

Transient localization from fractionalization: Vanishingly small energy transport in gapless quantum magnets

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In this work, we investigate zero-temperature energy transport in a gapless quantum magnet. We find that suppressed energy transport can arise at low frequencies due to transient localization from fractionalization, even in the absence of extrinsic defects or disorder. Concretely, we consider a Kitaev ladder model, whose spin degrees of freedom fractionalize into visons and spinons, in a uniform magnetic field. For moderate magnetic fields, visons are heavy and act as quasistatic disorder that induces transient localization of light spinons even in the translation-invariant model and at zero temperature, which strongly suppresses the residual transport.

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I. INTRODUCTION

The description of phases of matter and their response theory is typically formulated in terms of long-wavelength effective theories and provides a unifying framework for understanding emergent phenomena. For systems with fractionalized excitations, these descriptions are typically obtained from parton constructions and associated gauge theories, where the emergent gauge structure is assumed to be translationally invariant [1–3]. Within this framework, transport and thermodynamic properties are governed by the long-wavelength dynamics of emergent quasiparticles and gauge fluctuations.

Despite the success of this paradigm in capturing key qualitative aspects of many fractionalized phases [4–6], challenges arise when theoretical predictions are compared to several experiments. For example, gapless quantum spin liquids (QSL) with emergent Fermi surfaces are predicted to have thermodynamic signatures identical to a metal: a linear-in-temperature specific heat and thermal conductivity [7]. Several frustrated Mott insulators exhibit such linear specific heat down to the lowest temperatures [8–11]. Yet, various compounds, such as candidates of field-stabilized Kitaev QSL— α - RuCl_3 [12] and $\text{Na}_2\text{Co}_2\text{TeO}_6$ [13]; geometrically frustrated spin liquids on the triangular lattice like YbMgGaO_4 [14] and NaYbSe_2 [10,15]; and organic spin liquid candidates [16,17]—show a concomitant vanishing thermal conductivity at low temperatures, at variance with the theoretical expectation. Therefore,

refinements of the long-wavelength effective theories might be required to understand the experimental implications.

Here, we provide insights into this paradox by studying the zero-temperature energy transport of a gapless quantum magnet, which we find to be strongly suppressed at low frequencies. We argue that such a suppressed low-frequency response can arise intrinsically from fractionalization in the absence of extrinsic defects or disorder, and that the very mechanism which produces a finite spinon density at zero energy also suppresses the low-frequency energy conductivity. Concretely, we demonstrate the idea in a minimal model of the translation-invariant Kitaev ladder, in which a moderate uniform magnetic field drives a gapped spinon band insulator into an intermediate gapless phase where light spinons and heavy visons mutually interact [18]. The heavy visons imprint a quasistatic coherent randomness on the spinons, thereby localizing them transiently without quenched or extrinsic disorder [19–23]. As a consequence, the energy conductivity will be strongly suppressed as frequency is lowered. Asymptotically at low energy scales, the system will become conducting in the absence of any extrinsic perturbations. We therefore propose the transient localization (TL) of spinons induced by the heavy visons as a promising mechanism for a suppressed energy conductivity.

II. THE KITAEV LADDER

We start from a Kitaev ladder model of local spin- $\frac{1}{2}$ moments in the presence of a magnetic field:

$$H = H_{\text{ladder}} - h \sum_{j,l} (X_j^l + Y_j^l + Z_j^l), \quad (1)$$

$$H_{\text{ladder}} = \sum_{j+l \in \text{even}} (X_j^l X_{j+1}^l + Y_{j-1}^l Y_j^l + Z_j^l Z_{j+1}^l). \quad (2)$$

Here, the subscript j is the horizontal index along the ladder, and the superscript l a binary index labeling the lower

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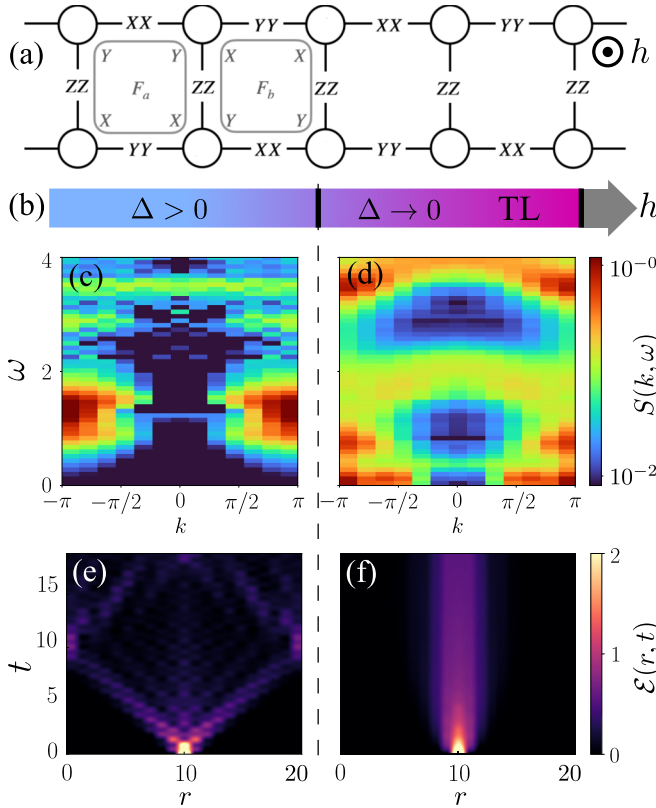


FIG. 1. Model and numerical results for field-induced dynamical spectrum and transient localization (TL) from fractionalization. (a) The Kitaev ladder model with local conservation laws F_a and F_b . (b) Increasing the magnetic field h drives a phase transition from the diffusive gapped spin liquid (blue) at $h \approx 0.4$, to a regime with TL and substantial low-energy modes near zero energy (purple), and then to the partially polarized paramagnet (gray) at $h \approx 0.9$. (c), (d) Spin spectral function at $h = 0.04$ (c) and $h = 0.80$ (d). (e), (f) Energy-density spreading after a local quench applied to the ground state for $h = 0.04$ (e) and $h = 0.80$ (f).

($l = 0$) or upper ($l = 1$) leg; see Fig. 1(a). We use the quasi-one-dimensional model to illustrate the mechanism where the localization due to emergent quasistatic coherent randomness is most easily visible numerically; we will argue for the validity of the concept beyond one dimension below. Similar to Kitaev's honeycomb model, H_{ladder} has extensive local symmetries given by the square faces $F_a = X_0^0 Y_0^1 X_1^0 Y_1^1$ and $F_b = Y_1^0 X_1^1 Y_2^0 X_2^1$, with $F_{a(b)} = 1$ in the ground state [18,24]. For $h \rightarrow 0$, elementary excitations of H_{ladder} are free Majorana fermions (referred to as spinons) and static Z_2 fluxes (visons) given by $F_{a(b)} = -1$. In contrast to the honeycomb model, the Majorana gap Δ in the ground state sector of H_{ladder} is finite; see the supplemental material (SM) [25]. A nonzero magnetic field breaks the exact local conservation laws. For a small-to-moderate h , the ground-state expectation of $F_{a(b)}$ decreases from 1.0 to ~ 0 [26–28], indicating an increasing density of flipped fluxes $\nu \sim -\frac{1}{2}(\langle F_{a(b)} \rangle - 1)$ to near half filling, as illustrated by the blue-to-purple gradient in the sketched phase diagram; Fig. 1(b). The finite flux density ν under moderate h allows us to put forward a simple approximate description of the ground state as a superposition of quasistatic vison

configurations. Since each typical configuration is disordered, the fermions are localized [29] up to a cut off timescale set by the weak tunneling between different vison configurations [19–23]. We next present numerical evidence to support this effective theory. We demonstrate the presence of TL by suppressed energy transport, and the simultaneous presence of emergent excitations near zero energy.

III. COEXISTENCE OF VANISHING GAP AND SLOW DYNAMICS

Although the frustrated quasi-one-dimensional ladder under the Zeeman field has no symmetry on the microscopic level, there can be a transition into an emergent floating gapless phase with incommensurate correlations before a polarized phase is entered [30–32]; see Fig. 1(b). This is reflected in the dynamical spin structure factor shown in Figs. 1(c) and 1(d), obtained by matrix product states (MPS) simulations [33,34]: while for small but finite field $h = 0.04$ there is an observable gap $\Delta > 0$ [Fig. 1(c)], at moderate field $h = 0.8$ there are concentrated low-energy modes around $k = \pm\pi$ points [Fig. 1(d)]. Qualitatively, this is similar to the emergence of the finite low-energy density of states of fermions due to thermal visons [35]; however, in this case, it is rather the quantum vison fluctuations that are responsible for the drastic change in the zero-temperature structure factor [28,36].

Now, we show the coexisting slow dynamics in this intermediate phase. The energy is the only conserved global charge of Eq. (1) at finite h . We therefore demonstrate the coexisting slow dynamics first by measuring the spreading of energy-density fluctuation $\mathcal{E}_r = h_r - \langle h_r \rangle$, where h_r is the local Hamiltonian of the r th unit cell. Since we are interested in the low-energy dynamics, the initial state $|0\rangle$ is prepared by perturbing the ground state $|0\rangle$ obtained from DMRG, locally by $P_{r_0} = Z_{r_0}^0 Z_{r_0}^1$, with r_0 the center of the ladder. Results are shown in Figs. 1(e) and 1(f), obtained by MPS time evolution from $|0\rangle$ (with $t_{\text{max}} \sim 20$ and bond dimensions up to 1000) [37]. For $h \rightarrow 0$, P_{r_0} is Z_2 gauge invariant, perturbing only Majorana fermions. Thus, there is a fast spread of energy density for $h = 0.04$ [Fig. 1(e)]. The intensity at the center of the ladder decays on a very short timescale set by $1/\text{bandwidth}$, and the correlation spreads as a lightcone; see the SM [25]. By contrast, for the moderate field $h = 0.8$, there is no obvious energy spreading beyond the early time dynamics [Fig. 1(f)]. The energy density is localized within a length scale of $\xi \sim 8$ unit cells [25]; and similar slow dynamics under moderate field can be seen in the correlation spreading [25]. This indicates that even a translation-invariant system can have ultraslow transport, thus, exhibiting TL [19–23] in the low energy sectors.

IV. SUPPRESSED ENERGY CONDUCTION

We provide further evidence for the TL by studying the energy current response. Using the definition of a local energy-density operator and the discrete continuity equation $\frac{d}{dt}\mathcal{E}(r, t) + [J_{r,r+1}(t) - J_{r-1,r}(t)] = 0$, where J is the energy

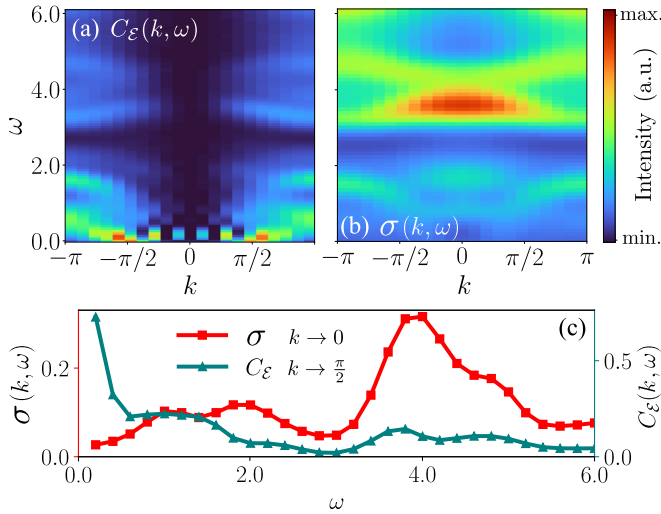


FIG. 2. Coexistence of a gapless energy-fluctuation spectrum and a gapped energy-current spectrum in the TL regime. (a) Dynamical energy correlation function $C_E(k, \omega)$ and (b) energy conductivity $\sigma(k, \omega)$ at $h = 0.8$. Intensities are plotted on a linear scale. (c) Right axis: Cut of $C_E(k, \omega)$ at $k \rightarrow \frac{\pi}{2}$ (teal); Left axis: cut of $\sigma(k, \omega)$ at $k \rightarrow 0$ (red).

current operator—see the SM [25]—we obtain

$$C_J(k, \omega) = \frac{\omega^2 C_E(k, \omega)}{2 - 2 \cos(k)}, \quad \sigma(k, \omega) \sim \frac{C_J(k, \omega)}{\omega}, \quad (3)$$

which relates the two correlation functions $C_E = \langle \mathcal{E}_k(\omega) \mathcal{E}_{-k}(\omega) \rangle$ and $C_J = \langle J_k(\omega) J_{-k}(\omega) \rangle$, reflecting respectively the spectral density of energy and current; and $\sigma(k, \omega)$ the energy conductivity [38]. Consistency is verified by confirming that the independently computed $C_J(k, \omega)$ coincides with $C_E(k, \omega)$ multiplied by $\omega^2/(2 - 2 \cos k)$, under different bond dimensions; see the SM [25]. Both $C_E(k, \omega)$ and $\sigma(k, \omega)$ at a moderate field $h = 0.8$ are shown in Fig. 2. C_E in Fig. 2(a) has a significant spectral weight near zero energy. This again confirms the suppressed gap under intermediate field, consistent with the spin spectral function in Fig. 1(d). However, in the numerically accessible energy scales, $\sigma(k, \omega)$, as well as $C_J(k, \omega)$, are suppressed at all momenta for small frequencies; see Figs. 2(b) and 2(c). Such a suppression in the DC limit of the conductivity in a gapless system is a hallmark of TL [39–41], in contrast to the Lorentzian peak in diffusive systems.

V. TRANSIENT LOCALIZATION BY QUASISTATIC VISONS

Having established the phenomenology numerically, we next discuss the underlying mechanism. Starting from zero field, H_{ladder} can be fermionized without introducing redundancies: $H_{\text{ladder}} = i \sum_{j+l \in \text{even}} (c_{j+1}^l c_j^l + c_{j-1}^l c_j^l + D_{jl} c_j^{l+1} c_j^l)$ with c the Majorana operator, and $D_{jk} = \pm 1$ a static local Z_2 field defined on vertical bonds. The ground-state gauge configuration is presented in the top row of Fig. 3(a), with D_{jk} on vertical bonds defined in consistency with a uniform π -flux configuration; see the SM [25]. The ground state is thus of the form $|0\rangle = |M_D\rangle \otimes |D\rangle$ with $|M_D\rangle$ the fermion sector

conditioned on the static fluxes given by $|D\rangle$. Upon turning on h , the ground state is no longer the exact π -flux ladder, rather, it consists of a superposition of multiple gauge-superselection sectors hybridized with the fermions. The mixed dynamics of visons and fermions is difficult to analyze analytically. However, both perturbation theory and numerical simulations find that the induced vison bandwidth by Zeeman terms is several orders of magnitude smaller than the Majorana bandwidth, i.e., the effective vison mass (m_v) $\gg$ fermion mass (m_f) [26,42,43] (see also the SM [25] for slow spreading of vison correlations). Thus, it allows for a quasistatic approximation of $|D\rangle$. The ground state sector of Eq. (1) at moderate h can be approximated by the following variational Ansatz [36] as a superposition of fluxes and fermions, capturing the interplay between both sectors while preserving translational invariance:

$$|0\rangle \sim \sum_D w_D(h) |M_D\rangle \otimes |D\rangle, \quad \sum_D |w_D(h)|^2 = 1. \quad (4)$$

Here $w_D(h)$ is a coefficient associated with a quasistatic random field $\{D_{jl}\}$ [bottom of Fig. 3(a)]. The disorder configurations $\{D_{jl}\}$ are determined from the flux filling factor ν at intermediate field and the static flux-flux correlation which form an incommensurate density wave in the gapless phase [26]. We have found numerically that the disorder correlations are negligible, allowing us to consider independent configurations $\{D_{jl}\}$ in our Ansatz.

The approximation of Eq. (4) immediately evokes two aspects: (i) it is formally similar to the setup of disorder-free localization, where the effective randomness originates from superposition [29,44], but here no auxiliary system is needed, since fractionalization itself provides the coherent disorder intrinsically. Within this Ansatz, fermions are found to retain a system size independent inverse-participation ratio (IPR) across the entire bandwidth; see Fig. 3(b). For the considered parameter the IPR converges to ~ 0.1 , consistent with the estimated $\xi \sim 8 \sim \text{IPR}^{-1}$ from our quench dynamics simulation; see the SM [25]. For a small but finite m_f/m_v present in our model, visons are slow but not static and the fermions exhibit localization on transient times. The key point here is that the system in the ground state and TL from fractionalization does neither require the preparation of a highly nonequilibrium initial state nor high temperatures [35,45]. We considered the ground state at finite fields, but a similar effect is expected to persist at low temperatures; while at temperatures much higher than the microscopic scales effective fractionalization ceases to exist, and conventional energy transport is expected [42,46].

(ii) A concomitant phenomenon of the TL is its emergent spectrum near zero energy. Each state $|M_D\rangle$ in the sum of Eq. (4) describes fermions in a background of random Z_2 fluxes. This setting produces a virtually gapless energy spectrum of a Majorana metal [47,48], consistent with our MPS results in Figs. 1(d) and 2(a). We emphasize that, unlike a conventional “classical” Majorana metal, where disorder is extrinsic, i.e., either quenched or thermally activated [35,49], the effective randomness in Eq. (1) emerges intrinsically from the fractionalization of its own degrees of freedom at low temperature, where one species affects the other. A consequence of the Ansatz with visons treated as quasistatic, Eq. (4), is

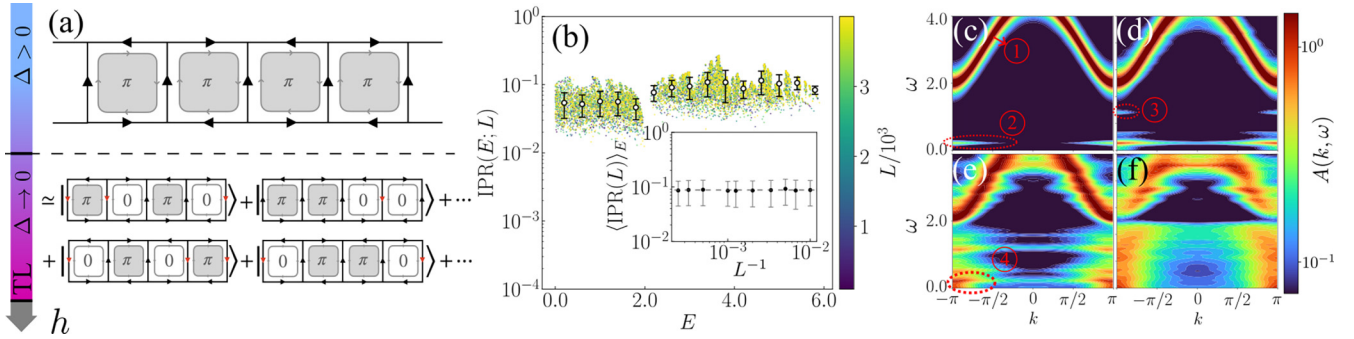


FIG. 3. Illustration and results of the effective theory, Eq. (4). (a) Top: ground-state sector with uniform π flux for $h \rightarrow 0$, black arrows denote the Z_2 gauge; bottom: field-induced superposition of quasistatic fluxes, with red arrows denoting the flipped links with respect to the π -flux ladder at $h \rightarrow 0$. (b) Energy-resolved IPR of the effective tight-binding model conditioned on a flux sector with $\nu = 0.5$ with uncorrelated disorder; the inset shows the size-independent IPR averaged over the whole bandwidth; see also the SM [25]. (c)–(f) Majorana spectral function for increasing probability $p = p(\nu)$ of flipped flux pairs: (c) $p = 0.04$, (d) $p = 0.1$, (e) $p = 0.3$, and (f) $p = 0.5$, resolved in average momentum. The circled labels denote contributions from: ① bulk spinon band, ② in-gap Majorana resonances, ③ higher-order Majorana resonances, and ④ emergent gapless spectral weight.

that within this Ansatz the spin spectrum coincides with the averaged Majorana dynamics over flux sectors; see the next section and Appendix A.

VI. FERMION AND SPIN SPECTRAL FUNCTIONS

Now we show that the Majorana spectral function evaluated with Eq. (4) reproduces key features of the spin structure factor obtained from MPS simulations. Consider randomly flipping flux pairs by changing the sign of D_{jl} on each vertical bond, illustrated by the red arrows in the bottom row of Fig. 3(a), with a probability p , which determines the flux density ν . At $p = 0$, one recovers the gapped Majorana band for a π -flux ladder. The emergence of Majorana modes near zero energy with increasing p is captured by the ensemble-averaged translational symmetry of $\{M_D\}$ in Eq. (4). For $0 < p \ll 1$, local in-gap Majorana resonances on flipped flux pairs have average separations $d \gg \xi$, so that the in-gap states remain sharp and degenerate [Fig. 3(c)]. As p increases, higher-order resonances emerge, which hybridize as $d \sim \xi$, lifting their degeneracy and broadening their spectral weight, ultimately producing Majorana modes near zero energy at sufficiently large p [Figs. 3(d) and 3(e)]. For a maximally random flipping of fluxes (quasistatic visons) given by $p = 0.5$, these in-gap Majorana resonances are merging into a continuum near zero energy; see Fig. 3(f).

The Majorana spectrum $A(k, \omega)$ in Fig. 3(f) with a flux filling $\nu \sim \frac{1}{2}$, as an effective theory to Eq. (1) under moderate h , should be compared to the numerical spin spectrum in Fig. 1(d). A more detailed argument regarding their mutual correspondence is presented in Appendix A. A good consistency is revealed: both the numerical result, Fig. 1(d), and the effective theory, Fig. 3(f), show a low-energy Majorana continuum due to coherent flux disorder; and a higher energy continuum due to the broadened bulk Majorana band, albeit with some deviations near the band edge around $\omega \sim 2$, and zero-frequency spectra extending to a finite area around $\pm\pi$, revealing the metallic nature of Majorana fermions.

VII. EXTENSIONS TO TWO DIMENSIONS

We demonstrated the TL mechanism in a quasi-one-dimensional ladder for a particularly controlled illustration. We will now discuss extensions to two dimensions, Majorana fermions moving in a static Z_2 gauge background exhibit weak antilocalization at zero energy, while the localization length decreases as $\xi(E) \sim E^{-1/2} \exp(\frac{1}{4} \ln^2 E)$ at finite energies [50]. Therefore, in the thermodynamic limit and for $m_f/m_v \rightarrow 0$, the extended states emerge only at $E = 0$, where they carry no energy and therefore do not contribute to energy transport; see also Appendix B. A quantitative difference is that in higher dimension the localization length is larger in general, but other than that the arguments for the one dimensional ladder should prevail. A finite vison time scale will again restore a finite energy conductivity and is expected to arise only in low-energy window. Above this regime, TL dominates and a small energy conductivity is expected as in the case of the Kitaev ladder. Analyzing the conductivity in two dimensions quantitatively is an interesting future direction.

VIII. CONCLUDING REMARKS

Using a Kitaev-ladder model of spins which fractionalize into Majorana fermions (spinons) and fluxes (visons), we showed that substantial low-energy fermion modes and highly suppressed energy transport can coexist in a translation-invariant system without extrinsic disorder. The essential mechanism is a form of coherent randomness that arises intrinsically from the mass-imbalanced fractionalization of the system's own local degrees of freedom: the heavy vison fluctuations around the ground state act as a self-generated, quasistatic disorder that localizes the Majorana fermions. A minimal Ansatz, Eq. (4), which treats the ground state as a superposition of static flux sectors, can account for the localization physics. The Ansatz lies beyond the reach of self-consistent mean fields, as self-averaged order parameters cannot capture the nonlocal interference responsible for localization [51]; see also the SM [25].

For future work it will be interesting to investigate whether transient localization arising from intrinsic, quantum-coherent disorder is a general phenomenon in gapless frustrated quantum magnets, not only in AFM Kitaev systems but also in other settings. For example, via a global rotation, FM extended-Kitaev models are equivalent to AFM extended-Kitaev models, which have been reported to host a field-induced gapless QSL phase smoothly connected to that of the pure AFM Kitaev model [52–54]. Besides Kitaev systems, intrinsic U(1) flux randomness has been discussed in triangular-lattice Dirac spin liquids [55], which enhances the spinon density of states at zero energy in close analogy to the Kitaev systems; hence, the same intrinsic mechanism may arise. For future work it will be pertinent to extend our studies to two-dimensional settings, develop novel methods to reach longer timescales/lower energies, and, in particular, finite temperatures.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [56].

APPENDIX A: SPIN AND MAJORANA SPECTRAL FUNCTIONS

In the main text we compared the Majorana spectral function using the quasistatic approximation, Eq. (4), and the dynamical spin structure factor. In this Appendix, we discuss this correspondence in detail, as well as the derivation of disorder-averaged Majorana spectral function. The dynamic spin correlation can be rewritten in terms of the fermion and flux basis. For simplicity, we present the correlation between Z_i and Z_j . Using Eq. (4), we decompose the dynamical spin correlation into the fermion and gauge sectors [57]:

$$\begin{aligned} \langle 0|Z_i(t)Z_j|0\rangle &= \sum_{D,D'} w_{D'}^* w_D e^{iE_0 t} \langle D' M_{D'} | Z_i e^{-iH t} Z_j | M_D D \rangle \\ &= \sum_{D,D'} w_{D'}^* w_D e^{iE_0 t} \langle D' M_{D'} | c_i e^{-iH_M t} c_j | M_D D_j \rangle \end{aligned}$$

$$\begin{aligned} &\simeq \sum_{D,D'} w_{D'}^* w_D e^{iE_0 t} \delta_{D',D_j} \langle M_{D'} | c_i e^{-iH_M t} c_j | M_D \rangle \\ &= \sum_D w_{D_{ij}}^* w_D e^{iE_0 t} \langle M_{D_{ij}} | c_i e^{-iH_M t} c_j | M_D \rangle, \end{aligned} \quad (\text{A1})$$

where D_j (D_{ij}) denotes the gauge configuration with j th (i - and j th) vertical bond(s) flipped, and in the third equality we assumed that in the quasistatic limit of $m_f \ll m_v$ the time evolution of fluxes is irrelevant on transient timescales. Thus, $\langle D_i | H | D_j \rangle \simeq \delta_{D_i, D_j} H_M^j$, with H_M^j the tight-binding Majorana model conditioned on the gauge sector D_j . This Ansatz is different from the Kitaev model at $h = 0$ where only the nearest neighbors contribute: the extensive superposition of flux configurations introduces long-range spin correlations. The evaluation of Eq. (A1) can be divided into two parts: (i) When i, j are nearest neighbors, the Z_2 link satisfies $D_{ij} = D$, hence the matrix elements in Eq. (A1) become $\langle M_D | c_i e^{-iH_M t} c_j | M_D \rangle$. This is similar to the nearest neighbor contribution in the integrable limit [57]. (ii) When i and j lie farther apart, the matrix elements of $\langle M_{D_{ij}} | c_i e^{-iH_M t} c_j | M_D \rangle$ are challenging to evaluate exactly without an explicit form for $|M_{D_{ij}}\rangle$. However, using the adiabatic approximation as a semiquantitative estimate [57,58], Eq. (A1) becomes the disorder-averaged spectral function:

$$\begin{aligned} \langle Z_i(t)Z_j \rangle &\simeq \sum_D |w_D|^2 e^{iE_0 t} \langle M_D | c_i e^{-iH_M t} c_j | M_D \rangle \\ &= \sum_D |w_D|^2 \langle M_D | c_i(t) c_j | M_D \rangle. \end{aligned} \quad (\text{A2})$$

The final result is therefore the average of the Majorana spectral functions over a random ensemble of gauge configurations with a fixed flux filling factor ν determined by $\langle F_{a(b)} \rangle \sim 0$ under moderate magnetic field. Correlations of X and Y spins can be done by the same token.

The approximate correspondence in Eq. (A2) between the dynamical spin structure factor on the left-hand side and the Majorana spectral function on the right-hand side allows us to directly test the proposed Ansatz by resolving the disordered eigenstates into a set of Bloch states of a clean system [36]. This allows us to compute spectral functions resolved in average quasimomenta due to the translation-invariance of the variational Ansatz. An eigenstate $|n\rangle$ of a disordered Hamiltonian is $|n\rangle = \sum_{r,s} \phi_n(r,s) |r,s\rangle$, $\phi_n(r,s) = \langle r,s | n \rangle$, where s labels the four sublattice sites in a unit cell and r the real-space index. For the translational invariant ladder, we diagonalize the 4×4 Bloch Hamiltonian $h(k)$ at momenta $k_m = 2\pi m/L$ ($m = 0, \dots, L-1$) and obtain four orthonormal eigenvectors $u_{\alpha k}$ ($\alpha = 1, \dots, 4$). The corresponding Bloch state is

$$|s, k\rangle = \frac{1}{\sqrt{L}} \sum_{r=0}^{L-1} \sum_{s=0}^3 u_{s,k} e^{ikr} |r, s\rangle, \quad (\text{A3})$$

with real-space Bloch wavefunction $\psi_{s,k}(r) = \frac{u_{s,k}}{\sqrt{L}} e^{ikr}$. The $4L$ Bloch states form a complete orthonormal set $\sum_{\alpha=1}^4 \sum_{k \in \text{BZ}} |s, k\rangle \langle s, k| = \mathbb{1}_{4L}$. This resolution of identity expands a generic Majorana state under quasistatic flux disorder in the translation-invariant Bloch basis:

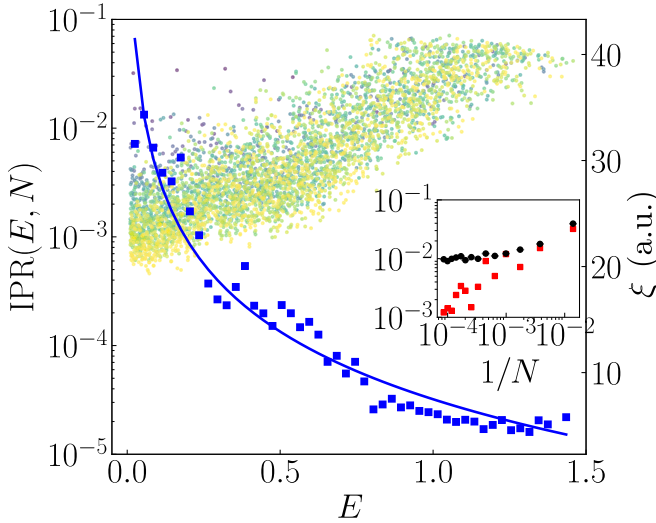


FIG. 4. Energy-resolved IPR (left axis) in a 2D honeycomb model with flux disorder at a fixed flux density for different system sizes. Purple-to-yellow colors of dots denote small ($N = 10^2$) to large lattice ($N = 10^4$). The inset shows the energy-averaged IPR of the states at $E > 0$ (black) and the state at $E = 0$ (red). The IPR at $E = 0$ is consistent with weak antilocalization. The correlation length $\xi(E)$ (right axis, blue squares) is consistent with Eq. (B1).

$|n\rangle = \sum_{s=0}^3 \sum_{k \in \text{BZ}} c_{s,k}^n |s, k\rangle$, where the coefficient $c_{s,k}^n = \langle s, k | n \rangle$ is given by

$$c_{s,k}^n = \sum_{r=0}^{L-1} \sum_{s=0}^3 \psi_{\alpha k}^*(r, s) \phi_n(r, s), \quad (\text{A4})$$

whose average over an ensemble of D contains all the information in the Majorana spectral function $A(k, \omega)$. This can be seen explicitly by averaging each real-space spectral function $A(r_i, r_j, \omega) = \sum_n \text{tr}_s \phi_n^*(r_i, s) \phi_n(r_j, s) \delta(\omega - E_n)$ over the center-of-mass position $R = (r_i + r_j)/2$, i.e.,

$\langle A(r_i, r_j, \omega) \rangle_R = \frac{1}{L} \sum_R A(r, R, \omega)$, which gives

$$\langle A(r_i, r_j, \omega) \rangle_R = \frac{1}{L} \sum_{n,s,k} e^{ikr} |c_{s,k}^n|^2 \delta(\omega - E_n). \quad (\text{A5})$$

Hence, averaging the norm square of Eq. (A4) suffices to give the average-momentum-resolved Majorana spectral function $A(k, \omega)$ as a good approximation for the spin spectral function; Eq. (A2). Using this result we can compare the spectral function obtained from Ansatz, Eq. (4), shown in Figs. 3(c)–3(f), and the dynamical spin structure factor shown in Fig. 1(d).

APPENDIX B: LOCALIZATION IN TWO DIMENSIONS

In the main text, we showed in Fig. 3(b) that the fermion localization length in a flux-disordered ladder remains a finite number at all energies. Here, we evaluate the IPR and energy-dependent localization length $\xi(E)$ in two-dimensional disordered models. In Fig. 4 we show the energy-resolved IPR in a 2D honeycomb model with fixed flux density for different system sizes. Our results suggest that the IPR does not scale with system size for states with $E > 0$. By contrast, the zero energy state is expected to have a diverging correlation length [50]. The energy-grouped IPR is also shown in the inset of Fig. 4, consistent with the $E = 0$ state being delocalized (red dots in the inset), while the averaged IPR for all $E > 0$ no longer scales with system size for large enough N (black dots in the inset).

For this model, the energy dependence of the localization length is

$$\xi(E) \sim \frac{1}{\sqrt{E}} \exp \left[\frac{1}{4} \ln^2 E \right], \quad (\text{B1})$$

which rapidly decreases with energy; see Ref. [50] and the references therein. We also present the numerically estimated $\xi(E)$ in blue markers in Fig. 4, which is consistent with Eq. (B1). Therefore, in the thermodynamic limit and for $m_f/m_v \rightarrow 0$, the only extended state is the particle-hole symmetric state exactly at $E = 0$, and all energy-carrying states at finite energies are localized [50].

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