

Buckling and flat bands in twisted bilayer graphene

Jannes van Poppelen* and Annica M. Black-Schaffer

Department of Physics and Astronomy, *Uppsala University*, Box 516, SE-75237 Uppsala, Sweden



(Received 17 October 2025; revised 30 January 2026; accepted 25 February 2026; published 26 March 2026)

Magic-angle twisted bilayer graphene (TBG) with its flat bands provides a rich platform for exploring emergent electronic orders. Similarly, periodically buckled monolayer graphene (MLG) has been proposed as a tunable alternative for realizing flat bands. Here, we investigate the combined effect of buckling and twisting in bilayer graphene. We find that periodic buckling in large-angle TBG initially enhances band flattening compared to monolayer graphene, but for sufficiently strong buckling, it instead increases the band dispersion. This occurs because of both the presence of interlayer coupling, which reduces the in-plane kinetic energy, and the opening of a gap at the Dirac point resulting from inversion-symmetry breaking. Additionally, we find that buckling-induced band flattening competes with twist-induced band flattening. While the former breaks sublattice symmetry, generating a sublattice polarization, the latter prefers to preserve it. This prevents buckling from generating even flatter bands at the magic angle. Nevertheless, we find that buckled TBG can exhibit flatter bands than pristine TBG over a wide range of twist angles, with a flatness similar to that of pristine magic-angle TBG.

DOI: [10.1103/gcbz-xhdg](https://doi.org/10.1103/gcbz-xhdg)

I. INTRODUCTION

Twisted bilayer graphene (TBG) is the prototypical moiré material. A slight rotational misalignment between two graphene layers has been shown to give rise to a rich variety of correlated phases, including the anomalous Hall effect [1], correlated insulating phases [2], unconventional superconductivity of still-uncertain origin [3–6], strange-metal behavior [7], and topological insulating phases [8]. It is the emergence of flat bands with a high density of states (DOS) at so-called “magic” twist angles [9–11] that strongly enhances the role of electronic interactions and thereby drives the emergence of the correlated phases. The electronic structure of TBG has also been shown to be controllable beyond the twist angle, with electrostatic doping and displacement fields shown to give rise to tunable flat bands [12–14]. An important next step is to explore whether it is possible to engineer even flatter bands and enhance the zero-energy DOS, or to extend the range of twist angles hosting flat bands, thereby further stabilizing and enriching the correlated phases that emerge in TBG.

A viable alternative proposed for realizing flat bands is provided by strained or buckled monolayer graphene (MLG). Here, structural deformations produce spatially varying strain fields that suppress the kinetic energy, thereby providing a route to flat band physics in a twist-free way [15–17]. Scanning tunneling microscopy/spectroscopy (STM/STS) measurements on buckled graphene superlattices have already observed the emergence of such flat bands [18],

and have further shown that the band structure is continuously tunable with buckling strength [19]. By depositing graphene on hBN or NbSe₂ [18], such buckled graphene superlattices are easily realizable and have been projected as ideal candidates for studying emergent electronic orders [20] and valley-dependent topology [21]. Given that buckling in monolayer graphene can give rise to flat bands with a high DOS, it is natural to consider the interplay of buckling and moiré physics in TBG.

Recent theoretical and experimental works have shown that free-standing TBG at small twist angles undergoes spontaneous corrugation [22–26]. Here, the twist-induced moiré patterns generate intrinsic shear stresses, which, under relaxation, cause the bilayer to undergo periodic buckling [25]. Likewise, the different stacking areas for all hexagonal van der Waals materials give rise to in-plane strain fields [27], which leads to in-plane relaxation [28]. These periodic deformations, involving both in-plane relaxation and out-of-plane corrugations, significantly affect the flat bands of TBG, increasing their bandwidth and modifying their separation from the valence and conduction bands [23,29,30]. In terms of external in-plane strain, homo/heterostrain has also already been extensively studied, and it has been shown that this can give rise to even flatter bands [31–34]. Despite these observations, the role of buckling in TBG, beyond natural corrugations and simple strain fields in moiré-driven flat bands, remains largely unexplored. Moreover, while recent work has revealed the coexistence of strain-induced confined states and pseudo-Landau levels (PLLs) [35], a systematic understanding of how buckling influences the twist-induced flat bands is still lacking.

In this work, we go beyond the current understanding by combining externally controlled periodic buckling and moiré physics to study their combined effect on the electronic structure of TBG using atom-resolved modeling. In particular, we consider TBG at both small and large twist angles, subject to a superlattice of buckles that has been shown to give rise

*Contact author: jannes.vanpoppelen@physics.uu.se

to tunable flat bands in monolayer graphene [18,19], thereby enhancing the flat band character of TBG. Experimentally, such buckled superlattices may be realized by depositing TBG on designed substrates [15,36].

At large twist angles, we demonstrate that TBG first exhibits greater band flattening than monolayer graphene for similar buckling, but eventually yields more dispersive bands for large enough buckles. By analyzing the symmetries of AA- and AB-stacked graphene in comparison to TBG, we find that the combined effects of buckling and twisting break the inversion symmetry that protects the Dirac-point degeneracy, thereby opening a gap at the Dirac points. This gap opening provides a general mechanism for band flattening around zero energy for larger angles and small to moderate buckling.

At the magic angle, we investigate the interplay between moiré-driven flat bands and buckling in TBG. In pristine magic-angle TBG, the flat bands exhibit a sublattice-symmetric charge distribution [37,38]. Buckling, however, couples asymmetrically to the two valleys in graphene through strain-induced gauge fields [39], which inherently breaks sublattice symmetry. As a consequence, the two flattening mechanisms, twist-induced moiré patterns and periodic buckling, cannot be additive, but instead compete more with one another at the magic angle. Therefore, the band flattening at large angles induced by buckling does not persist down to the magic angle, as the magic-angle moiré bands are already very narrow, but instead, buckling there leads to more dispersive bands.

We additionally quantify the emergence of buckling-induced flat bands to probe their potential as hosts for diverse correlated orders for nonmagic angles. We do this across a wide range of twist angles by using the integrated density of states (IDOS) near zero energy as a useful measure. Our results show that periodic buckling, with its strength serving as a tuning parameter, can, for a range of twist angles, more than double the IDOS compared to pristine TBG. Remarkably, near, but not at, the magic angle, we show that buckled TBG can rival pristine magic-angle TBG in its ability to support correlated phenomena. Interestingly, this establishes an intrinsic robustness of the flat bands of buckled TBG against twist-angle disorder.

The remainder of this work is organized as follows. We describe our model for TBG and the physics of periodic buckling in Sec. II. In Sec. III, we investigate the effect of buckling on TBG at large twist angles, and show that a broken inversion symmetry leads to band flattening. In Sec. IV, we study the effect of buckling at the TBG magic angle on the electronic structure. We finally consider intermediate twist angles in Sec. V and quantify the ability of buckled TBG to host flat bands using buckling strength as a tuning parameter. Finally, in Sec. VI, we summarize our results.

II. MODELING BUCKLING IN TWISTED BILAYER GRAPHENE

A. Twisted bilayer graphene

To capture the relevant physics, we describe TBG using the tight-binding Hamiltonian

$$\mathcal{H} = - \sum_{i,j} t(|\mathbf{r}_i - \mathbf{r}_j|) c_i^\dagger c_j, \quad (1)$$

where the hopping strengths t are given by a distance-dependent Slater-Koster parametrization [6,37,38,40]

$$t(|\mathbf{r}_i - \mathbf{r}_j|) = t_0 e^{(a_c - r_{ij})/\lambda} (1 - (\hat{\mathbf{r}}_{ij} \cdot \hat{\mathbf{z}})^2) + t_\perp e^{(d_0 - r_{ij})/\lambda} (\hat{\mathbf{r}}_{ij} \cdot \hat{\mathbf{z}})^2. \quad (2)$$

Here, $t_0 = 2.7$ eV and $t_\perp = 0.48$ eV are the in-plane and out-of-plane hopping strengths, respectively, and $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$. Furthermore, $a_c = 1.42$ Å is the intralayer carbon-carbon distance, $d_0 = 3.35$ Å is the equilibrium interlayer distance, and $\lambda = 0.184 a_c$ sets the decay length of the orbital overlap [6,38,40]. We omit spin degrees of freedom as we only consider spin-degenerate states. We restrict the in-plane hopping to nearest neighbors only, while for the interlayer coupling, we include interactions up to sixth-order nearest neighbors. This has been shown to reproduce the correct band structure for magic-angle TBG, including the emergence of flat bands and a finite band gap separating the moiré bands [6,38], and it also ensures compatibility with the selected buckling profile. Truncations of the interlayer coupling at shorter distances fail to reproduce the correct band structure, while higher-order terms have minimal effect on the band structure of pristine or buckled TBG. Monolayer graphene can be modeled using the same geometry and Hamiltonian but with no interlayer coupling ($t_\perp = 0$).

We define a commensurate moiré unit cell characterized by two integers p and q , for the Hamiltonian Eq. (1), with the twist angle between the two graphene layers given by $\cos \theta = (3q^2 - p^2)/(3q^2 + p^2)$ [41]. The twist angle θ is 0° ($(p, q) = (0, 1)$) for AA-stacked graphene and 60° ($(p, q) = (1, 1)$) for AB-stacked (Bernal-stacked) graphene. For all other angles studied in this work, we fix $p = 1$ and vary q . Furthermore, we assume q to be odd, as that yields smaller unit cells and is hence computationally more efficient. The size of the moiré unit cell is given by the moiré length $L_m = a/(2 \sin(\theta/2))$, where $a = \sqrt{3} a_c$ is the lattice constant of graphene [41]. At the so-called magic angle, here $\theta \approx 1.20^\circ$ generated by $(p, q) = (1, 55)$, the unit cell contains 9076 atoms and has a length $L_m \approx 47.6 a$. We further neglect atomic relaxation effects in free-standing TBG, which are known to modify the local stacking configuration by shrinking the AA-stacked regions [28,42–44]. This is motivated by our focus on periodically imposed buckling deformations, with buckling induced through external means, such as substrate engineering. This leads to effectively clamped geometries, where neglecting in-plane lattice relaxation is both a common and physically motivated approximation [35]. Within this approximation, the relative areas of the different stacking domains remain unchanged and the interlayer distance is kept constant. A fully consistent treatment incorporating structural relaxation after such buckling would require a complete structural optimization, incompatible with the current tight-binding model, and beyond the scope of this work.

We obtain the eigenenergies $E_{n,\mathbf{k}}$ and eigenvectors $\psi_{n,\mathbf{k}}$, with band index n and momentum $\mathbf{k}$, of the spectrum through sparse diagonalization of the Hamiltonian Eq. (1) and focus on the low-energy region. At the magic angle, the moiré pattern generates the characteristic flat band structure of TBG [37,45], as illustrated in Figs. 1(a) and 1(b). We identify the four lowest-energy bands as the moiré bands, indicated in red in

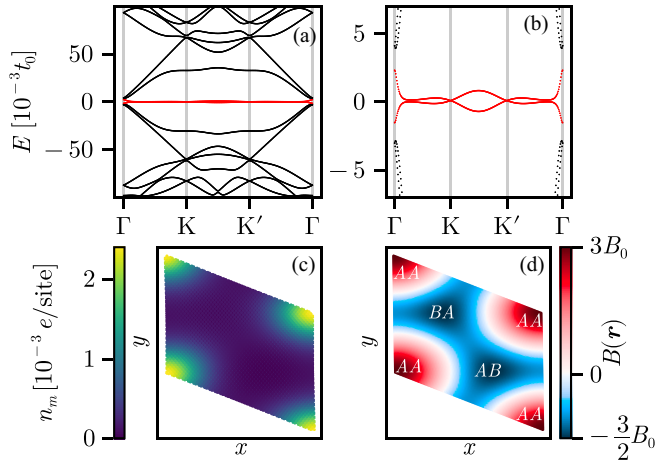


FIG. 1. Band structure of magic-angle TBG (a), with zoom-in on moiré bands highlighted in red (b), charge density of the moiré bands (c), and the buckling-induced PMF of Eq. (6) projected over the moiré unit cell (d). Indicated in (d) are the different stacking regions of TBG.

Figs. 1(a) and 1(b). We denote the bandwidth of the moiré bands as ΔE_m , chosen so that it captures the four moiré bands, but its value is not necessarily the same for every buckling configuration. We further resolve the moiré bands in real space by computing the site-resolved moiré charge density, defined as

$$\begin{aligned} n_m(\mathbf{x}, \Delta E_m) &= e \int_{\Delta E_m} \rho(\mathbf{x}, E) dE \\ &= e \sum_{E_{n,\mathbf{k}} \in \Delta E_m} |\psi_{n,\mathbf{k}}|^2, \end{aligned} \quad (3)$$

where e is the elementary charge. Here, ρ denotes the density of states (DOS). To obtain a smooth function, the discrete energies are broadened using a Lorentzian, mimicking finite-temperature effects. Computing $n_m(\mathbf{x}, \Delta E_m)$ for the four moiré bands, we see that they are primarily localized in the AA-stacked region, see Fig. 1(c), with electrons equally divided among both sublattices [37]. Closely related to the moiré charge density is the integrated DOS (IDOS), given by

$$\text{IDOS}(\Delta E_U) = \int_{\mathcal{M}} n(\mathbf{x}, \Delta E_U) d\mathbf{x}. \quad (4)$$

The spatial integral sums up the contribution of the charge density n of every site in the moiré unit cell $\mathcal{M}$ and thus the IDOS quantifies the available number of electronic states within an energy window ΔE_U . Here, we allow for ΔE_U not being the same as the moiré bandwidth ΔE_m and instead consider it to correspond to the Coulomb interaction strength projected onto the moiré bands. In this way, the IDOS becomes a measure of the effective flatness seen by the interactions in the system, thus quantifying the propensity for developing correlated phenomena, such as superconductivity and magnetism [4, 12, 13]. For TBG, this relevant energy scale ϵ_U is typically taken to lie between 20 and 30 meV [46, 47]. Here, we set $|\Delta E_U| = 27$ meV, which, for simplicity, corresponds to the interval $\Delta E_U = [-5, 5] \times 10^{-3} t_0$. We also check intervals corresponding to the extremal projected

Coulomb interaction strengths (20 and 30 meV) and find that the qualitative results remain unchanged. We note that there exist other measures of flatness than the IDOS, such as bandwidth or the effective mass of a band. While they are often adequate for well-separated, flat bands, here they fall short in capturing the contribution from multiple bands that are close in energy.

B. Buckling

Strain and buckling in monolayer graphene provide a powerful means to modulate the electronic properties [42, 48]. Such structural deformations alter the equilibrium atomic positions, thereby modifying the hopping strengths between atoms. In monolayer graphene, to lowest order, this breaks the symmetry of the three equivalent nearest-neighbor hoppings as $t_0 \mapsto t_i = t_0 + \delta t_i$, with $i \in (1, 2, 3)$, for the three nearest neighbors. A low-energy description of this transformation is obtained when it is plugged back into Eq. (1). The result is an effective pseudomagnetic vector potential $\mathbf{A}$ that couples to the electronic degrees of freedom, given by [20, 21, 39]

$$\mathbf{A} = (A_x, A_y) = \frac{1}{2ev_F} (\sqrt{3}(t_3 - t_2), t_2 + t_3 - 2t_1), \quad (5)$$

with $v_F = \frac{3}{2} t_0 a_c$ denoting the Fermi velocity in graphene. Similar to conventional gauge theories, the vector potential has an associated field given by $\mathbf{B}_{\text{PMF}} = \nabla \times \mathbf{A}$, known as the strain-induced pseudomagnetic field (PMF). Although it behaves like a magnetic field and, under the right conditions, even allows for (pseudo) Landau quantization of the electronic states with characteristic cyclotron orbits [15, 19], the PMF is fundamentally different from a real magnetic field. Importantly, real magnetic fields break time-reversal symmetry, whereas PMFs preserve it, but instead break valley (sublattice) symmetry [39, 49]. In graphene, it has been demonstrated that strain-induced PMFs can reach higher field strengths than are achievable using real magnetic fields, reaching strengths equivalent to 300 T [16].

In this work, we focus on a periodic buckling configuration that has been identified in monolayer graphene as particularly promising for the emergence of flat bands [19–21]. Notably, this buckling configuration can be continuously tuned, with the flatness of the bands increasing with the strength of PMF in monolayer graphene. Experimentally, this buckling pattern has been realized by depositing graphene on substrates such as hBN or NbSe₂, which naturally give rise to buckled superlattices [18]. Alternatively, a corrugated substrate can be engineered and positioned beneath the graphene layer to achieve a similar effect [36]. Here, we aim to combine the intrinsic moiré potential in TBG with the buckling-induced PMF generated by such substrate engineering and to study its effect on the electronic structure. We do not consider other buckles reported in earlier works [19], as they are unable to localize electrons and instead only scatter them, preventing the formation of flat bands, and thus making them unsuitable for our goal.

As we consider both buckling and moiré patterning, we must consider buckling periodic with the moiré pattern to be able to achieve a numerically viable setup.

The PMF, which is empirically derived [18], is given by

$$\mathbf{B}(\mathbf{r}) = B_0 \hat{\mathbf{z}} \sum_i \cos(\mathbf{b}_i \cdot (\mathbf{r} - \mathbf{r}_0)), \quad (6)$$

where B_0 is the strength of the induced PMF in Tesla, and the three different $\mathbf{b}_i$ correspond to the two reciprocal moiré unit cell lattice vectors, and their linear combination. Furthermore, $\mathbf{r}_0$ corresponds to the center of the buckle, serving as an additional parameter that allows us to shift the buckle center within the moiré unit cell. Setting $\mathbf{r}_0 = 0$ centers the buckle in the middle of the AA-stacked region. In Fig. 1(d), we illustrate such centered buckling in magic-angle TBG. The PMF attains its maximum in the center of the AA-stacked region, and its minima in the center of the AB/BA-stacked regions. It is important to note that the average induced PMF is zero. As a result, the electronic states arising from this buckling are Bloch bands rather than pLL [50]. Fundamentally, because the PMF varies rapidly, it cannot give rise to well-defined pLLs. For magnetic quantization effects to emerge, the PMF instead needs to remain constant on the length scale of the magnetic length [15,19]. Thus, we do not expect to see well-defined pLLs in this work. Unless stated otherwise, we assume $\mathbf{r}_0 = 0$, but we also explicitly investigate the nonzero $\mathbf{r}_0$.

For the PMF in Eq. (6), the deformation fields that generate it are unknown. As a consequence, we cannot directly modify the atomic positions and plug these into the Slater-Koster formalism of Eq. (1). However, following Refs. [18–21], we can still compute the changes to the in-plane hopping strengths δt_i by inserting the ansatz $\delta t_i = \alpha_i \sin(\mathbf{b}_i \cdot \mathbf{r})$ into Eq. (5) and fixing the α_i to match Eq. (6); for details, see Appendix A. In the end, the resulting changes to the three different nearest-neighbor hoppings, setting $\hbar = 1$, are given by

$$\delta t_i = \frac{-\sqrt{3}ev_F}{2\pi} L_m B_0 \sin(\mathbf{b}_i \cdot (\mathbf{r} - \mathbf{r}_0)). \quad (7)$$

This expression is overall consistent with earlier works [18–21].

Importantly, the modulation of the hopping amplitudes scales with both L_m , which varies across the different TBG systems studied here, and B_0 . Experimentally, the degree of lattice buckling controls bond length variations and, therefore, the magnitude of the hopping modulation. Since these modulations set the relevant energy scale in Eq. (1), keeping this scale fixed, or equivalently the shape of the buckle, requires that the product $L_m B_0$ remain constant. As a result, smaller systems, corresponding to TBG at large twist angles, demand significantly larger B_0 to produce comparable buckles. This scaling is physically intuitive, as a similar buckle in a larger moiré unit cell corresponds to a smoother buckling profile with lower curvature, and thus a weaker effective PMF [39].

Further, we note that Eqs. (6) and (7) are also directly applicable to monolayer graphene, because they only change the intralayer coupling. In TBG, the situation becomes more complex because buckling may also modulate the interlayer distance between the two graphene sheets, which is not captured by these two equations. To address this, we consider identical buckles in each layer, so that the effective interlayer distance remains constant, and the Slater-Koster parametrization in Eq. (1) remains valid. For substrate-induced buckling, this should be a good first approximation.

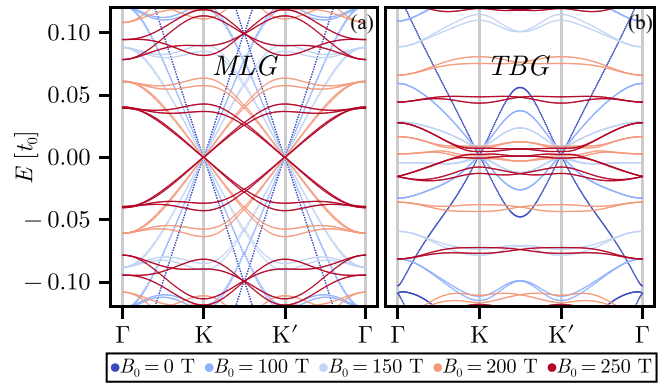


FIG. 2. Low-energy band structure of buckled monolayer graphene (a) and large-angle TBG at $\theta = 3.15^\circ$ (b) for different PMF strengths, with the same moiré unit cell in (a) and (b).

Taken together, we employ Eq. (7) to introduce controlled periodic buckling in TBG given in Eq. (1) from large twist angles, down to the magic angle. We aim to investigate how buckling interacts with the moiré physics of TBG, especially focusing on possibilities to flatten the low-energy bands. To estimate that we are in the correct parameter ranges, we compute the maximum modulations to the in-plane hopping using Eq. (7). The δt_i are bounded (from below) by $-\sqrt{3}ev_F L_m B_0 / 2\pi$. At the magic angle, where $L_m \approx 47.6a$, and using a PMF strength $B_0 = 50$ T, this suggests $\delta t_i \gtrsim -0.14$ eV. This represents the largest effective buckling strength considered in this work. This is smaller in magnitude than in the previous experimental works [18], which have investigated buckling in monolayer graphene at $L_m \approx 14$ nm and $B_0 = 120$ T, corresponding to $\delta t_i \gtrsim -0.40$ eV.

III. ENHANCED FLATTENING AND SYMMETRY BREAKING IN LARGE-ANGLE TBG

We start by investigating the effect of periodic buckling, centered on the AA-stacked region, on the low-energy electronic structure of large-angle TBG, choosing a representative twist angle $\theta = 3.15^\circ$. We first show that buckled large-angle TBG exhibits enhanced band flattening compared to monolayer graphene. We also compare the effects of buckling in AA- and AB-stacked bilayer graphene and thereby uncover the importance of broken inversion symmetry.

In Fig. 2, we plot the band structures of pristine and buckled graphene (a) and large-angle TBG (b) at different PMF strengths B_0 , using the same moiré unit cell for both systems. Pristine monolayer graphene and large-angle TBG have comparable Fermi velocities and other low-energy electronic properties, a direct consequence of large-angle TBG's nearly homogenous charge density [37]. However, significant differences emerge after buckling. In graphene, Fig. 2(a), buckling continuously and monotonously flattens the Dirac cones, which has been attributed to electronic confinement owing to the nonuniform strain [19]. This leads to a substantial reduction in the Fermi velocity, resulting in flatter bands. In large-angle TBG, there is an even stronger Fermi velocity reduction, resulting in an increased band flattening. We attribute this enhancement to the interlayer coupling in

TBG. In its absence, electrons in the individual monolayers tend to localize in regions of high PMF, tied to the buckle. However, when interlayer coupling is introduced, electrons are not only able to tunnel in-plane, but also between layers, effectively decreasing the in-plane kinetic energy. This enhances the in-plane electronic confinement and localization caused by the PMF, overall resulting in a flatter dispersion. This mechanism, in fact, resembles the band-flattening behavior observed in pristine TBG as the twist angle decreases towards the magic angle. In the latter case, it is the increasing size of the AA-stacked regions causing the moiré potential to grow, which promotes greater electron localization. Moreover, unlike monolayer graphene, the band flattening in TBG is not fully monotonic with increasing PMF strength B_0 . For TBG, buckling initially flattens the spectrum with increasing PMF strength, but, for example, for $B_0 = 250$ T, the resulting four central bands are more dispersive compared to $B_0 = 200$ T.

Furthermore, analogous to monolayer graphene [18,19], the continuum of bands away from the low-energy moiré bands in TBG transforms into a set of well-isolated bands at higher energies with increasing PMF strength. In fact, these higher-energy isolated flat bands are significantly flatter in large-angle TBG than in monolayer graphene. Unlike monolayer graphene, in TBG, these bands also sporadically hybridize with the lowest-energy moiré bands. We find that these higher-energy isolated flat bands emerge at all twist angles. They may be ideal candidates for hosting fractional Chern insulating states, as has been recently postulated in periodically strained MLG and buckled AB-stacked graphene [50,51]. We note that because of the spatially strongly varying PMF, these flat bands do not correspond to pLLs.

Additionally, we find in Fig. 2 that buckling in large-angle TBG gaps the spectrum at the Dirac points, K, K' , which does not occur in pristine TBG or in monolayer graphene. To highlight this, we compare large-angle TBG to AA- and AB-stacked bilayer graphene. Because of the small size of the unit cell in the latter two systems, we need to employ a strong PMF to reach effective lattice changes. For a fair comparison, where we take large-angle TBG at $B_0 = 100$ T as a reference, we can adjust the corresponding B_0 in bilayer graphene such that the product $L_m B_0$ is roughly constant. This, however, yields little buckling-induced change in the electronic structure of the low-energy band structure of either AA- or AB-stacked bilayer graphene. Instead, to induce noticeable changes and highlight the gap features, we apply a B_0 to AA- and AB-stacked graphene that is two and a half times stronger. We plot the resulting band structures of the pristine and buckled bilayers in Fig. 3. In both monolayer graphene Fig. 2(a) and the untwisted bilayers Figs. 3(a) and 3(b), the Dirac points remain protected by combined $C_{2z}\mathcal{T}$ symmetry [52], where the z axis is defined perpendicular to the two graphene sheets. The induced PMF couples with opposite sign to the two valleys in graphene. This breaks the sublattice symmetry and induces a spatially inhomogeneous mass term. However, this inhomogeneous mass term alone is not sufficient to lift the degeneracy at the Dirac point, since it does not break the C_{2z} inversion symmetry. Also, as mentioned in Sec. II, buckling never breaks $\mathcal{T}$. Thus, the degeneracy at the Dirac points is preserved for monolayer, and AA- and AB-stacked bilayer graphene. In contrast, in Fig. 3(c), the Dirac points are gapped

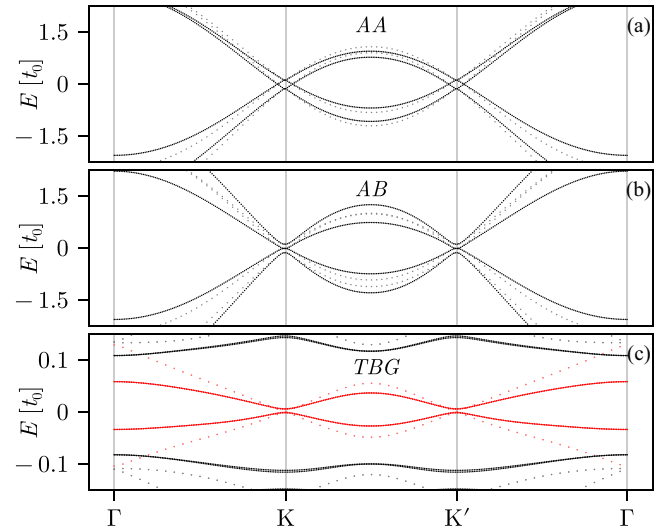


FIG. 3. Low-energy band structure for pristine and buckled AA-stacked bilayer graphene with $B_0 = 10\,000$ T and $L_m = a$ (a), AB-stacked bilayer graphene with $B_0 = 10\,000$ T and $L_m = a$ (b), and large-angle TBG at $\theta = 3.15^\circ$, corresponding to $L_m \approx 18.2 a$, with $B_0 = 100$ T (c) to keep that the product $L_m B_0$ roughly constant. Red bands are the moiré bands. Dotted bands correspond to pristine, nonbuckled bands. Note in (c) the smaller scale on the y axis.

with buckling for the twisted bilayers. Specifically, the slight layer misalignment in TBG, together with buckling, is enough to break inversion symmetry, and thus gap the spectrum [53]. This opening of a gap at the Dirac point is interesting on a fundamental level, but importantly, also has the consequence of both effectively flattening the four moiré bands. In fact, at large twist angles, the bandwidth at the center of the Brillouin zone Γ , where it reaches its maximum, is sufficiently large so that the emergence of the gap at the Dirac point enhances the overall flatness. This resembles the alignment or encapsulation of TBG with hBN [54]. We further note that the size of the gap does not grow monotonically with increasing strength of the PMF, something that would happen in the presence of a global sublattice-symmetry-breaking mass term. We attribute this to the nonlocal nature of the inhomogeneous mass term. Moreover, we verify that other buckling profiles, such as those employed in Refs. [19,49], also produce a similar gap at the Dirac points in TBG. We also investigate the role of the buckling center position $\mathbf{r}_0$ in large-angle TBG. However, because of the nearly uniform charge density in large-angle TBG, similar to that of monolayer graphene, we find that the position of the buckling center has only a minimal impact on the electronic structure. Aside from an occasional shift in energies, the qualitative features remain the same.

We conclude that large-angle TBG exhibits enhanced band flattening compared to monolayer graphene because of the presence of interlayer coupling. We interpret this as an enhancement of electronic localization in regions of strong PMF. Also, unlike monolayer graphene, the band flattening in TBG is not monotonic. Moreover, buckling in TBG lifts the degeneracy at the Dirac points by breaking the C_{2z} inversion symmetry, leading to flatter bands. In monolayer graphene and

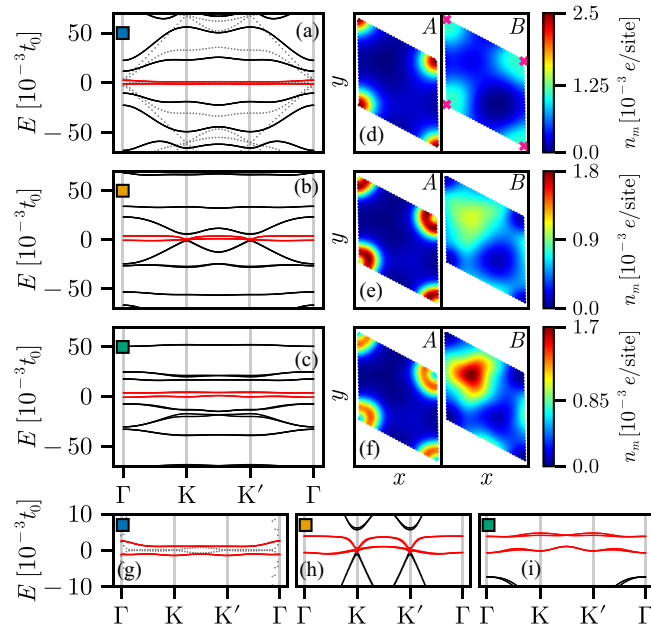


FIG. 4. Low-energy band structures (a)–(c), corresponding top-layer sublattice-resolved (A, B) moiré charge densities n_m (d)–(f), and zoom-in on low-energy bands (g)–(i) for buckled magic-angle TBG with the buckle centered in the AA-stacked region ($\mathbf{r}_0 = 0$) in both layers, indicated with pink crosses in (d), and buckling strengths $B_0 = 10$ T (a), (d), (g), 25 T (b), (e), (h), and 35 T (c), (f), (i). Red bands are the four energy bands that most resemble the moiré bands; gray bands in (a) are those of pristine magic-angle TBG.

AA- and AB-stacked bilayer graphene, the degeneracy at the Dirac point remains.

IV. COMPETING MECHANISMS AT THE MAGIC ANGLE

After discussing buckling in large-angle TBG, we turn to the magic angle, where TBG exhibits moiré-like physics rather than graphene-like physics. We buckle magic-angle TBG at multiple PMF strengths and analyze the resulting band structures and charge densities in Fig. 4. We first choose to center the buckle in the AA-stacked region $\mathbf{r}_0 = 0$, with the idea to enhance the electron confinement that is already there from twisting, but later also center the buckle in the AB-stacked region, which, at the magic angle, has a negligible charge density [37].

To investigate the effect of buckling on the electronic structure of magic-angle TBG, we plot in Fig. 4 both the band structures and (a)–(c) and sublattice-resolved moiré-charge densities (d)–(f) for increasing PMF strength. The charge densities are visualized in the top layer only, but are the same for the bottom layer. The most striking feature is the emergence of a sublattice charge polarization, causing a sublattice asymmetry in Figs. 4(d)–4(f). Already in Fig. 4(d), where the band structure is not so affected by the buckling [compare gray and black lines in Fig. 4(a)], there is an enhancement of charge localized around the AA-stacked region on one sublattice (A), while there is a depletion for the other sublattice B. Such sublattice polarization is also known to occur in monolayer graphene with the same buckling profile [18,19]. It will also

occur for other periodic buckling patterns because of how the PMF generally couples differently to the two valleys in graphene. In contrast to pristine magic-angle TBG, which has a sublattice symmetric charge density, buckling explicitly and inhomogeneously polarizes the sublattices. Hence, because of the difference in where these two flattening mechanisms want to localize the electrons, they become competing and are thus fundamentally incompatible with one another. As the buckling strength is further increased in Figs. 4(e) and 4(f), the sublattice polarization becomes more pronounced, but it is not enough to create isolated islands of electrons. It is worth mentioning that these electrons that are localized by the PMF do not correspond to pLLs, but are instead confined to regions with a strong PMF [19]. As a consequence of the pronounced sublattice polarization in buckled TBG, describing the system in terms of moiré physics becomes unfitting.

Focusing on the low-energy moiré band structure, we find that the buckling-induced PMF does not permit continuous flattening of the spectrum, either at the magic angle as seen in Figs. 4(a)–4(c), or at large twist angles, as discussed in Fig. 2(b), in contrast to monolayer graphene. Moreover, we find that the gap induced at the Dirac point, because of the breaking of the inversion symmetry, causes substantial changes to the moiré bands with buckling. Already at low to moderate PMF strengths in Fig. 4(a), the gap is clearly noticeable and, in contrast to large-angle TBG, the original bandwidth at Γ is now sufficiently narrow so that the gap instead increases the overall bandwidth, yielding a more dispersive band structure, compared to pristine TBG. Interestingly, for moderate PMF strength in Fig. 4(b), the conduction and valence bands even hybridize with the original four moiré bands. This roughly mimics the dispersion of large-angle buckled MLG, except for the four extra moiré bands in the middle.

When it comes to the continuum of bands away from the moiré bands, they gradually evolve into a set of isolated bands with increasing PMF strength as illustrated through Figs. 4(a)–4(c). As the PMF strength increases, these bands can, unlike graphene, occasionally hybridize with each other or the four original moiré bands; see e.g. Fig. 4(b). For strong PMF, where buckling is the dominant flattening mechanism, these isolated bands can become extremely flat, as shown in Fig. 4(c). We compare the bandwidth of these emergent flat bands to those of monolayer graphene with the same buckled moiré unit cell and PMF strengths (see Appendix B for details). We find that the TBG bandwidths are significantly smaller, often reduced by up to a factor of 2–3. As an example, at a large $B_0 = 35$ T, we find that the bandwidth of the first isolated band above the Fermi energy is about 70% smaller than in monolayer graphene.

Finally, we center the buckle in the AB-stacked region to investigate the importance of the location of the buckle. In Fig. 5, we again plot the band structure and charge density for the same buckling as in Fig. 4. Here, because of the AB-stacking at the center of the buckle, there is an asymmetry between the charge accumulated in the top and bottom layers. We focus on the top layer, since it turns out to be more susceptible to changes in the PMF strength. When we compute the charge density in the bottom layer, it initially looks similar to the top layer. For moderate to strong buckles, however,

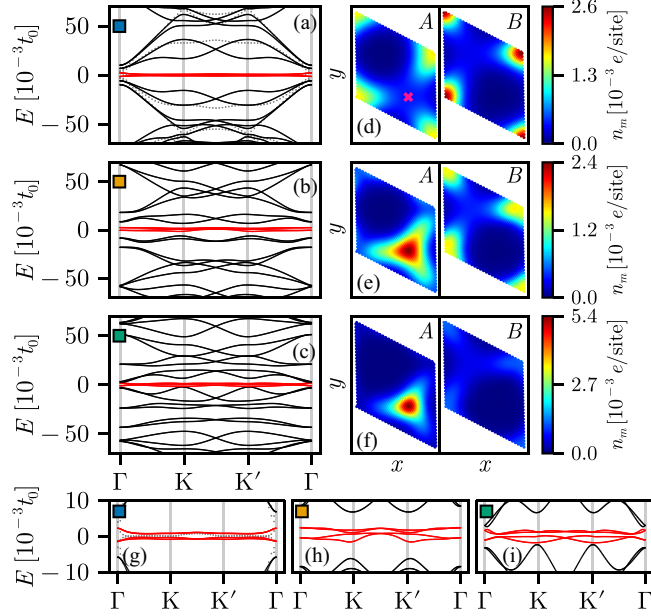


FIG. 5. Same as Fig. 4, but with buckle centered on the AB-stacked region, indicated with pink cross in (d).

electrons are mostly localized in the top layer, leaving almost no charge in the bottom layer. We attribute this layer dependence to the interplay between buckling-induced sublattice polarization and the different local stacking configurations in both layers.

For weak buckles in Fig. 5(a), we obtain a noticeable difference in the spectrum compared to Fig. 4(a). Here, the continuum of bands away from the four moiré bands initially becomes denser in energy, with more bands appearing in the same energy window, instead of transforming into a series of isolated bands. This continuum of bands is robust up to moderate PMF strengths, as illustrated in Fig. 5(b), but for large enough PMF strengths, shown in Fig. 5(c), it behaves similarly to the buckle centered in the AA-stacked region, where instead isolated bands start to emerge, although with larger bandwidth compared to that in Fig. 4(c). Unlike for the buckle centered in the AA-stacked region, the bands are, however, still not well isolated, and instead, the whole spectrum is more dispersive. This overall behavior can readily be explained by the fact that in pristine magic-angle TBG, electrons are primarily localized in the AA-stacked region; see Fig. 1(c). Consequently, a considerable perturbation in the AB-stacked region is required to induce noticeable changes in the electronic structure. This is similar to the behavior of atomic defects that also require a larger strength in the AB-region to evoke a noticeable change [38]. Still, several similarities to the AA-centered buckles remain. First, as shown by the sublattice-polarized charge densities in Figs. 5(d)–5(f), there is still competition between electronic localization driven by twisting and buckling. Second, inversion symmetry breaking persists, leading to a gap at the Dirac points. However, a key difference between Fig. 5(c) and Fig. 4(c) lies in the spatial localization of the electrons. For AB-region buckles, electrons are confined to the center of the buckle, whereas for AA-region buckles, they form a ring around the center in one

sublattice A, as also known to occur in monolayer graphene [18,19], and are also localized on the BA-stacked region in the other sublattice B.

We conclude that the flattening of the moiré bands because of twisting at the magic-angle in TBG and flattening because of the PMF fundamentally compete with each other, since the two flattening mechanisms localize electrons in different regions on different sublattices, making them fundamentally incompatible. This incompatibility persists regardless of the buckle's location. The main distinction between AA- and AB-buckled TBG is that the former exhibits flatter isolated bands, while the latter features a denser energy spectrum.

V. ENHANCED BAND FLATTENING CLOSE TO THE MAGIC ANGLE

After exploring both the large- and magic-angle limits of TBG, we finally focus on the intermediate regime near the magic angle, studying twist angles from the magic angle up to around 2° . In Secs. III and IV, we uncovered buckling in large- and magic-angle TBG can induce additional flatness into the low-energy bands. We are here interested in how suitable TBG is as a platform for hosting buckling-induced flat bands, as quantified by the IDOS introduced in Sec. II. This would open up the possibility of inducing electronic ordering, such as superconductivity and magnetism, also beyond the magic angle. Here, we focus on the low-energy regime around zero energy, using ΔE_U as the relevant energy regime to probe the IDOS relevant for correlation effects, but note that additional gating of TBG can also open for using higher-energy bands. However, while higher-energy bands can become well isolated and flat for large buckling, they generally only consist of two spin-degenerate bands, while there are four spin-degenerate (buckled deformed) moiré bands, thus providing more IDOS near zero energy. We introduce buckles in the center of the AA-stacked regions, since they give rise to flatter structures sooner than buckles centered in the AB-stacked region; see Sec. IV. From Eq. (7), we see that both the size of the moiré unit cell L_m and the PMF strength B_0 dictate the magnitude of the modulations to the in-plane hopping. We thus hold the product $L_m B_0$ constant across all twist angles, ensuring a fair comparison of the effect of buckling on the low-energy electronic structure. Furthermore, we use values of B_0 from 0 T to 50 T at the magic angle as the range of strengths at other twist angles, and denote these by B_M , with corresponding moiré length L_M . The corresponding B_0 at other twist angles is related through $B_0 = B_M L_M / L_m$.

We compute the IDOS for ΔE_U as a function of twist angle and buckling strength and compactly present the results in Fig. 6(a). For twist angles away from the magic angle, we see that buckling can significantly and consistently enhance the flatness of the spectrum. Here, the presence of buckles can more than double the IDOS of pristine TBG. For low B_M , the enhanced flatness is primarily driven by the gap opening at the Dirac points caused by inversion symmetry breaking, as illustrated by a prototypical low-energy band structure in Fig. 6(c) extracted at pink circle in Fig. 6(a), and also discussed in Sec. III. At moderate to large B_M and near the magic angle, an increased IDOS can result from hybridization between buckling-induced emergent flat bands and the moiré bands,

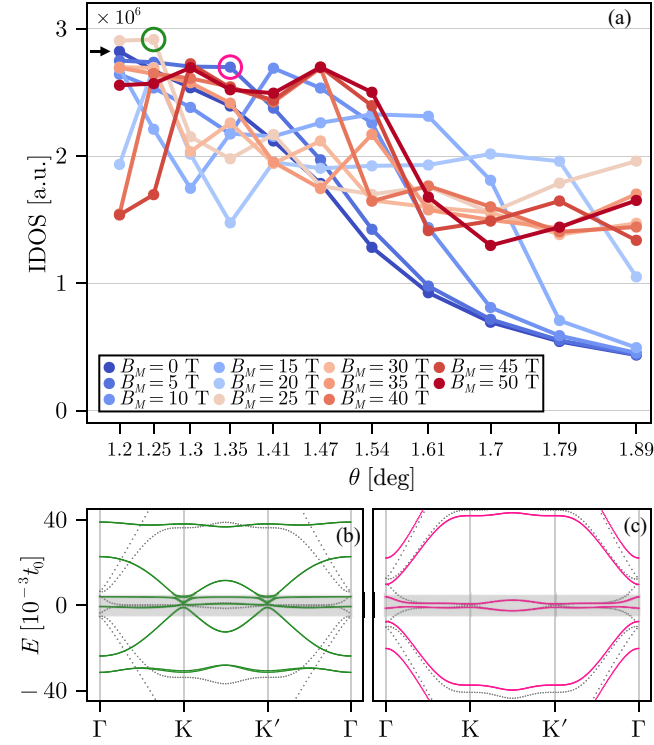


FIG. 6. IDOS as a function of twist angle θ (a) and prototypical bandstructures (b), (c), marked by matching colored circles in (a). Arrow indicates the IDOS for pristine magic-angle TBG. Light gray bands in (b) and (c) are band structures of pristine TBG at same twist angles. Dark gray boxes represent integration window set by ΔE_U .

consistent with our observations in Sec. IV and illustrated with the band structure in Fig. 6(b) extracted from the green circle in Fig. 6(a). However, while smaller twist angles progressively flatten the Dirac cones, buckling can cause the moiré bands to flatten less or even become more dispersive, and hence lead to a smaller IDOS than in the pristine case. Overall, this is because of the emergent competition between the two flattening mechanisms: the strong sublattice polarization induced by buckling negates the effect of twist-induced flattening. As a consequence, buckling does not monotonically increase the IDOS near the magic angle, implying that no clear trends of IDOS as a function of buckling strength exist. However, for slightly larger angles we do find that buckling becomes the dominant flattening mechanism, and in turn can transform the moiré bands into very flat bands, as observed in Fig. 4(c).

We conclude that buckling generally enhances band flatness at twist angles larger than the magic angle, compared to pristine TBG at the same angles. Near the magic angle, buckled TBG remains comparable to pristine magic-angle TBG in its ability to host flat bands. As a consequence, buckled TBG becomes much more robust to twist-angle disorder.

VI. CONCLUSIONS

In this work, we show how active buckling of TBG, achievable using, for example, substrate engineering, influences its low-energy electronic structure across a wide range of twist angles. Our work extends previous studies on buckled

monolayer graphene as a platform for strongly correlated electronic phases by introducing buckling into TBG. TBG itself is already an excellent candidate for realizing such phases at magic twist angles because of flat moiré bands emerging from twisting.

In the large-angle limit, we find that TBG's moiré bands can be continuously flattened by increasing buckling strength. This behavior is similar to that of monolayer graphene, but with two key differences. First, the combination of twist and buckle breaks the symmetry that protects the Dirac points, leading to the opening of a gap, which generates a slightly flatter spectrum. Second, the interlayer coupling in TBG allows for a greater degree of band flattening than is achieved in monolayer graphene for the same PMF strengths. Overall, we find that buckling can significantly enhance large-angle TBG's capacity to form flat bands. For example, for twist angles close to 2° , the integrated density of states (IDOS) in an energy range set by the effective Coulomb interaction projected onto the magic-angle flat bands can more than double for specific buckling strengths.

Close to the magic angle, we find that twist-induced band flattening and buckling-induced band flattening compete in how they localize electronic states. Specifically, buckling breaks sublattice symmetry, whereas it is opposed to twist-induced flattening. Buckling thus induces a sublattice polarization, which is absent in the pristine moiré bands, making the two mechanisms fundamentally incompatible. Moreover, compared to large-angle TBG, the initial bandwidth near Γ is now too narrow so that the gap generated by the inversion symmetry breaking at the Dirac points instead causes the bands to become overall more dispersive. Nonetheless, like in monolayer graphene, buckling in TBG can still develop a series of well-isolated bands at higher energy, and now with bandwidths often reduced by a factor of 2–3 compared to monolayer graphene. Moreover, in monolayer graphene, these bands remain well isolated, while in TBG, they occasionally hybridize with the moiré bands, especially close to the magic angle. This hybridization of additional flat bands, together with the gap opening, leads to the low-energy IDOS approaching or even slightly exceeding that of pristine magic-angle TBG for a range of PMFs and twist angles.

Our results establish buckled TBG as a versatile and robust platform for studying strongly correlated electronic phenomena, and can even enhance them due to a higher low-energy DOS. For example, buckling can easily counteract twist angle disorder by helping to keep a high flatness of the low-energy bands independent of the twist angle. Importantly, all effects considered here should be experimentally accessible. Previously, similar periodic buckling has been implemented in monolayer graphene [18], and strain-induced PMFs of several hundreds of Tesla have also been observed in monolayer graphene [16], much higher than required to reach the effects of buckling studied in this work.

ACKNOWLEDGMENTS

We thank L. Baldo and P. Holmval for fruitful discussions. This work was supported by the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (ERC-2022-CoG, Grant

Agreement No. 101087096). The calculations were enabled by resources provided by the National Academic Infrastructure for Supercomputing in Sweden (NAISS), partially funded by the Swedish Research Council through Grant Agreement No. 2022-06725.

DATA AVAILABILITY

The data that support the findings of this article are openly available [55].

APPENDIX A: DERIVATION OF HOPPING MODULATIONS

In Sec. II, we present in Eq. (7) a relationship describing the modulations in in-plane hopping strength because of an applied buckle. Here, we derive this expression in more detail. We start with the ansatz $\delta t_i = \alpha_i \sin(\mathbf{b}_i \cdot \mathbf{r})$ [20,21] and plug this into Eq. (5). The buckling-induced vector potential is related to the PMF as $\mathbf{B}_{\text{PMF}} = \partial_x \mathbf{A}_y - \partial_y \mathbf{A}_x$, which gives

$$\begin{aligned} \mathbf{B}_{\text{PMF}} = & \frac{1}{2ev_F} (\alpha_2 b_{2,x} \cos(\mathbf{b}_2 \cdot \mathbf{r}) + \alpha_3 b_{3,x} \cos(\mathbf{b}_3 \cdot \mathbf{r}) \\ & - 2\alpha_1 b_{1,x} \cos(\mathbf{b}_1 \cdot \mathbf{r})) + \frac{\sqrt{3}}{2ev_F} (\alpha_2 b_{2,y} \cos(\mathbf{b}_2 \cdot \mathbf{r}) \\ & - \alpha_3 b_{3,y} \cos(\mathbf{b}_3 \cdot \mathbf{r})). \end{aligned} \quad (\text{A1})$$

Next, we use the reciprocal moiré vectors

$$\begin{aligned} \mathbf{b}_1 &= \frac{2\pi}{L_m} \left(\frac{1}{\sqrt{3}}, -1 \right), \\ \mathbf{b}_2 &= \frac{2\pi}{L_m} \left(\frac{-2}{\sqrt{3}}, 0 \right), \\ \mathbf{b}_3 &= -\mathbf{b}_1 - \mathbf{b}_2 = \frac{2\pi}{L_m} \left(\frac{1}{\sqrt{3}}, 1 \right), \end{aligned} \quad (\text{A2})$$

and fix the α_i to match with Eq. (6). Coincidentally, one finds that all the α_i are equal. This leads to

$$\alpha_i = -\frac{\sqrt{3}ev_F}{2\pi} L_m B_0, \quad (\text{A3})$$

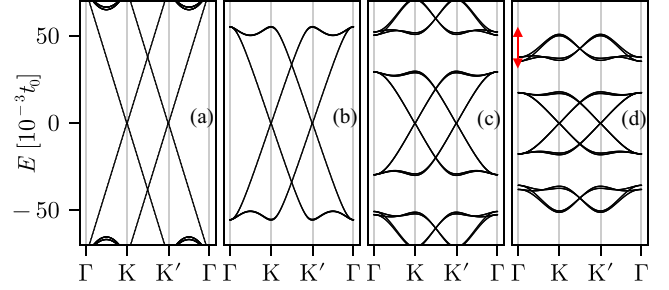


FIG. 7. Low-energy band structure of pristine (a) and buckled (b)–(d) monolayer graphene calculated using the magic-angle TBG moiré unit cell for the buckling periodicity for $B_0 = 0$ T (a), $B_0 = 10$ T (b), $B_0 = 25$ T (c), and $B_0 = 35$ T (d). Red arrow indicates the bandwidth of the isolated band discussed in the appendix text.

which finally yields Eq. (7) in the main text. We note that this expression differs by a factor of 1/2 from Refs. [20,21] and yields comparable spectra of works that follow a different ansatz if B_0 is rescaled to $B_0/4$ [18,19]. These prefactors do not influence our conclusions.

APPENDIX B: MONOLAYER GRAPHENE

In Figs. 4(a)–4(c), we find that the bandwidths of both the Dirac cone and emergent isolated bands in buckled graphene are larger than buckled TBG. Here, we complement this by plotting the effect of buckling on monolayer graphene in Fig. 7, which allows us to compare the various bandwidths directly. As an example, we find that the first set of well-isolated bands in buckled graphene at $B_0 = 35$ T has a bandwidth of around $16 \times 10^{-3} t_0$ (see red double-arrow), which is significantly larger than in magic-angle TBG, with a bandwidth of around $5 \times 10^{-3} t_0$ for the same first set of higher-energy isolated bands. Furthermore, as discussed in Sec. II, we find a continuous flattening of the Dirac cones, which has been observed in previous works [18,19], and a preserved Dirac point degeneracy owing to the absence of inversion-symmetry breaking.

- [1] C.-C. Tseng, X. Ma, Z. Liu, K. Watanabe, T. Taniguchi, J.-H. Chu, and M. Yankowitz, Anomalous Hall effect at half filling in twisted bilayer graphene, *Nat. Phys.* **18**, 1038 (2022).
- [2] Y. Cao, V. Fatemi, A. Demir, S. Fang, S. L. Tomarken, J. Y. Luo, J. D. Sanchez-Yamagishi, K. Watanabe, T. Taniguchi, E. Kaxiras, *et al.*, Correlated insulator behaviour at half-filling in magic-angle graphene superlattices, *Nature (London)* **556**, 80 (2018).
- [3] Y. Cao, V. Fatemi, S. Fang, K. Watanabe, T. Taniguchi, E. Kaxiras, and P. Jarillo-Herrero, Unconventional superconductivity in magic-angle graphene superlattices, *Nature (London)* **556**, 43 (2018).
- [4] M. Yankowitz, S. Chen, H. Polshyn, Y. Zhang, K. Watanabe, T. Taniguchi, D. Graf, A. F. Young, and C. R. Dean, Tuning

superconductivity in twisted bilayer graphene, *Science* **363**, 1059 (2019).

- [5] M. Oh, K. P. Nuckolls, D. Wong, R. L. Lee, X. Liu, K. Watanabe, T. Taniguchi, and A. Yazdani, Evidence for unconventional superconductivity in twisted bilayer graphene, *Nature (London)* **600**, 240 (2021).
- [6] T. Löthman, J. Schmidt, F. Parhizgar, and A. M. Black-Schaffer, Nematic superconductivity in magic-angle twisted bilayer graphene from atomistic modeling, *Commun. Phys.* **5**, 92 (2022).
- [7] Y. Cao, D. Chowdhury, D. Rodan-Legrain, O. Rubies-Bigorda, K. Watanabe, T. Taniguchi, T. Senthil, and P. Jarillo-Herrero, Strange metal in magic-angle graphene with near Planckian dissipation, *Phys. Rev. Lett.* **124**, 076801 (2020).

- [8] K. P. Nuckolls, M. Oh, D. Wong, B. Lian, K. Watanabe, T. Taniguchi, B. A. Bernevig, and A. Yazdani, Strongly correlated Chern insulators in magic-angle twisted bilayer graphene, *Nature (London)* **588**, 610 (2020).
- [9] E. Suárez Morell, J. D. Correa, P. Vargas, M. Pacheco, and Z. Barticevic, Flat bands in slightly twisted bilayer graphene: Tight-binding calculations, *Phys. Rev. B* **82**, 121407(R) (2010).
- [10] R. Bistritzer and A. H. MacDonald, Moiré bands in twisted double-layer graphene, *Proc. Natl. Acad. Sci. USA* **108**, 12233 (2011).
- [11] G. Tarnopolsky, A. J. Kruchkov, and A. Vishwanath, Origin of magic angles in twisted bilayer graphene, *Phys. Rev. Lett.* **122**, 106405 (2019).
- [12] L. A. Gonzalez-Arraga, J. L. Lado, F. Guinea, and P. San-Jose, Electrically controllable magnetism in twisted bilayer graphene, *Phys. Rev. Lett.* **119**, 107201 (2017).
- [13] T. M. R. Wolf, J. L. Lado, G. Blatter, and O. Zilberberg, Electrically tunable flat bands and magnetism in twisted bilayer graphene, *Phys. Rev. Lett.* **123**, 096802 (2019).
- [14] Y. Cao, D. Rodan-Legrain, O. Rubies-Bigorda, J. M. Park, K. Watanabe, T. Taniguchi, and P. Jarillo-Herrero, Tunable correlated states and spin-polarized phases in twisted bilayer-bilayer graphene, *Nature (London)* **583**, 215 (2020).
- [15] F. Guinea, M. I. Katsnelson, and A. Geim, Energy gaps and a zero-field quantum Hall effect in graphene by strain engineering, *Nat. Phys.* **6**, 30 (2010).
- [16] N. Levy, S. Burke, K. Meaker, M. Panlasigui, A. Zettl, F. Guinea, A. C. Neto, and M. F. Crommie, Strain-induced pseudo-magnetic fields greater than 300 tesla in graphene nanobubbles, *Science* **329**, 544 (2010).
- [17] X. Yuan, Z. Zhang, and S. Zhu, Strain engineering of flat bands in buckled graphene superlattices, *Phys. Rev. B* **111**, 235407 (2025).
- [18] J. Mao, *et al.*, Evidence of flat bands and correlated states in buckled graphene superlattices, *Nature (London)* **584**, 215 (2020).
- [19] S. P. Milovanović, M. Andjelković, L. Covaci, and F. M. Peeters, Band flattening in buckled monolayer graphene, *Phys. Rev. B* **102**, 245427 (2020).
- [20] A. L. Manesco, J. L. Lado, E. V. Ribeiro, G. Weber, and D. Rodrigues Jr, Correlations in the elastic Landau level of spontaneously buckled graphene, *2D Mater.* **8**, 015011 (2021).
- [21] A. L. Manesco and J. L. Lado, Correlation-induced valley topology in buckled graphene superlattices, *2D Mater.* **8**, 035057 (2021).
- [22] S. K. Jain, V. Juričić, and G. T. Barkema, Structure of twisted and buckled bilayer graphene, *2D Mater.* **4**, 015018 (2016).
- [23] T. Rakib, P. Pochet, E. Ertekin, and H. T. Johnson, Corrugation-driven symmetry breaking in magic-angle twisted bilayer graphene, *Commun. Phys.* **5**, 242 (2022).
- [24] J. Wang, A. Khosravi, A. Silva, M. Fabrizio, A. Vanossi, and E. Tosatti, Bending stiffness collapse, buckling, topological bands of freestanding twisted bilayer graphene, *Phys. Rev. B* **108**, L081407 (2023).
- [25] J. Wang and E. Tosatti, Universal moiré buckling of freestanding 2D bilayers, *Proc. Natl. Acad. Sci. USA* **121**, e2418390121 (2024).
- [26] G. Soares, R. R. Barreto, R. Nadas, K. Watanabe, T. Taniguchi, L. C. Campos, L. G. Cancado, and A. Malachias, Anisotropic strain observation in naturally occurring buckling on twisted bilayer graphene: A nano-Raman study, *J. Phys. Chem. C* **129**, 7508 (2025).
- [27] N. P. Kazmierczak, M. Van Winkle, C. Ophus, K. C. Bustillo, S. Carr, H. G. Brown, J. Ciston, T. Taniguchi, K. Watanabe, and D. K. Bediako, Strain fields in twisted bilayer graphene, *Nat. Mater.* **20**, 956 (2021).
- [28] N. N. Nam and M. Koshino, Lattice relaxation and energy band modulation in twisted bilayer graphene, *Phys. Rev. B* **96**, 075311 (2017).
- [29] P. Lucignano, D. Alfè, V. Cataudella, D. Ninno, and G. Cantele, Crucial role of atomic corrugation on the flat bands and energy gaps of twisted bilayer graphene at the magic angle $\theta = 1.08^\circ$, *Phys. Rev. B* **99**, 195419 (2019).
- [30] S.-C. Ho, C.-H. Chang, Y.-C. Hsieh, S.-T. Lo, B. Huang, T.-H.-Y. Vu, C. Ortix, and T.-M. Chen, Hall effects in artificially corrugated bilayer graphene without breaking time-reversal symmetry, *Nat. Electron.* **4**, 116 (2021).
- [31] J.-B. Qiao, L.-J. Yin, and L. He, Twisted graphene bilayer around the first magic angle engineered by heterostrain, *Phys. Rev. B* **98**, 235402 (2018).
- [32] L. Huder, A. Artaud, T. Le Quang, G. T. De Laissardière, A. G. Jansen, G. Lapertot, C. Chapelier, and V. T. Renard, Electronic spectrum of twisted graphene layers under heterostrain, *Phys. Rev. Lett.* **120**, 156405 (2018).
- [33] Y. Zhang, Z. Hou, Y.-X. Zhao, Z.-H. Guo, Y.-W. Liu, S.-Y. Li, Y.-N. Ren, Q.-F. Sun, and L. He, Correlation-induced valley splitting and orbital magnetism in a strain-induced zero-energy flatband in twisted bilayer graphene near the magic angle, *Phys. Rev. B* **102**, 081403(R) (2020).
- [34] S. Li, X. Shi, J. Li, C. He, T. Ouyang, C. Tang, and J. Zhong, Strain-induced flatbands in large-angle twisted bilayer graphene, *J. Appl. Phys.* **137**, 083902 (2025).
- [35] Z. Wu, Z. Zhan, J. A. Silva-Guillén, F. Guinea, M. I. Katsnelson, and S. Yuan, Evolution of the confined states in graphene nanobubbles, *Phys. Rev. B* **109**, 115420 (2024).
- [36] V. T. Phong and E. J. Mele, Boundary modes from periodic magnetic and pseudomagnetic fields in graphene, *Phys. Rev. Lett.* **128**, 176406 (2022).
- [37] G. Trambly de Laissardière, D. Mayou, and L. Magaud, Localization of Dirac electrons in rotated graphene bilayers, *Nano Lett.* **10**, 804 (2010).
- [38] L. Baldo, T. Löthman, P. Holmval, and A. M. Black-Schaffer, Defect-induced band restructuring and length scales in twisted bilayer graphene, *Phys. Rev. B* **108**, 125141 (2023).
- [39] M. A. Vozmediano, M. Katsnelson, and F. Guinea, Gauge fields in graphene, *Phys. Rep.* **496**, 109 (2010).
- [40] P. Moon and M. Koshino, Optical absorption in twisted bilayer graphene, *Phys. Rev. B* **87**, 205404 (2013).
- [41] S. Shallcross, S. Sharma, E. Kandelaki, and O. A. Pankratov, Electronic structure of turbostratic graphene, *Phys. Rev. B* **81**, 165105 (2010).
- [42] C. Si, Z. Sun, and F. Liu, Strain engineering of graphene: A review, *Nanoscale* **8**, 3207 (2016).
- [43] M. M. A. Ezzi, G. N. Pallewela, C. De Beule, E. J. Mele, and S. Adam, Analytical model for atomic relaxation in twisted moiré materials, *Phys. Rev. Lett.* **133**, 266201 (2024).
- [44] J. Kang and O. Vafek, Analytical solution for the relaxed atomic configuration of twisted bilayer graphene including heterostrain, *Phys. Rev. B* **112**, 125138 (2025).

- [45] E. Y. Andrei and A. H. MacDonald, Graphene bilayers with a twist, *Nat. Mater.* **19**, 1265 (2020).
- [46] B. A. Bernevig, Z.-D. Song, N. Regnault, and B. Lian, Twisted bilayer graphene. III. Interacting Hamiltonian and exact symmetries, *Phys. Rev. B* **103**, 205413 (2021).
- [47] C.-X. Liu and B. A. Bernevig, Nodal nematic superconductivity in multiple flat-band systems, *Phys. Rev. B* **111**, L020502 (2025).
- [48] Y. Hou, J. Zhou, M. Xue, M. Yu, Y. Han, Z. Zhang, and Y. Lu, Strain engineering of twisted bilayer graphene: The rise of strain-twistronics, *Small* **21**, 2311185 (2024).
- [49] D. Moldovan, M. Ramezani Masir, and F. M. Peeters, Electronic states in a graphene flake strained by a Gaussian bump, *Phys. Rev. B* **88**, 035446 (2013).
- [50] M. Fujimoto, D. E. Parker, J. Dong, E. Khalaf, A. Vishwanath, and P. Ledwith, Higher vortexability: Zero-field realization of higher Landau levels, *Phys. Rev. Lett.* **134**, 106502 (2025).
- [51] Q. Gao, J. Dong, P. Ledwith, D. Parker, and E. Khalaf, Untwisting moiré physics: Almost ideal bands and fractional Chern insulators in periodically strained monolayer graphene, *Phys. Rev. Lett.* **131**, 096401 (2023).
- [52] A. H. Castro Neto, F. Guinea, N. M. R. Peres, K. S. Novoselov, and A. K. Geim, The electronic properties of graphene, *Rev. Mod. Phys.* **81**, 109 (2009).
- [53] H. C. Po, L. Zou, A. Vishwanath, and T. Senthil, Origin of Mott insulating behavior and superconductivity in twisted bilayer graphene, *Phys. Rev. X* **8**, 031089 (2018).
- [54] M. Long, Z. Zhan, P. A. Pantaleón, J. Á. Silva-Guillén, F. Guinea, and S. Yuan, Electronic properties of twisted bilayer graphene suspended and encapsulated with hexagonal boron nitride, *Phys. Rev. B* **107**, 115140 (2023).
- [55] J. van Poppelen and A. M. Black-Schaffer, Buckling and flat bands in twisted bilayer graphene, Zenodo (2025), doi:10.5281/zenodo.18934500.