

Impact of spin-orbit coupling on electron correlation corrections to the density of states in anisotropic conductors

Bahruz Suleymanli ^{*}*Physics Department, Yıldız Technical University, 34220 Esenler, Istanbul, Türkiye*B. Tanatar [†]*Department of Physics, Bilkent University, 06800 Ankara, Türkiye*

(Received 26 November 2025; revised 12 June 2026; accepted 24 July 2026; published 18 August 2026)

We study Altshuler-Aronov-type interaction corrections to the single-particle density of states (DOS) in a strongly anisotropic two-dimensional conductor with an open Fermi surface and weak disorder, in the presence of coexisting Rashba and Dresselhaus spin-orbit couplings (SOCs) constrained to the longitudinal direction. The low-energy band consists of two warped sheets weakly tunnel-coupled transversely; the SOC splits the sheets into helicity branches with a fixed spin axis. Working in a Matsubara space, we compute the exchange contribution in the diffusion channel with dynamically screened Coulomb interaction and an impurity ladder. The resulting DOS anomaly exhibits a dimensional crossover governed by the transverse coupling scale ε_c . Close to the Fermi level ($|\varepsilon - \varepsilon_F| < \varepsilon_c$), the system behaves two dimensionally, featuring a logarithmic DOS dip whose magnitude is enhanced by intrinsic SOC. Further from the Fermi level ($|\varepsilon - \varepsilon_F| > \varepsilon_c$), the system behaves quasi-one-dimensionally, featuring a sharper square-root singularity whose amplitude is remarkably enhanced by the SOC. Notably, we identify a critical SOC strength at which these spin-orbit effects exactly cancel the electron-correlation correction, perfectly restoring the unperturbed density of states. Furthermore, increasing the SOC beyond this critical point inverts the sign of the anomaly entirely, yielding a positive DOS correction. This sign reversal fundamentally alters the energy dependence, such that at energies beyond $\varepsilon_F + \varepsilon_c$, the positive correction decays to smaller values as energy increases, opposite to the standard negative correction. This contrasting trend provides a distinct spectroscopic signature of SOC-modulated correlation effects.

DOI: [10.1103/physrevb.114.074202](https://doi.org/10.1103/physrevb.114.074202)

I. INTRODUCTION

Low-energy irregularities—most notably zero-bias anomalies—are frequently observed in modern low-dimensional conductors, including gate-tunable two-dimensional electron gases with Rashba spin-orbit coupling (SOC) [1–3], two-dimensional (2D) materials with proximity-induced SOC, such as graphene/transition metal dichalcogenide (TMD) heterostructures [4–8], 2D electron systems patterned by unidirectional lateral superlattices [9,10], self-assembled quasi-1D surface atomic chains [11–13], and ultrathin disordered metallic films [14,15]. However, in practice, distinct microscopic processes—Altshuler-Aronov (AA) interaction anomalies that suppress single-particle density-of-states (DOS) in a dimensionality-dependent way [16], Coulomb gaps in localized regimes [17], Luttinger liquid power laws in 1D [18], and Kondo resonances from magnetic moments [19]—can

yield nearly indistinguishable dependences on bias, temperature, and magnetic field, obscuring the identification of the underlying mechanism. To address this ambiguity, we develop a predictive framework that jointly treats strong band anisotropy, weak spin-independent disorder, and intrinsic SOC (Rashba/Dresselhaus) in an open-Fermi-surface 2D conductor.

Against this backdrop, we focus on the weak-disorder diffusive regime, characterized by $k_F l \gg 1$, with mean free path $l = v_F \tau$, where k_F denotes the Fermi wave number and v_F the Fermi velocity. In low-dimensional electronic systems, this regime can qualitatively reshape screening and localization. For instance, in 1D and quasi-1D conductors, it can induce localization phenomena that destabilize the clean-limit DOS. Similarly, in strongly anisotropic systems with Dirac or quasi-Dirac dispersions, weak disorder can generate a finite background DOS through quasilocalized states [20]. Such interplay is particularly relevant for open Fermi-surface geometries.

Beyond the generic low-dimensional platforms discussed above, strongly anisotropic 2D systems—which we model as a planar array of dislocation tubes with weak, spin-independent disorder—are not an abstraction but a realistic substructure in several state-of-the-art materials and devices. In III-nitride semiconductors (GaN, InN), parallel threading dislocations form arrays of quasi-1D conductive channels

^{*}Contact author: bahruz.suleymanli@gmail.com[†]Contact author: tanatar@fen.bilkent.edu.tr

that can dominate charge and heat transport [21,22]. Similar dislocation-guided “supermetallic” pathways have been identified and are now being engineered inside Si/SiGe MOS-FET channels [23]. In 2D materials, grain boundaries and AB–BA stacking lines in bilayer graphene and hexagonal boron nitride (hBN) realize ordered dislocation arrays that host robust 1D metallic modes [24]. Oxide heterostructures (e.g., SrTiO₃, LAO/STO bicrystals) similarly exhibit conduction along periodic dislocations and ferroelastic walls, with spacing set by crystal disorientation [25]. Finally, both theory and experiment on topological insulators and semimetals highlight helical 1D modes bound to dislocation lines, underscoring the technological relevance of our open-Fermi-surface, anisotropic model [26].

To isolate impurity effects in such low-dimensional settings, one works at temperatures low enough that thermal fluctuations do not wash out the electron’s accumulated phase during impurity scattering, thereby preserving quantum-interference corrections [27,28]. In this regime, electron-electron (e-e) interactions combine with elastic scattering to produce interference effects that renormalize single-particle and transport properties. The first weak-disorder calculations showed that, already at lowest order in perturbation theory, including e-e interactions yields a singular correction to the DOS at the Fermi level [29,30], with closely related consequences for kinetic and thermodynamic properties [31]. Higher-order treatments in the weak-disorder limit established that the form of these singularities is preserved while prefactors are renormalized [32,33]. Comparable conclusions were reached for structurally anisotropic 2D systems without SOC, providing a natural baseline for the present work where SOC is included explicitly [34,35].

Recent experiments and theoretical studies on e-e interactions in weakly disordered conductors consistently show a strong suppression of the DOS near the Fermi energy ε_F . Diagrammatic studies clarify how magnetic fields regularize the low-energy anomaly and modify scaling [36], while nonequilibrium Green’s function approaches extend e-e interaction corrections to nanoscale devices [37]. Scanning tunneling spectroscopy on strongly disordered ultrathin films, such as NbN and MoN, has directly observed the AA DOS suppression [38,39], and local DOS fluctuation measurements have linked the anomaly to quantum diffusion [40]. On the theory side, topological-insulator surface analyses separate e-e interaction from interference contributions [41], and states-conserving AA extensions show how spectral weight depleted near ε_F is recovered at higher energies [42]. More broadly, the interaction-disorder interplay has been connected to Friedel oscillation physics near the 2D metal-insulator transition [43], and even in the ballistic regime e-e interaction can reshape the DOS nonperturbatively [44].

In this work, we compute the AA interaction correction to the single-particle DOS for a strongly anisotropic 2D metal with an open Fermi surface, weak spin-independent disorder, and intrinsic Rashba/Dresselhaus SOC confined to the dispersive x direction. We evaluate the DOS correction from the exchange diagrams in the diffusive channel with dynamically screened Coulomb interaction within the random-phase approximation (RPA): the particle-hole bubble is dressed by the impurity ladder, and—after taking the spin trace at density

vertices—the ladder reduces to the charge/singlet diffuson. We neglect the corresponding Hartree contribution, as its requisite large momentum transfer makes it parametrically suppressed for the long-range Coulomb interaction considered here. The resulting parameter-explicit expression exhibits a clear dimensional crossover governed by the transverse tunneling t_y and the elastic time τ . The physical role of the SOC strictly depends on this dimensionality. While SOC increases the magnitude of the 2D logarithmic dip, it also remarkably enhances the 1D square-root anomaly. Furthermore, strong SOC in the 1D regime can invert the sign of the correction entirely, producing a positive DOS peak that decays at higher energies.

The rest of the paper is organized as follows. Section II introduces the main model—a strongly anisotropic 2D conductor with an open Fermi surface and intrinsic Rashba/Dresselhaus SOC along the dispersive x direction—and collects the essential ingredients for the DOS correction: the energy spectrum, the disorder model and mean free times (τ_1 , τ_2), and the explicit forms of the bare Green’s functions (in the helicity basis). Section III presents the diagrammatic calculation in the diffusive limit: we evaluate the exchange self-energy, compute the particle-hole bubbles, resum the impurity ladders to obtain the charge (singlet) diffuson $\theta(\mathbf{q}, i\varepsilon)$, and derive the dynamically screened Coulomb interaction $V(\mathbf{q}, i\varepsilon)$ within the RPA. Section IV provides analytic expressions for the DOS correction $\delta\rho(\varepsilon, T)$ in the 2D and 1D windows, identifies the crossover scale $\varepsilon_c \sim t_y^2\tau$. Section VI summarizes the main results and outlines proposed experimental tests. Finally, For completeness, the explicit derivation of the dynamically screened interaction is detailed in the Appendix.

II. MODEL AND FORMALISM FOR THE DOS CORRECTION

We consider a strongly anisotropic 2D electron system modeled as a planar array of edge-dislocation tubes (chains), each tube providing a quasi-1D conducting channel. Electrons propagate predominantly along the tubes, while transverse motion occurs only via nearest-neighbor tunneling with amplitude t_y between tubes separated by d . Intrinsic spin-orbit interaction acts only along the longitudinal direction and is taken as $H_{\text{SO}} = \alpha k_x \sigma_y - \beta k_x \sigma_x$, where α and β denote the Rashba and Dresselhaus SOC parameters, respectively [45,46]. The low-energy dispersion near ε_F is written as

$$\varepsilon_\sigma(\mathbf{k}) = \varepsilon_F + v_F(|k_x| - k_F) + \sigma\sqrt{\alpha^2 + \beta^2}k_x - 2t_y \cos(dk_y), \quad (1)$$

where k_x is the longitudinal quasimomentum (along the tubes), k_y is the transverse quasimomentum, and $\sigma = \pm$ labels the helicity. Note that the dispersion in Eq. (1) represents a spin-orbit-coupled realization of the well-established coupled-wire construction, a mathematical framework widely used to model 2D anisotropic and topological phases [47,48]. We work in the strong-anisotropy regime $v_F > \sqrt{\alpha^2 + \beta^2}$ and $2t_y/(v_F k_F) \ll 1$. Then the helicity-dependent longitudinal velocity $v_x^\sigma = \partial\varepsilon_\sigma/\partial k_x = v_F \text{sgn}(k_x) + \sigma\sqrt{\alpha^2 + \beta^2}$ remains positive on the right sheet ($k_x > 0$) and negative on the left

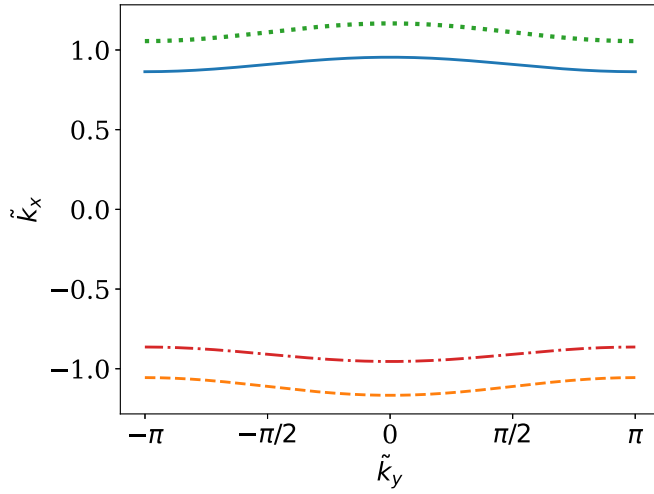


FIG. 1. Fermi surface for a strongly anisotropic 2D band. Here $\tilde{k}_x = k_x/k_F$ and $\tilde{k}_y = dk_y$ are dimensionless momenta; parameters used are $2t_y/(k_F v_F) = 0.05$ and $\sqrt{\alpha^2 + \beta^2}/v_F = 0.1$.

sheet ($k_x < 0$), preserving the natural chirality of the open Fermi lines. The overlap integral t_y decreases exponentially with d/a , where a is the tube radius, and—within the above inequalities—the Fermi surface consists of four warped lines (two sheets $\times$ two helicities), see Fig. 1. Throughout we assume the weak-disorder diffusive window $k_F l \gg 1$.

In this system, impurities are randomly distributed on the tubes, and are spin-independent [49]. Their potential is effective only within a given tube (no intertube scattering). Within the Born approximation, scattering decomposes into forward ($q \approx 0$) and backward ($q \approx 2k_F$) channels with rates

$$\tau_1^{-1} = \frac{c_{\text{imp}}}{v_F} U^2(0) \quad \text{and} \quad \tau_2^{-1} = \frac{c_{\text{imp}}}{v_F} U^2(2k_F), \quad (2)$$

respectively, where c_{imp} is the linear impurity concentration along a tube and $U(q)$ the Fourier transform of the impurity potential [50]. The total momentum-relaxation rate is $\tau^{-1} = \tau_1^{-1} + \tau_2^{-1}$, and the mean free path is $l = v_F \tau$.

In this setting, including e-e interactions, the one-particle DOS is obtained from the retarded Green's function as

$$\rho(\varepsilon, T) = -\frac{2}{\pi} \int \frac{d^2 k}{(2\pi)^2} \Im \left(\sum_s \text{Tr} (G_s^{-1}(\mathbf{k}, i\varepsilon_n \rightarrow \varepsilon + i0^+)) \right. \\ \left. - \Sigma_s(\mathbf{k}, i\varepsilon_n \rightarrow \varepsilon + i0^+) \right)^{-1}, \quad (3)$$

where the bare Matsubara Green function $G_s(\mathbf{k}, i\varepsilon_n)$ characterizes the noninteracting problem, $\Sigma_s(\mathbf{k}, i\varepsilon_n)$ is the self-energy, and $\varepsilon_n = \pi T(2n+1)$ are fermionic Matsubara frequencies, $n \in \mathbb{Z}$. The index $s = \pm$ labels the right ($k_x \approx +k_F$) and left ($k_x \approx -k_F$) open-Fermi-surface sheets (solid/dotted vs dashed/dashed-dotted lines in Fig. 1). For our strongly anisotropic spectrum with longitudinal Rashba/Dresselhaus SOC, the bare propagator can be written in a k -independent

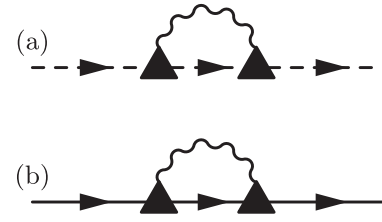


FIG. 2. Exchange diagrams in the diffusion channel that contribute to Σ_s in the first order of the e-e interaction. Dashed (a) and solid (b) arrowed lines represent the bare Green's functions for the $s = +1$ and $s = -1$ bands, respectively. The wavy line denotes the dynamically screened Coulomb interaction $V_s(\mathbf{q}, i\omega_m)$, while the filled triangles indicate the impurity ladders $\theta_s(\mathbf{q}, i\omega_m)$ corresponding to multiple impurity scatterings at the density vertices.

helicity basis as

$$G_s(\mathbf{k}, i\varepsilon_n) = \sum_{\sigma} (i\varepsilon_n - s v_F (k_x - s k_F) - \sigma \sqrt{\alpha^2 + \beta^2} k_x \\ + 2t_y \cos(dk_y) + i/2\tau \text{sgn}(\varepsilon_n))^{-1} g_{\sigma}, \quad (4)$$

where

$$g_{\sigma} = \frac{1}{2} \begin{pmatrix} 1 & \sigma \sqrt{\frac{\beta+i\alpha}{\beta-i\alpha}} \\ \sigma \sqrt{\frac{\beta-i\alpha}{\beta+i\alpha}} & 1 \end{pmatrix}. \quad (5)$$

In this study, we compute the correction to the DOS due to the e-e interaction, where from Eq. (3) for this purpose we find the correction to the DOS

$$\delta\rho(\varepsilon, T) = -\frac{2}{\pi} \int \frac{d^2 k}{(2\pi)^2} \Im \left(\sum_s \text{Tr} [G_s^2(\mathbf{k}, i\varepsilon_n \rightarrow \varepsilon + i0^+)] \right. \\ \left. \times \Sigma_s(\mathbf{k}, i\varepsilon_n \rightarrow \varepsilon + i0^+) \right). \quad (6)$$

Equation (6) represents the leading-order interaction correction. In the weak-disorder regime, higher-order terms are parametrically suppressed and merely renormalize the prefactors without altering the fundamental form of the singularity [32,33]. This correction can be computed by the conventional diagrammatic technique [50], and this is presented in detail in Sec. III.

III. IMPURITY VERTICES AND RENORMALIZATION OF THE COULOMB INTERACTION

The leading contribution of the e-e interaction self-energy $\Sigma_s(\mathbf{k}, i\varepsilon_n)$ to the one-particle DOS arises, to first order in the interaction, from the diffusion channel [31–34]. Therefore, when evaluating $\Sigma_s(\mathbf{k}, i\varepsilon_n)$ we sum the diagrams shown in Fig. 2. As already noted in Sec. I, the nontrivial quantum corrections driven by e-e interaction occur due to interference between multiple impurity scatterings and inelastic e-e processes. Multiple impurity scatterings are represented in Fig. 2 by filled triangles (impurity ladders). In this notation, dashed arrowed lines correspond to the bare Green's function with $s = +1$ [see Eq. (4)], while solid lines correspond to the bare Green's function with $s = -1$.

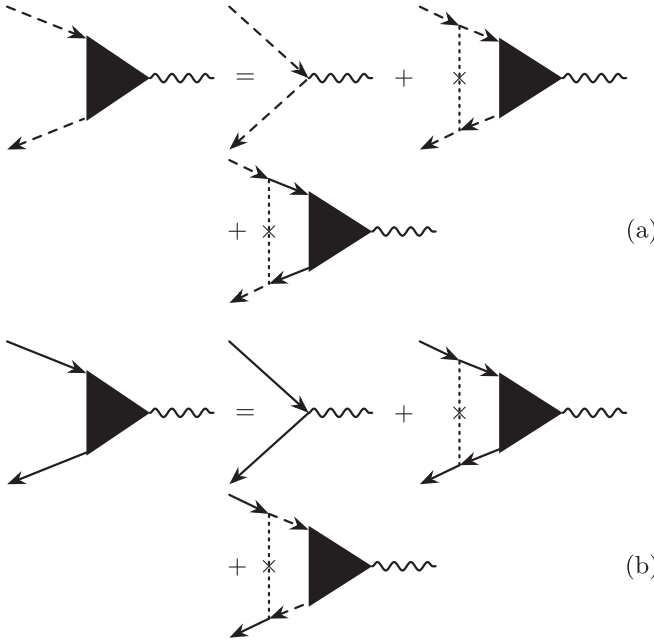


FIG. 3. Ladder approximation for calculating impurity vertices.

We compute $\Sigma_s(\mathbf{k}, i\varepsilon_n)$ as

$$\Sigma_s(\mathbf{k}, i\varepsilon_n) = T \sum_{\omega_m} \int \frac{d^2q}{(2\pi)^2} G_s(\mathbf{k} + \mathbf{q}, i(\varepsilon_n + \omega_m)) \times \theta_s^2(\mathbf{q}, i\omega_m) V_s(\mathbf{q}, i\omega_m), \quad (7)$$

where T is the temperature, $\omega_m = 2\pi Tm$ is the transferred Matsubara frequency, and $\mathbf{q}$ is the transferred momentum in the inelastic e-e scattering. The factor $\theta_s(\mathbf{q}, i\omega_m)$ denotes the impurity ladder (i.e., the dressed density vertex) represented by triangles in Fig. 2, while $V_s(\mathbf{q}, i\omega_m)$ is the screened Coulomb interaction, depicted by the wavy line in Fig. 2.

Here we outline the calculation of the impurity ladders. We work in the ladder approximation because we are in the weak-disorder regime, where diagrams with intersecting impurity lines are suppressed by the small parameter $(k_F l)^{-1} \ll 1$ [50]. The renormalization of the vertices by impurity lines is implemented via the graphical Dyson equations shown in Fig. 3: Fig. 3(a) corresponds to the impurity ladder relevant for the diagram in Fig. 2(a), and Fig. 3(b) to the one for Fig. 2(b). From Fig. 3 we can write

$$\theta_1(\mathbf{q}, i\omega_m) = 1 + \frac{dv_F}{\tau_1} I_1(\mathbf{q}, i\omega_m) \theta_1(\mathbf{q}, i\omega_m) + \frac{dv_F}{\tau_2} I_2(\mathbf{q}, i\omega_m) \theta_2(\mathbf{q}, i\omega_m), \quad (8)$$

$$\theta_2(\mathbf{q}, i\omega_m) = 1 + \frac{dv_F}{\tau_1} I_1(\mathbf{q}, i\omega_m) \theta_2(\mathbf{q}, i\omega_m) + \frac{dv_F}{\tau_2} I_2(\mathbf{q}, i\omega_m) \theta_1(\mathbf{q}, i\omega_m), \quad (9)$$

where $\tau_{1,2}$ are defined in Eq. (2), and $I_{1,2}$ are the elementary bubbles [displayed in

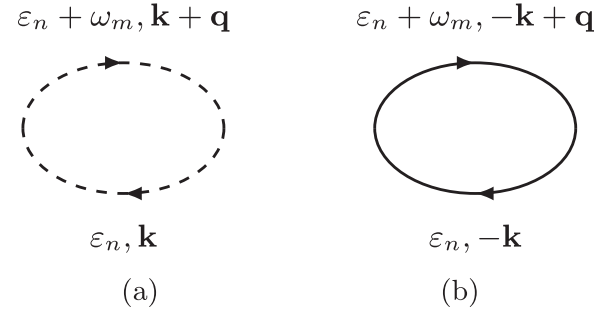


FIG. 4. Elementary bubbles.

Fig. 4(a) and 4(b), respectively]. We begin by evaluating I_1 [Fig. 4(a)]:

$$I_1(\mathbf{q}, i\omega_m) = \int \frac{d^2k}{(2\pi)^2} \text{Tr}[G_1(\mathbf{k} + \mathbf{q}, i(\varepsilon_n + \omega_m)) \times G_1(\mathbf{k}, i\varepsilon_n)], \quad (10)$$

where $G_1(\mathbf{k}, i\varepsilon_n)$ taken from Eq. (4) for $s = +1$. From Eqs. (5) and (10) we find

$$\text{Tr}[g_\sigma g_{\sigma'}] = \delta_{\sigma\sigma'}. \quad (11)$$

Moreover, using the residue theorem one sees that $I_1 = 0$ whenever $\varepsilon_n(\varepsilon_n + \omega_m) > 0$ [51]. Therefore, in what follows we restrict to $\varepsilon_n(\varepsilon_n + \omega_m) < 0$ and treat the two cases separately: (i) $\varepsilon_n < 0$ with $\varepsilon_n + \omega_m > 0$ and (ii) $\varepsilon_n > 0$ with $\varepsilon_n + \omega_m < 0$. Evaluating each case explicitly yields

$$I_1(\mathbf{q}, i\omega_m) = \sum_{\sigma} \frac{\tau}{dv_F^{\sigma}} (1 - |\omega_m| \tau - i l^{\sigma} q_x \text{sgn}(\omega_m) - (l^{\sigma} q_x)^2 - 4\tau^2 t_y^2 (1 - \cos(dq_y))), \quad (12)$$

where $v_F^{\sigma} = v_F + \sigma \sqrt{\alpha^2 + \beta^2}$ and $l^{\sigma} = v_F^{\sigma} \tau$. In deriving Eq. (12) we use the diffusive small-parameter conditions $|\omega_m| \tau \ll 1$, $q_x l^{\sigma} \ll 1$, and $t_y \tau \ll 1$. Physically, these inequalities mean that within the inelastic time scale $\sim 1/|\omega_m|$, and before a transverse tunneling event, an electron undergoes many elastic impurity scatterings, while the longitudinal momentum transfer q_x remains small enough that the motion stays diffusive. Note that these inequalities also establish the validity limits for all calculations in this study.

For $I_2(\mathbf{q}, i\omega_m)$ we proceed analogously, now using the bare Green's function of Eq. (4) with $s = -1$, which yields

$$I_2(\mathbf{q}, i\omega_m) = I_1^*(\mathbf{q}, i\omega_m). \quad (13)$$

Combining Eq. (13) with Eqs. (8) and (9) we obtain

$$\theta_1(\mathbf{q}, i\omega_m) = (1 + (\tau_2^{-1} - \tau_1^{-1}) dv_F I_1^*) \times (1 - 2\tau_1^{-1} dv_F \Re(I_1) - \tau^{-1} (\tau_2^{-1} - \tau_1^{-1}) d^2 v_F^2 I_1 I_1^*)^{-1}. \quad (14)$$

Finally, substituting Eq. (12) into Eq. (14) gives

$$\theta_1(\mathbf{q}, i\omega_m) = \sum_{\sigma} \frac{\theta_{11}(\mathbf{q}, i\omega_m)}{\theta_{12}(\mathbf{q}, i\omega_m)}, \quad (15)$$

where

$$\begin{aligned} \theta_{11}(\mathbf{q}, i\omega_m) &= 1 + \tau(\tau_2^{-1} - \tau_1^{-1}) \frac{v_F}{v_F^\sigma} \\ &\times (1 - |\omega_m|\tau + il^\sigma q_x \text{sgn}(\omega_m) \\ &- (l^\sigma q_x)^2 - 4\tau^2 t_y^2 (1 - \cos(dq_y))), \end{aligned} \quad (16)$$

$$\begin{aligned} \theta_{12}(\mathbf{q}, i\omega_m) &= 1 - 2\tau\tau_1^{-1} \frac{v_F}{v_F^\sigma} (1 - |\omega_m|\tau \\ &- 4\tau^2 t_y^2 (1 - \cos(dq_y))) + \tau(\tau_2^{-1} - \tau_1^{-1}) \\ &\times \left(\frac{v_F}{v_F^\sigma}\right)^2 (1 - 2|\omega_m|\tau - (l^\sigma q_x)^2 \\ &- 8\tau^2 t_y^2 (1 - \cos(dq_y))) \end{aligned} \quad (17)$$

and

$$\theta_2(\mathbf{q}, i\omega_m) = \theta_1^*(\mathbf{q}, i\omega_m). \quad (18)$$

We now compute the screened Coulomb interaction $V_s(\mathbf{q}, i\omega_m)$. Since the bare Coulomb potential is long-ranged, its renormalization by electronic polarization must be taken into account [52]. Employing the RPA is necessary here to capture this collective screening and prevent unphysical divergences as $q \rightarrow 0$. The Dyson equation for the dynamically screened interaction can be written as

$$V_s(\mathbf{q}, i\omega_m) = V_0(\mathbf{q}) + V_0(\mathbf{q})\Pi_s(\mathbf{q}, i\omega_m)V_s(\mathbf{q}, i\omega_m), \quad (19)$$

where the bare 2D Coulomb kernel is $V_0(\mathbf{q}) = 2\pi e^2/\sqrt{q_x^2 + q_y^2}$ [31,34], and $\Pi_s(\mathbf{q}, i\omega_m)$ is the polarization operator, evaluated as

$$\begin{aligned} \Pi_s(\mathbf{q}, i\omega_m) &= T \int \frac{d^2k}{(2\pi)^2} \sum_{\varepsilon_n} \text{Tr}[G_s(\mathbf{k} + \mathbf{q}, i(\varepsilon_n + \omega_m)) \\ &\times G_s(\mathbf{k}, i\varepsilon_n)] \theta_s(\mathbf{q}, i\omega_m). \end{aligned} \quad (20)$$

Note that when computing $\Pi_s(\mathbf{q}, i\omega_m)$, we perform the Matsubara sum over ε_n before the $\mathbf{k}$ integration. If one integrates over $\mathbf{k}$ first, the result would be nonzero only for $\varepsilon_n(\varepsilon_n + \omega_m) < 0$, incorrectly discarding other frequency sectors. Consequently, the Matsubara sum naturally splits into three parts,

$$\sum_{\varepsilon_n} = \sum_{\varepsilon_n=-\omega_m}^0 + \sum_{\varepsilon_n=0}^{\infty} + \sum_{\varepsilon_n=-\omega_m}^{-\infty}, \quad (21)$$

and only the first sum $\sum_{\varepsilon_n=-\omega_m}^0$ acquires impurity-ladder corrections through $\theta_s(\mathbf{q}, i\omega_m)$; for the other two sums one has $\theta_s = 1$ because the corresponding bubbles $I_{1,2}$ vanish. Evaluating each sum using

$$T \sum_{\varepsilon_n} \rightarrow \frac{1}{2\pi i} \int_C d\varepsilon \frac{\text{sgn } \varepsilon}{\exp((\varepsilon - \varepsilon_F)/T) + 1} \quad (22)$$

and then performing the $\mathbf{k}$ integration via the residue theorem [53] (see the Appendix for explicit details of the integration contours and derivations), we obtain

$$V_s(\mathbf{q}, i\omega_m) = 2\pi e^2 \left(|\mathbf{q}| + 2\pi e^2 \sum_{\sigma} \rho_0^{\sigma} \frac{V_1^{\sigma}(\mathbf{q})}{|\omega_m| + V_1^{\sigma}(\mathbf{q})} \right)^{-1}, \quad (23)$$

where

$$V_1^{\sigma}(\mathbf{q}) = \frac{(v_F^{\sigma})^2 \tau_2}{2} q_x^2 + 4\tau t_y^2 (1 - \cos(dq_y)), \quad (24)$$

and $\rho_0^{\sigma} = 2/(\pi v_F^{\sigma} d)$.

IV. CORRECTION TO THE DOS

Substituting Eqs. (7), (15), (18), and (23) into Eq. (6) yields the exchange-diagram correction of Fig. 2 to the DOS. For the Matsubara sum $\sum_{\omega_m}$ appearing in Eq. (7), we convert the ω_m sum to a contour integral and then to an integral along the imaginary axis; at this point we set $i\varepsilon_n \rightarrow \varepsilon$ (analytic continuation). Because $\varepsilon_n = \pi T(2n+1)$ and $\omega_m = 2\pi Tm$, the integration contour does not pass through the origin; therefore we introduce $\varepsilon_l = \omega_m + \varepsilon_n$ (with $l = n+m$). When replacing the sum by an integral, we use the standard identity for fermionic sums, $\text{Res}[\tanh(\varepsilon/2T)] = 2T$ [53]. As a result, from Eq. (6) we obtain

$$\begin{aligned} \delta\rho(\varepsilon, T) &= -2e^2 \sum_{\sigma} \rho_0^{\sigma} \int d\omega \frac{\sinh \frac{\varepsilon+\omega}{T}}{\cosh \frac{\varepsilon+\omega}{T} - \cosh \frac{\varepsilon-\omega}{T}} \\ &\times \Im \left(\int \frac{d^2q}{(2\pi)^2} (-i\omega + V_1^{\sigma}(\mathbf{q}))^{-1} \right. \\ &\times \left. (-i\omega|\mathbf{q}| + 2\pi e^2 \rho_0^{\sigma} V_1^{\sigma}(\mathbf{q}))^{-1} \right). \end{aligned} \quad (25)$$

Substituting Eq. (24) in Eq. (25) explicitly, changing variables, and specifying the integration domains, we find

$$\delta\rho(\varepsilon, T) = - \sum_{\sigma} \frac{\tau}{\pi^3 d l_2^{\sigma}} \int_0^1 \frac{d\xi}{\xi^2} \frac{\Im(A(\xi)) \sinh \frac{\varepsilon+\xi/\tau}{T}}{\cosh \frac{\varepsilon+\xi/\tau}{T} - \cosh \frac{\varepsilon-\xi/\tau}{T}}, \quad (26)$$

where

$$\begin{aligned} A(\xi) &= \int_0^1 dx \int_0^1 dy (-i + c_1 x^2 + c_2 y^2)^{-1} \\ &\times (-i\sqrt{c_3^2 x^2 + c_4^2 y^2} + c_1 x^2 + c_2 y^2)^{-1}, \end{aligned} \quad (27)$$

and $\xi = \omega\tau$, $x = l_2^{\sigma} q_x/2$, $y = dq_y/2$, $c_1 = 2\tau\tau_2^{-1}\xi^{-1}$, $c_2 = 8t_y^2\tau^2\xi^{-1}$, $c_3 = (2\pi e^2 \rho_0^{\sigma} l_2^{\sigma})^{-1}$ and $c_4 = (\pi e^2 \rho_0^{\sigma} d)^{-1}$. As is evident, to compute the DOS correction we first perform the $\mathbf{q}$ integration—this defines $A(\xi)$ as in Eq. (27)—and then integrate over ω ; all steps are carried out below. Our calculations show that the ξ dependence of $A(\xi)$ changes with c_2 , so we analyze the two asymptotic regimes separately, $c_2 \gg 1$ and $c_2 \ll 1$, in the subsections that follow.

(1) $c_2 \gg 1$ (i.e., $\omega \ll 8t_y^2\tau$). We pass to polar coordinates, $x = r \cos \varphi$ and $y = r \sin \varphi$, so that Eq. (27) takes the form

$$A(\xi) = \frac{1}{2} \int_0^{\pi} d\varphi \Phi_1^4(\varphi) \int_0^1 dr (r - i\Phi_2(\varphi))^{-1} (r^2 - i\Phi_1^2(\varphi))^{-1}, \quad (28)$$

where

$$\Phi_1(\varphi) = (c_1 \cos^2 \varphi + c_2 \sin^2 \varphi)^{-1/2}, \quad (29)$$

$$\Phi_2(\varphi) = \Phi_1^2(\varphi) \sqrt{c_3^2 \cos^2 \varphi + c_4^2 \sin^2 \varphi}. \quad (30)$$

From Eq. (28) we obtain $A(\xi)$ by integrating Eq. (28) first over r and then over φ . Integrating Eq. (28) with respect to r yields

$$\begin{aligned} & \int_0^1 dr (r - i\Phi_2)^{-1} (r^2 - i\Phi_1^2)^{-1} \\ &= \frac{1}{\Phi_1^4 + \Phi_2^4} \left[\frac{\Phi_2^2 - i\Phi_1^2}{4} \ln \left(\frac{1 + \Phi_1^{-4}}{(1 + \Phi_2^{-2})^2} \right) \right. \\ &+ \frac{i\Phi_2^2 + \Phi_1^2}{2} \left(\arctan \left(\frac{1}{\Phi_1^2} \right) - 2 \arctan \left(\frac{1}{\Phi_2} \right) \right) \\ &+ 2^{-5/2} \frac{\Phi_2(\Phi_1^2 + \Phi_2^2) + i\Phi_2(\Phi_2^2 - \Phi_1^2)}{\Phi_1} \\ &\times \ln \left(\frac{1 - \sqrt{2}\Phi_1 + \Phi_1^2}{1 + \sqrt{2}\Phi_1 + \Phi_1^2} \right) \\ &- 2^{-3/2} \frac{\Phi_2(\Phi_1^2 - \Phi_2^2) - i\Phi_2(\Phi_2^2 + \Phi_1^2)}{\Phi_1} \\ &\times \left(\arctan \left(\frac{1}{\sqrt{2}\Phi_1 - 1} \right) \right. \\ &\left. \left. + \arctan \left(\frac{1}{\sqrt{2}\Phi_1 + 1} \right) \right) \right]. \quad (31) \end{aligned}$$

Substituting Eq. (31) back into Eq. (28) allows us to perform the φ integration. In the regime $c_2 \gg 1$, Eq. (31) simplifies substantially; inserting this simplification into Eq. (28) we arrive at an expression for $\Im(A(\xi))$:

$$\Im(A(\xi)) = -\frac{1}{2} \int_0^\pi d\varphi \Phi_1^2(\varphi) \ln \left(\frac{\Phi_2(\varphi)}{\Phi_1(\varphi)} \right). \quad (32)$$

Using Eqs. (29) and (30) in Eq. (32), evaluation of the integral gives

$$\Im(A(\xi)) = -\frac{1}{2} \frac{\pi}{\sqrt{c_1 c_2}} \ln \left(\frac{1}{2} \left(\frac{c_3}{\sqrt{c_1}} + \frac{c_4}{\sqrt{c_2}} \right) \right). \quad (33)$$

Finally, substituting Eq. (33) into Eq. (26) and integrating over ξ , we obtain

$$\begin{aligned} \delta\rho(\varepsilon, T) &= -\frac{\ln(|\tau(\max\{\varepsilon, T\} - \varepsilon_F)|)}{8\pi^2 d \sqrt{\tau \tau_2} t_y (v_F^2 - (\alpha^2 + \beta^2))} \\ &\times (v_F [\ln(|\tau(\max\{\varepsilon, T\} - \varepsilon_F)|) + 4\Lambda_1] \\ &+ \sqrt{\alpha^2 + \beta^2} \Lambda_2), \quad (34) \end{aligned}$$

where

$$\Lambda_1 = \ln \frac{v_F^2 - (\alpha^2 + \beta^2) + 4d\sqrt{\tau \tau_2^{-1} t_y} (v_F + d\sqrt{\tau \tau_2^{-1} t_y})}{128e^4 \tau^2 t_y^2}, \quad (35)$$

$$\Lambda_2 = 4 \ln \frac{v_F - \sqrt{\alpha^2 + \beta^2} + d\sqrt{\tau \tau_2^{-1} t_y}}{v_F + \sqrt{\alpha^2 + \beta^2} + d\sqrt{\tau \tau_2^{-1} t_y}}. \quad (36)$$

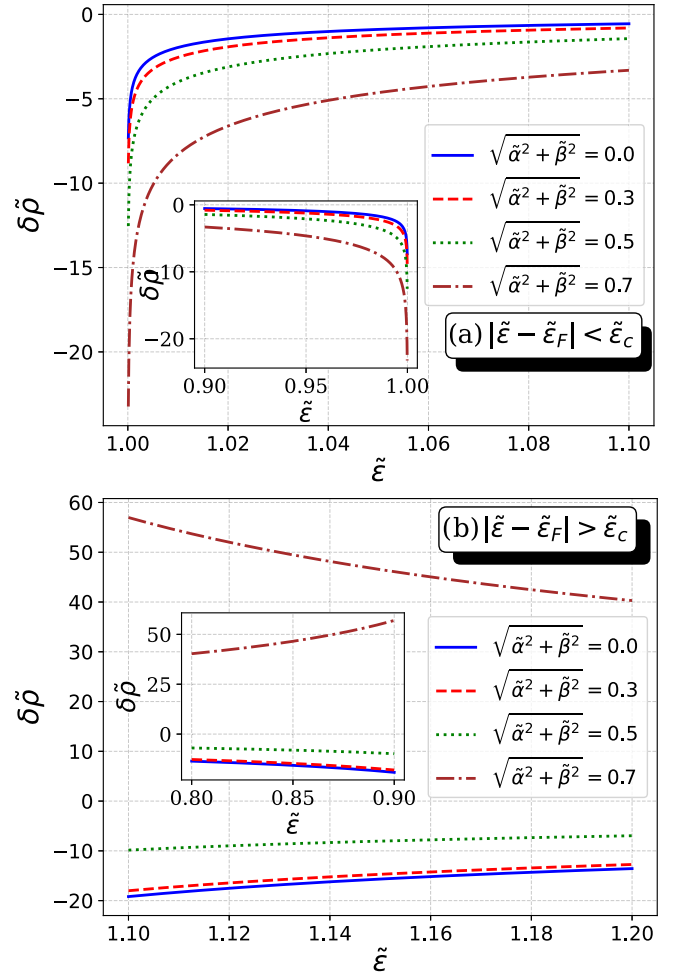


FIG. 5. Effects of Rashba and Dresselhaus SOC on the interaction-induced correction to the density of states in strongly anisotropic 2D systems. (a) The $|\varepsilon - \varepsilon_F| < \varepsilon_c$ regime, plotted using Eq. (34). (b) The $|\varepsilon - \varepsilon_F| > \varepsilon_c$ regime, plotted using Eq. (41). Here, the following dimensionless quantities are used: $\tilde{\varepsilon} = \varepsilon\tau$, $\tilde{\varepsilon}_{F,c} = \varepsilon_{F,c}\tau$, $\delta\tilde{\rho}(\tilde{\varepsilon}) = \delta\rho(\tilde{\varepsilon})v_F d$, $\tilde{\alpha} = \alpha/v_F$, and $\tilde{\beta} = \beta/v_F$. The Fermi energy and the critical energy are chosen as $\varepsilon_F = 1/\tau$ and $\varepsilon_c = 1/(10\tau)$. All curves are plotted in the strictly zero-temperature limit ($T = 0$) to isolate the pure energy dependence of the anomalies.

Equation (34) shows that there is a critical energy $\varepsilon_c = 8t_y^2\tau$ such that for $|\varepsilon - \varepsilon_F| < \varepsilon_c$ the DOS exhibits the two-dimensional, logarithmic singularity at the Fermi level. Spin-orbit coupling does not shift this critical scale but suppresses the overall amplitude of the anomaly. This behavior is illustrated in Fig. 5(a) for the energy range $(\tilde{\varepsilon}_F, \tilde{\varepsilon}_F + \tilde{\varepsilon}_c)$, while the corresponding behavior for the range $(\tilde{\varepsilon}_F - \tilde{\varepsilon}_c, \tilde{\varepsilon}_F)$ is shown in the inset. Here, $\tilde{\varepsilon}_{F,c} = \varepsilon_{F,c}\tau$.

(2) $c_2 \ll 1$ (i.e., $\omega \gg 8t_y^2\tau$). To treat this regime, we first extract $\Im(A(\xi))$ from Eq. (27):

$$\Im(A(\xi)) = \int_0^1 dx \int_0^1 dy \frac{(c_1 x^2 + c_2 y^2)(1 + \sqrt{c_3^2 x^2 + c_4^2 y^2})}{((c_1 x^2 + c_2 y^2)^2 + c_3^2 x^2 + c_4^2 y^2)(1 + \sqrt{c_1^2 x^2 + c_2^2 y^2})}. \quad (37)$$

Specializing Eq. (37) to $c_2 \ll 1$, we integrate over x on the interval $(0, 1/\sqrt{c_1})$ and then perform the y -integration, which yields

$$\Im(A(\xi)) \sim \frac{1}{\sqrt{c_1}c_4\xi^2} \left(\frac{2}{9} + \frac{\pi}{2} + \frac{\ln c_4}{3} \right). \quad (38)$$

On the complementary interval $(1/\sqrt{c_1}, 1)$, the integration yields

$$\Im(A(\xi)) \sim \frac{1}{\sqrt{c_1}c_4\xi^2} \left(\frac{\pi}{6} - 2 + \ln c_4 \right). \quad (39)$$

Hence, we can write

$$\Im(A(\xi)) = \frac{1}{3\sqrt{c_1}c_4\xi^2} \left(2\pi - \frac{16}{3} + 4 \ln c_4 \right) + O(c_2), \quad (40)$$

where $O(c_2)$ denotes terms that are proportional to c_2 and thus parametrically small in this limit. Finally, substituting Eq. (40) into Eq. (26) and carrying out the remaining integration, we obtain

$$\delta\rho(\varepsilon) = -\frac{4e^2\Lambda_3}{(v_F^2 - (\alpha^2 + \beta^2))^2 |\tau(\varepsilon - \varepsilon_F)|^{1/2}}, \quad (41)$$

where

$$\Lambda_3 = (v_F^2 + (\alpha^2 + \beta^2)) \left(2 + \ln \left(\frac{\pi^2(v_F^2 - (\alpha^2 + \beta^2))}{16e^4} \right) \right) + 2v_F\sqrt{\alpha^2 + \beta^2} \ln \left(\frac{v_F - \sqrt{\alpha^2 + \beta^2}}{v_F + \sqrt{\alpha^2 + \beta^2}} \right). \quad (42)$$

As seen from Eq. (41), in the regime $|\varepsilon - \varepsilon_F| > \varepsilon_c$ the interaction-induced DOS correction crosses over to the stronger, one-dimensional square-root-type singularity. In Fig. 5(b), this correction is plotted for energies greater than $\tilde{\varepsilon}_F + \tilde{\varepsilon}_c$, while the inset displays energies smaller than $\tilde{\varepsilon}_F - \tilde{\varepsilon}_c$. As evident from Fig. 5(b), the dimensionless interaction correction, defined from Eq. (41) as $\delta\tilde{\rho} = v_F d\delta\rho$, attains larger values with increasing Rashba and Dresselhaus spin-orbit interactions (SOIs). This indicates that for $|\varepsilon - \varepsilon_F| > \varepsilon_c$, the DOS correction not only crosses over from a 2D logarithmic to a 1D square-root singularity, but the underlying mechanism of the SOIs also changes. Specifically, the SOIs, which suppress the DOS correction in the $|\varepsilon - \varepsilon_F| < \varepsilon_c$ regime, cause it to take larger values when $|\varepsilon - \varepsilon_F| > \varepsilon_c$. Furthermore, Fig. 5(b) reveals that under the influence of the SOIs, $\delta\tilde{\rho}$ can change its sign and assume positive values. The dependence of $\delta\tilde{\rho}$ on $\tilde{\varepsilon}$ also changes character at these positive values. Namely, for energies greater than $\tilde{\varepsilon}_F + \tilde{\varepsilon}_c$, $\delta\tilde{\rho}$ increases at higher energies when $\delta\tilde{\rho} < 0$. However, when $\delta\tilde{\rho}$ is positive, it decreases with increasing energy. A similar behavior is valid for the energies smaller than $\tilde{\varepsilon}_F - \tilde{\varepsilon}_c$ shown in the inset of Fig. 5(b). It is also useful to emphasize that, since Eqs. (34) and (41) are asymptotic expressions obtained from the same integral representation in the opposite limits $c_2 \gg 1$ and $c_2 \ll 1$, respectively, they describe the DOS correction sufficiently below and sufficiently above the crossover scale, but do not constitute a uniform description of the crossover region. Consequently, exact matching at $\tilde{\varepsilon} = \tilde{\varepsilon}_F \pm \tilde{\varepsilon}_c$ is not imposed in Figs. 5(a) and 5(b).

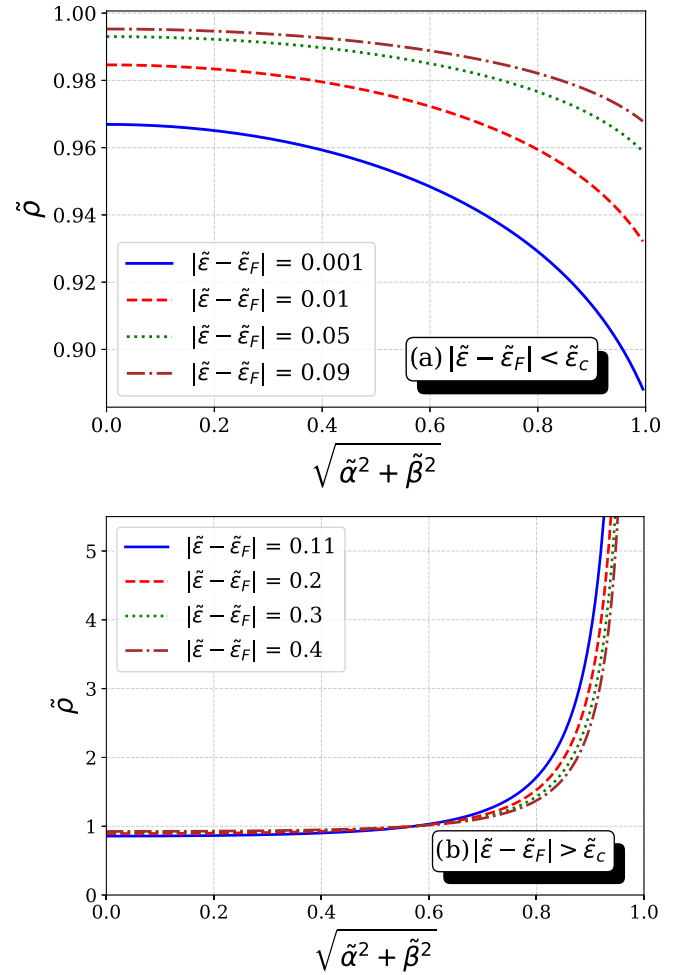


FIG. 6. Evolution of the dimensionless density of states per transverse conducting channel $\tilde{\rho}$ with effective spin-orbit coupling strength $\sqrt{\tilde{\alpha}^2 + \tilde{\beta}^2}$ for various energy distances $|\tilde{\varepsilon} - \tilde{\varepsilon}_F|$ from the Fermi level. Here, the critical energy is chosen as $\varepsilon_c = 1/(10\tau)$.

V. RENORMALIZED DOS AND PERTURBATIVE VALIDITY

Determining the DOS correction allows us to calculate the renormalized single-particle DOS per transverse conducting channel, defined as $\rho = \rho_0 + \delta\rho/N$, where $\rho_0 = \sum_{\sigma} \rho_0^{\sigma}$ is the unperturbed DOS per transverse channel, while $\delta\rho$ denotes the interaction-induced correction obtained from Eq. (6), in which the transverse degrees of freedom are included through the integration over k_y . The results for the dimensionless renormalized DOS per transverse conducting channel, $\tilde{\rho} = \rho/\rho_0$, are plotted as a function of the effective spin-orbit coupling strength, $\sqrt{\tilde{\alpha}^2 + \tilde{\beta}^2}$, in Fig. 6(a) for the $|\varepsilon - \varepsilon_F| < \varepsilon_c$ regime, and in Fig. 6(b) for the $|\varepsilon - \varepsilon_F| > \varepsilon_c$ regime, assuming $N = 100$ transverse channels. As seen in Fig. 6(a), within the $|\varepsilon - \varepsilon_F| < \varepsilon_c$ window, $\tilde{\rho}$ increases as the energy deviates further from the Fermi level. However, increasing the SOC strength strictly suppresses these values. Conversely, Fig. 6(b) demonstrates that in the quasi-1D regime, $\tilde{\rho}$ is enhanced by the SOC. In this regime, for weak SOC ($\sqrt{\tilde{\alpha}^2 + \tilde{\beta}^2} < 0.57$), $\tilde{\rho}$ continues to grow as $|\varepsilon - \varepsilon_F|$ increases. Remarkably, at the

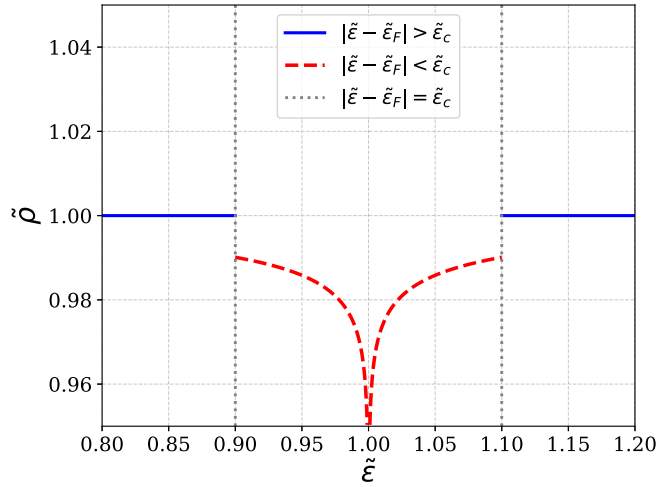


FIG. 7. Dimensionless density of states per transverse conducting channel $\tilde{\rho}$ as a function of energy at the critical SOC strength $\sqrt{\tilde{\alpha}^2 + \tilde{\beta}^2} = 0.57$. The Fermi energy and the critical energy are chosen as $\varepsilon_F = 1/\tau$ and $\varepsilon_c = 1/(10\tau)$.

critical SOC strength $\sqrt{\tilde{\alpha}^2 + \tilde{\beta}^2} = 0.57$, the dimensionless DOS in the quasi-1D regime is restored to unity. This implies that the SOC perfectly compensates for the electron-electron interaction, effectively driving the correlation correction to zero. At this critical value of the SOC strength, the variation of $\tilde{\rho}$ with $\tilde{\epsilon}$ is shown separately in Fig. 7. For stronger SOC's ($\sqrt{\tilde{\alpha}^2 + \tilde{\beta}^2} > 0.57$), while the overall magnitude of $\tilde{\rho}$ rises above unity (becoming a positive anomaly), the energy dependence inverts, where for a fixed $\sqrt{\tilde{\alpha}^2 + \tilde{\beta}^2}$, $\tilde{\rho}$ now decays to smaller values as $|\varepsilon - \varepsilon_F|$ increases.

Finally, it is important to note that the logarithmic divergence at the Fermi level requires an explicit lower-energy cutoff for the perturbative DOS correction. For a given SOC strength $\sqrt{\alpha^2 + \beta^2}$, this cutoff is the minimum allowed distance from the Fermi level and will be denoted by $|\varepsilon - \varepsilon_F|_{\min}$. The perturbative result is meaningful only while the correction remains small compared with the background DOS:

$$\left| \frac{\delta\rho(|\varepsilon - \varepsilon_F|, \sqrt{\alpha^2 + \beta^2})}{N\rho_0(\sqrt{\alpha^2 + \beta^2})} \right| \ll 1. \quad (43)$$

This condition is consistent with the known limitation of the AA perturbative DOS correction, where a resummation of the Dyson equation is required to remove the unphysical divergence when the correction becomes large near the Fermi level [54]. For each value of $\sqrt{\alpha^2 + \beta^2}$, $|\varepsilon - \varepsilon_F|_{\min}$ is determined from

$$\left| \frac{\delta\rho(|\varepsilon - \varepsilon_F|_{\min}, \sqrt{\alpha^2 + \beta^2})}{N\rho_0(\sqrt{\alpha^2 + \beta^2})} \right| = \eta_0, \quad (44)$$

where $\eta_0 = 0.1$ is used in the numerical estimate. Hence,

$$|\varepsilon - \varepsilon_F| < |\varepsilon - \varepsilon_F|_{\min} \quad (45)$$

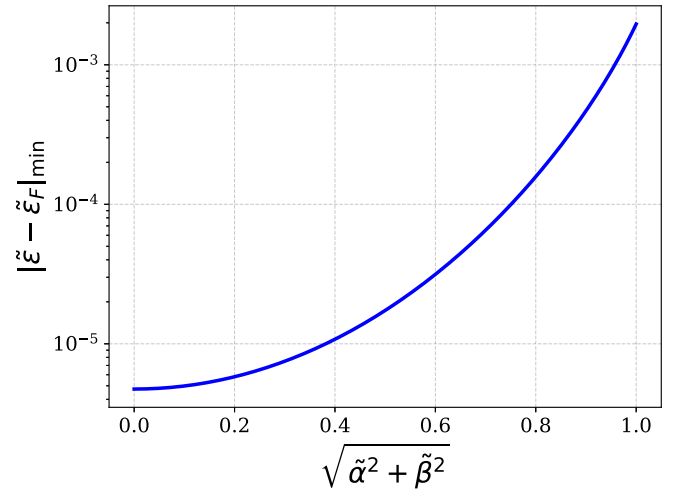


FIG. 8. SOC dependence of the lower energy cutoff $|\tilde{\epsilon} - \tilde{\epsilon}_F|_{\min}$ for the perturbative validity of the DOS correction.

is outside the perturbative domain, while the valid low-energy region is

$$|\varepsilon - \varepsilon_F|_{\min} < |\varepsilon - \varepsilon_F| < \varepsilon_c. \quad (46)$$

As shown in Fig. 8, the lower cutoff $|\tilde{\epsilon} - \tilde{\epsilon}_F|_{\min}$ increases monotonically with the SOC strength $\sqrt{\tilde{\alpha}^2 + \tilde{\beta}^2}$. This confirms that stronger SOC widens the excluded energy region around the Fermi level, where the logarithmic correction becomes nonperturbative.

VI. CONCLUSIONS

We have analyzed the AA interaction correction to the single-particle DOS in a strongly anisotropic 2D metal modeled as a planar array of quasi-1D tubes with weak, spin-independent disorder and intrinsic Rashba/Dresselhaus SOC acting along the longitudinal direction. Within an exchange-only, diffusive framework, we resummed the impurity ladder (charge/singlet diffusion) and used the RPA screening to obtain a compact result exhibiting a dimensional crossover governed by $\varepsilon_c \sim t_y^2\tau$. For energies well below this scale, the interaction-induced DOS anomaly has a two-dimensional, logarithmic singularity. For energies well above it, the anomaly crosses into the one-dimensional regime with the stronger square-root-type singularity. Crucially, we have demonstrated that the impact of intrinsic SOC's strictly depends on this effective dimensionality. Close to the Fermi level, SOC deepens the two-dimensional logarithmic DOS dip by increasing the magnitude of the negative interaction correction. Further from the Fermi level, the physical role of the SOC remarkably reverses, acting instead to prominently enhance the amplitude of the sharper square-root singularity. Furthermore, we found that strong SOC in this quasi-1D regime can invert the sign of the anomaly entirely, yielding a positive DOS correction. This sign reversal fundamentally alters the energy dependence of the spectrum. Unlike the standard negative correction, which recovers at higher energies, the SOC-induced positive correction decays to smaller values as energy increases beyond $\varepsilon_F + \varepsilon_c$. Consequently, the total

DOS can be tuned via SOC from a suppressed dip to a fully restored or enhanced state.

Our predictions are directly testable in anisotropic spin-orbit-coupled thin films such as 2D Te, where strong and gate-tunable SOC has already been established experimentally [55,56]. The most direct probe is low-temperature tunneling spectroscopy, since the differential conductance dI/dV tracks the single-particle DOS and is known to resolve AA zero-bias anomalies [14,57]. In the present case, one should observe (i) a low-bias logarithmic DOS dip, (ii) a crossover at $eV \sim \varepsilon_c$ to a sharper quasi-1D anomaly, and (iii) a systematic SOC-driven evolution in which the low-energy dip is weakened, the finite-bias anomaly is enhanced, and both vanish at a critical SOC strength before changing sign. Observation of this sequence would provide a clear spectroscopic signature of SOC-modulated interaction effects in anisotropic diffusive conductors.

ACKNOWLEDGMENT

B.T. acknowledges support from the Turkish Academy of Sciences (TUBA) under Grant No. AD-2026.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX: DERIVATION OF THE DYNAMICALLY SCREENED INTERACTION

In this Appendix, we detail the analytic continuation and momentum integration required to evaluate the dynamically screened interaction $V_s(\mathbf{q}, i\omega_m)$. To this end, we rewrite Eq. (19) as

$$V_s(\mathbf{q}, i\omega_m) = \frac{V_0(\mathbf{q})}{1 - V_0(\mathbf{q})\Pi_s(\mathbf{q}, i\omega_m)}, \quad (\text{A1})$$

where $\Pi_s(\mathbf{q}, i\omega_m)$ is defined by Eq. (20). To properly evaluate Eq. (20), the Matsubara summation over ε_n must be performed before the $\mathbf{k}$ integration to avoid discarding nonzero frequency sectors. By converting the discrete sum into a contour integral via Eq. (22), the branch cuts of the Green's functions at $\text{Im}(\varepsilon) = 0$ and $\text{Im}(\varepsilon) = -\omega_m$ naturally decompose the polarization operator into three distinct analytic contributions, as illustrated in Fig. 9:

$$\Pi_s(\mathbf{q}, i\omega_m) = \Pi_s^{(C_1)} + \Pi_s^{(C_2)} + \Pi_s^{(C_3)}. \quad (\text{A2})$$

For the outer contours C_2 (where $\varepsilon_n > 0$) and C_3 (where $\varepsilon_n < -\omega_m$), the product of the Green's functions is strictly retarded-retarded and advanced-advanced, respectively. In these frequency sectors, the elementary impurity bubbles vanish, meaning there are no vertex corrections ($\theta_s = 1$). Performing the momentum and energy integrations via the residue theorem in these unrenormalized sectors captures the

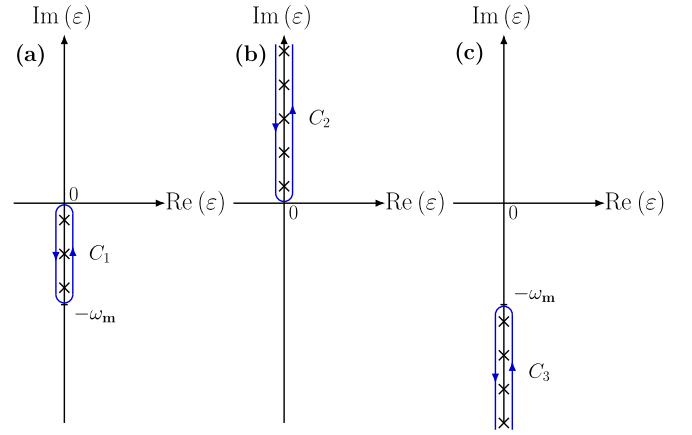


FIG. 9. Integration contours in the continuous complex frequency plane used to evaluate the Matsubara sum for the polarization operator. Crosses represent discrete fermionic Matsubara frequencies $i\varepsilon_n$.

bare, static compressibility of the 2D gas:

$$\Pi_s^{(C_2)}(\mathbf{q}, i\omega_m) + \Pi_s^{(C_3)}(\mathbf{q}, i\omega_m) = - \sum_{\sigma} \rho_0^{\sigma}. \quad (\text{A3})$$

Conversely, the central contour C_1 strictly encloses the region $-\omega_m \leq \varepsilon_n \leq 0$, where the Green's functions take the retarded-advanced form. It is exclusively in this anomalous sector that the vertex acquires the macroscopic impurity-ladder correction $\theta_s \neq 1$. To evaluate $\Pi_s^{(C_1)}$, we convert the momentum integral into an integral over the energy dispersion ξ . Closing the contour in the upper half-plane encloses the single pole of the retarded Green's function at $\xi_{\text{pole}} = \varepsilon + \omega_m - \Delta\xi_{\mathbf{q}}^{\sigma} + \frac{i}{2\tau}$. Evaluating this via the residue theorem, applying angular averaging over the Fermi surface, and dressing the result with the diffusive ladder θ_s , we obtain the dynamic correction:

$$\Pi_s^{(C_1)}(\mathbf{q}, i\omega_m) = \sum_{\sigma} \rho_0^{\sigma} \frac{|\omega_m|}{|\omega_m| + V_1^{\sigma}(\mathbf{q})}, \quad (\text{A4})$$

where $V_1^{\sigma}(\mathbf{q})$ is the anisotropic diffusion operator defined in Eq. (24). Summing all three contour contributions [Eqs. (A3) and (A4)] yields the exact, total polarization operator:

$$\begin{aligned} \Pi_s(\mathbf{q}, i\omega_m) &= - \sum_{\sigma} \rho_0^{\sigma} + \sum_{\sigma} \rho_0^{\sigma} \frac{|\omega_m|}{|\omega_m| + V_1^{\sigma}(\mathbf{q})} \\ &= - \sum_{\sigma} \rho_0^{\sigma} \frac{V_1^{\sigma}(\mathbf{q})}{|\omega_m| + V_1^{\sigma}(\mathbf{q})}. \end{aligned} \quad (\text{A5})$$

Finally, substituting this dressed polarization operator and the bare Coulomb kernel $V_0(\mathbf{q}) = 2\pi e^2/|\mathbf{q}|$ into the Dyson Eq. (A1), we divide the numerator and denominator by $V_0(\mathbf{q})$ to obtain

$$V_s(\mathbf{q}, i\omega_m) = \left(\frac{|\mathbf{q}|}{2\pi e^2} + \sum_{\sigma} \rho_0^{\sigma} \frac{V_1^{\sigma}(\mathbf{q})}{|\omega_m| + V_1^{\sigma}(\mathbf{q})} \right)^{-1}. \quad (\text{A6})$$

Factoring out $(2\pi e^2)^{-1}$ immediately recovers Eq. (23) of the main text, completing the derivation.

- [1] J. B. Miller, D. M. Zumbühl, C. M. Marcus, Y. B. Lyanda-Geller, D. Goldhaber-Gordon, K. Campman, and A. C. Gossard, Gate-controlled spin-orbit quantum interference effects in lateral transport, *Phys. Rev. Lett.* **90**, 076807 (2003).
- [2] A. D. Caviglia, M. Gabay, S. Gariglio, N. Reyren, C. Cancellieri, and J.-M. Triscone, Tunable Rashba spin-orbit interaction at oxide interfaces, *Phys. Rev. Lett.* **104**, 126803 (2010).
- [3] L. Guo, Y. Yan, R. Xu, J. Li, and C. Zeng, Zero-bias conductance peaks effectively tuned by gating-controlled Rashba spin-orbit coupling, *Phys. Rev. Lett.* **126**, 057701 (2021).
- [4] E. Mariani, L. I. Glazman, A. Kamenev, and F. von Oppen, Zero-bias anomaly in the tunneling density of states of graphene, *Phys. Rev. B* **76**, 165402 (2007).
- [5] Z. Wang, D.-K. Ki, H. Chen, H. Berger, A. H. MacDonald, and A. F. Morpurgo, Strong interface-induced spin-orbit interaction in graphene on WS₂, *Nat. Commun.* **6**, 8339 (2015).
- [6] T. Wakamura, F. Reale, P. Palczynski, S. Guéron, C. Mattevi, and H. Bouchiat, Strong anisotropic spin-orbit interaction induced in graphene by monolayer WS₂, *Phys. Rev. Lett.* **120**, 106802 (2018).
- [7] B. Fülöp, A. Márffy, S. Zihlmann, M. Gmitra, E. Tóvári, B. Szentpéteri, M. Kedves, K. Watanabe, T. Taniguchi, J. Fabian, C. Schönenberger, P. Makk, and S. Csonka, Boosting proximity spin orbit coupling in graphene/WSe₂ heterostructures via hydrostatic pressure, *npj 2D Mater. Appl.* **5**, 82 (2021).
- [8] L. Sun, L. Rademaker, D. Mauro, A. Scarfato, Á. Pásztor, I. Gutiérrez-Lezama, Z. Wang, J. Martinez-Castro, A. F. Morpurgo, and C. Renner, Determining spin-orbit coupling in graphene by quasiparticle interference imaging, *Nat. Commun.* **14**, 3771 (2023).
- [9] A. Endo and Y. Iye, Origin of positive magnetoresistance in small-amplitude unidirectional lateral superlattices, *Phys. Rev. B* **72**, 235303 (2005).
- [10] A. Endo, S. Katsumoto, and Y. Iye, Commensurability oscillations in the Hall resistance of unidirectional lateral superlattices, *Phys. Rev. B* **103**, 235303 (2021).
- [11] C. Blumenstein, J. Schäfer, S. Mietke, S. Meyer, A. Dollinger, M. Lochner, X. Y. Cui, L. Patthey, R. Matzdorf, and R. Claessen, Atomically controlled quantum chains hosting a Tomonaga-Luttinger liquid, *Nat. Phys.* **7**, 776 (2011).
- [12] C. Tegenkamp, D. Lükermann, H. Pfñür, B. Slomski, G. Landolt, and J. H. Dil, Fermi nesting between atomic wires with strong spin-orbit coupling, *Phys. Rev. Lett.* **109**, 266401 (2012).
- [13] J. Park, S. W. Jung, M.-C. Jung, H. Yamane, N. Kosugi, and H. W. Yeom, Self-assembled nanowires with giant Rashba split bands, *Phys. Rev. Lett.* **110**, 036801 (2013).
- [14] N. Ossi, L. Bitton, D. B. Gutman, and A. Frydman, Zero-bias anomaly in a two-dimensional granular insulator, *Phys. Rev. B* **87**, 115137 (2013).
- [15] Z. Ovadyahu, Interaction-induced spatial correlations in a disordered glass, *Phys. Rev. B* **105**, 235101 (2022).
- [16] B. L. Altshuler and A. G. Aronov, Zero bias anomaly in tunnel resistance and electron-electron interaction, *Solid State Commun.* **30**, 115 (1979).
- [17] A. L. Efros and B. I. Shklovskii, Coulomb gap and low temperature conductivity of disordered systems, *J. Phys. C* **8**, L49 (1975).
- [18] M. Bockrath, D. H. Cobden, J. Lu, A. G. Rinzier, R. E. Smalley, L. Balents, and P. L. McEuen, Luttinger-liquid behaviour in carbon nanotubes, *Nature (London)* **397**, 598 (1999).
- [19] E. Turco, M. Aapro, S. C. Ganguli, N. Krane, R. Drost, N. Sobrino, A. Bernhardt, M. Juríček, R. Fasel, P. Ruffieux, P. Liljeroth, and D. Jacob, Demonstrating Kondo behavior by temperature-dependent scanning tunneling spectroscopy, *Phys. Rev. Res.* **6**, L022061 (2024).
- [20] J. H. Pixley, D. A. Huse, and S. Das Sarma, Rare-region-induced avoided quantum criticality in disordered three-dimensional Dirac and Weyl semimetals, *Phys. Rev. X* **6**, 021042 (2016).
- [21] H. Li, R. Hanus, C. A. Polanco, A. Zeidler, G. Koblmüller, Y. K. Koh, and L. Lindsay, GaN thermal transport limited by the interplay of dislocations and size effects, *Phys. Rev. B* **102**, 014313 (2020).
- [22] Y. Yao, S. Huang, R. Cao, Z. Zhang, *et al.*, Dislocation-assisted electron and hole transport in GaN epitaxial layers, *Nat. Commun.* **16**, 6448 (2025).
- [23] M. Reiche and M. Kittler, Electronic and optical properties of dislocations in silicon, *Crystals* **6**, 74 (2016).
- [24] L.-J. Yin, H. Jiang, J.-B. Qiao, and L. He, Direct imaging of topological edge states at a bilayer graphene domain wall, *Nat. Commun.* **7**, 11760 (2016).
- [25] S. Thiel, C. W. Schneider, L. F. Kourkoutis, D. A. Muller, N. Reyren, A. D. Caviglia, S. Gariglio, J.-M. Triscone, and J. Mannhart, Electron scattering at dislocations in LaAlO₃/SrTiO₃ interfaces, *Phys. Rev. Lett.* **102**, 046809 (2009).
- [26] H. Xue, D. Jia, Y. Ge, Y. j. Guan, Q. Wang, S.-q. Yuan, H.-x. Sun, Y. D. Chong, and B. Zhang, Observation of dislocation-induced topological modes in a three-dimensional acoustic topological insulator, *Phys. Rev. Lett.* **127**, 214301 (2021).
- [27] G. Bergmann, Weak localization in thin films: A time-of-flight experiment with conduction electrons, *Phys. Rep.* **107**, 1 (1984).
- [28] A. Altland, B. Simons, and M. Zirnbauer, Theories of low-energy quasi-particle states in disordered d-wave superconductors, *Phys. Rep.* **359**, 283 (2002).
- [29] B. L. Altshuler and A. Aronov, Contribution to the theory of disordered metals in strongly doped semiconductors, *Zh. Eksp. Teor. Fiz.* **77**, 2028 (1979) [*Sov. Phys. JETP* **50**, 968 (1979)].
- [30] B. L. Altshuler, A. G. Aronov, and P. A. Lee, Interaction effects in disordered Fermi systems in two dimensions, *Phys. Rev. Lett.* **44**, 1288 (1980).
- [31] B. Altshuler and A. Aronov, Electron-electron interaction in disordered conductors, in *Electron-Electron Interactions in Disordered Systems*, edited by A. Efros and M. Pollak, Modern Problems in Condensed Matter Sciences, Vol. 10 (Elsevier, Amsterdam, 1985), Chap. 1, pp. 1–153.
- [32] A. M. Finkel'shtein, Influence of Coulomb interaction on the properties of disordered metals, *Zh. Eksp. Teor. Fiz.* **84**, 168 (1983) [*Sov. Phys. JETP* **57**, 97 (1983)].
- [33] H. Fukuyama, Interaction effects in the weakly localized regime of two- and three-dimensional disordered systems, in *Electron-Electron Interactions in Disordered Systems*, edited by A. Efros and M. Pollak, Modern Problems in Condensed Matter Sciences, Vol. 10 (Elsevier, Amsterdam, 1985), Chap. 2, pp. 155–230.

- [34] Y. A. Firsov, E. P. Nakhmebov, V. N. Prigodin, and W. Weller, Corrections from electron-electron interaction to the one-particle density of states in strongly anisotropic two-dimensional systems, *Physica Status Solidi B* **141**, 111 (1987).
- [35] Y. A. Firsov, E. P. Nakhmedov, V. N. Prigodin, and W. Weller, Low-temperature conductivity of a weakly disordered, strongly anisotropic two-dimensional system with electron-electron interaction. Application to germanium bicrystals and to surface superlattices, *Physica Status Solidi B* **145**, 625 (1988).
- [36] S. Caliskan and M. Kumru, High-order perturbation corrections to the density of states of disordered metals in a magnetic field, *Phys. Rev. B* **85**, 205148 (2012).
- [37] C. Zhou and H. Guo, Altshuler-Aronov effects in nonequilibrium disordered nanostructures, *Phys. Rev. B* **100**, 045413 (2019).
- [38] C. Carbillet, V. Cherkez, M. A. Skvortsov, M. V. Feigel'man, F. Debontridder, L. B. Ioffe, V. S. Stolyarov, K. Ilin, M. Siegel, D. Roditchev, T. Cren, and C. Brun, Spectroscopic evidence for strong correlations between local superconducting gap and local Altshuler-Aronov density of states suppression in ultrathin NbN films, *Phys. Rev. B* **102**, 024504 (2020).
- [39] M. Kuzmiak, M. Kopčák, F. Košuth, V. Vaňo, J. Haniš, T. Samuely, V. Latyshev, O. Onufriienko, V. Komanický, J. Kačmarčík, M. Žemlička, M. Gmitra, P. Szabó, and P. Samuely, Disorder- and magnetic field-tuned fermionic superconductor-insulator transition in MoN thin films: Transport and scanning tunneling microscopy, *Phys. Rev. B* **108**, 184511 (2023).
- [40] M. Lizée, M. Stosiek, I. Burmistrov, T. Cren, and C. Brun, Local density of states fluctuations in a two-dimensional superconductor as a probe of quantum diffusion, *Phys. Rev. B* **107**, 174508 (2023).
- [41] G. Shi, F. Gao, Z. Li, R. Zhang, I. Gornyi, D. Gutman, and Y. Li, Quantum corrections to the magnetoconductivity of surface states in three-dimensional topological insulators, *Nat. Commun.* **14**, 2596 (2023).
- [42] A. Mošková and M. Moško, States-conserving density of states for Altshuler-Aronov effect: Heuristic derivation, *Solid State Commun.* **284-286**, 56 (2018).
- [43] Y. Huang and S. Das Sarma, Electronic transport, metal-insulator transition, and Wigner crystallization in transition metal dichalcogenide monolayers, *Phys. Rev. B* **109**, 245431 (2024).
- [44] J. Rollbühler and H. Grabert, Tunneling density of states of the interacting two-dimensional electron gas, *Phys. Rev. Lett.* **91**, 166402 (2003).
- [45] C. L. Romano, S. E. Ulloa, and P. I. Tamborenea, Level structure and spin-orbit effects in quasi-one-dimensional semiconductor nanostructures, *Phys. Rev. B* **71**, 035336 (2005).
- [46] B. Suleymanli, E. Nakhmedov, F. Tatardar, and B. Tanatar, The diagrammatic method of Berezinskii for one-dimensional disordered wire with spin-orbit interaction, *Physica E* **146**, 115550 (2023).
- [47] C. L. Kane, R. Mukhopadhyay, and T. C. Lubensky, Fractional quantum Hall effect in an array of quantum wires, *Phys. Rev. Lett.* **88**, 036401 (2002).
- [48] J. Klinovaja and D. Loss, Topological edge states and fractional quantum Hall effect from umklapp scattering, *Phys. Rev. Lett.* **111**, 196401 (2013).
- [49] A. A. Burkov, A. S. Núñez, and A. H. MacDonald, Theory of spin-charge-coupled transport in a two-dimensional electron gas with Rashba spin-orbit interactions, *Phys. Rev. B* **70**, 155308 (2004).
- [50] A. Abrikosov, L. Gorkov, and I. Dzyaloshinski, *Methods of Quantum Field Theory in Statistical Physics* (Dover, New York, 2012).
- [51] G. B. Arfken, H. J. Weber, and F. E. Harris, Complex variable theory, in *Mathematical Methods for Physicists (Seventh Edition)*, edited by G. B. Arfken, H. J. Weber, and F. E. Harris (Academic, Boston, 2013), Chap. 11, pp. 469–550.
- [52] C. Hamaguchi, *Basic Semiconductor Physics* (Springer, Berlin, 2009).
- [53] A. Altland and B. Simons, *Condensed Matter Field Theory* (Cambridge University Press, Cambridge, 2010).
- [54] E. Şaşıoğlu, S. Çalışkan, and M. Kumru, Critical behavior of density of states near Fermi energy in low-dimensional disordered metals, *Phys. Rev. B* **79**, 035123 (2009).
- [55] C. Niu, G. Qiu, Y. Wang, Z. Zhang, M. Si, W. Wu, and P. D. Ye, Gate-tunable strong spin-orbit interaction in two-dimensional tellurium probed by weak antilocalization, *Phys. Rev. B* **101**, 205414 (2020).
- [56] B. Geldiyev, M. Ünzelmann, P. Eck, T. Kißlinger, J. Schusser, T. Figgemeier, P. Kagerer, N. Tezak, M. Krivenkov, A. Varykhalov, *et al.*, Strongly anisotropic spin and orbital Rashba effect at a tellurium – noble metal interface, *Phys. Rev. B* **108**, L121107 (2023).
- [57] L. Liu, J. Niu, L. Xiang, J. Wei, D.-L. Li, J.-F. Feng, X.-F. Han, X.-G. Zhang, and J. M. D. Coey, Symmetry-dependent electron-electron interaction in coherent tunnel junctions resolved by measurements of zero-bias anomaly, *Phys. Rev. B* **90**, 195132 (2014).