

Local classical correlations between physical electrons in Hubbard systems

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We demonstrate that the local *nonfreeness*, an unbiased measure of correlation between electrons at a single lattice site, can be computed as the mutual information between local natural spin orbitals. This leads us to prove a general result: local electron correlations in Hubbard-type models that conserve the orbital- and spin-resolved electron number are fully classical since the local reduced density matrix is separable in the natural basis and no quantum correlations beyond entanglement are present. Finally, we compare different theoretical descriptions of magnetic and nonmagnetic states, showing that local classical correlations are drastically influenced by nonlocal processes. These results confirm the relation between local classical correlations within an open system and nonlocal entanglement and they provide a clear path for the study of the relationship between traditional quantum resources and the nonfreeness in terms of experimentally accessible quantities.

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I. INTRODUCTION

The concept of strongly correlated electrons is widely used to identify a broad class of systems that display a variety of phases and regimes arising from the electron-electron interactions. Given their rich and tunable phenomenology, strongly correlated electron systems have been identified as promising platforms for quantum technologies, which have assumed a central role in the second quantum revolution. However, a quantitative measure of strong particle correlations that can differentiate the quantum and classical contributions is still elusive, and different estimates can be used in different frameworks. Some of these estimates are not uniquely defined as one can use different reference states or orbital bases. For instance, in variational methods like density-functional theory or quantum Monte Carlo, the *correlation energy* (energy difference with respect to a Slater determinant) is a natural candidate [1–4], whereas configuration interaction schemes naturally ascribe correlations to the number of necessary Slater determinants [5–7]. In diagrammatic methods one can define the functional distance between the self-energy and/or two-particle vertex and their uncorrelated mean-field (Hartree-Fock) counterparts [8–12]. Finally, tensor network methods rely on the notion of bond dimension [13,14]. The latter is deeply rooted in the framework of quantum information theory in Fock space, which we will refer to as *orbital information theory* in the following, as it is concerned with the correlations and entanglement between (groups of)

single-particle states (orbitals), rather than the correlations between the constituent physical particles [15–20].

Building on the ideas recently introduced in different communities [21,22], here we propose an unbiased measure of the correlations contained in a single electronic orbital (or intraorbital correlations) as it is the mutual information between its two *natural* spin orbitals that define the single bipartition of the single-orbital density matrix into eigenstates of the single-electron density matrix (up and down, when the spin is quantized along the z direction). The recipe is readily generalized including N local orbitals by considering a $2N$ partition into the $2N$ natural spin orbitals. Here we focus on the $N = 1$ case for its simplicity and relevance for the study of many systems in which the one-electron density matrix does not mix the atomic orbitals, including orbital-selective phases and interaction-resilient metals [23–31].

We first prove that, under completely general circumstances, for a state that conserves the total (orbital-resolved [32]) density and magnetization, all local intraorbital correlations are *effectively classical*, as neither entanglement [33–39] nor quantum discord [40–48] between natural spin orbitals is contained in the local reduced density matrix (RDM). Then we prove that our measure of intraorbital classical correlations vanishes for any Hartree-Fock state and it coincides with the single-orbital *nonfreeness*, a well-known quantitative measure of correlations between physical electrons [49–52]. This leads us to a rigorous and unbiased measure of local classical correlations which is based on simple quantities that are experimentally accessible in quantum simulators.

We follow this direction studying the half-filled Hubbard model on the square and honeycomb lattices. Comparing local embedding methods with numerically exact approaches, we show that the local classical nonfreeness captures the physics of strong correlations in an unbiased way and, remarkably, it is clearly sensitive to nonlocal quantum fluctuations and correlations, as expected from the quantum information theory of open systems [45,53].

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II. LOCAL SPIN-ORBITAL CORRELATIONS

Let us consider a lattice model of itinerant fermions in which the orbital-resolved density and magnetization are conserved, as realized, e.g., in the single-band Hubbard model [54], its $SU(N)$ versions [55], and the density-density version of the Kanamori Hamiltonian [56], where the rotational symmetry is partially broken. In light of the abelian nature of such symmetries, we can write the RDM describing a single lattice site ρ_{loc} so as to commute with the local, *orbital-resolved*, density and magnetization. The explicit derivation can be found in the Appendix. Considering an arbitrary number of orbitals, labeled as $i = 1, 2, \dots, N$, the orbital-resolved conservation of spin densities $\langle n_{i,\sigma} \rangle$ ensures a diagonal form also for the full N -orbital local density matrix ρ_{loc} , involving expectation values of $2N$ -particle operators, of the form $\langle n_{1,\sigma} n_{2,\sigma'} \dots n_{N,\sigma''} \rangle$. It follows that the local RDM can be written as a mixture of products of pure states

$$\rho_{\text{loc}} = \sum_p p(|\psi_{1,\uparrow}^p\rangle\langle\psi_{1,\uparrow}^p| \otimes |\psi_{1,\downarrow}^p\rangle\langle\psi_{1,\downarrow}^p| \otimes \dots \otimes |\psi_{N,\uparrow}^p\rangle\langle\psi_{N,\uparrow}^p| \otimes |\psi_{N,\downarrow}^p\rangle\langle\psi_{N,\downarrow}^p|), \quad (1)$$

which are separable with respect to the multipartition in the $2N$ spin orbitals [34–37,57] and do not contain any global multipartite quantum discord [40–48]. Hence all local spin-orbital correlations are determined by the classical probability $\{p\}$. Remarkably, they can be accurately measured by the multipartite mutual information between the $2N$ separable subsystems [22].

To facilitate the contact with experiments, we now specialize the result to a single orbital i , addressing its bipartite spin-orbital correlations in terms of easy to compute and measurable quantities. For $SU(N)$ systems, where all flavors are inherently equivalent, this would give a fairly complete picture of local classical correlations, while for more complex systems with nonequivalent orbitals (as, e.g., orbital-selective phases) one would evaluate the correlation of physically distinct orbitals separately. As a direct consequence of Eq. (1), the RDM describing a single local orbital i takes the following diagonal form [58–62]:

$$\rho_i = \sum_{\ell=\emptyset,\uparrow,\downarrow,\uparrow\downarrow} p_\ell |\ell\rangle\langle\ell|, \quad (2)$$

where the p_ℓ coefficients define a classical probability distribution for the four local states $|\emptyset\rangle = |0\rangle \otimes |0\rangle$, $|\uparrow\rangle = |1\rangle \otimes |0\rangle$, $|\downarrow\rangle = |0\rangle \otimes |1\rangle$, and $|\uparrow\downarrow\rangle = |1\rangle \otimes |1\rangle$, expressed in the natural occupation basis $|n_\uparrow\rangle \otimes |n_\downarrow\rangle$ [63]. We can explicitly express ρ_i in terms of the local doping fraction, magnetization and double occupancy, respectively, $\delta_i = 1 - \langle n_{i,\uparrow} \rangle - \langle n_{i,\downarrow} \rangle$, $m_i = \langle n_{i,\uparrow} \rangle - \langle n_{i,\downarrow} \rangle$, and $\mathcal{D}_i = \langle n_{i,\uparrow} n_{i,\downarrow} \rangle$, evaluated on the i th orbital

$$\rho_i = \begin{pmatrix} \delta_i + \mathcal{D}_i & 0 & 0 & 0 \\ 0 & \frac{1 - \delta_i + m_i}{2} - \mathcal{D}_i & 0 & 0 \\ 0 & 0 & \frac{1 - \delta_i - m_i}{2} - \mathcal{D}_i & 0 \\ 0 & 0 & 0 & \mathcal{D}_i \end{pmatrix}. \quad (3)$$

Since Eq. (2) describes a classical mixture of product states, hence a separable state with respect to the bipartition into the two spin orbitals, these latter are not entangled for any value of the parameters [34–37]. Furthermore, the diagonal structure of Eq. (2) in the natural basis guarantees that all bipartite correlations between the two spin orbitals are only due to the probability distribution $\{p_\ell\}$. Hence, they should be considered classical as there is no (bipartite) quantum discord [40–46].

In direct analogy to the multipartite case, local classical correlations contained in a single orbital are quantified by the *intraorbital mutual information* between the selected local spin orbitals [21] (i.e., between the $\uparrow$ and $\downarrow$ single-electron states on the same orbital)

$$I(\uparrow:\downarrow) = s(\rho_{i,\uparrow}) + s(\rho_{i,\downarrow}) - s(\rho_i), \quad (4)$$

where $s(\rho) = -\text{tr} \rho \log(\rho)$ is the von Neumann entropy of a state ρ , and $\rho_{i,\uparrow}$ and $\rho_{i,\downarrow}$ are the two local spin-orbital density matrices, defined by the partial traces

$$\begin{aligned} \rho_{i,\uparrow} = \text{tr}_\downarrow \rho_i &= \begin{pmatrix} \frac{1 + \delta_i - m_i}{2} & 0 \\ 0 & \frac{1 - \delta_i + m_i}{2} \end{pmatrix}, \\ \rho_{i,\downarrow} = \text{tr}_\uparrow \rho_i &= \begin{pmatrix} \frac{1 + \delta_i + m_i}{2} & 0 \\ 0 & \frac{1 - \delta_i - m_i}{2} \end{pmatrix}. \end{aligned} \quad (5)$$

The mutual information $I(\uparrow:\downarrow)$ gives, in fact, an upper bound for all possible correlation functions between the two spin orbitals [64], namely,

$$I(\uparrow:\downarrow) \geq \frac{(\langle \mathcal{O}_\uparrow \otimes \mathcal{O}_\downarrow \rangle_{\rho_i} - \langle \mathcal{O}_\uparrow \rangle_{\rho_{i,\uparrow}} \langle \mathcal{O}_\downarrow \rangle_{\rho_{i,\downarrow}})^2}{2\|\mathcal{O}_\uparrow\|^2\|\mathcal{O}_\downarrow\|^2}, \quad (6)$$

for any pair $\mathcal{O}_\uparrow, \mathcal{O}_\downarrow$ of generic operators acting on the Hilbert spaces of subsystems associated to the two spin and with $\|\mathcal{O}_\circ\|$ denoting the Euclidean operator norm of $\mathcal{O}_\circ$ (its largest singular value).

So far, we established that two (or more) local spin orbitals are never entangled in Hubbard-like models that conserve the orbital-resolved density and magnetization and we provided a general and accessible recipe to quantify their classical correlations in terms of the intraorbital mutual information. We end the section by showing that in the Hartree-Fock mean-field theory also these intraorbital correlations vanish. This is easily proven from the fact that the double occupancy is factorized in the product of single-particle expectation values, namely, $\mathcal{D}_i^{\text{HF}} = \langle n_{i,\uparrow} \rangle \langle n_{i,\downarrow} \rangle = \frac{1}{4}(1 - \delta_i + m_i)(1 - \delta_i - m_i)$, which plugged into Eq. (3) gives $\rho_i^{\text{HF}} = \rho_{i,\uparrow} \otimes \rho_{i,\downarrow}$ corresponding to a vanishing intraorbital mutual information. Indeed, from a quantum information geometry point of view [20,21,33], the mutual information is defined as the distance between the quantum state ρ_{AB} and the closest product state $\rho_{\text{A}} \otimes \rho_{\text{B}}$ for any partition into the A and B subsystems. This implies that the Hartree-Fock approximation to ρ_i corresponds to the optimal product state with respect to the bipartition in local natural orbitals. In this sense Eq. (4) provides a notion of statistical distance between a given ρ_i and the Hartree-Fock solution.

III. EQUIVALENCE TO LOCAL NONFREENESS

A fundamental quantity in the framework of electronic structure theory is the nonfreeness [49–52], an unbiased quantitative measure of correlations between electrons, defined as the quantum relative entropy [20,21,33] between a given state ρ and the set of free states ρ^{free} , i.e., the Gibbs ensemble associated to the closest Hartree-Fock theory [65]

$$\mathfrak{N}(\rho) = \min_{\{\rho^{\text{free}}\}} S(\rho || \rho^{\text{free}}). \quad (7)$$

Remarkably, the minimization in Eq. (7) can be performed analytically [49–51], characterizing the unique closest free state as to have the same one-electron density matrix $\gamma_\rho = \langle c_{i,\sigma}^\dagger c_{j,\sigma'} \rangle_\rho$ as the many-electron state ρ . The resulting closed expression is given as [22]

$$\mathfrak{N}(\rho) = s(\gamma_\rho) + s(\mathbb{1} - \gamma_\rho) - s(\rho). \quad (8)$$

Despite its success in quantifying in a reasonable and well-defined way the correlations between electrons [51,52], the nonfreeness cannot be easily connected to the quantum information theory of many-body systems. The latter, in fact, assumes the subsystems of interest to be distinguishable whereas it is well known that individual quantum particles are, in general, indistinguishable, especially in condensed matter settings. For this reason, it has been long believed that the interparticle correlations measured by the nonfreeness could not be split in quantum and classical parts as done, for instance, in the framework of orbital information theory [20]. However, a rather general argument was recently proposed stating that the minimization procedure in Eq. (7) is equivalent to minimize the mutual information between spin orbitals, with respect to all possible orbital bases. The resulting optimal single-electron basis corresponds to the *natural* spin orbitals of the given system, i.e., the eigenvectors of γ_ρ [22]. While the general proof is rather involved, as it assumes that the given ρ satisfies the fermionic parity superselection rule [66–73] and exploits results from majorization theory [74,75], we can explicitly show the equivalence of $I(\uparrow : \downarrow)$ [Eq. (4)] and $\mathfrak{N}(\rho_i)$ [Eq. (8)], leveraging the symmetry properties of the single-orbital density matrix ρ_i in Eq. (2). Given that $s(\rho_i)$ enters both expressions additively, we are left to prove that $s(\rho_{i,\uparrow}) + s(\rho_{i,\downarrow}) = s(\gamma_{\rho_i}) + s(\mathbb{1} - \gamma_{\rho_i})$. Given

$$\gamma_{\rho_i} = \begin{pmatrix} \langle n_{i,\uparrow} \rangle & 0 \\ 0 & \langle n_{i,\downarrow} \rangle \end{pmatrix} = \begin{pmatrix} \frac{1-\delta_i+m_i}{2} & 0 \\ 0 & \frac{1-\delta_i-m_i}{2} \end{pmatrix}, \quad (9)$$

$$\mathbb{1} - \gamma_{\rho_i} = \begin{pmatrix} \frac{1+\delta_i-m_i}{2} & 0 \\ 0 & \frac{1+\delta_i+m_i}{2} \end{pmatrix},$$

a direct comparison of all diagonal entries in Eqs. (5) and (9) completes the proof. Given that ρ_i contains only classical correlations between the natural orbitals underlying the definition of $I(\uparrow : \downarrow)$, we conclude that, whenever the orbital-resolved spin density is conserved, the local nonfreeness measures only classical correlations. In the more general case, one can exploit the bipartition into natural spin orbitals to minimize the quantum relative entropy with respect to separable states or pseudoclassical/pointer states [40–46], instead of product/free states, and so find measures of entanglement and quantum discord between physical electrons in the state ρ_i .

In other words, the power of the $I(\uparrow : \downarrow) \equiv \mathfrak{N}(\rho_i)$ correspondence lies in the definition of an optimal bipartition into single-electron states and so the access to particle correlations within the well-developed framework of orbital quantum information theory [15–18,20].

In the more general case of a local RDM ρ_{loc} consisting of N charge and magnetization conserving orbitals, one can easily generalize the proof, without invoking any superselection rule: the symmetry properties of ρ_{loc} ensure that the single spin-orbital density matrices $\rho_{i,\sigma}$ are all diagonal, generalizing Eq. (5), and their eigenvalues correspond to the corresponding natural occupations $\langle n_{i,\sigma} \rangle$ and $1 - \langle n_{i,\sigma} \rangle$. Hence, the multipartite mutual information between all spin orbitals [22] matches the full local nonfreeness

$$\mathfrak{N}(\rho_{\text{loc}}) \equiv \sum_{i=1}^N [s(\rho_{i,\uparrow}) + s(\rho_{i,\downarrow})] - s(\rho_{\text{loc}}), \quad (10)$$

proving the classical nature of local interparticle correlations also at the multipartite level.

IV. MOTT-HUBBARD TRANSITION

Having defined a rigorous and general measure of local classical correlations between electrons, Eq. (4), in the following we showcase its power to provide further insight and understanding about strongly correlated fermions. We consider the most characteristic fingerprint of electron-electron correlations: the interaction-driven transition between a metal and a Mott insulator in the single-band half-filled Hubbard model [76–81].

The Hubbard model on a square lattice has an antiferromagnetic (AFM) ground state for every value of U . However, we start from the paramagnetic (PM) solution in which magnetism is inhibited to single out the intrinsic effects of the local repulsion. We address this problem within a local quantum embedding, which describes the effect of correlations in term of a local self-energy: the ghost rotational invariant slave bosons (gRISB) method (equivalent to the ghost-Gutzwiller) [82,83]. While the RISB [84–87] method only captures the low-energy itinerant properties of correlated electrons, gRISB includes a description of high-energy excitations by introducing extra auxiliary degrees of freedom, and produces results of accuracy comparable to dynamical mean-field theory (DMFT) [81] at a much smaller computational cost [88–95]. In the present context, we note that gRISB faithfully represents the local RDM [87,96], and hence it allows us to compute straightforwardly the intraorbital mutual information in Eq. (4).

In the upper panel of Fig. 1 we show the quasiparticle weight Z as a function of the ratio U/t for the square lattice at zero temperature. Z is maximum for $U = 0$ and it decreases with increasing U up to a critical value U_{c2} , where it vanishes continuously, signaling a metal-insulator transition. Starting from large U and reducing the interaction strength, we recover a metal for $U \leq U_{c1}$, where Z abruptly jumps to a finite value. The whole scenario is in good agreement with the well established picture from DMFT [81].

We now focus on the new insight brought by $I(\uparrow : \downarrow)$. This measure of correlations vanishes in the noninteracting limit

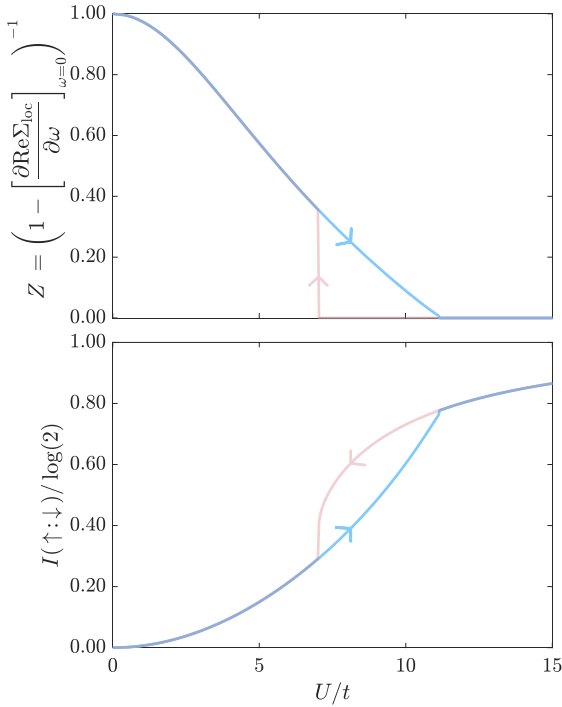


FIG. 1. Local quasiparticle weight (top) and local nonfreeness (bottom) across the paramagnetic Mott-Hubbard transition on the square lattice, at zero temperature, in the gRISB approximation. The arrows depict the increasing- U and decreasing- U solutions.

and grows with U , approaching in the strong-coupling regime ($U \gg t$) the value of $\log(2) = 1$ bit, i.e., the maximum possible value for classical correlations between two qubits. $I(\uparrow:\downarrow) = \log(2)$ is reached only in the atomic limit $t = 0$, where the double occupancy is exactly zero. Similarly to Z , also the local classical correlations display a hysteresis in the region $U_{c1} \leq U \leq U_{c2}$ and both solutions are singular at their respective critical interactions. We stress that for $U > U_{c2}$ $I(\uparrow:\downarrow)$ is close to the maximum value of 1 bit, revealing the strongly correlated nature of the paramagnetic Mott-Hubbard insulator. Recent work [21,72] proposed a *quasilocal* (short-range nonlocal) quantum origin to these maximal local classical correlations, in the form of superselected entanglement between nearest-neighboring sites.

V. ANTIFERROMAGNETIC STATE

We proceed by studying the AFM ground state on the square lattice. In Fig. 2 we present the local magnetization m_i (top), the local double occupancy $\mathcal{D}_i$ (center), and our measure of local classical correlations $I(\uparrow:\downarrow)$ (bottom), comparing results from RISB, gRISB, and numerically exact results for the spin- $\frac{1}{2}$ isotropic Heisenberg model ($\text{XXX}_{\frac{1}{2}}$) [97,98], corresponding to the large- U limit of the half-filled Hubbard model. The value of $I(\uparrow:\downarrow)$ in the $\text{XXX}_{\frac{1}{2}}$ model is computed by inserting the values for m_i and $\mathcal{D}_i$ into Eqs. (3) to (5).

The first observation is that while both the RISB and gRISB calculations share a dome-like structure in $I(\uparrow:\downarrow)$, with the noninteracting and the strong-coupling limits giving

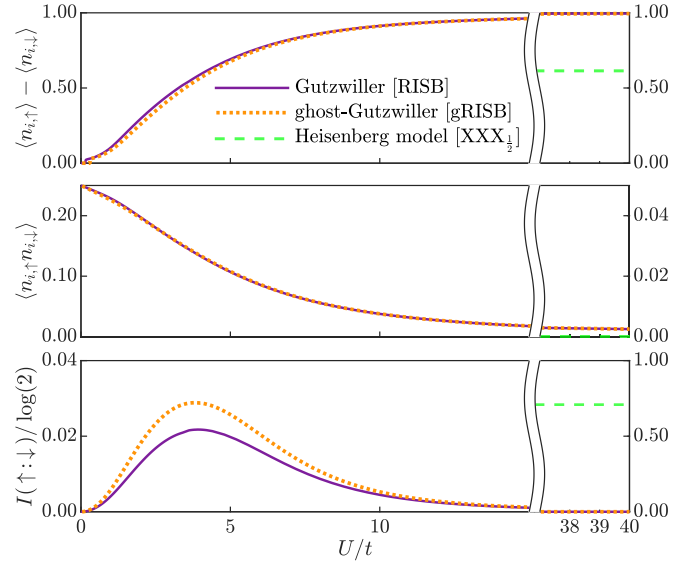


FIG. 2. Local magnetization (top), local double occupancy (center) and local nonfreeness (bottom) in the ground state of the Hubbard model on the square lattice, comparing the RISB approximation and the gRISB approximation ($N_{\text{ghost}} = 2$). The dashed green line in the strong-coupling regime marks the exact solution of the isotropic Heisenberg model [97].

a local free state, the Heisenberg model calculations provide a large value of $I(\uparrow:\downarrow)$ at large- U , not far from that of the paramagnetic Mott-Hubbard insulator (see Fig. 1). We can understand this difference in terms of the behavior of m_i and $\mathcal{D}_i$, the only ingredients to compute the local nonfreeness in our formulation. The double occupancy $\mathcal{D}_i$ is correctly found as small but finite in gRISB, as opposed to the zero value of the Heisenberg model. On the other hand, m_i is substantially reduced in the Heisenberg model with respect to RISB and gRISB, as a result of nonlocal effects [97–103], absent in (g)RISB. This is remarkably reflected also in $I(\uparrow:\downarrow)$, a quantity based on measurements defined on the local (natural) spin orbitals. Thus, local correlations are clearly influenced by nonlocal quantum processes, as expected for any open subsystem embedded in a nondegenerate quantum ground state, as its local classical correlations must be understood as originating from (nonlocal) entanglement with its environment [45,53].

We complete our analysis considering the honeycomb lattice, where the noninteracting system is a semimetal with Dirac cones. Hence, a finite critical value of U is necessary to obtain an AFM state [108–113]. Also in this case, by inhibiting the Néel order, a paramagnetic transition between a Dirac semimetal and a Mott-Hubbard insulator can be realized [114,115]. In Fig. 3 we present results for both PM and AFM transitions, comparing solutions from the gRISB and DMFT approaches, the spin- $\frac{1}{2}$ isotropic Heisenberg model ($\text{XXX}_{\frac{1}{2}}$) on the honeycomb lattice (referring to numerically exact results reported in [98]), and, finally, sign-free auxiliary-field quantum Monte Carlo (AFQMC) data for the low-temperature regime of the Hubbard model on the honeycomb lattice, taken from [108]. We find that gRISB and DMFT results are substantially equivalent for both transitions, except for a slight discrepancy in the large- U degree of correlation, inside

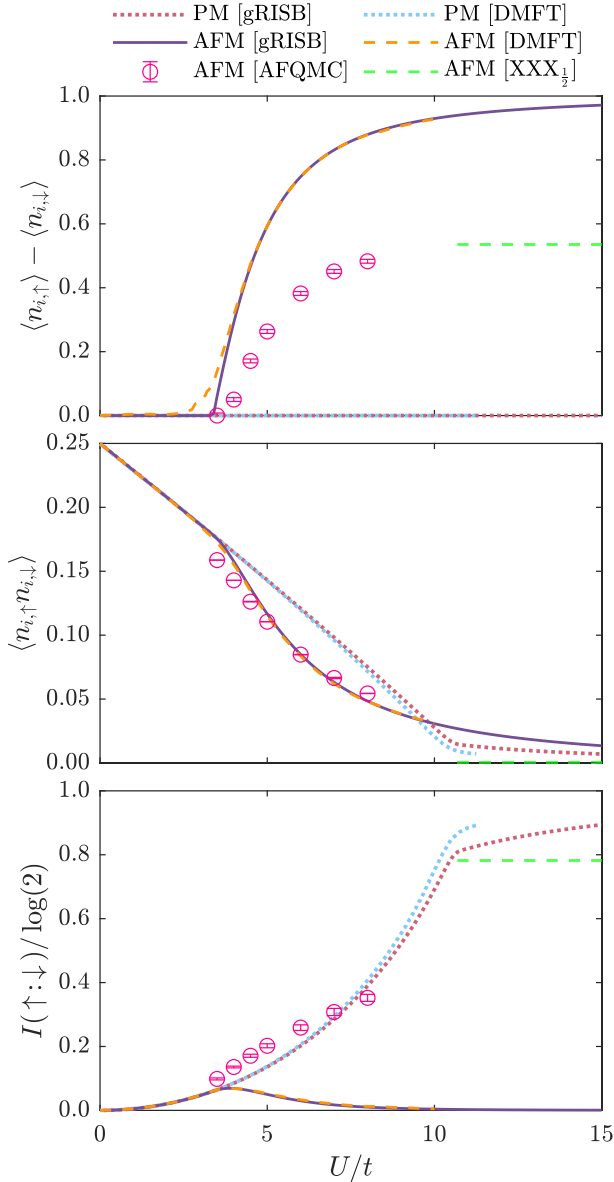


FIG. 3. Local magnetization (top), local double occupancy (center), and local nonfreeness (bottom) on the honeycomb lattice, comparing the gRISB approximation ($N_{\text{ghost}} = 2$), the DMFT and AFQMC data. DMFT data are generated with the EDIPack library [104–107]. The AFQMC data are taken, with permission, from [108]. The error bars for the local mutual information are computed with linear propagation of the standard deviation. The dashed green line in the strong-coupling regime marks the numerically exact solution of the isotropic Heisenberg model on the honeycomb lattice, as reported in [98].

the PM Mott insulator [116]. On the other hand, while the AFQMC double occupancy in the large- U regime is close to that of local embedding methods, the staggered magnetization is remarkably different and compatible with the Heisenberg prediction. We can thus expect that $I(\uparrow:\downarrow)$ would also asymptotically approach the very high degree of correlation of the Heisenberg limit, which for the honeycomb lattice is of the order of magnitude of the paramagnetic Mott insulator (see the bottom panel in Fig. 3).

Our results show that quantum embedding methods based on a single-site do not fully capture the substantial local classical correlations of the AFM ground state both on the square and on the honeycomb lattice. The AFM local RDM turns out to be uncorrelated like in a Hartree-Fock (Slater) magnetic state. This may appear in contrast with nontrivial properties of the Mott antiferromagnet in DMFT [108,117,118], which indeed recovers the correct strong-coupling behavior of the ordering temperature, in contrast with a simple Hartree-Fock treatment. This behavior is, however encoded in the nontrivial properties of the excited states, in particular in the paramagnetic states found at high energy or temperature in DMFT, while the ground state is essentially trivial and uncorrelated, at large U .

VI. SUMMARY AND OUTLOOK

We provided an alternative way to evaluate the local nonfreeness in the framework of orbital quantum information theory. Remarkably, this allows to classify the electron-electron correlations in classical and quantum contributions.

For the Hubbard model (and closely related models where the orbital-resolved density and magnetization are conserved) this leads us to prove a remarkable and completely general result, namely that the local reduced density matrix describes only classical correlations between electrons, as it cannot contain electron-electron entanglement or any kind of quantum correlation beyond entanglement. The resulting measure of local classical correlations is shown to be uniquely defined and to vanish for any single-particle Hartree-Fock solution.

We apply this result to highlight some fundamental differences between local embedding descriptions of the paramagnetic and antiferromagnetic phases of the Hubbard model. The paramagnetic Mott phase is characterized by a large degree of correlation which approaches the asymptotic maximal value in the large- U limit. This effect is indeed well captured by quantum embedding methods that include only a local self-energy, such as DMFT and gRISB. On the other hand, the same embedding methods find a weakly correlated local density matrix in the AFM state, in sharp contrast with the large correlations predicted by numerically exact theories, in the large- U regime. This implies that nonlocal self-energy effects (neglected in the local embedding) are crucial to increase the local classical correlations between electrons. Treating a single site as an open system, our results provide a concrete and nontrivial example of how local classical correlations can originate from the nonlocal correlations with its quantum environment (i.e., the rest of the lattice) [45,53].

We highlight that our measure of local classical correlations depends only on fundamental and simple local quantities. As such, it is readily available in many theoretical methods including nonlocal self-energies [108,119–122] that would allow to shed light on the precise nonlocal origin of the classical correlations. The intraorbital mutual information is also experimentally accessible in analog quantum simulators, based on ultracold atoms [31,55,123], as well as digital quantum circuits [124–126]. The latter represent one of the main proposals for the exploitation of strongly correlated electron systems on the path to achieve a practical quantum advantage with current hardware [127,128]. In this respect, the recent

interest in alternative quantum many-body resources [129] has unveiled the *nonstabilizerness* [130,131] and the *non-gaussianity* (a resource-theory homologue of the nonfreeness [132–134]) as crucial ingredients for generating high quantum complexity in fermionic systems [135–137]. Therefore, the $I(\uparrow:\downarrow) \equiv \mathfrak{N}(\rho_i)$ correspondence not only provides an efficient scheme to estimate the degree of quantum advantage inherent to strong correlations, but also paves the way towards a better understanding of the intricate relationship between *a priori* distinct many-body resources.

Finally, we highlight that realistic descriptions of strongly correlated *materials* often call for the inclusion of multiorbital interaction terms that go beyond orbital-resolved conservation laws [56,138,139]. Our findings suggest that, besides the improved accuracy, these additional terms may introduce local quantum correlations and entanglement, modifying the physical picture of the underlying quantum states *at a qualitative level*. A future, quantitative, assessment of these aspects is prospected to be very relevant for the multidisciplinary efforts towards both an improved fundamental understanding of strong correlations in solids and the development of materials-based quantum technologies.

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DATA AVAILABILITY

Some of the data that support the findings of this article are openly available [140]. RAW data for the AFQMC solutions reported in Fig. 3 are not publicly available. These data are available from the authors of [108] upon reasonable request.

APPENDIX: LOCAL SYMMETRIES FROM GLOBAL CONSERVATION OF ADDITIVE QUANTITIES

Here we prove the fundamental and general result that the global conservation of density and magnetization reflect on the form of the local density matrix, thereby implying the classical character of the correlations. Let us consider an *additive* globally conserved quantity $\mathcal{O} = \sum_i \langle \mathcal{O}_i \rangle$, where i spans a set of orbitals in the lattice. For instance, we can always write the total density and the total magnetization in the Hubbard model as, respectively, $n = \sum_i \langle n_i \rangle$ and $m = \sum_i \langle m_i \rangle$, where $\langle m_i \rangle = \langle n_{i,\uparrow} \rangle - \langle n_{i,\downarrow} \rangle$. Equivalently, we can consider the total spin-dependent densities $n_\sigma = \sum_i \langle n_{i,\sigma} \rangle$.

Similarly, we can write the total flavor-resolved density of a $SU(N)$ Hubbard model [55] as a sum over all lattice sites of the local flavor density $n_v = \sum_i \langle n_{i,v} \rangle$, or the total spin- and orbital-resolved density of a density-density Kanamori Hamiltonian [56] as $n = \sum_i \langle n_{i,v,\sigma} \rangle$.

All these quantities are described by *local* operators $\mathcal{O}_i$, in the form

$$\langle \mathcal{O} \rangle = \langle \mathcal{O}_1 \otimes \mathbb{1}_2 \otimes \cdots \rangle + \langle \mathbb{1}_1 \otimes \mathcal{O}_2 \otimes \mathbb{1}_3 \otimes \cdots \rangle + \cdots \quad (\text{A1})$$

If $\langle \mathcal{O} \rangle$ is conserved on the lattice, the associated unitary group $U_{\mathcal{O}} = \exp(i\alpha\mathcal{O})$ is factorized on local subspaces

$$U_{\mathcal{O}} = U_{\mathcal{O}_1} \otimes U_{\mathcal{O}_2} \otimes \cdots = \exp(i\alpha\mathcal{O}_1) \otimes \exp(i\alpha\mathcal{O}_2) \cdots \quad (\text{A2})$$

It directly follows that the conservation of $\langle \mathcal{O} \rangle$, on the global state ρ , implies the conservation of $\langle \mathcal{O}_i \rangle$ on the local single-orbital reduced density matrix ρ_i , namely, $U_{\mathcal{O}} \rho U_{\mathcal{O}}^\dagger = \rho \implies U_{\mathcal{O}_i} \rho_i U_{\mathcal{O}_i}^\dagger = \rho_i$. Hence, ρ_i (or $\rho_{i,v}$ or $\rho_{i,v,\sigma}$) must be diagonal over the local density and spin/flavors, proving Eqs. (1) and (2) of the main text.

- [1] E. Wigner, On the interaction of electrons in metals, *Phys. Rev.* **46**, 1002 (1934).
- [2] R. G. Parr and W. Yang, Density-functional theory of the electronic structure of molecules, *Annu. Rev. Phys. Chem.* **46**, 701 (1995).
- [3] M. Nekovee, W. M. C. Foulkes, and R. J. Needs, Quantum Monte Carlo analysis of exchange and correlation in the strongly inhomogeneous electron gas, *Phys. Rev. Lett.* **87**, 036401 (2001).
- [4] J. Quintanilla and C. Hooley, The strong-correlations puzzle, *Phys. World* **22**, 32 (2009).
- [5] C. Mejuto-Zaera, N. M. Tubman, and K. B. Whaley, Dynamical mean field theory simulations with the adaptive sampling configuration interaction method, *Phys. Rev. B* **100**, 125165 (2019).
- [6] D. Werner, J. Lotze, and E. Arrigoni, Configuration interaction based nonequilibrium steady state impurity solver, *Phys. Rev. B* **107**, 075119 (2023).
- [7] B. Ganoe and J. Shee, On the notion of strong correlation in electronic structure theory, *Faraday Discuss.* **254**, 53 (2024).
- [8] A. Amaricci, J. C. Budich, M. Capone, B. Trauzettel, and G. Sangiovanni, First-order character and observable signatures of topological quantum phase transitions, *Phys. Rev. Lett.* **114**, 185701 (2015).
- [9] E. A. Nowadnick, J. P. Ruf, H. Park, P. D. C. King, D. G. Schlom, K. M. Shen, and A. J. Millis, Quantifying electronic correlation strength in a complex oxide: A combined DMFT and arpes study of LaNiO_3 , *Phys. Rev. B* **92**, 245109 (2015).
- [10] X. Dong, E. Gull, and A. J. Millis, Quantifying the role of antiferromagnetic fluctuations in the superconductivity of the doped Hubbard model, *Nat. Phys.* **18**, 1293 (2022).
- [11] P. Chalupa, P. Gunacker, T. Schäfer, K. Held, and A. Toschi, Divergences of the irreducible vertex functions in correlated metallic systems: Insights from the Anderson impurity model, *Phys. Rev. B* **97**, 245136 (2018).

- [12] Y. Yu, S. Isakov, E. Gull, K. Held, and F. Krien, Unambiguous fluctuation decomposition of the self-energy: Pseudogap physics beyond spin fluctuations, *Phys. Rev. Lett.* **132**, 216501 (2024).
- [13] J. I. Cirac, D. Pérez-García, N. Schuch, and F. Verstraete, Matrix product states and projected entangled pair states: Concepts, symmetries, theorems, *Rev. Mod. Phys.* **93**, 045003 (2021).
- [14] C. Carisch and O. Zilberberg, Efficient separation of quantum from classical correlations for mixed states with a fixed charge, *Quantum* **7**, 954 (2023).
- [15] L. Amico, R. Fazio, A. Osterloh, and V. Vedral, Entanglement in many-body systems, *Rev. Mod. Phys.* **80**, 517 (2008).
- [16] N. Laflorencie, Quantum entanglement in condensed matter systems, *Phys. Rep.* **646**, 1 (2016).
- [17] L. Henderson and V. Vedral, Classical, quantum and total correlations, *J. Phys. A: Math. Gen.* **34**, 6899 (2001).
- [18] M.-C. Bañuls, J. I. Cirac, and M. M. Wolf, Entanglement in fermionic systems, *Phys. Rev. A* **76**, 022311 (2007).
- [19] F. Benatti, R. Floreanini, F. Franchini, and U. Marzolino, Entanglement in indistinguishable particle systems, *Phys. Rep.* **878**, 1 (2020).
- [20] L. Ding, S. Knecht, Z. Zimborás, and C. Schilling, Quantum correlations in molecules: From quantum resourcing to chemical bonding, *Quantum Sci. Technol.* **8**, 015015 (2023).
- [21] G. Bellomia, Quantum information insights into strongly correlated electrons, Ph.D. thesis, SISSA, 2024.
- [22] D. Aliverti-Piuri, K. Chatterjee, L. Ding, K. Liao, J. Liebert, and C. Schilling, What can quantum information theory offer to quantum chemistry? *Faraday Discuss.* **254**, 76 (2024).
- [23] L. de' Medici, A. Georges, and S. Biermann, Orbital-selective mott transition in multiband systems: Slave-spin representation and dynamical mean-field theory, *Phys. Rev. B* **72**, 205124 (2005).
- [24] L. de' Medici, Hund's coupling and its key role in tuning multiorbital correlations, *Phys. Rev. B* **83**, 205112 (2011).
- [25] L. de' Medici, J. Mravlje, and A. Georges, Janus-faced influence of Hund's rule coupling in strongly correlated materials, *Phys. Rev. Lett.* **107**, 256401 (2011).
- [26] L. de' Medici, G. Giovannetti, and M. Capone, Selective Mott physics as a key to iron superconductors, *Phys. Rev. Lett.* **112**, 177001 (2014).
- [27] A. Isidori, M. Berović, L. Fanfarillo, L. de' Medici, M. Fabrizio, and M. Capone, Charge disproportionation, mixed valence, and Janus effect in multiorbital systems: A tale of two insulators, *Phys. Rev. Lett.* **122**, 186401 (2019).
- [28] L. Del Re and M. Capone, Selective insulators and anomalous responses in three-component fermionic gases with broken SU(3) symmetry, *Phys. Rev. A* **98**, 063628 (2018).
- [29] A. Richaud, M. Ferraretto, and M. Capone, Interaction-resistant metals in multicomponent Fermi systems, *Phys. Rev. B* **103**, 205132 (2021).
- [30] A. Richaud, M. Ferraretto, and M. Capone, Mimicking multiorbital systems with SU(N) atoms: Hund's physics and beyond, *Condens. Matter* **7**, 18 (2022).
- [31] D. Tusi, L. Franchi, L. F. Livi, K. Baumann, D. Benedicto Orenes, L. Del Re, R. E. Barfknecht, T.-W. Zhou, M. Inguscio, G. Cappellini, M. Capone, J. Catani, and L. Fallani, Flavour-selective localization in interacting lattice fermions, *Nat. Phys.* **18**, 1201 (2022).
- [32] For a multiband state we require the density and magnetization in each band to be conserved independently.
- [33] V. Vedral, The role of relative entropy in quantum information theory, *Rev. Mod. Phys.* **74**, 197 (2002).
- [34] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information: 10th Anniversary Edition* (Cambridge University Press, Cambridge, England, 2012).
- [35] A. Peres, Separability criterion for density matrices, *Phys. Rev. Lett.* **77**, 1413 (1996).
- [36] P. Horodecki, Separability criterion and inseparable mixed states with positive partial transposition, *Phys. Lett. A* **232**, 333 (1997).
- [37] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Quantum entanglement, *Rev. Mod. Phys.* **81**, 865 (2009).
- [38] O. Gühne and G. Tóth, Entanglement detection, *Phys. Rep.* **474**, 1 (2009).
- [39] M. B. Plenio and S. Virmani, An introduction to entanglement measures, *Quantum Info. Comput.* **7**, 1 (2007).
- [40] H. Ollivier and W. H. Zurek, Quantum discord: A measure of the quantumness of correlations, *Phys. Rev. Lett.* **88**, 017901 (2001).
- [41] S. Luo, Quantum discord for two-qubit systems, *Phys. Rev. A* **77**, 042303 (2008).
- [42] B. Dakić, V. Vedral, and Č. Brukner, Necessary and sufficient condition for nonzero quantum discord, *Phys. Rev. Lett.* **105**, 190502 (2010).
- [43] K. Modi, T. Paterek, W. Son, V. Vedral, and M. Williamson, Unified view of quantum and classical correlations, *Phys. Rev. Lett.* **104**, 080501 (2010).
- [44] K. Modi, A. Brodutch, H. Cable, T. Paterek, and V. Vedral, The classical-quantum boundary for correlations: Discord and related measures, *Rev. Mod. Phys.* **84**, 1655 (2012).
- [45] W. H. Zurek, Decoherence, einselection, and the quantum origins of the classical, *Rev. Mod. Phys.* **75**, 715 (2003).
- [46] M. Horodecki, P. Horodecki, R. Horodecki, J. Oppenheim, A. Sen(De), U. Sen, and B. Synak-Radtke, Local versus nonlocal information in quantum-information theory: Formalism and phenomena, *Phys. Rev. A* **71**, 062307 (2005).
- [47] C. C. Rulli and M. S. Sarandy, Global quantum discord in multipartite systems, *Phys. Rev. A* **84**, 042109 (2011).
- [48] G. De Chiara and A. Sanpera, Genuine quantum correlations in quantum many-body systems: A review of recent progress, *Rep. Prog. Phys.* **81**, 074002 (2018).
- [49] A. D. Gottlieb and N. J. Mauser, New measure of electron correlation, *Phys. Rev. Lett.* **95**, 123003 (2005).
- [50] A. D. Gottlieb and N. J. Mauser, Properties of nonfreeness: An entropy measure of electron correlation, *Int. J. Quantum Inf.* **05**, 815 (2007).
- [51] K. Held and N. J. Mauser, Physics behind the minimum of relative entropy measures for correlations, *Eur. Phys. J. B* **86**, 328 (2013).
- [52] K. Byczuk, J. Kuneš, W. Hofstetter, and D. Vollhardt, Quantification of correlations in quantum many-particle systems, *Phys. Rev. Lett.* **108**, 087004 (2012).
- [53] C. Artiago, T. Klein Kvorning, D. Aceituno Chávez, L. Herviou, and J. H. Bardarson, Universal characterization of quantum many-body states through local information, *Phys. Rev. Lett.* **134**, 190401 (2025).

- [54] J. Hubbard, Electron correlations in narrow energy bands, *Proc. R. Soc. Lond. A* **276**, 238 (1963).
- [55] C. Honerkamp and W. Hofstetter, Ultracold fermions and the SU(N) Hubbard model, *Phys. Rev. Lett.* **92**, 170403 (2004).
- [56] J. Kanamori, Electron correlation and ferromagnetism of transition metals, *Prog. Theor. Phys.* **30**, 275 (1963).
- [57] W. Dür, J. I. Cirac, and R. Tarrach, Separability and distillability of multiparticle quantum systems, *Phys. Rev. Lett.* **83**, 3562 (1999).
- [58] D. D. Su, X. Dai, and N. H. Tong, Local entanglement entropy at the mott metal-insulator transition in infinite dimensions, *Mod. Phys. Lett. B* **27**, 1350034 (2013).
- [59] C. Walsh, P. Sémon, D. Poulin, G. Sordi, and A.-M. S. Tremblay, Local entanglement entropy and mutual information across the Mott transition in the two-dimensional Hubbard model, *Phys. Rev. Lett.* **122**, 067203 (2019).
- [60] C. Walsh, P. Sémon, D. Poulin, G. Sordi, and A.-M. S. Tremblay, Entanglement and classical correlations at the doping-driven Mott transition in the two-dimensional Hubbard model, *PRX Quantum* **1**, 020310 (2020).
- [61] C. Walsh, M. Charlebois, P. Sémon, G. Sordi, and A.-M. S. Tremblay, Information-theoretic measures of superconductivity in a two-dimensional doped Mott insulator, *Proc. Natl. Acad. Sci. USA* **118**, e2104114118 (2021).
- [62] G. Roósz, A. Kauch, F. Bippus, D. Wieser, and K. Held, Two-site reduced density matrix from one- and two-particle Green's functions, *Phys. Rev. B* **110**, 075115 (2024).
- [63] The natural spin orbitals are the eigenstates of any spin projection, e.g., the S_z basis, in the Hubbard model.
- [64] M. M. Wolf, F. Verstraete, M. B. Hastings, and J. I. Cirac, Area laws in quantum systems: Mutual information and correlations, *Phys. Rev. Lett.* **100**, 070502 (2008).
- [65] Alternatively, one can define free states as those for which the Wick theorem holds for computing any N -body expectation value. Hence, the closest free state is the best one-body approximation of the N -body state.
- [66] G. C. Wick, A. S. Wightman, and E. P. Wigner, The intrinsic parity of elementary particles, *Phys. Rev.* **88**, 101 (1952).
- [67] G. C. Wick, A. S. Wightman, and E. P. Wigner, Superselection rule for charge, *Phys. Rev. D* **1**, 3267 (1970).
- [68] D. Giulini, Superselection rules, [arXiv:0710.1516](https://arxiv.org/abs/0710.1516).
- [69] N. Friis, A. R. Lee, and D. E. Bruschi, Fermionic-mode entanglement in quantum information, *Phys. Rev. A* **87**, 022338 (2013).
- [70] N. Friis, Reasonable fermionic quantum information theories require relativity, *New J. Phys.* **18**, 033014 (2016).
- [71] A. Galler and P. Thunström, Orbital and electronic entanglement in quantum teleportation schemes, *Phys. Rev. Res.* **3**, 033120 (2021).
- [72] G. Bellomia, C. Mejuto-Zaera, M. Capone, and A. Amaricci, Quasilocal entanglement across the Mott-Hubbard transition, *Phys. Rev. B* **109**, 115104 (2024).
- [73] T. Shen, H. Barghathi, A. Del Maestro, and B. M. Rubenstein, Disentangling the physics of the attractive Hubbard model as a fully interacting model of fermions via the accessible and symmetry-resolved entanglement entropies, *Phys. Rev. B* **109**, 195119 (2024).
- [74] M. A. Nielsen and G. Vidal, Majorization and the interconversion of bipartite states, *Quantum Info. Comput.* **1**, 76 (2001).
- [75] A. W. Marshall, I. Olkin, and B. C. Arnold, *Inequalities: Theory of Majorization and Its Applications* (Springer, New York, 2011).
- [76] N. F. Mott, The basis of the electron theory of metals, with special reference to the transition metals, *Proc. Phys. Soc. A* **62**, 416 (1949).
- [77] N. F. Mott, Metal-insulator transition, *Rev. Mod. Phys.* **40**, 677 (1968).
- [78] C. Castellani, C. D. Castro, D. Feinberg, and J. Ranninger, New model Hamiltonian for the metal-insulator transition, *Phys. Rev. Lett.* **43**, 1957 (1979).
- [79] W. F. Brinkman and T. M. Rice, Single-particle excitations in magnetic insulators, *Phys. Rev. B* **2**, 1324 (1970).
- [80] D. Vollhardt, Normal ^3He : An almost localized Fermi liquid, *Rev. Mod. Phys.* **56**, 99 (1984).
- [81] A. Georges, G. Kotliar, W. Krauth, and M. J. Rozenberg, Dynamical mean-field theory of strongly correlated fermion systems and the limit of infinite dimensions, *Rev. Mod. Phys.* **68**, 13 (1996).
- [82] N. Lanatà, T.-H. Lee, Y.-X. Yao, and V. Dobrosavljević, Emergent Bloch excitations in Mott matter, *Phys. Rev. B* **96**, 195126 (2017).
- [83] M. S. Frank, T.-H. Lee, G. Bhattacharyya, P. K. H. Tsang, V. L. Quito, V. Dobrosavljević, O. Christiansen, and N. Lanatà, Quantum embedding description of the Anderson lattice model with the ghost Gutzwiller approximation, *Phys. Rev. B* **104**, L081103 (2021).
- [84] F. Lechermann, A. Georges, G. Kotliar, and O. Parcollet, Rotationally invariant slave-boson formalism and momentum dependence of the quasiparticle weight, *Phys. Rev. B* **76**, 155102 (2007).
- [85] J. Bünenmann and F. Gebhard, Equivalence of Gutzwiller and slave-boson mean-field theories for multiband Hubbard models, *Phys. Rev. B* **76**, 193104 (2007).
- [86] A. Isidori and M. Capone, Rotationally invariant slave bosons for strongly correlated superconductors, *Phys. Rev. B* **80**, 115120 (2009).
- [87] T. Ayral, T.-H. Lee, and G. Kotliar, Dynamical mean-field theory, density-matrix embedding theory, and rotationally invariant slave bosons: A unified perspective, *Phys. Rev. B* **96**, 235139 (2017).
- [88] T.-H. Lee, N. Lanatà, and G. Kotliar, Accuracy of ghost rotationally invariant slave-boson and dynamical mean-field theory as a function of the impurity-model bath size, *Phys. Rev. B* **107**, L121104 (2023).
- [89] D. Guerci, M. Capone, and M. Fabrizio, Exciton Mott transition revisited, *Phys. Rev. Mater.* **3**, 054605 (2019).
- [90] C. Mejuto-Zaera and M. Fabrizio, Efficient computational screening of strongly correlated materials: Multiorbital phenomenology within the ghost Gutzwiller approximation, *Phys. Rev. B* **107**, 235150 (2023).
- [91] S. Giuli, C. Mejuto-Zaera, and M. Capone, Altermagnetism from interaction-driven itinerant magnetism, *Phys. Rev. B* **111**, L020401 (2025).
- [92] A. M. Tagliente, C. Mejuto-Zaera, and M. Fabrizio, Revealing spinons by proximity effect, *Phys. Rev. B* **111**, 125110 (2025).
- [93] I. Pasqua, A. M. Tagliente, G. Bellomia, B. Monserrat, M. Fabrizio, and C. Mejuto-Zaera, Band structure picture for topology in strongly correlated systems with the ghost Gutzwiller ansatz, *Phys. Rev. B* **113**, 155133 (2026).

- [94] P. W. Anderson, Localized magnetic states in metals, *Phys. Rev.* **124**, 41 (1961).
- [95] D. Guerci, M. Capone, and N. Lanatà, Time-dependent ghost Gutzwiller nonequilibrium dynamics, *Phys. Rev. Res.* **5**, L032023 (2023).
- [96] S. Giuli, Efficient and effective quantum embedding methods for strongly correlated materials and models, Ph.D. thesis, SISSA, 2025.
- [97] A. W. Sandvik, Classical percolation transition in the diluted two-dimensional $s = \frac{1}{2}$ Heisenberg antiferromagnet, *Phys. Rev. B* **66**, 024418 (2002).
- [98] E. V. Castro, N. M. R. Peres, K. S. D. Beach, and A. W. Sandvik, Site dilution of quantum spins in the honeycomb lattice, *Phys. Rev. B* **73**, 054422 (2006).
- [99] J. D. Reger and A. P. Young, Monte Carlo simulations of the spin-1/2 Heisenberg antiferromagnet on a square lattice, *Phys. Rev. B* **37**, 5978 (1988).
- [100] P. Horsch and W. von der Linden, Spin-correlations of the spin-1/2 Heisenberg antiferromagnet on a square lattice, *Physica C* **153–155**, 1285 (1988).
- [101] S. Tang and J. E. Hirsch, Peierls instability in the two-dimensional half-filled Hubbard model, *Phys. Rev. B* **37**, 9546 (1988).
- [102] M. Gross, E. Sánchez-Velasco, and E. Siggia, Ground-state properties of the two-dimensional antiferromagnetic Heisenberg model, *Phys. Rev. B* **39**, 2484 (1989).
- [103] J. D. Reger, J. A. Riera, and A. P. Young, Monte Carlo simulations of the spin-1/2 Heisenberg antiferromagnet in two dimensions, *J. Phys.: Condens. Matter* **1**, 1855 (1989).
- [104] M. Caffarel and W. Krauth, Exact diagonalization approach to correlated fermions in infinite dimensions: Mott transition and superconductivity, *Phys. Rev. Lett.* **72**, 1545 (1994).
- [105] M. Capone, L. de' Medici, and A. Georges, Solving the dynamical mean-field theory at very low temperatures using the Lanczos exact diagonalization, *Phys. Rev. B* **76**, 245116 (2007).
- [106] A. Amaricci, L. Crippa, A. Scazzola, F. Petocchi, G. Mazza, L. de Medici, and M. Capone, EDIPack: A parallel exact diagonalization package for quantum impurity problems, *Comput. Phys. Commun.* **273**, 108261 (2022).
- [107] L. Crippa, I. Krivenko, S. Giuli, G. Bellomia, A. Kowalski, F. Petocchi, A. Scazzola, M. Wallerberger, G. Mazza, L. de Medici, G. Sangiovanni, M. Capone, and A. Amaricci, Next-generation EDIPack: A Lanczos-based package for quantum impurity models featuring general broken-symmetry phases, flexible bath topologies and multi-platform interoperability, *SciPost Phys. Codebases* **58** (2025), Codebase release 5.3 for EDIPack, 58-r5.3 (2025).
- [108] M. Raczkowski, R. Peters, T. T. Phung, N. Takemori, F. F. Assaad, A. Honecker, and J. Vahedi, Hubbard model on the honeycomb lattice: From static and dynamical mean-field theories to lattice quantum Monte Carlo simulations, *Phys. Rev. B* **101**, 125103 (2020).
- [109] S. Sorella and E. Tosatti, Semi-metal-insulator transition of the Hubbard model in the honeycomb lattice, *Europhys. Lett. (EPL)* **19**, 699 (1992).
- [110] W. Wu, Y.-H. Chen, H.-S. Tao, N.-H. Tong, and W.-M. Liu, Interacting Dirac fermions on honeycomb lattice, *Phys. Rev. B* **82**, 245102 (2010).
- [111] S. Sorella, Y. Otsuka, and S. Yunoki, Absence of a spin liquid phase in the Hubbard model on the honeycomb lattice, *Sci. Rep.* **2**, 992 (2012).
- [112] S. R. Hassan and D. Sénéchal, Absence of spin liquid in nonfrustrated correlated systems, *Phys. Rev. Lett.* **110**, 096402 (2013).
- [113] W. Wu and A.-M. S. Tremblay, Phase diagram and Fermi liquid properties of the extended Hubbard model on the honeycomb lattice, *Phys. Rev. B* **89**, 205128 (2014).
- [114] S. A. Jafari, Dynamical mean-field study of the Dirac liquid, *Eur. Phys. J. B* **68**, 537 (2009).
- [115] M.-T. Tran and K. Kuroki, Finite-temperature semimetal-insulator transition on the honeycomb lattice, *Phys. Rev. B* **79**, 125125 (2009).
- [116] However, the implied observation that gRISB could capture less correlations than DMFT, with two ghost orbitals, appears to be in good agreement with some conjectures on the relationship between the two methods [88].
- [117] G. Sangiovanni, A. Toschi, E. Koch, K. Held, M. Capone, C. Castellani, O. Gunnarsson, S.-K. Mo, J. W. Allen, H.-D. Kim, A. Sekiyama, A. Yamasaki, S. Suga, and P. Metcalf, Static versus dynamical mean-field theory of Mott antiferromagnets, *Phys. Rev. B* **73**, 205121 (2006).
- [118] C. Taranto, G. Sangiovanni, K. Held, M. Capone, A. Georges, and A. Toschi, Signature of antiferromagnetic long-range order in the optical spectrum of strongly correlated electron systems, *Phys. Rev. B* **85**, 085124 (2012).
- [119] T. A. Maier, M. Jarrell, T. Pruschke, and M. H. Hettler, Quantum cluster theories, *Rev. Mod. Phys.* **77**, 1027 (2005).
- [120] G. Rohringer, H. Hafermann, A. Toschi, A. A. Katanin, A. E. Antipov, M. I. Katsnelson, A. I. Lichtenstein, A. N. Rubtsov, and K. Held, Diagrammatic routes to nonlocal correlations beyond dynamical mean-field theory, *Rev. Mod. Phys.* **90**, 025003 (2018).
- [121] F. Bippus, J. Krsnik, M. Kitatani, L. Akšamović, A. Kauch, N. Barišić, and K. Held, Entanglement in the pseudogap regime of cuprate superconductors, *Phys. Rev. B* **112**, L081110 (2025).
- [122] F. Bippus, A. Kauch, G. Roósz, C. Mayrhofer, F. Assaad, and K. Held, Two-site entanglement in the two-dimensional Hubbard model, *Phys. Rev. B* **113**, 035152 (2026).
- [123] E. Cocchi, L. A. Miller, J. H. Drewes, C. F. Chan, D. Pertot, F. Brennecke, and M. Köhl, Measuring entropy and short-range correlations in the two-dimensional Hubbard model, *Phys. Rev. X* **7**, 031025 (2017).
- [124] D. Wecker, M. B. Hastings, N. Wiebe, B. K. Clark, C. Nayak, and M. Troyer, Solving strongly correlated electron models on a quantum computer, *Phys. Rev. A* **92**, 062318 (2015).
- [125] E. T. Campbell, Early fault-tolerant simulations of the Hubbard model, *Quantum Sci. Technol.* **7**, 015007 (2022).
- [126] A. Kan and B. C. B. Symons, Resource-optimized fault-tolerant simulation of the Fermi-Hubbard model and high-temperature superconductor models, *npj Quantum Inf.* **11**, 138 (2025).
- [127] J. Lemieux, G. Duclos-Cianci, D. Sénéchal, and D. Poulin, Resource estimate for quantum many-body ground-state preparation on a quantum computer, *Phys. Rev. A* **103**, 052408 (2021).

- [128] N. Yoshioka, T. Okubo, Y. Suzuki, Y. Koizumi, and W. Mizukami, Hunting for quantum-classical crossover in condensed matter problems, [npj Quantum Inf. **10**, 45 \(2024\)](#).
- [129] E. Chitambar and G. Gour, Quantum resource theories, [Rev. Mod. Phys. **91**, 025001 \(2019\)](#).
- [130] M. Howard and E. Campbell, Application of a resource theory for magic states to fault-tolerant quantum computing, [Phys. Rev. Lett. **118**, 090501 \(2017\)](#).
- [131] Z.-W. Liu and A. Winter, Many-body quantum magic, [PRX Quantum **3**, 020333 \(2022\)](#).
- [132] X. Lyu and K. Bu, Fermionic Gaussian testing and non-Gaussian measures via convolution, [arXiv:2409.08180](#).
- [133] P. Sierant, P. Stornati, and X. Turkeshi, Fermionic magic resources of quantum many-body systems, [PRX Quantum **7**, 010302 \(2026\)](#).
- [134] L. Bittel, A. A. Mele, J. Eisert, and L. Leone, Optimal trace-distance bounds for free-fermionic states: Testing and improved tomography, [PRX Quantum **6**, 030341 \(2025\)](#).
- [135] M. Collura, J. D. Nardis, V. Alba, and G. Lami, The quantum magic of fermionic Gaussian states [Quantum **10**, 2036 \(2025\)](#).
- [136] A. A. Mele and Y. Herasymenko, Efficient learning of quantum states prepared with few fermionic non-Gaussian gates, [PRX Quantum **6**, 010319 \(2025\)](#).
- [137] A. Paviglianiti, L. Lumia, E. Tirrito, A. Silva, M. Collura, X. Turkeshi, and G. Lami, Emergence of generic entanglement structure in doped matchgate circuits, [Phys. Rev. Lett. **136**, 020403 \(2026\)](#).
- [138] A. E. Antipov, I. S. Krivenko, V. I. Anisimov, A. I. Lichtenstein, and A. N. Rubtsov, Role of rotational symmetry in the magnetism of a multiorbital model, [Phys. Rev. B **86**, 155107 \(2012\)](#).
- [139] C. P. Koçer, K. Haule, G. L. Pascut, and B. Monserrat, Efficient lattice dynamics calculations for correlated materials with DFT + DMFT, [Phys. Rev. B **102**, 245104 \(2020\)](#).
- [140] G. Bellomia, Local quantum embedding data supporting [arXiv:2506.18709] (1.0.0) [Data set], Zenodo (2025), <https://doi.org/10.5281/zenodo.17655291>.