transition currents, and N-domain evolution.

## II. SAMPLES

$\text{MgB}_2$  thin films were grown by molecular beam epitaxy under ultra-high-vacuum conditions in a chamber with a base pressure in the  $10^{-10}$  Torr range [35]. The films were grown on c-cut sapphire substrates. Prior to film growth, the substrates were annealed at 1000 °C to obtain clean, well-ordered surfaces. During deposition, the chamber pressure was maintained at approximately  $10^{-8}$  Torr. Magnesium was evaporated from an effusion cell, while boron was evaporated using an electron-beam gun. Evaporation rates were controlled using a quartz crystal microbalance. Reflection high-energy electron diffraction (RHEED) was performed *in*

*situ* to monitor the crystalline quality of the films. The deposition temperature  $T_s$  was monitored using either a pyrometer or a thermocouple attached to the sample holder. After systematically varying  $T_s$  and the deposition rates, optimal growth was achieved at  $T_s \approx 370$  °C with Mg and B rates of 1 and 0.1 Å/s, respectively, yielding an effective Mg:B ratio of 3:1 to compensate for the higher reevaporation rate of Mg [36–38]. Lowering the deposition temperature by a few tens of degrees forms Mg clusters from insufficient desorption, while its increasing induces multiphase polycrystalline growth. *In situ* x-ray photoemission spectroscopy confirmed that the  $\text{MgB}_2$  surface is free of oxygen and carbon.

For this study we used one textured and one single-crystal  $\text{MgB}_2$  film, each with a thickness  $d$  of 20 nm, hereafter referred to as sample T and sample S, respectively. This film thickness was chosen for two reasons. First, it is sufficiently large for vortex dynamics to be governed by both the film-buffer interface and the film microstructure, rather than by interfacial effects alone, while still enabling comparison with our previous FFI studies on 15-nm-thick NbC [5] and MoSi [23] films. Second, although 20 nm is somewhat thicker than the 5- to 15-nm films typically used for SSPDs [2,39], it remains close to the upper end of this range.

The single-crystal film was grown on a 4-nm-thick  $\text{MgO}$  (111) buffer layer, deposited at 900 °C on the sapphire substrate prior to  $\text{MgB}_2$  deposition. The chemical stability of  $\text{MgO}$  and the small lattice mismatch of approximately 3.7% between  $\text{MgO}$  (111) and  $\text{MgB}_2$  (0001) enable the growth of epitaxial, single-crystal  $\text{MgB}_2$  films [35,38]. In contrast, direct growth of  $\text{MgB}_2$  on the sapphire substrate yielded a textured film. Both films were then capped *in situ* with an 8-nm-thick gold layer to prevent oxidation. The layer structure of the fabricated stacks is illustrated in the insets of Figs. 3(a) and 3(b).

Figure 1 shows RHEED diffraction patterns and high-resolution transmission electron microscopy (HRTEM) images of a single-crystal (a) and a textured (b)  $\text{MgB}_2$  film. Note that Fig. 1(b) presents an HRTEM image of a 50-nm-thick textured sample, rather than the 20-nm-thick sample T, in order to better highlight the columnar growth and its expansion through the film thickness. The ordered arrays of diffraction spots and streaks in the RHEED patterns indicate well-ordered growth for both films. For sample S, in-plane rotation by 30° in Fig. 1(a) reveals a difference in the RHEED pattern, pointing to a fixed epitaxial relationship of the film with the substrate. In contrast, the invariance of the spots upon in-plane rotation for sample T in Fig. 1(b) indicates a textured film with a common  $c$ -axis orientation of crystallites perpendicular to the film plane.

The HRTEM image of sample S in Fig. 1(a) confirms ordered film growth with the epitaxial relationship [1-210]  $\text{Al}_2\text{O}_3$  (0001)/[2-1-1]  $\text{MgO}$  (111)/[10-10]  $\text{MgB}_2$  (0001). The  $c$ -axis parameter of the  $\text{MgB}_2$  hexagonal lattice is 3.54 Å, close to the bulk value [40]. Overall, the  $\text{MgO}/\text{MgB}_2$  interface exhibits low contrast, indicating a gradual variation of the film/buffer layer properties. Its rms roughness over a 100 nm scan is approximately 2 nm, with characteristic lateral contrast variations on length scales of 6–19 nm, as determined by Fourier analysis of the black-contrast profile along the film-buffer interface. The  $\text{MgB}_2/\text{Al}_2\text{O}_3$  interface

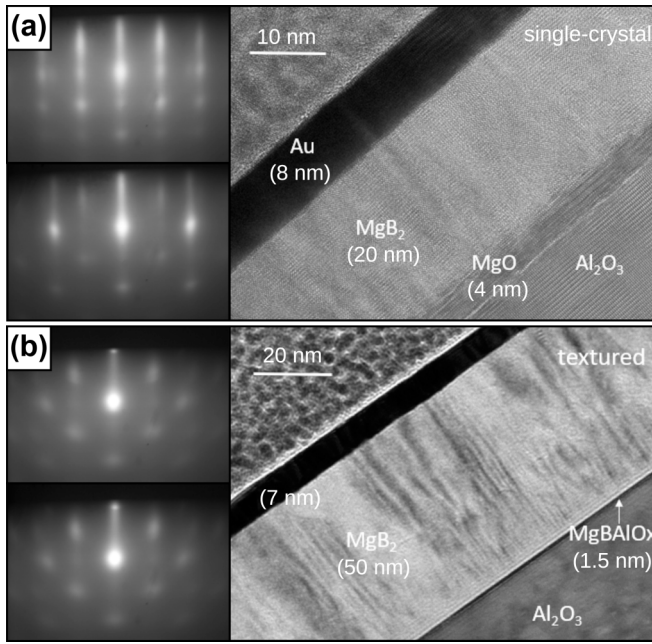


FIG. 1. RHEED patterns obtained upon sample in-plane rotation by 30° (left panels) and HRTEM images (right panels) for the single-crystal (a) and textured (b) MgB<sub>2</sub> films.

in sample T, shown in Fig. 1(b), appears much sharper. The textured MgB<sub>2</sub> film shows columnar structures with diameters below 1 nm near the Al<sub>2</sub>O<sub>3</sub>/MgB<sub>2</sub> interface and their diameters increase up to about 5 nm at the MgB<sub>2</sub>/Au interface. The film's polycrystallinity is attributed to the large lattice mismatch (>10%) between the two materials and to the presence of an intermixed layer at the Al<sub>2</sub>O<sub>3</sub>/MgB<sub>2</sub> interface [41–44]. This atomically smooth amorphous intermediate layer is about 1.5 nm thick, and energy-dispersive x-ray (EDX) spectroscopy (not shown) reveals that it consists of a mixture of Al, O, Mg, and B atoms. Above the interfacial layers, EDX confirms the MgB<sub>2</sub> stoichiometry for both samples. The rms roughness at the MgB<sub>2</sub>/Au interface does not exceed 1 nm over 100 nm for either film.

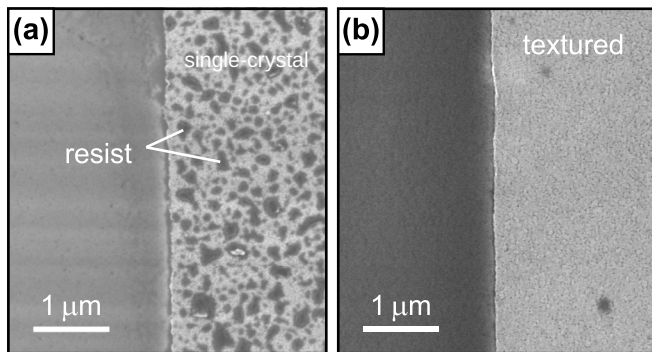


FIG. 2. SEM images of the regions near the strip edges for the single-crystal (a) and textured (b) MgB<sub>2</sub> films. The darker contrast regions visible on the film surface in panel (a) correspond to residual resist remaining after fabrication.

Figure 2 presents scanning electron microscope (SEM) images of the regions near the strip edges for the single-crystal (a) and textured (b) MgB<sub>2</sub> films. Both strips exhibit an rms edge roughness of less than 10 nm over line scans of 5 μm, with no significant differences observed between the samples.

### III. RESISTANCE MEASUREMENTS

Both films were patterned into standard four-probe geometries using conventional UV lithography for electrical resistance measurements. The resulting microstrips have a width  $W$  of 10 μm and a voltage-lead separation  $L$  of 28 μm. The full layer stacks were first patterned and then etched down to the substrate by ion beam milling, with the milled thickness monitored using an ion mass spectrometer. In a second step, Ti(10 nm)/Au(150 nm) contacts were patterned and deposited using an e-beam evaporator. Resistance measurements were performed in a Quantum Design PPMS with magnetic field perpendicular to the film plane.  $I$ - $V$  curves were acquired in the current-driven regime with up-sweeps of the dc current.

#### A. Superconducting parameters

The temperature dependence of the resistivity,  $\rho(T)$ , for both films is shown in Fig. 3. At 30 K, sample S has a resistivity of  $\rho_{30\text{ K}} = 16 \mu\Omega\text{cm}$ , while for sample T it is  $30 \mu\Omega\text{cm}$ . The residual resistivity ratios, defined as the ratio of the resistivity at 300 K to that at 30 K, are 1.4 and 1.25 for samples S and T, respectively. The insets in Fig. 3 display the  $\rho(T)$  curves through the superconducting transition, with  $T_c = 25.0$  K for sample S and 21.6 K for sample T. The transition temperatures were determined using the 50% resistance criterion. The transition widths  $\Delta T_c$  are 0.6 K for sample S and 0.8 K for sample T. The higher resistivity and lower  $T_c$  are ascribed to the polycrystalline structure of sample T.

Applying a magnetic field broadens the superconducting transition and systematically shifts it to lower temperatures. Near  $T_c$ , the temperature dependence of the upper critical field  $B_{c2}(T)$  exhibits two slopes  $dB_{c2}/dT = -1.78$  T/K and  $dB_{c2}/dT = -0.54$  T/K for sample S, see Fig. 3(c). For sample T, these values are  $dB_{c2}/dT = -1.16$  T/K and  $dB_{c2}/dT = -0.28$  T/K. These slopes correspond to electron diffusion coefficients  $D$  of  $0.62$  cm<sup>2</sup>/s and  $2.04$  cm<sup>2</sup>/s for sample S, and to  $0.97$  cm<sup>2</sup>/s and  $3.99$  cm<sup>2</sup>/s for sample T. The diffusion coefficients were deduced using the relation  $D = -1.097(dB_{c2}/dT)^{-1}|_{T=T_c}$  [6]. The two slopes in  $B_{c2}(T)$  for MgB<sub>2</sub> were observed previously [45] and associated with the two-gap nature of the superconducting order parameter [46]. Near  $T_c$ , both bands contribute, and the more isotropic  $\pi$ -band with higher diffusivity plays a stronger role [31–33]. At lower temperatures, the  $\sigma$ -band with stronger pairing and lower diffusivity dominates. In the following, measurements were performed well below  $T_c$ , in the regime where the  $\sigma$  band dominates vortex dynamics and critical currents. The coherence lengths at zero temperature are estimated [23] as  $\xi(0) = \sqrt{\hbar D / 1.76 k_B T_c} = 3.3$  nm and 6 nm for sample S and 4.4 nm and 9 nm for sample T. The magnetic penetration depth is estimated as  $\lambda(0) = 1.05 \times 10^{-3} \sqrt{\rho_{30\text{ K}} / T_c} \approx 90$  nm, with the Pearl length  $\Lambda(0) = 2\lambda^2(0)/d \approx 800$  nm for sample S. For sample T, we obtain  $\lambda(0) = 125$  nm and  $\Lambda \approx 1.5$  μm.

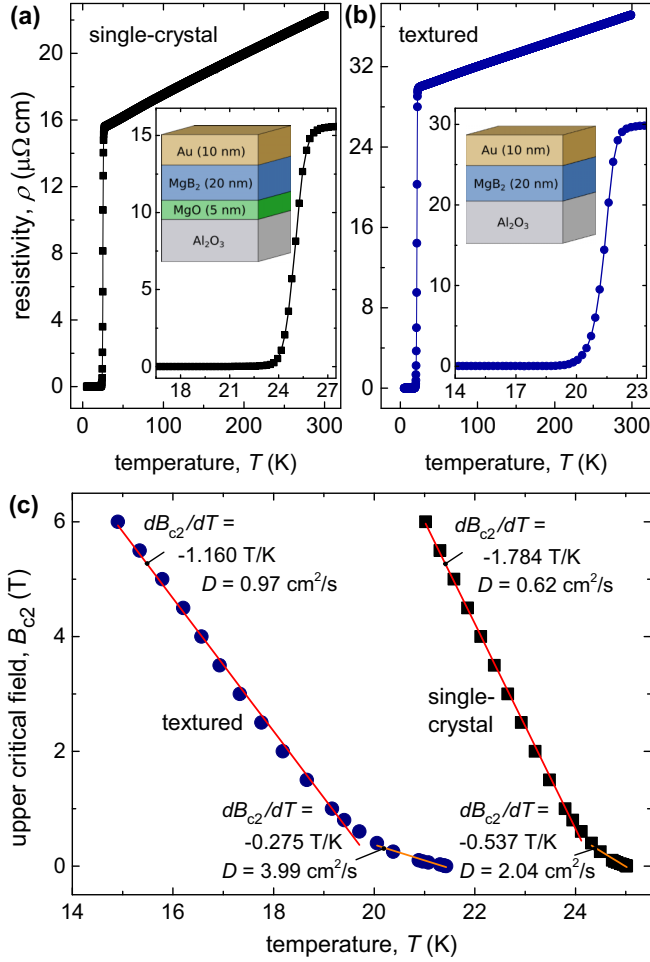


FIG. 3. Temperature dependence of the resistivity for the single-crystal (a) and textured (b) MgB<sub>2</sub> films. Insets: Stack structures and enlarged view of the superconducting transitions. (c) Temperature dependence of the upper critical field for both samples. Symbols: experimental data; solid lines: linear fits.

Thus, our strips are thin, with  $d \ll \lambda(0)$ , and wide, with  $\xi \ll \Lambda(0) \ll W$ .

### B. Arrhenius analysis

In order to quantify the pinning potential that prevents vortex motion, an Arrhenius analysis of the resistivity has been performed. At small transport currents, the resistivity follows an Arrhenius law,  $\rho = \rho_0 \exp(-U_{\text{eff}}/k_B T)$ , where  $\rho_0$  is a prefactor and  $U_{\text{eff}}$  is the effective pinning activation energy [47,48]. The analysis was carried out for magnetic fields up to 6 T. The deduced activation energies for both films are presented in Fig. 4, with an Arrhenius plot for sample S shown in the inset of Fig. 4(a). The systematically lower  $U_{\text{eff}}$  values for sample T—reduced by a factor of two to three compared to sample S—indicate weaker pinning, as smaller energy barriers suffice to activate vortex motion. At first glance, this observation may appear counterintuitive, since the single-crystal film is expected to have a low density of structural defects. However, structural perfection alone does not guarantee weak overall pinning, which also arises

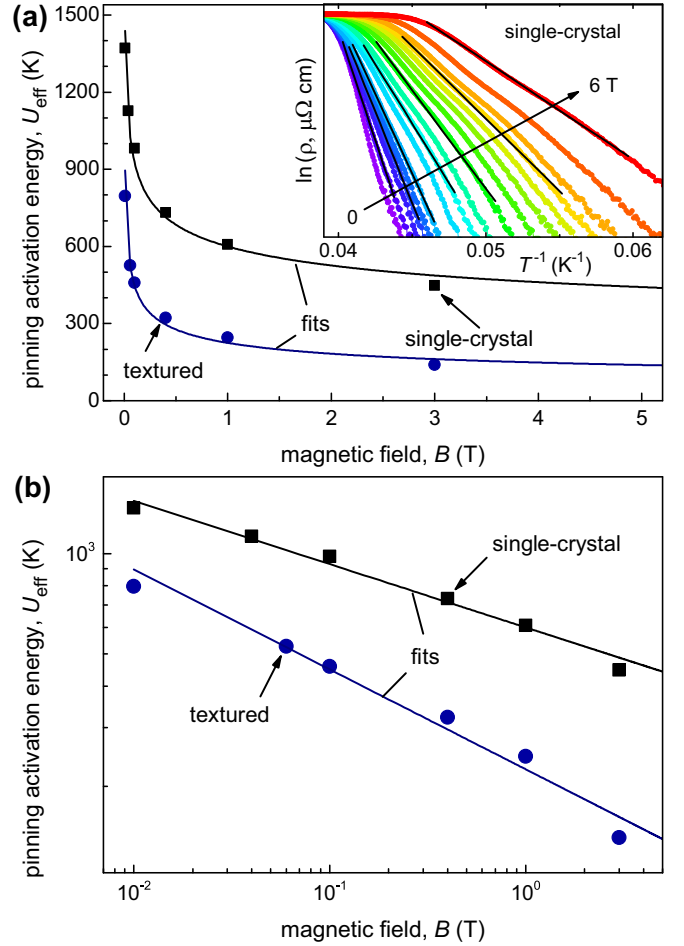


FIG. 4. Pinning activation energy for the single-crystal and textured MgB<sub>2</sub> films as a function of the applied magnetic field. Symbols: Experimental data; solid lines: fits to  $U_{\text{eff}} = U_0 B^{-n}$  with  $U_0 = 225$  K and  $n = 0.3$  for sample T and  $U_0 = 600$  K and  $n = 0.19$  for sample S with (a) linear scaling and (b) logarithmic scaling. Inset in (a): Representative Arrhenius plot from which the activation energies of the single-crystal film were deduced using linear fits at the corresponding magnetic fields.

from volume defects, geometrical constrictions, edge quality, thickness variations, and suppression of the superconducting order parameter at the film-substrate and film-capping interfaces [7,30,49].

Since the postfabrication SEM inspection revealed no noticeable differences in edge morphology between the two samples, see Fig. 2, edge effects are unlikely to account for the observed differences in  $U_{\text{eff}}$ . Given that the well-ordered crystal structure of sample S implies weak volume pinning and the strip width and edge quality are the same for both samples, the dominant pinning contribution should be sought at the film-substrate and film-capping interfaces. The transition temperature  $T_c$  of sample S is higher than that of sample T, while the transition widths are nearly identical for both samples, see Figs. 3(a) and 3(b). Therefore, possible differences in the MgB<sub>2</sub>/Au interface (see Fig. 1) are unlikely to be the origin of the stronger pinning in sample S. The spatial variations in contrast along the MgB<sub>2</sub>/MgO interface in sample S suggest corresponding variations in the structural and,

hence, superconducting properties. These variations extend over characteristic length scales of approximately 6–19 nm, exceeding the coherence length. This observation suggests that the larger spacing between individual pinning centers reduces the overlap of their pinning potentials, allowing them to act as stronger pinning sites in sample S.

The HRTEM image in Fig. 1(b) reveals columnar boundaries in the textured film. The presumed enhanced electron scattering at these boundaries is consistent with the nearly twofold higher resistivity of sample T compared to sample S. Accordingly, the superconducting order parameter is expected to be strongly reduced at the column boundaries and, in general, this order-parameter variation should give rise to vortex pinning. However, the width of the column walls is much smaller than the superconducting coherence length, which sets the typical radius of a vortex core [50]. Furthermore, the diameters of the columns in Fig. 1(b) range from a relatively uniform  $\sim 0.8$  nm near the  $\text{Al}_2\text{O}_3/\text{MgB}_2$  interface to a more nonuniform distribution of 0.5–4 nm at a thickness of 20 nm (corresponding to the  $\text{MgB}_2/\text{Au}$  interface in sample T). Thus, in the textured sample each vortex interacts simultaneously with multiple columnar defects, resulting in partial averaging of individual pinning forces and a reduced effective pinning strength [7]. For a 2D collective pinning regime, the effective activation energy is expected to scale as  $U_{\text{eff}} \propto U_0 B^{-n}$  with  $n = 0.5$  [51]. In contrast, strong pinning generally results in a much weaker field dependence, although no universal exponent is predicted [52]. The best fits to our experimental data in Fig. 4 yield  $U_0 = 225$  K and  $n = 0.3$  for sample T and  $U_0 = 600$  K and  $n = 0.19$  for sample S. While both exponents are smaller than the value expected for 2D collective pinning, the smaller exponent and larger activation energy of sample S indicate a stronger individual-pinning regime than in sample T.

### C. Current-voltage curves

Figure 5 shows the  $I$ - $V$  curves for both films in magnetic fields up to 2 T at  $0.25T_c$ . At low fields, the  $I$ - $V$  curves of sample S display a zero-voltage plateau with a well-defined critical current which decreases from  $I_c \approx 12$  mA to about 2 mA, with increase of the magnetic field from 0 to 2 T. Here  $I_c$  is deduced using a voltage criterion of 100 nV. The steepness of the  $I$ - $V$  curves ensures that small variations in the voltage criterion have little effect on the deduced  $I_c$ . The resistive transition in sample T occurs at a factor-of-two smaller currents, in-line with the low pinning activation energies deduced from the Arrhenius analysis in Fig. 4. We note that the larger  $B_{c2}$  of sample S may contribute to its higher critical current scale and delayed onset of dissipation. Nevertheless, we believe that these differences cannot be attributed to  $B_{c2}$  alone, as the samples also exhibit variations in vortex pinning, current distribution, and microstructural weak links.

At low magnetic fields, the  $I$ - $V$  curves of both films exhibit multiple jumps in the resistive transition. Specifically, up to six jumps are observed in the  $I$ - $V$  curve of sample S in Fig. 5(b), while two jumps appear in the  $I$ - $V$  curve of sample T [Fig. 5(d)]. These voltage jumps correspond to maxima in the  $dV/dI$  curves shown in the insets of Figs. 5(b) and 5(d), where a running average over three neighboring points was applied

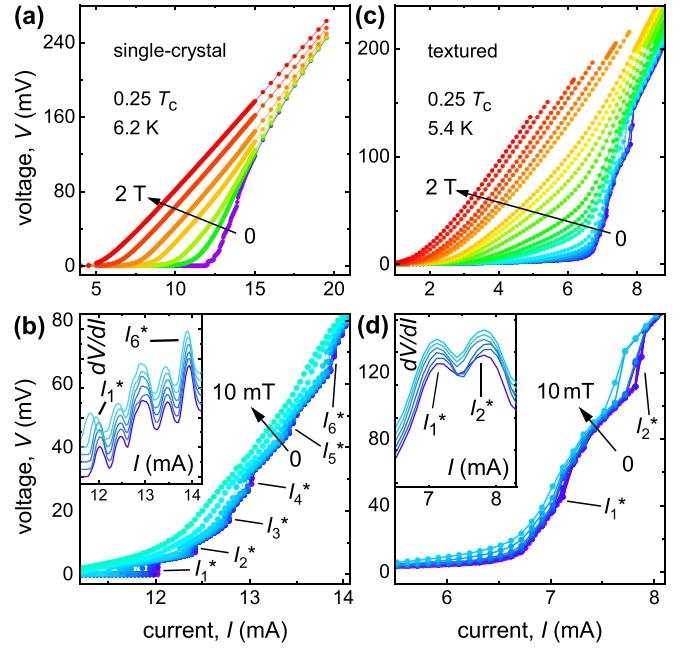


FIG. 5.  $I$ - $V$  curves of the single-crystal [(a) and (b)] and textured [(c) and (d)]  $\text{MgB}_2$  films in magnetic fields ranging from 0 to 2 T. Panels [(b) and (d)] show multiple voltage steps (indicated by lines) in the  $I$ - $V$  curves and the corresponding maxima in the differential resistance (insets) for samples S and T, respectively, at magnetic fields below 10 mT. The currents  $I_1^*$ ,  $I_2^*$ , and  $I_6^*$  in the insets correspond to the voltages which are discussed in the main text. For clarity, the derivative curves in the insets are vertically offset by 5 mV/mA for each magnetic field.

to smooth the derivative data. As the magnetic field increases, the steps in the  $I$ - $V$  curves become smeared and disappear above 40 and 50 mT for samples S and T, respectively.

## IV. DISCUSSION

### A. Multiple voltage transitions

We first summarize the major experimental findings. First, the textured film exhibits weaker vortex pinning, with the high-resistive state appearing at roughly half the current of the single-crystal film. Second, the  $I$ - $V$  curves of both samples exhibit multiple voltage transitions at low magnetic fields, with a larger number of steps in the single-crystal film. The origin of the different pinning strengths was already discussed in Sec. III B. We now turn to a discussion of the multiple voltage transitions.

As the transport current exceeds a depinning threshold, one generally expects a nonlinear transition to a quasilinear flux-flow regime, characterized by a smooth rise of the  $I$ - $V$  curve due to fast vortex motion, followed by an abrupt jump to a highly resistive state as a consequence of FFI [11]. Our observations in Fig. 5 go beyond this simple scenario for spatially uniform systems. Indeed, in the presence of stronger random pinning, the FFI jump to the high-resistive state is generally expected at lower voltages, corresponding to smaller currents and lower vortex velocities compared to the case of weaker pinning [25,28]. This earlier onset of FFI arises from

a less ordered vortex lattice. The resulting broad distribution of vortex velocities leads to a significant difference between the average vortex velocity, associated with the measured dc voltage, and the maximal vortex velocity within the vortex ensemble, which is responsible for triggering the FFI [15]. We note that the current  $I_1^*$  at which the first jump in the  $I$ - $V$  curve occurs for sample S [see the inset of Fig. 5(b)] depends on the magnetic field, suggesting a possible relation to vortex dynamics. In contrast, the currents corresponding to all subsequent transitions in sample S, as well as both transitions in sample T, are much less field dependent [see the insets in Figs. 5(c) and 5(d)], suggesting a nonvortex origin or at least a nonglobal-FFI origin. Furthermore, using formally  $v^* = V^*/(BL)$  to estimate the vortex velocity at the onset of the voltage jumps yields  $v^* \approx 180$  m/s for sample S at  $B = 10$  mT at the first jump  $I_1^*$  [see Fig. 5(b)]. At the last jump ( $I_6^*$ ), this value increases to 220 km/s. For sample T, the first jump ( $I_1^*$ ) gives  $v^* \approx 140$  km/s at 10 mT while the second jump ( $I_2^*$ ) yields 382 km/s. The  $v^*$  value for sample S is notably smaller, while the  $v^*$  values for sample T are at least one order of magnitude larger than typical FFI velocities [5,14,23]. Thus, one must consider alternative mechanisms for the resistive transition.

In general, multiple jumps in the  $I$ - $V$  curves can appear in the presence of phase-slip centers (PSCs) or phase-slip lines (PSLs), the latter being two-dimensional analogs of PSCs [21]. PSCs are typically observed in narrow films with width  $W$  on the order of  $\xi$ , whereas PSLs can occur in wider superconducting films where both  $\xi$  and  $\lambda$  are smaller than  $W$  [12]. The oscillations of the superconducting order parameter along a PSL may be nonuniform, propagating as waves—so-called kinematic vortices—that carry the singularities of the order parameter across the film [53]. Accordingly, multiple steps in the  $I$ - $V$  curves in the phase-slip regime are usually observed in superconductors with relatively large  $\xi$  and in geometries where several, but not too many, PSCs or PSLs fit between the voltage leads [21]. In practice, for uniform systems, the PSC or PSL mechanism is usually inferred when the distinct linear segments of the  $I$ - $V$  curve, extrapolated to intersect the current axis, cross at a common point which corresponds to the excess current. This excess current reflects the contribution of the supercurrent that persists even when a finite voltage is present due to phase-slip dynamics. In our study, the strip length of 28  $\mu\text{m}$  is likely too long and the coherence length  $\xi(T) \simeq 10$  nm is too small for the observation of steps arising solely from PSLs without the involvement of Abrikosov vortices. As both strips in this work were fabricated with identical geometries, follow-up studies could examine how the  $I$ - $V$  curves depend on device length, as devices of different lengths may host different numbers of hotspots and exhibit distinct step structures.

The role of Abrikosov vortices in the formation of PSLs can be elucidated as follows. Namely, channels of fast vortex motion in localized regions of the superconducting strip may lead to the formation of PSLs, commonly referred to in this context as vortex rivers [53,54]. As the transport current increases, the number of vortex rivers grows, resulting in a multistep  $I$ - $V$  curve. Previous modeling, based on the generalized TDGL equation coupled with a heat-balance equation, has shown that vortex rivers can evolve into  $N$

domains when heat removal from the sample is insufficient [55]. The individual resistance steps then correspond to different numbers of  $N$  domains, which increase with increasing transport current. The plausibility of attributing the multistep resistive transitions in both samples to  $N$  domains can be further analyzed on the basis of Eq. (1). In this equation, the heat-removal coefficient for our samples is unknown. As an upper-bound estimate, we use the experimentally deduced value  $h = 0.27 \text{ W K}^{-1} \text{ cm}^{-2}$  for a  $\text{Al}_2\text{O}_3/\text{Nb}$  interface [15], which exhibits efficient heat removal. Substituting the experimental data for sample S at 10 mT into Eq. (1) yields  $j_{\text{eq}} = 409 \text{ kA/cm}^2$ . This value is much smaller than  $j^* = 6.9 \text{ MA/cm}^2$  at which the first jump in the  $I$ - $V$  occurs. The definition of the current(s)  $I^*$  associated with  $j^*$  is shown in Fig. 5(b). For sample T,  $j_{\text{eq}} = 382 \text{ kA/cm}^2$  and  $j^* = 3.9 \text{ MA/cm}^2$ . These estimates place our measurements within the regime where  $j_{\text{eq}} \ll j^*$ . The large separation between  $j_{\text{eq}}$  and  $j^*$  strongly supports a scenario in which normal-domain growth governs the resistive transitions. This condition is naturally satisfied for all other jumps occurring at higher currents in the  $I$ - $V$  curves of both films.

## B. TDGL modeling

To elucidate the spatiotemporal dynamics of the superconducting order parameter during the resistive transition, we performed numerical simulations based on the generalized TDGL equation [56,57]. The TDGL equation is valid close to  $T_c$ . Far from  $T_c$ , the TDGL equation does not reproduce the physics in the vortex core quantitatively, but it still describes the spatiotemporal evolution of vortex matter qualitatively [58]. At low temperatures ( $\sim 0.25T_c$ ) and low magnetic fields, both gaps in  $\text{MgB}_2$  are fully open [46], and the material behaves as a single superconducting condensate. Accordingly, we adopted a strong interband-coupling approximation in the TDGL modeling [59,60], where fast Cooper-pair tunneling locks the order parameters of all bands, rendering the system effectively single-band for vortex dynamics and allowing a single-band TDGL description. In two dimensions, the generalized TDGL equation reads

$$\frac{u}{\sqrt{1 + \gamma^2 |\psi|^2}} \left( \frac{\partial}{\partial t} + i\varphi + \frac{\gamma^2}{2} \frac{\partial |\psi|^2}{\partial t} \right) \psi = (\nabla - i\mathbf{A})^2 \psi + (\epsilon - |\psi|^2) \psi, \quad (2)$$

where  $\psi(\mathbf{r}, t) = |\psi|e^{i\theta}$  is the superconducting order parameter with the ratio  $u = \pi^4/14\zeta(3) \approx 5.79$  [ $\zeta(x)$ : Riemann zeta function] of relaxation times for its amplitude and phase  $\theta$  in the dirty-limit regime. The parameter  $\gamma = 2\tau_E \Delta_0/\hbar$  involves the inelastic electron-phonon scattering time  $\tau_E$  and the zero-field superconducting gap  $\Delta_0$ .  $\varphi(\mathbf{r}, t)$  is the electric scalar potential and  $\mathbf{A}$  the vector potential.  $\epsilon(\mathbf{r}) \in [-1, 1]$  is a real-valued parameter that locally modulates the superconducting order parameter [61] and is used to model vortex pinning sites. It is defined as  $\epsilon(\mathbf{r}) = 1 - \Delta\epsilon$ , where  $\Delta\epsilon$  characterizes the local order-parameter suppression strength. The values  $\Delta\epsilon = 0.3$  for sample S and  $\Delta\epsilon = 0.6$  for sample T were chosen phenomenologically to describe the distinct defect landscapes considered in the simulations, namely localized pinning regions for sample S and a sparse network of connected defect

TABLE I. Material parameters used in the simulations.

| Parameter                        | Denotation                                                       | Value  |
|----------------------------------|------------------------------------------------------------------|--------|
| Clean-limit coherence length     | $\xi_0$                                                          | 9 nm   |
| Electron mean free path          | $l$                                                              | 3 nm   |
| Coherence length                 | $\xi(0) = 0.85 \sqrt{\xi_0 l}$                                   | 5 nm   |
| London penetration depth         | $\lambda(0)$                                                     | 100 nm |
| Inelastic scattering coefficient | $\gamma$                                                         | 10     |
| Relative temperature             | $T/T_c$                                                          | 0.3    |
| Coherence length                 | $\xi = \frac{\xi(0)}{\sqrt{1-T/T_c}}$                            | 5.2 nm |
| Penetration depth                | $\lambda = \frac{0.615 \lambda(0) \sqrt{\xi_0}}{\sqrt{1-T/T_c}}$ | 127 nm |

channels for sample T. These values are not intended to represent the absolute macroscopic pinning strength of the samples, which depends on both the local defect potential and the spatial arrangement of defects; rather, they are used to examine how distinct defect landscapes lead to different vortex dynamics.

The total current density satisfies the continuity condition  $\nabla \cdot (\mathbf{j}_s + \mathbf{j}_n) = 0$  and involves the supercurrent  $\mathbf{j}_s = \text{Im}[\psi^*(\nabla - i\mathbf{A})\psi]$  and normal  $\mathbf{j}_n = -\nabla\varphi - \partial\mathbf{A}/\partial t$  densities. The electric potential  $\varphi(\mathbf{r}, t)$  satisfies the Poisson equation  $\nabla^2\varphi = \nabla \cdot \text{Im}[\psi^*(\nabla - i\mathbf{A})\psi] - \nabla \cdot \partial\mathbf{A}/\partial t$ . At the strip edges, where vortices enter and exit, superconductor-vacuum boundary conditions are imposed as  $\mathbf{n} \cdot (\nabla - i\mathbf{A})\psi = 0$  and  $\mathbf{n} \cdot \nabla\varphi = 0$ , where  $\mathbf{n}$  is the unit vector normal to the interface. At the superconductor–normal-metal interfaces supplying the transport current  $j_{\text{ext}}$ , the boundary conditions read  $\psi = 0$  and  $\mathbf{n} \cdot \nabla\varphi = j_{\text{ext}}$ .

In Eq. (2), the order-parameter amplitude is normalized to the superconducting carrier density,  $|\psi|^2 = n_s$ . Lengths are expressed in units of the coherence length  $\xi$ , and time is scaled by  $\tau_0 = \mu_0\sigma\lambda^2$ , where  $\sigma$  is the normal-state conductivity and  $\lambda$  the London penetration depth. The vector potential  $\mathbf{A}$  is scaled by  $\xi B_{c2}$ . The external current density  $j_{\text{ext}}$  and electric potential  $\varphi$  are scaled by  $4\xi B_{c2}/(\mu_0\lambda^2)$  and  $\xi j_0/\sigma$ , respectively. The maximum mesh size was  $\xi/2$ , and the material parameters are listed in Table I. The simulations were performed for a  $200 \times 1000 \text{ nm}^2$  (width  $\times$  length) domain which is smaller than the experimental device dimensions ( $\sim 10 \times 28 \mu\text{m}^2$ ), as required by the computational cost of TDGL calculations. The chosen dimensions capture the relevant vortex length scales and defect configurations governing the local dynamics. Since the films are in the thin-film limit ( $d \ll \lambda$ ), vortices are effectively rigid along the thickness direction, and columnar defects in sample T are represented by an effective 2D defect landscape. The simulation results are summarized next.

We first consider the  $I$ - $V$  curves of a film in which the order parameter varies slightly on a lateral scale of  $\sim 3\xi$  [see the inset in Fig. 6(a)] and which corresponds to sample S. At subcritical transport currents, the  $I$ - $V$  curves exhibit a zero-voltage response, with vortices pinned in regions of the strongest order-parameter suppression [regime a in Figs. 6(a) and 6(b)]. At magnetic fields up to about 300 mT, all  $I$ - $V$  curves display multiple steps, which become progressively smeared at higher fields (not shown). This smearing is

attributed to the increased rigidity of the vortex lattice at higher vortex densities [50]. As the transport current increases, the system evolves through a sequence of dynamic states (denoted b–h), illustrated in Figs. 6(a) and 6(b) for the  $I$ - $V$  curve at 50 mT. Owing to the difference in the widths of the superconducting strip in the model and in the experiment, this field corresponds to approximately 5 mT for sample S in Figs. 5(a) and 5(b). The simulations suggest that the first voltage step is associated with the formation of one or more channels of fast-moving vortices [regime b in Figs. 6(a) and 6(b)]. At such high velocities, these vortex chains form vortex rivers (PSLs) [12,21]. Subsequent voltage transitions correspond to states with an increasing number of vortex rivers [regimes c–e in Figs. 6(a) and 6(b)]. With further increase of the transport current, the system evolves toward states involving N-domain nucleation within the strip (regime f), followed by the formation and growth of additional domains (regime g), until the entire strip transitions to the normal state (regime h). In previous simulations including a heat-balance equation, such transitions were associated with the competition between Joule heating and heat removal to the substrate [16,23]. In the present TDGL simulations, thermal evolution is not included. A realistic thermal description of spatially nonuniform systems is challenging, as it requires additional thermal parameters whose values are poorly constrained experimentally.

We now proceed to the analysis of the  $I$ - $V$  curves for the film with a network of channels of width  $\sim \xi$  with suppressed order parameter, as illustrated in the inset in Fig. 6(c) and which corresponds to sample T. While the net pinning in this case is collective and weak, we assume that there might be some areas which allow for vortex channeling. Simultaneously, on the basis of structural characterization of film T we assume that the density of such channels is rather small. The simulated  $I$ - $V$  curve exhibits a zero-voltage response up to the depinning currents which are a factor of two smaller than those for sample S. In this regime a in Figs. 6(c) and 6(d) the vortices are pinned to the nodes of the pinning sites network.

When the transport current exceeds the depinning threshold, a low-resistive regime emerges, characterized by a slightly S-shaped global  $I$ - $V$  response with superimposed small voltage steps. Because overheating is neglected in the TDGL simulations, these steps are expected to be smeared in the experimental  $I$ - $V$  curves. The corresponding vortex configurations evolve from a single vortex river to multiple rivers with increasing current [regimes b–g in Figs. 6(c) and 6(d)], with the texture stabilizing these channels by providing multiple pathways. In this way, the TDGL simulations reproduce the essential features of the experimental  $I$ - $V$  curves and indicate that the multiple voltage transitions are associated with distinct dynamical regimes of vortex rivers and normal domains in the superconducting strip.

Finally, we recall that the TDGL modeling was performed within the strong interband-coupling approximation [59,60]. In  $\text{MgB}_2$ , the  $\sigma$  and  $\pi$  bands possess distinct superconducting parameters and different sensitivities to structural defects. The  $\sigma$  band, characterized by strong electron-phonon coupling, is primarily affected by large structural defects. In contrast, the  $\pi$  band, with weaker electron-phonon coupling and a smaller superconducting gap, is more sensitive to disorder [62]. Consequently, structural defects have a more pronounced

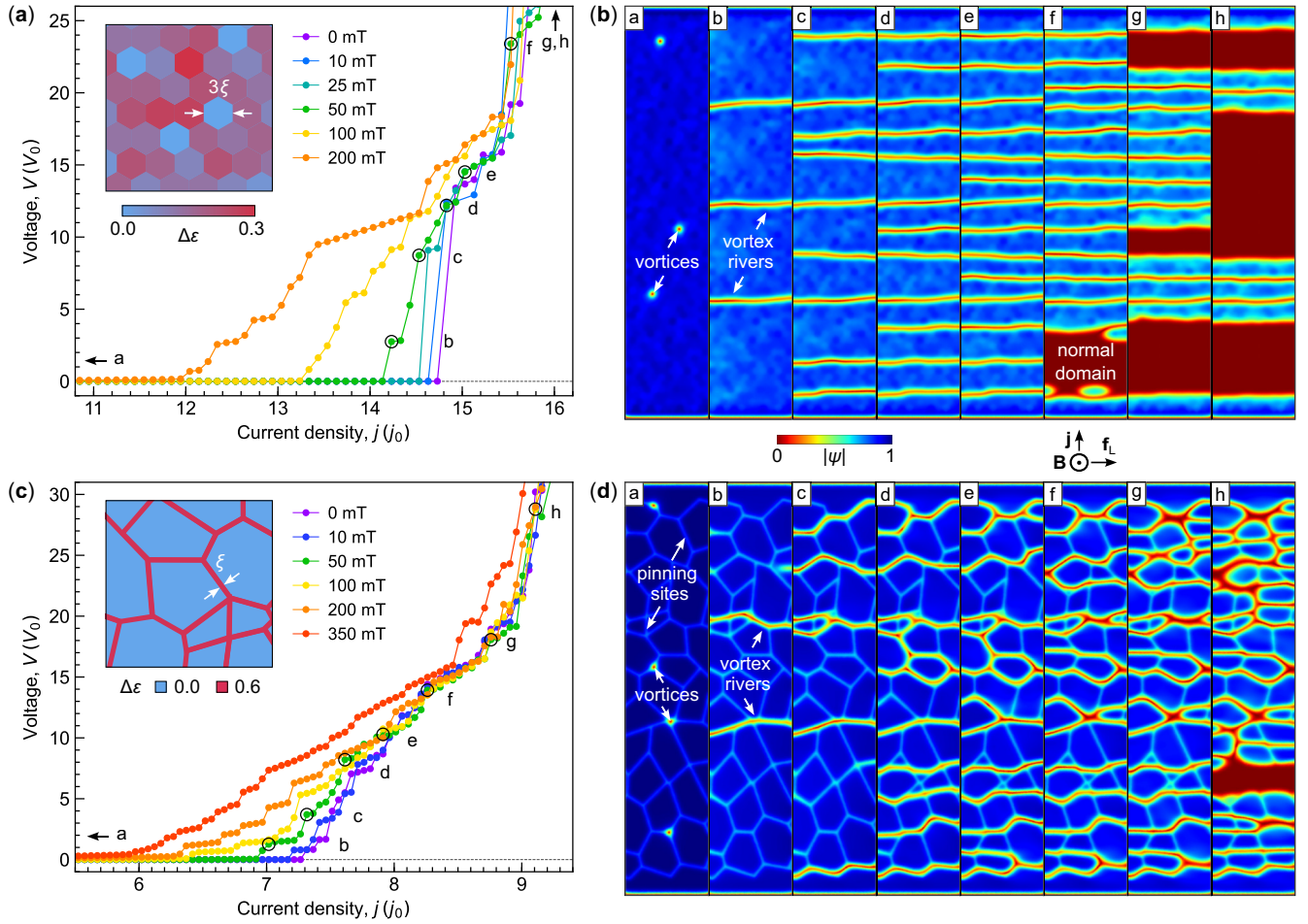


FIG. 6. TDGL simulations of the current-driven resistive transition. Panels (a) and (c) show the  $I$ - $V$  curves, while panels (b) and (d) display snapshots of the superconducting order-parameter magnitude  $|\psi|$  at representative points along the corresponding  $I$ - $V$  curves. The insets in (a) and (c) illustrate the spatial distribution of  $|\psi|$  in the simulations, with the parameter  $\Delta\epsilon$  controlling the local suppression of the order parameter. The current density  $j$  is expressed in units of  $j_0 = 4 \times 10^{-2} \xi B_{c2} / (\mu_0 \lambda^2)$ , where  $B_{c2} = \Phi_0 / (2\pi \xi^2)$  is the upper critical field, and the voltage  $V$  is in units of  $V_0 = \xi j_0 / \sigma$ , with  $\sigma$  being the normal-state conductivity.

impact on the  $\pi$  band, where scattering more strongly perturbs superconductivity. Extending the modeling to explicitly account for the two-gap nature of  $\text{MgB}_2$  is therefore expected to yield a broader distribution of pinning strengths, as well as a richer variety of vortex pathways and coexisting dynamical states.

### C. Impact of heat removal

We note that our TDGL simulations were performed without accounting for heat removal, which would require solving the TDGL equations in conjunction with the heat equation. Consequently, the present model does not quantitatively capture hotspot formation and the thermal evolution associated with high-current transitions. Nevertheless, the simulations identify the role of the defect landscape in governing vortex dynamics and the onset of dissipation. Previous simulations [5,16,23] suggest that accounting for a finite heat-removal rate is expected to (i) reduce the current range over which vortex rivers persist in favor of regimes dominated by N domains, (ii) accelerate the growth of N domains, and (iii) promote the smearing of small voltage steps.

At the same time, the presence of multiple steps in the  $I$ - $V$  curve of both films indicates that superconductivity is not destroyed throughout the entire sample simultaneously. Instead, several distinct dynamic states exist, each with a different resistance. The dissipation generated in these states does not drive the sample into the normal state, which is consistent with a balance between dissipation and heat removal, although thermal effects are not explicitly modeled here. The greater number of voltage transitions in sample S, occurring at currents roughly twice as high as in sample T, suggests that dissipative states remain stabilized over a wider current range in sample S, which may be related to more efficient heat removal. A possible explanation is that the epitaxial nature of the  $\text{Al}_2\text{O}_3/\text{MgO}/\text{MgB}_2$  stack may facilitate phonon transmission into the substrate. On the contrary, the large lattice mismatch at the  $\text{Al}_2\text{O}_3/\text{MgB}_2$  interface and the 1.5-nm-thick interfacial amorphous  $\text{MgAlBO}_x$  layer for the textured film in Fig. 1(b), may introduce additional acoustic impedance mismatch and increase the thermal boundary resistance [63,64], thereby reducing heat transfer efficiency. Such effects could contribute to a phonon bottleneck [18] and may promote the transition to the normal state.

We note that experimental observation of different temperature rises in the two samples would provide support for this interpretation. In principle, such measurements could be performed using an approach similar to that of Ref. [65], but they are not feasible with our current experimental setup. Therefore, the above discussion represents a possible interpretation of the observed differences rather than a determination of the thermal properties of the two samples.

#### D. Relevance for applications

Our results offer insights for optimizing superconducting devices based on  $\text{MgB}_2$ , where precise control of dissipation at high transport currents is required. For instance,  $\text{MgB}_2$  films have recently attracted interest as SSPDs operating at a 1.55  $\mu\text{m}$  wavelength over a broad temperature range up to 20 K [2,39]. These devices are typically fabricated from thinner films (5–10 nm) than those used in our study and benefit from a small electron diffusion coefficient, high switching (critical) current, low kinetic inductance, and good sample uniformity. For our film S, the electron diffusion coefficient  $D \approx 0.6 \text{ cm}^2/\text{s}$  and the sheet kinetic inductance  $L_k^\square \approx 0.5 \text{ pH}/\square$ , estimated from the London expression  $L_k^\square = \mu_0 \lambda(0)^2/d$ , are comparable to reported detector-grade values of  $D \sim 0.5 \text{ cm}^2/\text{s}$  and  $L_k^\square \simeq 1.5 \text{ pH}/\square$  [39], as expected from the inverse thickness scaling of  $L_k^\square$ . This indicates that our single-crystal films are intrinsically compatible with low- $L_k$  nanowire operation. Accordingly, film S with the MgO buffer appears particularly attractive for  $\text{MgB}_2$ -based devices operated near the critical current, owing to its higher dissipation-onset current and improved uniformity. Device-level figures of merit, including switching stability and noise, depend strongly on nanowire geometry, film thickness, and device architecture, and represent possible topics for future work. In contrast, the stronger pinning in textured films may be beneficial for applications requiring enhanced vortex immobilization, such as microwave transmission lines, where reduced vortex motion may contribute to lower dissipation [49]. More generally, our results demonstrate that controlling both the film microstructure and interfaces provides an effective route for tailoring the current-carrying capability and dissipation behavior of  $\text{MgB}_2$  thin films.

#### V. CONCLUSION

In summary, we have investigated how interfacial and volumetric defects influence the resistive transition in

20-nm-thick  $\text{MgB}_2$  films. Interfacial roughness associated with the MgO buffer layer produces stronger vortex pinning than the weaker pinning landscape arising from slight misalignment of columnar defects in textured films, resulting in a higher current for the onset of dissipation in the single-crystal film. The distinct  $I$ - $V$  characteristics, including multiple voltage jumps, cannot be explained by a conventional global flux-flow instability but are instead consistent with a sequence of dynamic states involving vortex motion and the nucleation and growth of normal domains, as supported by time-dependent Ginzburg-Landau simulations. Although differences in heat removal may contribute to the observed variations between the samples, further experimental investigation is required to establish their role. These results demonstrate that controlling both defect landscapes and film interfaces provides a pathway for tailoring vortex dynamics and dissipation behavior in  $\text{MgB}_2$  thin films.

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#### DATA AVAILABILITY

The data that support the findings of this article are openly available [66].

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