

Revisiting the J_1 – J_2 Heisenberg model on a triangular lattice: Quasidegenerate ground states and phase competition

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 (Received 19 March 2026; revised 24 August 2026; accepted 28 August 2026; published 5 October 2026)

It is generally believed that the spin- $\frac{1}{2}$ triangular-lattice J_1 – J_2 Heisenberg model hosts a quantum spin liquid in the intermediate regime between the 120° and stripe ordered phases. Density matrix renormalization group studies on cylinders have consistently found two nearly degenerate ground states, commonly interpreted as distinct topological sectors. Using state-of-the-art matrix product state simulations on YC6 cylinders, we compare the static and dynamical properties of these two sectors at $J_2/J_1 = 0.125$. Noticeable differences appear already in static correlations; moreover, high-resolution dynamical structure factors reveal qualitatively distinct low-energy excitations. These results suggest that the two ground states cannot be understood as merely topologically distinct sectors of a gapped $\mathbb{Z}_2$ spin liquid.

DOI: [10.1103/cljk-jcq3](https://doi.org/10.1103/cljk-jcq3)

I. INTRODUCTION

The spin- $\frac{1}{2}$ antiferromagnetic Heisenberg model on a triangular lattice (TLHAF) has long served as a paradigmatic platform for studying frustrated quantum magnetism. In his pioneering work, Anderson argued that the TLHAF with only nearest-neighbor (NN) coupling could have a magnetically disordered ground state due to geometric frustration [1]. Subsequent numerical studies, however, established that the ground state of the TLHAF with NN coupling instead has a coplanar 120° magnetic long-range order [2–7].

Introducing further competing interactions can enhance quantum fluctuations and potentially destabilize the 120° magnetic order. In particular, adding a next-nearest-neighbor (NNN) antiferromagnetic coupling leads to the so-called J_1 – J_2 Heisenberg model:

$$H = J_1 \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \mathbf{S}_i \cdot \mathbf{S}_j, \quad (1)$$

where $\langle i,j \rangle$ and $\langle\langle i,j \rangle\rangle$ denote NN and NNN pairs on the triangular lattice, respectively. For small J_2/J_1 , the 120° magnetic order remains robust, whereas at sufficiently large J_2/J_1 , the system develops a collinear stripe order [8,9]. Between these two magnetically ordered phases, extensive numerical studies [10–23] have identified an intermediate nonmagnetic regime in the approximate range $0.07 \lesssim J_2/J_1 \lesssim 0.15$,

exhibiting signatures of a quantum spin liquid (QSL) [24–27]. A schematic phase diagram is shown in Fig. 1.

The nature of this putative QSL remains under active debate. Two prominent candidates have been proposed: a gapped $\mathbb{Z}_2$ spin liquid [14,22,28–30] and a gapless U(1) Dirac spin liquid [17,18,23,31–38]. Since QSLs do not break global symmetries and lack local order parameters, distinguishing these scenarios using numerical methods is inherently challenging. Interestingly, several independent density matrix renormalization group (DMRG) [39,40] studies on finite-width cylinders have consistently reported the presence of two nearly degenerate ground states [14,18,33]. This observation has often been interpreted as being consistent with the presence of topological (or flux) sector structure [18,33]: on a finite cylinder, different topological sectors may manifest as nearly degenerate ground states separated by small energy splittings. These states are commonly referred to as even and odd sectors [18,33]. In a gapped topological phase, such sectors are expected to become exactly degenerate in the thermodynamic limit and indistinguishable by any local observable in the bulk. However, similar near degeneracies may also arise in gapless spin liquids or from finite-size effects. Therefore, whether the observed near degeneracy unambiguously signals topological order remains unsettled.

In this work, we employ large-scale matrix product state (MPS) simulations to shed light on the nature of the even- and odd-sector ground states on a width-6 cylinder at $J_2/J_1 = 0.125$. The latter is a representative point within the proposed nonmagnetic regime and coincides with the transition point of the corresponding classical model [8]. We demonstrate that the two sectors exhibit clear differences in local observables. Qualitative differences appear already in static correlations: (i) the equal-time structure factor exhibits substantially different intensity profiles across the Brillouin zone, and (ii) the NN bond correlations pattern in the odd sector is noticeably more isotropic. Most importantly, their low-energy dynamical

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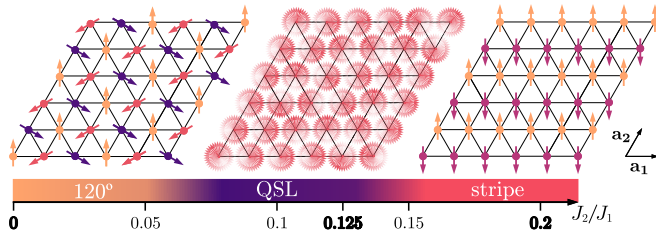


FIG. 1. A schematic phase diagram of the J_1 – J_2 Heisenberg model on a triangular lattice as a function of J_2/J_1 , as conjectured from previous numerical studies [10–23]. For $J_2 = 0$ the model exhibits coplanar 120° order, transitioning into the QSL candidate region around $J_2/J_1 \approx 0.07$, followed by a stripe-ordered phase at larger values of coupling.

responses are drastically different. While the even sector exhibits comparable spectral intensity at the K and M points in the Brillouin zone, the odd sector concentrates most of its spectral weight at the K point, reminiscent of the proximate 120° ordered phase. These results indicate that the two ground states cannot be understood as arising from the topological sectors of a gapped $\mathbb{Z}_2$ spin liquid. Instead, our findings call for a careful reassessment of the origin of the two-sector structure in this parameter regime.

II. GROUND STATES ON A CYLINDER

We study the Hamiltonian (1) on a triangular lattice wrapped onto a finite cylinder. We denote by L_y (L_x) the number of spins along the circumference (length) of the cylinder. Throughout this work we adopt the YC geometry [14], imposing a periodic boundary condition along $\mathbf{a}_2$ by identifying sites at $\mathbf{r}$ and $\mathbf{r} + L_y \mathbf{a}_2$, with primitive vectors $\mathbf{a}_1 = (1, 0)$ and $\mathbf{a}_2 = (1/2, \sqrt{3}/2)$ (see Fig. 1). Unless stated otherwise, the calculations are performed on YC6-36 cylinders, corresponding to $L_y = 6$ and $L_x = 36$. In the following, we set the energy unit as $J_1 = 1$.

The ground-state MPSs are obtained using single-site DMRG with controlled bond expansion [41]. We use the QSpace tensor library [42–44] to exploit the non-Abelian $SU(2)$ symmetry of the Hamiltonian (1). The simulations are performed until the targeted MPS reaches at least $D^* = 3000$ kept $SU(2)$ multiplets, after which it is compressed down to $D^* = 1000$ for subsequent use. We find this strategy to be optimal for the TLHAF, particularly in the QSL regime where convergence is hindered by substantial frustration and entanglement.

Previous studies of the TLHAF on cylindrical geometry have consistently found two nearly degenerate ground states in the QSL candidate phase [14,15,29,33,45], whose properties, at large, remained unclear. When considering infinitely long cylinders, for example, by means of iDMRG, direct simulations yielded only the higher-energy state (the even sector) [33]. The ground state in the odd sector was then reached through flux insertion [18,33], which explicitly breaks the $SU(2)$ symmetry.

It is not obvious how to systematically obtain both states while maintaining the $SU(2)$ symmetry of the microscopic Hamiltonian (1). In their DMRG study of finite-size even- L_y

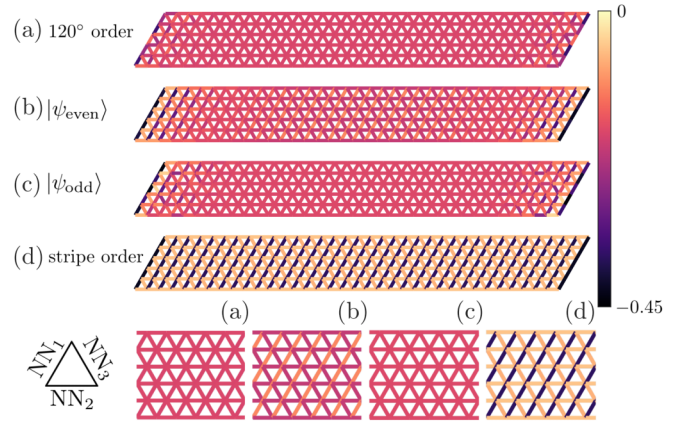


FIG. 2. Nearest-neighbor spin-spin correlation $S_{(i,j)}$ on the YC6-36 cylinder computed for (a) 120° ordered state at $J_2 = 0$, (b) even- and (c) odd-sector states at $J_2/J_1 = 0.125$, and (d) stripe-ordered state at $J_2/J_1 = 0.2$. Bond strengths are represented via a colormap, with the same scale applied globally across all panels to allow direct comparison. The bottom row contains insets of the four central rings of each cylinder respectively.

cylinders, Zhu and White [14] reported that, for sufficiently large L_x and at a certain number of kept states, the MPS ground state dynamically falls into the odd sector. Similarly, when running DMRG in the QSL regime for YC6-36, we observe a sudden drop in energy while performing the sweeping routine for $D^* = 2048$ (see Sec. S-1 of the Supplemental Material [46]). This behavior is reproducible across multiple independent DMRG simulations with randomized initial states, demonstrating its robustness. Conversely, for shorter cylinders ($L_x = 18$), our ground-state optimization routine does not exhibit any such abrupt changes.

Throughout this work, we will refer to the states before and after the drop as $|\psi_{\text{even}}\rangle$ and $|\psi_{\text{odd}}\rangle$, respectively. While it is not immediately clear how they are related, we note that the energy of $|\psi_{\text{odd}}\rangle$ is lower by 1.1%, and the nature of NN spin-spin correlations changes drastically across the drop (see Fig. 2). Moreover, we find that the two states are nearly orthogonal, with a fidelity $\mathcal{F} = |\langle \psi_{\text{even}} | \psi_{\text{odd}} \rangle|$ of order $\sim 10^{-4}$. Motivated by these observations, in the following sections, we examine the static and dynamical properties of the even and odd sectors in greater detail, comparing them with each other and with results obtained for the ordered phases.

III. STATIC CORRELATIONS

Figure 3 displays the equal-time structure factor (ETSF)

$$\chi(\mathbf{k}) = \frac{1}{\mathcal{L}} \sum_{i,j} e^{-i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle \quad (2)$$

obtained from ground-state MPSs on the YC6-36 cylinder. Here, $\mathcal{L} = L_y \times L_x$ denotes the total number of sites in the system.

In the 120° ordered phase at $J_2 = 0$ [Fig. 3(a)], the ETSF shows sharp isolated maxima at the corners of the Brillouin zone, corresponding to K and K' points. This is consistent

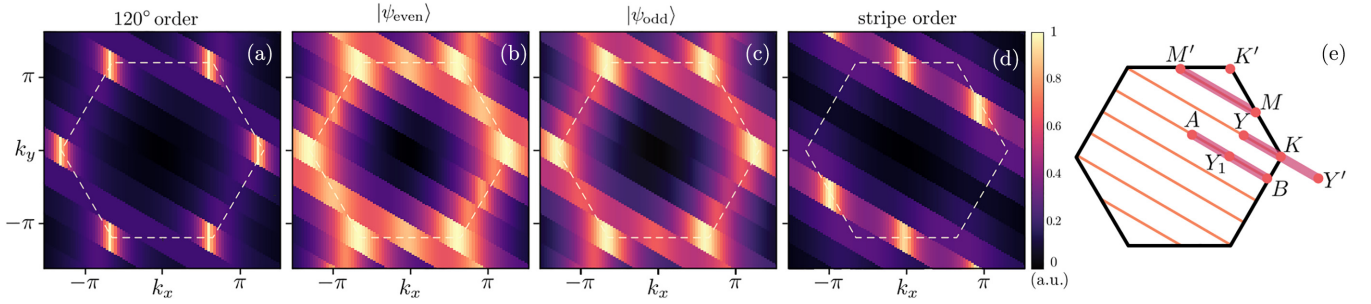


FIG. 3. The equal-time structure factor $\chi(\mathbf{k})$ extracted from ground-state MPSs of the YC6-36 cylinder. Panels (a) and (d) correspond to $J_2 = 0$ (120° order) and $J_2/J_1 = 0.2$ (stripe order), respectively. Results for even (b) and odd (c) ground states in the QSL candidate phase are plotted side by side for comparison. Each panel is normalized independently to its own maximum intensity to highlight the relative distribution of spectral weight across different coupling regimes. The dashed white lines indicate the boundaries of the first Brillouin zone, which we also plot separately in panel (e), marking high symmetry points and the cuts considered in our DSF computations (see Fig. 4).

with the presence of long-range magnetic order in the ground state, which manifests through high-intensity Bragg peaks. In the stripe-ordered phase at $J_2/J_1 = 0.2$ [Fig. 3(d)], sharp features instead emerge at the M and M' points, with little to no intensity across the remainder of the Brillouin zone, characteristic of stripe formation.

For the study of the QSL candidate phase, we set $J_2/J_1 = 0.125$, which lies within the putative QSL region (see also Fig. 1). In contrast to magnetically ordered states, U(1) Dirac QSLs are expected to host a broad distribution of correlations across the Brillouin zone, up to anisotropies induced by the cylindrical geometry. This scenario is most consistent with the even-sector ETSF [Fig. 3(b)], where a significant softening of the sharp features is observed at the K and M points, compared to the ordered phases. Moreover, these points exhibit comparable intensity, which is compatible with expectations for a U(1) Dirac QSL.

For the odd-sector ETSF [Fig. 3(c)], the correlations likewise become broader near the corners of the Brillouin zone, albeit less prominently than in the even sector. The maxima at the M points are less pronounced in comparison, in particular those located at $\pm(\pi, \pi/\sqrt{3})$.

We now turn to the NN spin-spin correlations $S_{(i,j)} = \langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle$, shown in Fig. 2. In the bulk of the cylinder, the bond strength patterns for $|\psi_{\text{even}}\rangle$ [Fig. 2(b)] and $|\psi_{\text{odd}}\rangle$ [Fig. 2(c)] are strikingly different. The former exhibits a staggering pattern, similar to the one observed in the stripe-ordered phase [Fig. 2(d)]. However, the difference in amplitude between the three distinct bonds is much smaller, with the bond strength for NN_1 differing significantly from NN_2 and NN_3 (see Table I). Conversely, for the odd sector, the correlations appear to be isotropic and close in value to those obtained in the 120° ordered phase [Fig. 2(a)], with substantial differences arising only at edges. The above analysis of the static properties of the four states suggests that the two QSL ground states exhibit notable differences in their local observables. In particular, $|\psi_{\text{odd}}\rangle$ appears to share certain features with the ordered phase at $J_2 = 0$.

IV. DYNAMIC CORRELATIONS

To further elucidate the differences between the states discussed above, we compute the dynamical structure factor

(DSF)

$$S(\mathbf{k}, \omega) = \langle \Psi_g | \mathbf{S}_{\mathbf{k}}^\dagger \delta(\omega - H + E_g) \mathbf{S}_{\mathbf{k}} | \Psi_g \rangle, \quad (3)$$

using the tangent space Krylov (TaSK) method [47] with DMRG ground states as input.

In this approach, we approximate resolvents by projecting the Hamiltonian to the tangent space of the ground-state MPS, denoted by $\mathbb{V}^\perp$. The explicit vector-space structure of $\mathbb{V}^\perp$ allows us to construct a Krylov basis through iterative application of the projected Hamiltonian $H^\perp$ to the initial state $|\Psi_0\rangle = \mathbf{S}_{\mathbf{k}} | \Psi_g \rangle$,

$$\mathcal{K}(\Psi_0) = \text{span}\{|\Psi_0\rangle, H^\perp |\Psi_0\rangle, \dots, (H^\perp)^{\mathcal{N}_{\text{kr}}} |\Psi_0\rangle\}, \quad (4)$$

where $\mathcal{N}_{\text{kr}}$ is the number of Krylov steps. This is achieved via a stable realization of the Lanczos algorithm, meaning that at every iteration, all vectors are mutually orthogonal down to numerical precision. In comparison to time-evolution-based tensor network methods, when computing dynamical correlators, TaSK achieves high resolution at low frequencies much more efficiently.

The real discrete frequencies and spectral weights $\{\omega_\alpha, S_\alpha\}$, approximating $S(\mathbf{k}, \omega)$, are extracted from the matrix representation of $H^\perp$ in the Krylov basis. To obtain smooth spectra, we broaden them using a continued fraction expansion scheme, which conserves the total spectral weight and the lowest frequency moments of the DSF (see Sec. S-2 B of the Supplemental Material of Ref. [47]). The discrete data and broadened curves corresponding to other choices of the broadening parameters can be found in the Supplemental Material [46].

TABLE I. Nearest-neighbor spin-spin correlations evaluated on the central ring of the YC6-36 cylinder for four different ground states at $J_2/J_1 = 0, 0.125$, and 0.2 , respectively. NN_i ($i = 1, 2, 3$) denotes a distinct pair of neighboring spins on a triangular lattice, as depicted in Fig. 2.

	120° order	$ \psi_{\text{even}}\rangle$	$ \psi_{\text{odd}}\rangle$	Stripe order
NN_1	−0.1874	−0.1124	−0.1804	−0.3763
NN_2	−0.1839	−0.2113	−0.1821	−0.0542
NN_3	−0.1837	−0.2111	−0.1821	−0.0733

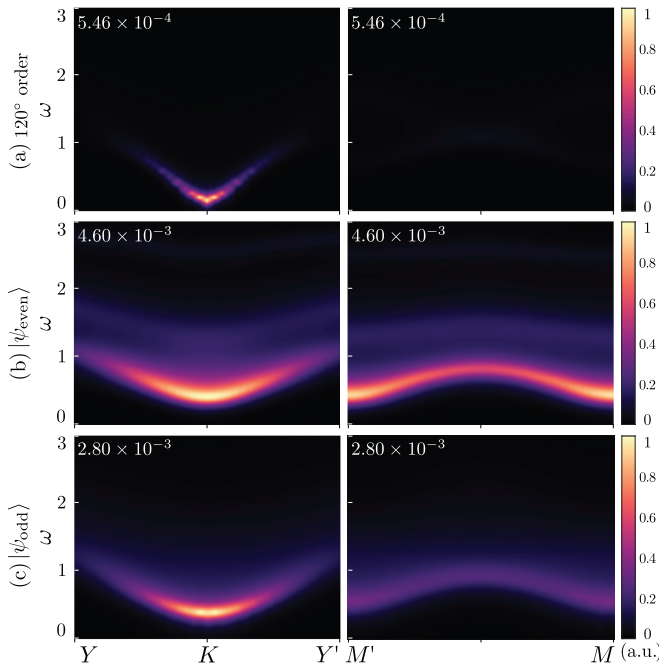


FIG. 4. The dynamical structure factor computed for (a) the 120° ordered phase at $J_2 = 0$ and (b), (c) the putative QSL phase at $J_2/J_1 = 0.125$ along the momentum cuts indicated in Fig. 3(e). The normalization factor $1/S_{\max}(\mathbf{k}, \omega)$ is provided in the top left corner of each panel.

Figure 4 shows the dynamical structure factor computed with TaSK for the 120° ordered phase [Fig. 4(a)] and QSL candidate phase [Figs. 4(b) and 4(c)]. We focus on two momentum cuts containing K and M points [see Fig. 3(e)]. To enable a fair comparison of spectral intensities, each row is normalized to its maximum value $S_{\max}(\mathbf{k}, \omega)$ among all considered momentum points. For line cuts through the K and M point as well as a discussion of results obtained for the $A - B$ path, including the conjectured magnon minimum at the Y_1 point [48], see the Supplemental Material [46].

In the 120° ordered phase [Fig. 4(a)], the DSF exhibits a sharp magnon Goldstone mode, gapless at the K point up to finite-size effects imposed by cylindrical geometry. At the centers of the Brillouin zone edges (M and M' points), the spectral weight is significantly suppressed and has rotonlike minima (not visible in this normalization; see also Fig. S-2 in the Supplemental Material [46]). Overall, our results at $J_2 = 0$ are in conclusive agreement with previous theoretical and numerical studies [19,33,49,50].

Comparing the DSFs of $|\psi_{\text{even}}\rangle$ [Fig. 4(b)] and $|\psi_{\text{odd}}\rangle$ [Fig. 4(c)] in the QSL regime, we find that the two states exhibit similar spectral profiles, distinct from that of the 120° ordered state. However, significant differences arise in the distribution of spectral weight: for $|\psi_{\text{odd}}\rangle$, it is mostly concentrated at the K point, whereas for $|\psi_{\text{even}}\rangle$ it is distributed more uniformly throughout the Brillouin zone, peaking at the K and M points with comparable intensity. In agreement with the ETSF results, these observations are consistent with the even sector potentially being a $U(1)$ Dirac QSL [19,33], while the nature of the odd sector remains, at large, unresolved, albeit exhibiting signatures of a proximate 120° state. We

also note that for the Dirac QSL the DSF should be gapless at the M points (since the spinon Dirac cones are expected to occur at $\pm Q = \pm(\pi/2, \pi/2\sqrt{3})$) [33,38]. The finite gap, observed in Figs. 4(b) and 4(c) is attributed to the finite circumference of the cylinder, and we expect it to close in the $L_y \rightarrow \infty$ limit.

V. SUMMARY AND OUTLOOK

Our work provides compelling evidence that the “even” and “odd” sectors of the TLHAF model, thus far thought to be only topologically distinct, exhibit substantial differences in their locally observable properties on the YC6 cylinder. This result rules out the mere emergence of quasidegenerate ground states as a signature of a gapped $\mathbb{Z}_2$ QSL. Instead, we find that the even-sector ground state could be interpreted as the $U(1)$ Dirac QSL, while the odd-sector ground state, which appears to be the stable configuration for this geometry, shares many low-energy features with the magnetically ordered phase at $J_2 = 0$. The latter finding has also been reported in recent time-evolution studies of the same model [33,38], which focused on the odd sector.

Several questions, therefore, remain open: (i) Is the true ground state always in the odd sector, i.e., the state with more isotropic correlations, or could the two states trade places as system size and geometry are varied? (ii) Does the odd-sector ground state represent a QSL, whose nature is different from $U(1)$ Dirac, or is it a proximate 120° state? Providing definitive answers will require systematic, large-scale DMRG studies on wider cylinders and different geometries.

More generally, unambiguous identification of spectral features in the QSL regime remains a major challenge, particularly in tensor-network studies. On this front, additional insights could be gained by extracting dynamical correlations from MPS representations of parton wave functions [51–56] corresponding to candidate QSL Ansätze. Such calculations can be readily carried out within the TaSK framework. Exploring this direction in detail, however, lies beyond the scope of the present work and is left for future study.

Note added. Recently, we became aware of Ref. [57], which likewise compares the static and dynamical properties of two nearly degenerate ground states of the TLHAF in the intermediate J_2/J_1 regime, on cylinders of width 6–9. While both their and our works identify two nearly degenerate states and discuss their distinct features, the present work emphasizes a complementary perspective based on $SU(2)$ -symmetric MPS simulations paired with the tangent space Krylov approach, which enables a direct evaluation of the dynamical structure factor within the MPS tangent-space framework.

ACKNOWLEDGMENTS

We thank Eric Andrade, Carlo de Castro Bellinati, Andreas Gleis, Jheng-Wei Li, Wei Li, Bruce Normand, Frank Pollmann, Marc Ritter, and Wei Zhu for helpful discussions, and Markus Drescher for sharing the quasiparticle ansatz data. This work was supported in part by the Deutsche Forschungsgemeinschaft under Grants INST No. 86/1885-1 FUGG and LE No. 3883/2-2, and Germany’s Excellence Strategy EXC-2111 (Project No. 390814868). It is part of the Munich

Quantum Valley, supported by the Bavarian state government with funds from the Hightech Agenda Bayern Plus. The National Science Foundation supported JvD in part under Grant No. PHY-1748958.

DATA AVAILABILITY

The simulation data including plotting scripts are available via the zenodo repository at Ref. [58].

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