

Spin-photon entanglement dynamics under cavity-delayed feedback

Noam Tur¹ and Ido Schwartz^{2*}

¹The Physics Department and the Solid State Institute, *Technion-Israel Institute of Technology*, 32000 Haifa, Israel

 (Received 29 August 2025; revised 15 February 2026; accepted 20 February 2026; published 23 March 2026)

We analytically study the interaction between a single confined spin with a four-level Π configuration in an optical cavity and a single photon. The spin's ground and excited manifolds are each split into two levels coupled by circularly polarized optical transitions. Starting from the spin in an excited state, we analyze the spontaneous emission of the photon, which leaves the spin and photon entangled, and then follow the subsequent time evolution of the system under delayed feedback. We examine how photon reabsorption depends on energy splittings in both the ground and excited spin states, and how the feedback delay influences the degree of spin-photon entanglement. Finally, we identify the conditions that maximize this entanglement after photon reabsorption and re-emission.

DOI: [10.1103/f4vf-nc9t](https://doi.org/10.1103/f4vf-nc9t)

I. INTRODUCTION

The ability to control quantum entanglement between matter and light in general, and between single spins and single photons in particular, is essential for advancing quantum information processing and communication. This capability forms the basis for quantum technologies such as quantum key distribution [1], quantum repeaters [2], and quantum computing [3].

Over the past two decades, significant progress has been made in generating spin-photon and photon-photon entanglement using single-photon sources [4]. More recently, one-dimensional cluster states, a special class of spin-multiphoton entangled states, have also been demonstrated using self-assembled quantum dots [5–9] and cold atoms [10,11]. Cluster states are considered a crucial resource for future technologies such as measurement-based quantum computation [12] and quantum repeater protocols [2,13–15]; however, cluster states of higher dimensionality are required for these applications.

In a recent, most relevant proposal by Pichler *et al.* [16], a protocol for generating a two-dimensional photonic cluster state using delayed quantum feedback is outlined. The proposal encodes the photonic qubit's $|0\rangle$ and $|1\rangle$ states in the presence or absence of a single photon in a temporal array of two-dimensional connectivity. The increased connectivity is achieved by delaying photons from a one-dimensional cluster state and reflecting them back to the emitter [17], where their interaction forms additional entanglement links between photons that are not sequentially emitted. The interaction of a two-level emitter with a delayed, back-reflected photon was

previously investigated in detail by Alber *et al.* [18] and by Dorner and Zoller [19].

Instead, here we propose encoding photonic qubits in polarization, with right- and left-handed circular polarization states representing the qubit basis $|0\rangle$ and $|1\rangle$. Polarization encoding avoids the need for number-resolving detection during state growth and allows deterministic, passive routing inside the cavity. It also enables coherent and deterministic control through the emitter's optical selection rules, which is not directly accessible in presence-absence encoding. Its main drawback is sensitivity to birefringence and polarization-mode dispersion in fibers, which must be actively stabilized.

In this approach, the internal degrees of freedom of the emitter and its optical selection rules must be explicitly considered in addition to the dipole-field interaction [19]. For this goal, we present here an analytical investigation of a four-level system that stands in the center of many protocols [17,20,21] and that has been used to generate a one-dimensional cluster state [5]. As an emitter, we investigate a single spin with circular polarization optical selection rules. The temporal evolution of the spin-photon state in these systems can be controlled by timing the excitation with the spin phase precession, which is induced either by the spin's fine structure (for integer spins like excitons), and/or by the Zeeman interaction with an externally applied magnetic field (for half-integer spins, like electrons, or holes).

We analyze delayed quantum feedback, focusing on the temporal and polarization dependence of photon reabsorption and the resulting spin-photon entanglement.

II. PHYSICAL SYSTEM

A schematic illustration of a single emitter in a cavity is presented in Fig. 1(a). The emitter's energy levels and optical selection rules are presented in Fig. 1(b). The energy difference between the ground and excited state is $\hbar\omega_0$, where the state e_0 is coupled to g_0 via a right-handed circular polarization and the state e_1 is coupled to g_1 via a left-handed circular polarization. Applying an external field or a fine struc-

*Contact author: Idosch@technion.ac.il

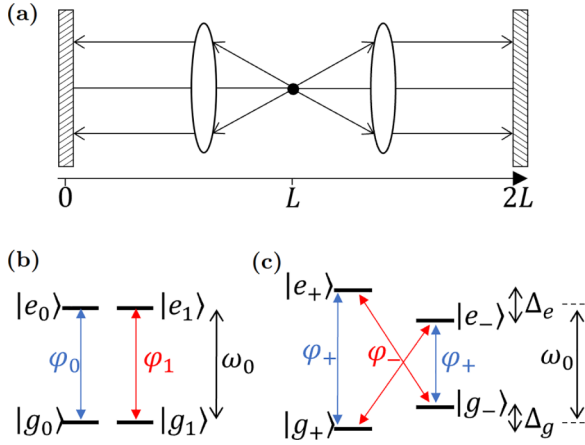


FIG. 1. A single emitter in a cavity. (a) A schematic of the emitter in a cavity showing that all the emitted radiation is reflected back from the mirrors to the emitter. (b) Energy levels and optical selection rules of the emitter in the $\{|0\rangle, |1\rangle\}$ basis. The system contains two polarization-orthogonal optical transitions. (c) Energy splitting and the corresponding optical selection rules in the system's $\{|+\rangle, |-\rangle\}$ basis.

ture splitting may remove the degeneracy between the ground and excited states with frequency differences Δ_g and Δ_e , respectively. The corresponding eigenstates are $|g_{\pm}\rangle = |g_0\rangle \pm |g_1\rangle$ and $|e_{\pm}\rangle = |e_0\rangle \pm |e_1\rangle$, while the optical transitions between the ground and excited eigenstates are linearly polarized, as illustrated in Fig. 1(c).

In the following, we analytically solve for the time evolution of the system using Schrodinger's equation, assuming an initial condition of a generally polarized excited state, $|\psi\rangle_{\text{init}} = b_{e0}(t=0)|e_0\rangle + b_{e1}(t=0)|e_1\rangle$.

A. The model

We consider the case schematically illustrated in Fig. 1(a), where an emitter is centered between two mirrors separated by a distance $2L$. For simplicity, we assume that all EM modes reach the mirrors and are perfectly reflected back to the emitter. We note that the solution is linear for partial reflection of the EM radiation [19] and that similar behavior is expected for a directional emitter near a single mirror.

The coupling between the emitter and the photon is described by two one-dimensional fields with standing-wave modes, one for each polarization. This leads to the Hamiltonian

$$H = H_0 - d(E_0^\dagger(z=L)|g_0\rangle\langle e_0| + E_1^\dagger(z=L)|g_1\rangle\langle e_1| + \text{H.c.}) + \hbar\left(\frac{\Delta_g}{2}|g_0\rangle\langle g_1| + \frac{\Delta_e}{2}|e_1\rangle\langle e_0| + \text{H.c.}\right), \quad (1)$$

where

$$H_0 = \hbar\omega_0(|e_0\rangle\langle e_0| + |e_1\rangle\langle e_1|) + \sum_k \hbar\omega_k(a_{k0}^\dagger a_{k0} + a_{k1}^\dagger a_{k1}),$$

and

$$E_{0/1}(z) = i \sum_k \alpha_{k0/1} \sin(kz) a_{k0/1}, \quad k = \frac{\pi n}{2L}$$

and d is the dipole matrix element, $\omega_k = c|k|$, and $a_{k0/1}^\dagger, a_{k0/1}$ are creation and annihilation operators for mode $k0/1$. We note that in the eigenbasis $\{|g_{\pm}\rangle\}$ and $\{|e_{\pm}\rangle\}$ the splittings Δ_g and Δ_e appear as diagonal energy shifts $\pm\Delta_{g,e}/2$; expressing the spin Hamiltonian in the circular (selection-rule) basis $\{|g_0\rangle, |g_1\rangle\}$ and $\{|e_0\rangle, |e_1\rangle\}$ therefore yields the off-diagonal terms proportional to Δ_g and Δ_e . Since in the limit $L \rightarrow \infty$, the modes are continuous, we can change the summation, $\sum_k$, with integration [22]:

$$H_0 = \hbar\omega_0(|e_0\rangle\langle e_0| + |e_1\rangle\langle e_1|) + \int dk \hbar\omega_k(a_{k0}^\dagger a_{k0} + a_{k1}^\dagger a_{k1})$$

and

$$E_{0/1}(z) = i \int dk \alpha_{k0/1} \sin(kz) a_{k0/1}.$$

The factors $\alpha_{k0/1}$ are functions of ω ($\propto \sqrt{\omega}$) and we assume that they remain almost unchanged in the small frequency range near the optical transition.

Taking the Hilbert space to be a product of the emitter's state and the photon state, the only possible states are of the form:

$$|\text{emitter}\rangle \otimes |\varphi\rangle,$$

where

$$|\text{emitter}\rangle = \{|e_0\rangle, |e_1\rangle, |g_0\rangle, |g_1\rangle\} \\ |\varphi\rangle = \{|0\rangle, |k_0\rangle, |k_1\rangle\},$$

and a general pure state is expressed as the sum of 6 elements:

$$|\psi(t)\rangle = b_{e0}(t)|e_0, 0\rangle + b_{e1}(t)|e_1, 0\rangle \\ + \int dk b_{g0,k0}(t)|g_0, k_0\rangle + \int dk b_{g0,k1}(t)|g_0, k_1\rangle \\ + \int dk b_{g1,k0}(t)|g_1, k_0\rangle + \int dk b_{g1,k1}(t)|g_1, k_1\rangle, \quad (2)$$

where b_i ($i \in \{e0, e1, (g0, k0), (g0, k1), (g1, k0), (g1, k1)\}$) are probability amplitudes. The first two terms represent excited states of the emitter with no excitation of the EM field, while the other four terms represent a ground state of the emitter with a single excitation of the EM field. Using Schrodinger's equation one gets a set of differential equations for the probability amplitudes. In the rotating frame, $b_{e0/1}(t) \rightarrow b_{e0/1}(t)e^{-i\omega_0 t}$ and $b_{g0/1,k0/1} \rightarrow b_{g0/1,k0/1}e^{-i\omega_k t}$, the set of equations are

$$\dot{b}_{e0}(t) = -\frac{d}{\hbar} \int dk \alpha_{k0} \sin(kL) e^{-i(\omega_k - \omega_0)t} b_{g0,k0}(t) \\ - i\frac{\Delta_e}{2} b_{e1}(t), \quad (3a)$$

$$\dot{b}_{e1}(t) = -\frac{d}{\hbar} \int dk \alpha_{k1} \sin(kL) e^{-i(\omega_k - \omega_0)t} b_{g1,k1}(t) \\ - i\frac{\Delta_e}{2} b_{e0}(t), \quad (3b)$$

$$\dot{b}_{g0,k0}(t) = \frac{d}{\hbar} \alpha_{k0} \sin(kL) e^{i(\omega_k - \omega_0)t} b_{e0}(t) - i \frac{\Delta_g}{2} b_{g1,k0}(t), \quad (3c)$$

$$\dot{b}_{g0,k1}(t) = -i \frac{\Delta_g}{2} b_{g1,k1}(t), \quad (3d)$$

$$\dot{b}_{g1,k0}(t) = -i \frac{\Delta_g}{2} b_{g0,k0}(t), \quad (3e)$$

$$\dot{b}_{g1,k1}(t) = \frac{d}{\hbar} \alpha_{k1} \sin(kL) e^{i(\omega_k - \omega_0)t} b_{e1}(t) - i \frac{\Delta_g}{2} b_{g0,k1}(t). \quad (3f)$$

B. The solution

The system of ODEs is solved by first applying the Wigner-Weisskopf approximation (the factors $\alpha_{k0/1}$ are constant), and then performing the Laplace transform and formal integration. The full derivation of the solution can be found in Appendix. With the definition of the spontaneous decay rate [23], $\gamma = d^2 \pi \alpha_{k0}^2 / \hbar^2 c$, and using a general superposition of the excited state for the initial condition, $|\psi(0)\rangle = \cos(\theta/2)|e_0\rangle + \sin(\theta/2)e^{i\phi}|e_1\rangle$ the solutions for the probability amplitudes of the excited states for the time interval $t \in [0, 2\tau]$ are given by

$$\begin{aligned} b_{e0}(t) &= \frac{c_+}{2} e^{-\frac{(\gamma+i\Delta_e)}{2}t} + \frac{c_+}{2} \cos(\Delta_g \tau) \gamma e^{i\omega_0 \tau} \\ &\quad \times e^{-\frac{(\gamma+i\Delta_e)}{2}(t-\tau)} (t-\tau) \Theta(t-\tau) \\ &\quad + \frac{c_-}{2} e^{-\frac{(\gamma-i\Delta_e)}{2}t} + \frac{c_-}{2} \cos(\Delta_g \tau) \gamma e^{i\omega_0 \tau} \\ &\quad \times e^{-\frac{(\gamma-i\Delta_e)}{2}(t-\tau)} (t-\tau) \Theta(t-\tau), \\ b_{e1}(t) &= \frac{c_+}{2} e^{-\frac{(\gamma+i\Delta_e)}{2}t} + \frac{c_+}{2} \cos(\Delta_g \tau) \gamma e^{i\omega_0 \tau} \\ &\quad \times e^{-\frac{(\gamma+i\Delta_e)}{2}(t-\tau)} (t-\tau) \Theta(t-\tau) \\ &\quad - \frac{c_-}{2} e^{-\frac{(\gamma-i\Delta_e)}{2}t} - \frac{c_-}{2} \cos(\Delta_g \tau) \gamma e^{i\omega_0 \tau} \\ &\quad \times e^{-\frac{(\gamma-i\Delta_e)}{2}(t-\tau)} (t-\tau) \Theta(t-\tau), \end{aligned} \quad (4)$$

where $c_{\pm} = \cos(\theta/2) \pm \sin(\theta/2)e^{i\phi}$ and $\tau = \frac{2L}{c}$.

C. Results: Reabsorption

We start with the simplest system settings, where $\Delta_e = \Delta_g = 0$ and the initial condition is $|\psi(0)\rangle = |e_0\rangle$. In this case, the two ground states and the two excited states are degenerate, and there is no coherent phase evolution between them. Therefore the system behaves like a single emitter (the circular selection rules are not important), as shown in Fig. 2(a). The solution is similar to solutions of previous works [19,24,25], where the maximal re-excited state population amplitude of $|e_0\rangle$ is $P_{e0}^{\max} \simeq 0.54$ in the first reabsorption. We further note that for $|\psi(0)\rangle = |e_0\rangle$ and $\Delta_g = \Delta_e = 0$, the dynamics is confined to the subspace spanned by $\{|e_0, 0\rangle, |g_0, k_0\rangle\}$, so the problem reduces to the standard two-level Wigner-Weisskopf model with delayed feedback [18,19].

In general, the solution is a sum of terms, as seen in Eq. (A3), where a given term is delayed by τ from the previous term due to the reabsorption of the emitted photon after a

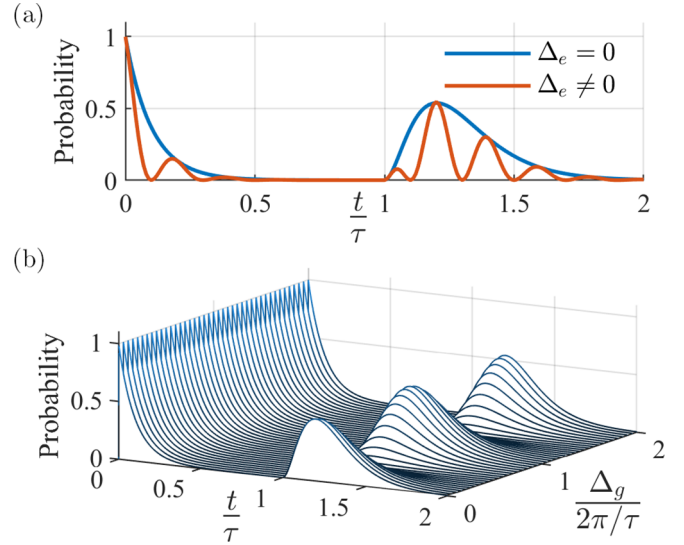


FIG. 2. Excited state's population probability as a function of time for different scenarios with fixed initial condition of $|e_0\rangle$, and $\gamma = \frac{10}{\tau}$. (a) The blue line shows the excited state's population probability with $\Delta_g = 0$ and $\Delta_e = 0$, and the orange line with $\Delta_g = 0$ and $\Delta_e = \frac{10\pi}{\tau}$. (b) Precession between the ground states only ($\Delta_e = 0$), for different ratios between the precession period $\frac{\Delta_g}{2\pi}$ and the delay time τ .

round trip to the mirror. The first term of the sum describes spontaneous emission, while the second term describes the first reabsorption and consequent emission process that starts at $t = \tau$. Here, we are interested only in the first two terms, i.e., spontaneous emission and the first reabsorption. A fast removal of the mirrors (between the first and second round trip) ensures that these conditions can be met in experiments.

Introducing an energy splitting Δ_e between the excited states results in precession between them. The precession is related to terms of the form $e^{\pm i \frac{\Delta_e}{2} t}$ in Eq. (4), and its influence on the probability to be in the state $|e_0\rangle$ is shown in Fig. 2(a) where the oscillations appear with the expected period $\frac{2\pi}{\Delta_e}$. We note that for an eigenstate initial condition, $|\psi(0)\rangle = |e_{\pm}\rangle$, there is no precession. Similarly, introducing energy splitting between the ground states results in a precession between them. However, this precession affects the overall absorption of the returning photon by the emitter, as can be seen in the term $\cos(\Delta_g \tau)$ in Eq. (4). By defining the precession time $T_p = \frac{2\pi}{\Delta_g}$, one can see that for $\tau = n \cdot T_p$ the polarization of the ground state matches the polarization of the returning photon and the coupling is maximal. In contrast, for $\tau = (n + \frac{1}{2}) \cdot T_p$, the polarization of the emitter is orthogonal to the polarization of the photon and the coupling is zero. Figure 2(b) shows the excited state population P_{e0} for different relations between the delay time τ and the precession rate Δ_g , for an initial condition of $|\psi(0)\rangle = |e_0\rangle$, without precession between the excited states, i.e., $\Delta_e = 0$. Figures 3(a) and 3(b) show the population $P_{e0/1}$ of the excited states for initial state $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|e_0\rangle + i|e_1\rangle)$ for a case with precession both between the excited and between the ground states (Δ_g varies, and $\Delta_e = \frac{10\pi}{\tau}$). For completeness, we present in Supplemental Material [26] (Fig. S1) a systematic comparison of all

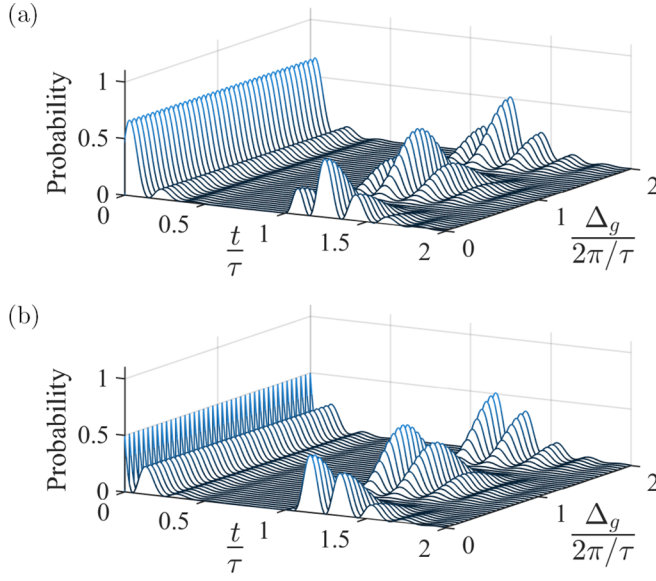


FIG. 3. Excited state's population as a function of time for an initial state of $\frac{1}{\sqrt{2}}(|e_0\rangle + i|e_1\rangle)$, $\gamma = \frac{10}{\tau}$, and $\Delta_e = \frac{10\pi}{\tau}$. (a) Population of $|e_0\rangle$ and (b) Population of $|e_1\rangle$. One can see that during the emission the population transfers from $|e_0\rangle$ to $|e_1\rangle$ and vice versa, and that there are still nulls in the re-excitation for specific ratios between the ground precession period $\frac{\Delta_g}{2\pi}$ and the photon's roundtrip time τ .

four combinations of ground- and excited-manifold splittings, $(\Delta_g, \Delta_e) \in \{(0, 0), (0, \Delta_e), (\Delta_g, 0), (\Delta_g, \Delta_e)\}$, for the initial state $|e_0\rangle$.

D. Results: Emitter-photon entanglement

In this section, we extend our analysis to examine how the phase of oscillations between the emitter's ground states is related to the photon's polarization by looking at the system's density matrix. Setting the initial condition to a superposition between the excited states leads to an entanglement between the emitter's ground states and the emitted photon's polarization. The full system dynamics is described in Appendix, providing information on all of the EM modes and the emitter's state. However, the description of this state involves an infinite number of elements given in the full wave function:

$$\begin{aligned} |\psi_{\text{inf}}(t)\rangle &= b_{e0}(t)|e_0, 0\rangle + b_{e1}(t)|e_1, 0\rangle \\ &+ \sum_k b_{g0,k0}(t)|g_0, k_0\rangle + b_{g0,k1}(t)|g_0, k_1\rangle \\ &+ b_{g1,k0}(t)|g_1, k_0\rangle + b_{g1,nk}(t)|g_1, k_1\rangle \end{aligned}$$

and the resulting infinite density matrix

$$\rho_{\text{inf}}(t) = |\psi_{\text{inf}}(t)\rangle\langle\psi_{\text{inf}}(t)|.$$

To look only at the emitter's and the photon's polarization degrees of freedom, we perform a partial trace on all the EM modes of the density matrix. The result is a reduced density matrix of 6×6 that retains the emitter state and the photon polarization, but discards intermode coherence:

$$\rho(t) = \text{Tr}_k(\rho_{\text{inf}}(t)),$$

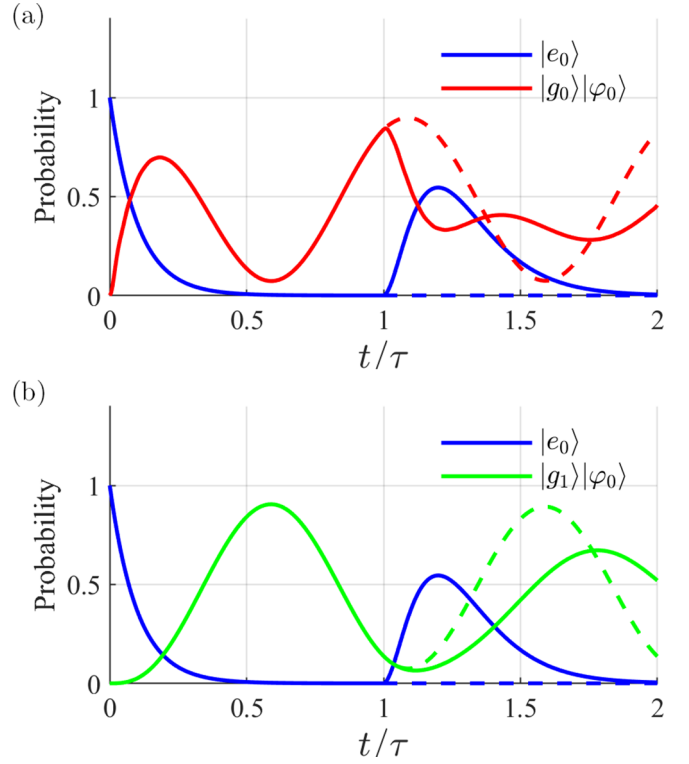


FIG. 4. Excited state and photon-ground states time evolution and phase accumulation during reabsorption ($\Delta_g = \frac{2\pi}{\tau}$, $\Delta_e = 0$). [(a) and (b)] States population for an initial state of $|e_0\rangle$. The blue line stands for $|e_0\rangle$ population, the red stands for $|g_0\rangle \otimes |\varphi_0\rangle$, and the green stands for $|g_1\rangle \otimes |\varphi_0\rangle$. Dashed lines show the time evolution without re-excitation (no mirror). One can see the precession between $|g_0\rangle$ and $|g_1\rangle$ while the photonic state remains unchanged, and that the re-excitation changes the relative phase between these states.

where $\text{Tr}_k(\dots)$ is a partial trace on the EM mode index k .

Figure 4(a) shows state probabilities of $|e_0\rangle\langle e_0|$, $|g_0\rangle\langle g_0| \otimes |\varphi_0\rangle\langle\varphi_0|$, and Fig. 4(b) shows $|e_0\rangle\langle e_0|$, $|g_1\rangle\langle g_1| \otimes |\varphi_0\rangle\langle\varphi_0|$ of such a density matrix for an initial condition of $|e_0\rangle$ with precession between ground states only ($\Delta_g = \frac{2\pi}{\tau}$). This choice of initial condition ensures that the photon's polarization state is $|\varphi_0\rangle$ and therefore that there is no entanglement between the emitter and the photon. The reabsorption affects the phase of oscillations and the overall purity of the density matrix. Figure 4 demonstrates the phase change of $|g_0\rangle|\varphi_0\rangle$ and $|g_1\rangle|\varphi_0\rangle$ (solid lines) relative to the case without reabsorption (dashed lines).

By initiating the system in a superposition of the excited states: $\frac{1}{\sqrt{2}}(|e_0\rangle + |e_1\rangle)$, an entanglement occurs between the emitter and the emitted photon. For the 4×4 density matrices shown below, we further project onto the ground-manifold subspace $\{|g_0\rangle, |g_1\rangle\}$ and condition on the presence of exactly one photon in the selected time interval. This projection discards correlations with the excited manifold and contributes to the observed reduction in purity after feedback. We note that, in principle, temporal or spatial filtering can select a specific photonic mode and thus partially restore the purity of the reduced state, since such filtering effectively projects the field onto a well-defined wave-packet mode rather than

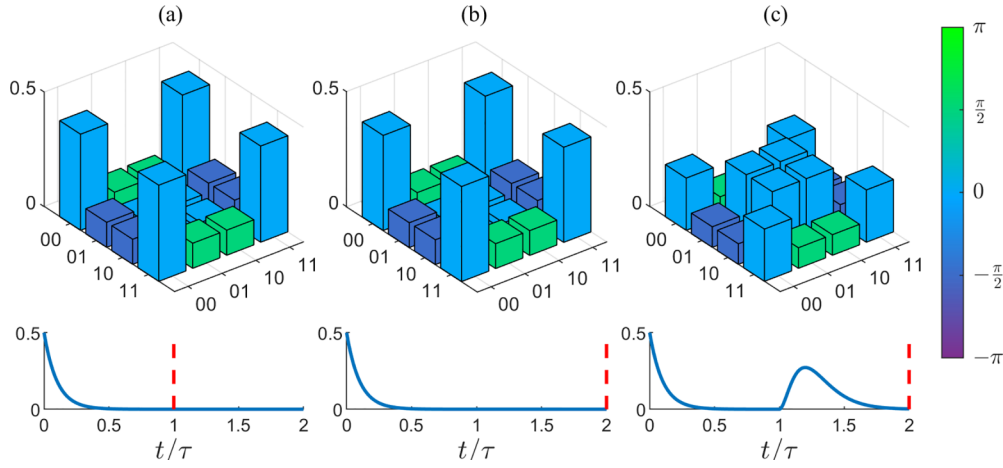


FIG. 5. 4×4 density matrices of the emitter's ground state and the emitted photon polarization at various stages of the process (with $\Delta_g = \frac{2\pi}{\tau}$ and $\Delta_e = 0$). The density matrix basis is $|g_i, \varphi_j\rangle$, $i, j = 0, 1$. The stage in the process is shown in the lower panel of each subfigure. (a) The density matrix immediately before the photon is returned to the emitter ($t = \tau$), where the system is in a Bell state [27]. (b) The density matrix at $t = 2\tau$ without reabsorption. The density matrix is identical to the one in (a). (c) The density matrix at $t = 2\tau$ after one reabsorption and re-emission occurs. Here, the system evolves into a mixed state of two Bell states, demonstrating the effect of reabsorption on the entanglement structure.

summing over all longitudinal modes. However, in the present work, we focus on the intrinsic emitter-feedback dynamics and therefore do not assume any mode-selective filtering.

Fig. 5 presents the 4×4 density matrices representing the system's quantum state at different stages of the photon reabsorption process. These normalized density matrices describe the joint state of the emitter's ground state and the polarization of the emitted photon. The leftmost density matrix corresponds to the state immediately before the photon returns to the emitter ($t = \tau$). It can be observed that this state is an entangled Bell state [27]. The middle and rightmost density matrices represent the state after the photon is re-emitted ($t = 2\tau$), for two distinct scenarios: without reabsorption (middle) and with reabsorption (right). In the case without reabsorption, the density matrix remains unchanged, but when reabsorption occurs, the system evolves into a mixed state of two Bell states, as seen from the changes in the density matrix. This shows the impact of the reabsorption process on the entanglement structure of the system. A corresponding systematic comparison for the initial superposition states $\frac{1}{\sqrt{2}}(|e_0\rangle + |e_1\rangle)$ and $\frac{1}{\sqrt{2}}(|e_0\rangle + i|e_1\rangle)$, including all four detuning combinations, is provided in Ref. [26] (Figs. S2 and S3, respectively).

The purity and negativity [28] (a measure of the entanglement) of this case are presented in Fig. 6(a), where a drop in both of them is observed due to reabsorption. For the presented case, the parameters are set as $\gamma = \frac{10}{\tau}$, $\Delta_g = \frac{2\pi}{\tau}$, and $\Delta_e = 0$. To further explore the behavior of the entanglement, Figs. 6(b) and 6(c) provides color maps of the negativity at $t = 2\tau$ (after single re-emission) for various values of γ , Δ_g , and Δ_e . The color maps reveal that the degree of entanglement, quantified by the negativity, is highly sensitive to the values of these parameters. Specifically, maximizing entanglement requires increasing the decay rate γ . Furthermore, Fig. 6(b) reflects the periodic dependence of the negativity on Δ_g , which arises from the relationship between the precession rate and the round trip time. Figure 6(c) shows a decrease in

the negativity for increasing values of Δ_e and for decreasing values of γ . The additional structure in the map (i.e., the periodic decrease in negativity while changing Δ_e) arises due to the accumulation of time intervals with varying photon polarization state during emission, where the relative phase accumulation between the ground and excited states as well as the ratio between Δ_e and γ determines the exact structure. Freely speaking, the negativity is influenced by the phase accumulation at a given time interval.

III. DISCUSSION

We have examined photon emission and reabsorption from a confined spin emitter with internal structure, placed at a large distance from the cavity reflecting mirrors. The system consists of two ground states coupled to two excited states via optical transitions with circular polarization selection rules, forming an optical Π system. Its dynamics were modeled using a Hamiltonian description with time-delayed feedback.

In the absence of energy splittings between the spin eigenstates, the model reproduces the results previously obtained for a single non-degenerate emitter in a cavity [19,24,25], yielding a maximum reabsorption probability of about $P_e^{\max} \simeq 0.54$. This relatively low reabsorption efficiency originates from the asymmetric temporal profile of the spontaneously emitted photon. While a time-reversal mirror [29–31] could, in principle, restore full reabsorption, such devices have not yet been demonstrated for single photons. A more practical approach is to control the temporal shape of the optical transition—by engineering the photonic environment or tuning the emitter's resonance relative to the cavity mode [32]—which can increase the reabsorption efficiency.

Energy splittings in the spin eigenstates introduce additional dynamical processes. A splitting in the ground states induces spin precession, so the timing of the photon round trip relative to the precession period determines the reabsorption probability. Similarly, splittings in the excited states imprint

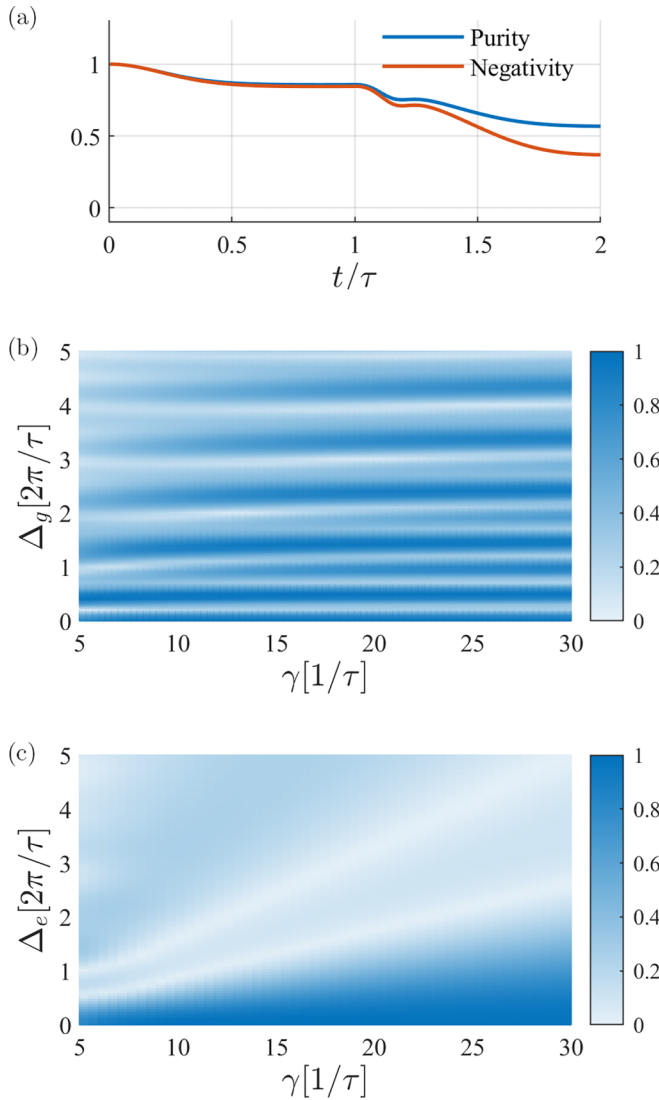


FIG. 6. Purity and negativity with various system parameters. (a) The purity (blue) and negativity (orange) of the 4×4 density matrix constructed by the emitter and the photon polarizations for $\gamma = \frac{10}{\tau}$, $\Delta_g = \frac{2\pi}{\tau}$, and $\Delta_e = 0$. The negativity is a measure of the “amount of entanglement” [28]. It can be seen that the reabsorption lowers both the purity and the negativity. [(b) and (c)] Negativity colormaps of the quantum state at time $t = 2\tau$, for various values of the decay rate γ and the ground-state precession rate Δ_g ($\Delta_e = 0$) (b) or the excited-state precession rate Δ_e ($\Delta_g = 0$) (c).

precession phases on the spin-photon correlations. In both cases, while the full emitter-photon state remains pure if all electromagnetic modes are retained, in practice, it is impossible to separate the different photonic modes. Summing over them is equivalent to tracing out part of the system and reduces the purity of the spin-photon state. The extent of this purity loss depends on the ratios Δ_g/γ and Δ_e/γ : slower decay rates amplify the reduction in coherence.

Photon emission from a superposition of excited states entangles the spin with the photon’s polarization. The resulting density matrix depends on the oscillation phases of the emitter’s eigenstates and, as in the case of purity, its structure is determined by Δ_g/γ and Δ_e/γ . When reabsorption occurs,

the situation becomes more complex. The joint state can be described as a combination of two Bell-state contributions: the first, originating from the initial photon emission, remains unaffected, while the second acquires an additional phase due to the finite reabsorption and re-emission process. As a result, both the purity and the degree of entanglement of the emitter-photon state are further reduced.

These effects have important implications for protocols that rely on photon reabsorption to implement controlled spin-photon gates required for extending a one-dimensional cluster state into a two-dimensional photonic cluster state. Our results show that the limited reabsorption probability, the loss of purity induced by spin precession during decay, and the phase picked up over the photon’s finite return time all reduce the achievable gate fidelity. Under these constraints, we identify a favorable regime for achieving enhanced gate fidelity: (i) synchronized ground-manifold precession, $\Delta_g = 2\pi m/\tau$ ($m \in \mathbb{Z}$), (ii) $\gamma \gg \Delta_g$, and (iii) $\gamma \gg \Delta_e$. In the synchronized case $\Delta_g = 2\pi/\tau$ and setting $\Delta_e = \Delta_g$, increasing the decay rate relative to the splittings rapidly improves the joint-state quality: $\gamma = 10\Delta_g$ yields negativity $>90\%$ and purity $>95\%$; $\gamma = 20\Delta_g$ yields negativity $>97\%$ and purity $>99\%$; and $\gamma = 30\Delta_g$ yields negativity $>99\%$ and purity $>99.5\%$. Thus, fast emission relative to the internal splittings, together with synchronized precession, enables near-ideal gate fidelity.

Although we identify a favorable regime, any protocol that uses reabsorption as part of the entangling step must still account for these constraints. Mitigating them will require enhanced control of the photon’s temporal mode, for example, through external shaping pulses applied during the photon’s return, time-reversal-based techniques, or photonic-structure engineering that symmetrizes the emission; in some architectures, post-selection can be used to isolate high-fidelity events. Such advances will be essential for realizing high-fidelity spin-photon interfaces and, ultimately, scalable two-dimensional cluster-state generation.

In summary, our analysis shows that while photon reabsorption from a spin emitter is fundamentally limited by the asymmetric temporal profile of spontaneous emission, internal dynamics such as spin precession further reduce purity and entanglement in the spin-photon state. These effects must be carefully accounted for in any practical protocol, particularly for quantum gates based on reabsorption. Progress in photon shaping and cavity engineering will therefore be key to overcoming these challenges and enabling high-fidelity spin-photon interfaces for quantum technologies.

ACKNOWLEDGMENT

This research was supported by the Israel Science Foundation (Grant No. 2479/24).

DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX: SOLUTION OF THE DIFFERENTIAL EQUATIONS

For solving the system of differential equations (3), we start by noting that there is a coupling between the equations, and therefore, the first step should be to uncoupling them. Looking at Eqs. (3c)–(3f) one can see that the coupling is between (3c) and (3e) and between (3d) and (3f). Both sub-systems have the same form; therefore, one can solve either of the two and deduce the solution for the other. We choose to solve (3c) and (3e).

Applying a derivative on (3c) leads to

$$\ddot{b}_{g0,k0}(t) = \frac{d}{\hbar} \alpha_{k0} \sin(kL) \frac{d}{dt} (e^{i(\omega_k - \omega_0)t} b_{e0}(t)) - i \frac{\Delta_g}{2} \dot{b}_{g1,k0}(t),$$

and substituting (3e) into the last term yields

$$\ddot{b}_{g0,k0}(t) + \frac{\Delta_g^2}{4} b_{g0,k0}(t) = \frac{d}{\hbar} \alpha_{k0} \sin(kL) \frac{d}{dt} (e^{i(\omega_k - \omega_0)t} b_{e0}(t)).$$

With the following notation changes: $y = b_{g0,k0}(t)$, $\Omega_0^2 = \frac{\Delta_g^2}{4}$, $A(k) = \frac{d}{\hbar} \alpha_{k0} \sin(kL)$, and $f(t) = (e^{i(\omega_k - \omega_0)t} b_{e0}(t))$ the equation can be rewritten as

$$\ddot{y} + \Omega_0^2 y = A(k) \frac{d}{dt} f(t).$$

Applying the Laplace transform [33] on the equation leads to

$$Y(s) = \frac{sy(0) + \dot{y}(0)}{s^2 + \Omega_0^2} + A(k) \frac{sF(s) - 1}{s^2 + \Omega_0^2},$$

where $Y(s)$ stands for the Laplace transform of $y(t)$, and in the last terms, the initial condition $b_{eR}(0) = 1$ was used. The other initial conditions are

$$y(0) = b_{g0,k0}(0) = 0,$$

$$\dot{y}(0) = \dot{b}_{g0,k0}(0) = \frac{d}{\hbar} \alpha_{k0} \sin(kL) b_{e0}(0) - i \Omega_0 b_{g1,k0}(0)$$

$$= \frac{d}{\hbar} \alpha_{k0} \sin(kL)$$

$$= A(k).$$

Introducing the initial condition to the Laplace transform leads to

$$Y(s) = A(k) \frac{sF(s)}{s^2 + \Omega_0^2}.$$

Next, applying an inverse Laplace transform and using the convolution theorem results in

$$\begin{aligned} b_{g0,k0}(t) &= A(k) \mathcal{L}^{-1} \left\{ \frac{sF(s)}{s^2 + \Omega_0^2} \right\} \\ &= A(k) \mathcal{L}^{-1} \left\{ \frac{s}{s^2 + \Omega_0^2} \right\} * \mathcal{L}^{-1} \{F(s)\} \\ &= A(k) \cos(\Omega_0 t) * f(t) \\ &= \frac{d}{\hbar} \alpha_{k0} \sin(kL) \cdot \cos(\Omega_0 t) * (e^{i(\omega_k - \omega_0)t} b_{e0}(t)) \\ &= \frac{d}{\hbar} \alpha_{k0} \sin(kL) \int_0^t dt' \cos\left(\frac{\Delta_g}{2} t'\right) e^{i(\omega_k - \omega_0)(t-t')} b_{e0}(t-t'). \end{aligned} \quad (A1)$$

Introducing this result into Eq. (3a) leads to

$$\dot{b}_{e0}(t) = -\frac{d^2}{\hbar^2} \int dk \alpha_{k0}^2 \sin^2(kL) e^{-i(\omega_k - \omega_0)t} \int_0^t dt' \cos\left(\frac{\Delta_g}{2} t'\right) e^{i(\omega_k - \omega_0)(t-t')} b_{e0}(t-t') - i \frac{\Delta_e}{2} b_{e1}(t).$$

The last step in simplifying the equation is using $kL = \frac{\omega_k \tau}{2}$ and applying the Wigner-Weisskopf approximation, leading to

$$\begin{aligned} \dot{b}_{e0}(t) + i \frac{\Delta_e}{2} b_{e1}(t) &= -\frac{d^2}{\hbar^2} \alpha_{k0}^2 \int_{-\infty}^{\infty} dk \sin^2(kL) \int_0^t dt' \cos\left(\frac{\Delta_g}{2} t'\right) e^{-i(\omega_k - \omega_0)t'} b_{e0}(t-t') \\ &= -\frac{d^2}{4\hbar^2} \alpha_{k0}^2 \int_{-\infty}^{\infty} dk (2 - e^{i\tau\omega_0} e^{i\tau(\omega_k - \omega_0)} - e^{-i\tau\omega_0} e^{-i\tau(\omega_k - \omega_0)}) \int_0^t dt' \cos\left(\frac{\Delta_g}{2} t'\right) e^{-i(\omega_k - \omega_0)t'} b_{e0}(t-t') \\ &= -\frac{d^2 \pi}{2\hbar^2 c} \alpha_{k0}^2 \int_0^t dt' [2\delta(t') - e^{i\tau\omega_0} \delta(t' - \tau) - e^{-i\tau\omega_0} \delta(t' + \tau)] \cos\left(\frac{\Delta_g}{2} t'\right) b_{e0}(t-t'). \end{aligned}$$

In a similar manner, Eqs. (3b), (3d), and (3f) are used to solve for b_{e1} , and by defining the decay rate $\gamma = \frac{d^2 \pi \alpha_{k0}^2}{2\hbar^2 c}$ and calculating the integral over the delta functions, results in two coupled equations:

$$\dot{b}_{e0}(t) = -\frac{\gamma}{2} b_{e0}(t) + \frac{\gamma}{2} e^{i\tau\omega_0} \cos(\Delta_g \tau) b_{e0}(t - \tau) \Theta(t - \tau) - i \Delta_e b_{e1}(t),$$

$$\dot{b}_{e1}(t) = -\frac{\gamma}{2} b_{e1}(t) + \frac{\gamma}{2} e^{i\tau\omega_0} \cos(\Delta_g \tau) b_{e1}(t - \tau) \Theta(t - \tau) - i \Delta_e b_{e0}(t).$$

Changing basis to

$$\begin{aligned} b_{e+}(t) &= b_{e0}(t) + b_{e1}(t), \\ b_{e-}(t) &= b_{e0}(t) - b_{e1}(t) \end{aligned}$$

leads to the following pair of uncoupled equations:

$$\begin{aligned} \dot{b}_{e+}(t) &= \left(-\frac{\gamma}{2} - i\Delta_e\right)b_{e+}(t) \\ &\quad + \frac{\gamma}{2}e^{i\tau\omega_0}\cos(\Delta_g\tau)b_{e+}(t-\tau)\Theta(t-\tau), \\ \dot{b}_{e-}(t) &= \left(-\frac{\gamma}{2} + i\Delta_e\right)b_{e-}(t) \\ &\quad + \frac{\gamma}{2}e^{i\tau\omega_0}\cos(\Delta_g\tau)b_{e-}(t-\tau)\Theta(t-\tau). \end{aligned}$$

These equations are separated, and their solution is given by

$$\begin{aligned} b_{e+}(t) &= c_+ \sum_{n=0}^{\infty} \frac{(\gamma \cos(\Delta_g\tau))^n}{n!} e^{i\omega_0 n\tau} e^{-\frac{\gamma+i\Delta_e}{2}(t-n\tau)} \\ &\quad \times (t-n\tau)^n \Theta(t-n\tau), \\ b_{e-}(t) &= c_- \sum_{n=0}^{\infty} \frac{(\gamma \cos(\Delta_g\tau))^n}{n!} e^{i\omega_0 n\tau} e^{-\frac{\gamma-i\Delta_e}{2}(t-n\tau)} \\ &\quad \times (t-n\tau)^n \Theta(t-n\tau). \end{aligned} \quad (\text{A2})$$

These are two independent delayed differential equations (DDEs), and for an initial condition of a general superposition

$$|\psi(0)\rangle = \cos\left(\frac{\theta}{2}\right)|e_0\rangle + \sin\left(\frac{\theta}{2}\right)e^{i\phi}|e_1\rangle,$$

one can calculate the initial condition in the new basis to get

$$\begin{aligned} b_{e+}(0) &= c_+ = \cos\left(\frac{\theta}{2}\right) + \sin\left(\frac{\theta}{2}\right)e^{i\phi}, \\ b_{e-}(0) &= c_- = \cos\left(\frac{\theta}{2}\right) - \sin\left(\frac{\theta}{2}\right)e^{i\phi}. \end{aligned}$$

The full solution is achieved by changing back to the original basis:

$$\begin{aligned} b_{e0}(t) &= \frac{c_+}{2} \sum_{n=0}^{\infty} \frac{(\gamma \cos(\Delta_g\tau))^n}{n!} e^{i\omega_0 n\tau} e^{-\frac{\gamma+i\Delta_e}{2}(t-n\tau)} \\ &\quad \times (t-n\tau)^n \Theta(t-n\tau), \\ &\quad + \frac{c_-}{2} \sum_{n=0}^{\infty} \frac{\gamma \cos(\Delta_g\tau))^n}{n!} e^{i\omega_0 n\tau} e^{-\frac{\gamma-i\Delta_e}{2}(t-n\tau)} \\ &\quad \times (t-n\tau)^n \Theta(t-n\tau), \\ b_{e1}(t) &= \frac{c_+}{2} \sum_{n=0}^{\infty} \frac{\gamma \cos(\Delta_g\tau))^n}{n!} e^{i\omega_0 n\tau} e^{-\frac{\gamma+i\Delta_e}{2}(t-n\tau)} \\ &\quad \times (t-n\tau)^n \Theta(t-n\tau), \\ &\quad - \frac{c_-}{2} \sum_{n=0}^{\infty} \frac{\gamma \cos(\Delta_g\tau))^n}{n!} e^{i\omega_0 n\tau} e^{-\frac{\gamma-i\Delta_e}{2}(t-n\tau)} \\ &\quad \times (t-n\tau)^n \Theta(t-n\tau). \end{aligned} \quad (\text{A3})$$

1. Solving the ground state population

Using Eq. (A1), one can implement the above solution for the excited state and receive a solution for the ground state:

$$\begin{aligned} b_{g0,k0}(t) &= \frac{d}{\hbar} \alpha_{k0} \sin(kL) \int_0^t dt' \cos\left(\frac{\Delta_g}{2}t'\right) \\ &\quad \times e^{i(\omega_k - \omega_0)(t-t')} b_{e0}(t-t'). \end{aligned}$$

It is important to note that, due to boundary conditions, k takes discrete values given by $k = \frac{n\pi}{2L}$, where $n \in \mathbb{N}$. Although we effectively sum over integer indices n , we retain the notation k for clarity and consistency with previous expressions. In particular, we observe that $\sin(kL) = \sin(\frac{n\pi}{2})$ which implies that $\sin(kL)$ takes on values only in the set $\{0, \pm 1\}$. Applying the Wigner-Weisskopf approximation yields

$$\begin{aligned} b_{g0,k0}(t) &= \sqrt{\frac{\gamma}{\pi}} \sin(kL) \int_0^t dt' \cos\left(\frac{\Delta_g}{2}t'\right) \\ &\quad \times e^{i(\omega_k - \omega_0)(t-t')} b_{e0}(t-t'). \end{aligned}$$

Using Eq. (3) one can also get a close formula for

$$b_{g1,k0}(t) = -i \frac{\Delta_g}{2} \int_0^t b_{g0,k0}(t') dt'.$$

In a similar way, a closed formula is also achieved for $b_{g1,k1}(t)$ and $b_{g0,k1}(t)$.

2. Constructing the emitter-photon density matrix

The solutions of the differential equations allow us to write the general wave function of the system as a function of time as

$$\begin{aligned} |\psi(t)\rangle &= b_{e0}(t)|e_0, 0\rangle + b_{e1}(t)|e_1, 0\rangle \\ &\quad + \sum_k b_{g0,k0}(t)|g_0, k_0\rangle + b_{g0,k1}(t)|g_0, k_1\rangle \\ &\quad + b_{g1,k0}(t)|g_1, k_0\rangle + b_{g1,k1}(t)|g_1, k_1\rangle, \end{aligned}$$

and using this one can construct an infinite density matrix of the system

$$\rho_{\text{inf}}(t) = |\psi(t)\rangle\langle\psi(t)|$$

in order to refer to the finite system of the emitter and the photon polarization states only, one can apply partial tracing on the photonic mode space, and the result will be a 6×6 density matrix

$$\rho(t) = \text{Tr}_k(\rho_{\text{inf}}(t)).$$

Finally, to examine the entanglement between the photon and the emitter's polarization, it is sufficient to consider the 4×4 block of the density matrix corresponding to the ground states. To interpret this block as a valid two-qubit density matrix, it is normalized accordingly.

- [1] A. K. Ekert, Quantum cryptography based on Bell's theorem, *Phys. Rev. Lett.* **67**, 661 (1991).
- [2] H.-J. Briegel, W. Dür, J. I. Cirac, and P. Zoller, Quantum repeaters: The role of imperfect local operations in quantum communication, *Phys. Rev. Lett.* **81**, 5932 (1998).
- [3] P. Kok, W. J. Munro, K. Nemoto, T. C. Ralph, J. P. Dowling, and G. J. Milburn, Linear optical quantum computing with photonic qubits, *Rev. Mod. Phys.* **79**, 135 (2007).
- [4] A. Imamoglu, D. D. Awschalom, G. Burkard, D. P. DiVincenzo, D. Loss, M. Sherwin, and A. Small, Quantum information processing using quantum dot spins and cavity QED, *Phys. Rev. Lett.* **83**, 4204 (1999).
- [5] I. Schwartz, D. Cogan, E. R. Schmidgall, Y. Don, L. Gantz, O. Kenneth, N. H. Lindner, and D. Gershoni, Deterministic generation of a cluster state of entangled photons, *Science* **354**, 434 (2016).
- [6] D. Istrati, Y. Pilnyak, J. C. Loredó, C. Antón, N. Somaschi, P. Hilaire, H. Ollivier, M. Esmann, L. Cohen, L. Vidro, C. Millet, A. Lemaître, I. Sagnes, A. Harouri, L. Lanco, P. Senellart, and H. S. Eisenberg, Sequential generation of linear cluster states from a single photon emitter, *Nat. Commun.* **11**, 5501 (2020).
- [7] D. Cogan, Z.-E. Su, O. Kenneth, and D. Gershoni, Deterministic generation of indistinguishable photons in a cluster state, *Nat. Photonics* **17**, 324 (2023).
- [8] N. Coste, D. Fioretto, N. Belabas, S. Wein, P. Hilaire, R. Frantzeskakis, M. Gundin, B. Goes, N. Somaschi, M. Morassi, *et al.*, High-rate entanglement between a semiconductor spin and indistinguishable photons, *Nat. Photonics* **17**, 582 (2023).
- [9] Z.-E. Su, B. Taitler, I. Schwartz, D. Cogan, I. Nassar, O. Kenneth, N. H. Lindner, and D. Gershoni, Continuous and deterministic all-photonic cluster state of indistinguishable photons, *Rep. Prog. Phys.* **87**, 077601 (2024).
- [10] A. Reiserer and G. Rempe, Cavity-based quantum networks with single atoms and optical photons, *Rev. Mod. Phys.* **87**, 1379 (2015).
- [11] P. Thomas, L. Ruscio, O. Morin, and G. Rempe, Efficient generation of entangled multiphoton graph states from a single atom, *Nature (London)* **608**, 677 (2022).
- [12] R. Raussendorf and H. J. Briegel, A one-way quantum computer, *Phys. Rev. Lett.* **86**, 5188 (2001).
- [13] M. Zwerger, W. Dür, and H. J. Briegel, Measurement-based quantum repeaters, *Phys. Rev. A* **85**, 062326 (2012).
- [14] K. Azuma, K. Tamaki, and H.-K. Lo, All-photonic quantum repeaters, *Nat. Commun.* **6**, 6787 (2015).
- [15] D. Buterakos, E. Barnes, and S. E. Economou, Deterministic generation of all-photonic quantum repeaters from solid-state emitters, *Phys. Rev. X* **7**, 041023 (2017).
- [16] H. Pichler, S. Choi, P. Zoller, and M. D. Lukin, Universal photonic quantum computation via time-delayed feedback, *Proc. Natl. Acad. Sci.* **114**, 11362 (2017).
- [17] Y. Shi and E. Waks, Deterministic generation of multidimensional photonic cluster states using time-delay feedback, *Phys. Rev. A* **104**, 013703 (2021).
- [18] G. Alber, Photon wave packets and spontaneous decay in a cavity, *Phys. Rev. A* **46**, R5338 (1992).
- [19] U. Dorner and P. Zoller, Laser-driven atoms in half-cavities, *Phys. Rev. A* **66**, 023816 (2002).
- [20] N. H. Lindner and T. Rudolph, Proposal for pulsed on-demand sources of photonic cluster state strings, *Phys. Rev. Lett.* **103**, 113602 (2009).
- [21] D. Cogan, O. Kenneth, N. H. Lindner, G. Peniakov, C. Hopfmann, D. Dalacu, P. J. Poole, P. Hawrylak, and D. Gershoni, Depolarization of electronic spin qubits confined in semiconductor quantum dots, *Phys. Rev. X* **8**, 041050 (2018).
- [22] X.-P. Feng and K. Ujihara, Quantum theory of spontaneous emission in a one-dimensional optical cavity with two-side output coupling, *Phys. Rev. A* **41**, 2668 (1990).
- [23] M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge University Press, 1997).
- [24] M. Stobińska, G. Alber, and G. Leuchs, Perfect excitation of a matter qubit by a single photon in free space, *Europhys. Lett.* **86**, 14007 (2009).
- [25] Y. Wang, J. Minář, L. Sheridan, and V. Scarani, Efficient excitation of a two-level atom by a single photon in a propagating mode, *Phys. Rev. A: At. Mol. Opt. Phys.* **83**, 063842 (2011).
- [26] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/f4vf-nc9t> for a systematic comparison of how all four combinations of ground- and excited-manifold splittings influence the system dynamics.
- [27] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, 2010).
- [28] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Quantum entanglement, *Rev. Mod. Phys.* **81**, 865 (2009).
- [29] V. Bacot, M. Labousse, A. Eddi, M. Fink, and E. Fort, Time reversal and holography with spacetime transformations, *Nat. Phys.* **12**, 972 (2016).
- [30] T. R. Jones, A. V. Kildishev, M. Segev, and D. Peroulis, Time-reflection of microwaves by a fast optically-controlled time-boundary, *Nat. Commun.* **15**, 6786 (2024).
- [31] H. Moussa, G. Xu, S. Yin, E. Galiffi, Y. Ra'di, and A. Alù, Observation of temporal reflection and broadband frequency translation at photonic time interfaces, *Nat. Phys.* **19**, 863 (2023).
- [32] E. Peinke, T. Sattler, G. M. Torelly, P. L. Souza, S. Perret, J. Bleuse, J. Claudon, W. L. Vos, and J. M. Gérard, Tailoring the properties of quantum dot-micropillars by ultrafast optical injection of free charge carriers, *Light Sci. Appl.* **10**, 215 (2021).
- [33] M. L. Boas, *Mathematical Methods in the Physical Sciences* (John Wiley & Sons, Hoboken, NJ, 2006).