

Five-terminal quantized transconductance originating from symmetric quantum fluctuations

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We consider the renormalization of transport in a quantum contact by the external electromagnetic environment and show that weak quantum fluctuations from a symmetric environment can lead to a quantized transconductance in five-terminal contacts. This mechanism does not work for fewer terminals. We present an implementation of the environment and investigate several example quantum contacts where the quantization takes place.

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Quantum transport in nanostructures is conveniently described in terms of scattering of electron waves coming from and going to macroscopic terminals [1]. Interactions between electrons can, however, dramatically alter scattering. At low-energy scales the most important contribution comes from interactions with the voltage fluctuations provided by the external electromagnetic environment, with the coupling strength characterized by the dimensionless impedance $z \equiv (G_Q/2)Z(\omega)$, $G_Q \equiv e^2/\pi\hbar$. The effect of interaction was first described in the context of tunnel junctions [2–5]. At $z \rightarrow \infty$, it leads to the Coulomb blockade, i.e., complete insulation of the contacts. Most interestingly, it was shown that the blockade persists at $z \ll 1$ in the limit of small energy scales $E \sim \max(k_B T, eV)$, as defined by the temperature T and voltage V , across the junction. This behavior at low energies was later confirmed for two-terminal quantum contacts of arbitrary transparency [6–8].

In the scattering paradigm, the effect of electron-electron interactions in two-terminal contacts at $z \ll 1$ can be accounted for through the renormalization of the transmission coefficient T_i of channel i . Starting from an upper energy cutoff, E_{cut} , quantum fluctuations act to iteratively rescale T_i . The resulting renormalization equation expresses the energy dependence of the effective transmission coefficients [7,8],

$$\frac{dT_i}{d\xi} = -2zT_i(1 - T_i), \quad (1)$$

$\xi = \ln(E_{\text{cut}}/E)$. The only stable fixed point of this flow equation is the insulating point where all $T_i = 0$. Thus the equation describes the gradual suppression of transmission as $\xi \rightarrow \infty$ ($E \rightarrow 0$). The equivalent equation for a general z is not known for arbitrary transmissions.

Recently, we extended the study of environment-induced renormalization to multiterminal quantum contacts [9]. Remarkably, we found that quantum fluctuations coming from

environments that break time-reversal symmetry can lead to quantized transconductance (QTC) at low energies. The quantization arises from an integer number of fully transmitting chiral channels. While the effect is phenomenologically similar to the integer quantum Hall effect [10], it is of a different origin. QTC has also been predicted in junctions of three strongly interacting quantum wires described by the Tomonaga-Luttinger (TL) liquid theory [11]. The renormalization in TL models is qualitatively similar but quantitatively different from that in the environment model. There are also other proposals to realize quantized transconductance based on superconducting devices [12–14].

In this Letter, we show that electromagnetic environments where time reversibility is preserved, can also give rise to quantized transconductance. We show that the minimum number of terminals for this is five. We design a concrete environment suitable for this effect and demonstrate the QTC for several illustrative quantum contacts. The time-reversal symmetry breaking required for QTC comes from the contact properties. In our examples, the symmetry is broken by the magnetic flux penetrating the contact.

The setup we consider is shown in Fig. 1(a). A five-terminal quantum contact is connected to a time-reversible electromagnetic environment, which is generally described by a *symmetric* 5×5 impedance matrix $\hat{Z}(\omega)$. We assume that the environment is Ohmic, so that the dimensionless impedance matrix $\hat{z} \equiv (G_Q/2)\text{Re}\hat{Z}$ does not depend on frequency in a wide frequency range. In addition, we assume small $z \ll 1$, and find that the effective energy-dependent scattering amplitudes between channels i and j obey the following renormalization equation [9],

$$\frac{ds_{ij}}{d\xi} = z_{ji}s_{ij} - \sum_{k,l} z_{kl}s_{ik}(\hat{s}^\dagger)_{kl}s_{lj}, \quad (2)$$

where, without loss of generality, we assume a separate terminal for each channel.

The renormalization equation admits the following fixed points, $\hat{s}_{ij}^{(0)} = e^{i\theta_i^l} P_{ij} e^{i\theta_j^l}$, where $\hat{P}$ is a permutation matrix (that is, one with exactly one $P_{ij} = 1$ nonzero element in each row and each column) and θ_i^l, θ_j^l are arbitrary phases. The stability

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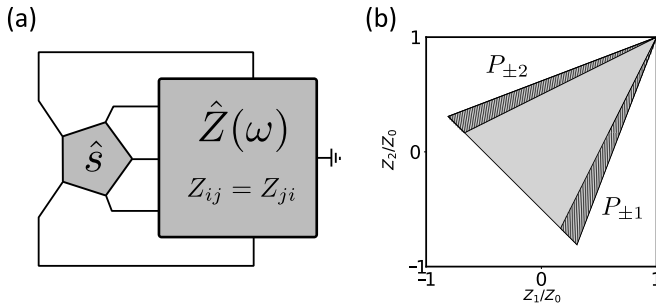


FIG. 1. (a) The setup under consideration. A five-terminal quantum contact, characterized by the scattering matrix $\hat{s}$, is connected to a time-reversible electromagnetic environment described by the frequency-dependent 5×5 symmetric impedance matrix $\hat{Z}(\omega)$. The high-frequency fluctuations in the environment can develop QTC in the contact. To uncouple the contact and environment at very low frequencies, which is required for transport measurements, the connection is realized via large capacitors not shown in the figure. (b) The region of positively defined impedance matrices (gray) overlapped by the region of stability (hatched) for $P_{\pm 1}$ and $P_{\pm 2}$ in the space of matrix elements Z_i . The stability region corresponds to amplifying Ohmic environments.

condition for a permutation fixed point reads

$$(\hat{z}\hat{P})_{ii} + (\hat{z}\hat{P})_{jj} - (\hat{z}\hat{P})_{ij} - (\hat{z}\hat{P})_{ji} > 0 \quad \forall i, j. \quad (3)$$

In addition, the impedance matrix must be positively defined (i.e., the environment is stable and passive). The two conditions are always met for the trivial insulating point $\hat{P} = \hat{1}$. In Ref. [9] we found nontrivial stable permutation points for an environment with broken time-reversal symmetry ($\hat{z}$ is asymmetric), which correspond to QTC. In this Letter, we examine the realization of QTC points for a time-reversible environment with a symmetric $\hat{z}$.

Permutation matrices can be decomposed into disjoint cycles, so that condition (3) is imposed on i, j within the same or different cycles. We found that cycles of length 2 are always unstable, whereas cycles of length 3 and 4 are stable only if $\hat{z}$ is asymmetric, so that only cycles of length 5 or more can be stable in symmetric environments. In what follows, we concentrate on cycles of length 5 in five-terminal contacts. There are many possible design choices for a 5×5 $\hat{z}$. For simplicity, we stick to a cyclically symmetric environment, such that $z_{i,i \pm n} \equiv z_n$, $\forall i$ ($n < 3$), so that the environment is completely characterized by three parameters $Z_{0,1,2}$. For such an environment, we show that there are four potentially stable QTC fixed points. Two of them, denoted by $P_{\pm 1}$, correspond to transmission between nearest-neighbor terminals, either in the clock- or counterclockwise direction, so the channels draw a pentagon. The other two, $P_{\pm 2}$, correspond to transmission between next-nearest-neighbor terminals, so the channels draw a five-pointed star.

Let us map the stability conditions [Fig. 1(b)] for $Z_{1,2}/Z_0$. The trivial reasoning on the positivity of $\hat{z}$ involving one terminal gives $Z_0 > 0$, and involving two terminals gives $|Z_{1,2}| < Z_0$. This yields a square in Fig. 1(b). A more accurate consideration involving more terminals gives three conditions on the three eigenvalues of $\hat{z}$, $Z_0 + 2Z_1 + 2Z_2 > 0$, $Z_0 \pm (\sqrt{5} \mp 1)/2 Z_1 \mp (\sqrt{5} \pm 1)/2 Z_2 > 0$, that carve a gray

triangle within the square. The nontrivial stability conditions (3) for the QTC fixed points described read

$$\begin{aligned} P_{\pm 1} : \quad & 2Z_1 > Z_0 + Z_2, \\ P_{\pm 2} : \quad & 2Z_2 > Z_0 + Z_1. \end{aligned} \quad (4)$$

These two conditions define the mutually exclusive hatched regions of interest on two sides of the triangle.

Let us discuss the realization of such environment. In general, the environment must be (a) symmetric, (b) amplifying, (c) passive, (d) broadband, and (e) must allow for low-frequency measurements on the contact. Criteria (c) and (d) are required for the applicability of the renormalization technique. Criterion (e) can be met by separating the environment and contact with large capacitor filters. The important criterion is (b). Most circuits assembled from passive elements are nonamplifying at low frequency: If a current is run between two contacts, the voltages of all other contacts lie between the voltages of the two. This is in contradiction with the general stability conditions (3) for nontrivial points. However, criterion (b) can be met by using (on-chip) transformer networks [9,15]. In the Supplemental material [16], we specify an implementation that comprises five resistors and 25 transformers realizing a general five-terminal cyclically symmetric environment.

With the environment implemented, we can achieve stable QTC fixed points. However, the insulating fixed point is also stable. Upon renormalization, an “initial” high-energy 5×5 scattering matrix $\hat{s}_{\text{in}}$ flows in the limit of low energy to either one of the two stable QTC points or the insulating point. To find out the probability for a scattering matrix to flow to the QTC points, we draw them at random from the circular unitary ensemble, and solve the flow equation numerically. The cyclically symmetric impedance matrix we made use of in this and all following numerical calculations was chosen to sit well within the stability region of the corresponding QTC points. For $P_{\pm 1}$, we work with the environment $\mathcal{E}_1$ where $Z_0, Z_1, Z_2 = z(1.57, 1.05, 0.26)$, and for $P_{\pm 2}$, we work with $\mathcal{E}_2$ where $Z_{1,2}$ are swapped.

We found the probability to be of the order of 10^{-5} . While a rather discouraging result, it is easy to understand why the probability is so small. Five terminals are crucial for the effect, and the matrices where most scattering is dominated by fewer terminals should flow to the insulating point. We show that simple engineering of the initial scattering matrix suffices to achieve the QTC fixed points.

First, the scattered waves should ideally be uniformly distributed among all five terminals. This implies that the initial scattering matrix must not have large reflection amplitudes for the conducting channels. Second, time reversibility must be broken: Since this is preserved by the flow equation, time-reversible $\hat{s}_{\text{in}}$ all flow to the insulating point. The first requirement is fulfilled by a design that makes possible propagation from any terminal to any other. The second requirement implies scattering trajectories sensitive to magnetic flux, that is, loops. There must also be sufficient probability to travel across such trajectories.

The required contacts can be implemented in a two-dimensional electron gas (2DEG) hosted in semiconductor heterostructures, an approach pioneered in Ref. [17]. By

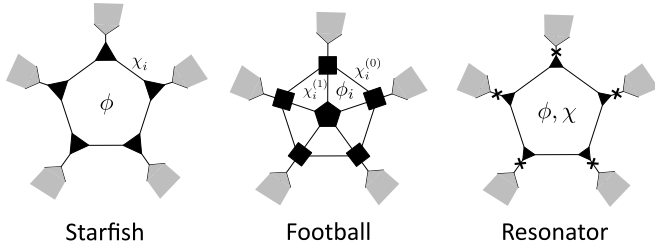


FIG. 2. Three setups proposed to demonstrate the QTC fixed points are built from beam splitters forming loops. Gray polygons represent the terminals, lines represent the propagation channels, and black n th-order polygons represent the beam splitters, corresponding to a unitary $n \times n$ matrix. In the starfish setup, electrons follow the loop unless exiting to the terminals underway. In the football setup, the central beam splitter provides more chances to explore the loops. The beam splitters in the resonator have a low entrance transparency, which is indicated by crosses. If a resonance condition is achieved, this guarantees electron trapping in the loop, so that it can explore all terminals.

using top electrodes to deplete the 2DEG, one can define a scattering region, while additional gates provide control over the scattering potential, and thus the scattering matrix, and dynamical phases [18]. This approach enables the realization of complex devices based on 2DEG-defined beam splitters [19], as well as loops [18,20,21] and multiterminal geometries [22,23]. Beyond 2DEG-based devices, similar structures can be realized using suspended graphene [24], or through more exotic approaches involving nanowires [25].

Based on these principles, we design three different setups building 5×5 scatterers from beam splitters realizing the loops. We dub them the starfish, football, and resonator (Fig. 2).

The starfish setup is composed of five 3×3 fully symmetric beam splitters, with a scattering matrix,

$$\hat{s}_3 = \frac{2}{3} \begin{pmatrix} -1/2 & 1 & 1 \\ 1 & -1/2 & 1 \\ 1 & 1 & -1/2 \end{pmatrix}. \quad (5)$$

The splitters are connected to form a loop that encloses the flux $\Phi = (\hbar/e)\phi$. Electrons propagating from the splitter i to $i+1$ (from $i+1$ to i) acquire a magnetic phase $\phi/5$ ($-\phi/5$), and a dynamical phase χ_i . The resulting scattering matrix $\hat{s}_{\text{in}}(\phi, \{\chi_i\})$ arises from interference between many scattering events as the electron travels through the nanostructure, and, crucially, is a function of the phases. We numerically solve the renormalization equation starting with $\hat{s}_{\text{in}}$ to the limit of low energies and determine the fixed point it converges to. This allows us to draw the convergence regions for the QTC fixed points in the space of ϕ and χ . The results are shown in Fig. 3. Figures 3(a) and 3(c) show the convergence regions to $P_{\pm 1}$ for the environment $\mathcal{E}_1$, while Figs. 3(b) and 3(d) illustrate the convergence to $P_{\pm 2}$, for $\mathcal{E}_2$. In Figs. 3(a) and 3(b), we set all dynamical phases to be the same, $\chi_i \equiv \chi$. At zero flux, $\chi = \pi/2$ minimizes the reflection coefficient. The convergence regions are therefore centered around $\chi = \pi/2$. As flux increases, the asymmetry of $\hat{s}_{\text{in}}$ favors either P_1 or P_{-1} . In Fig. 3(b), the convergence regions are narrower, since in the starfish setup the transmission to the nearest-neighbor ter-

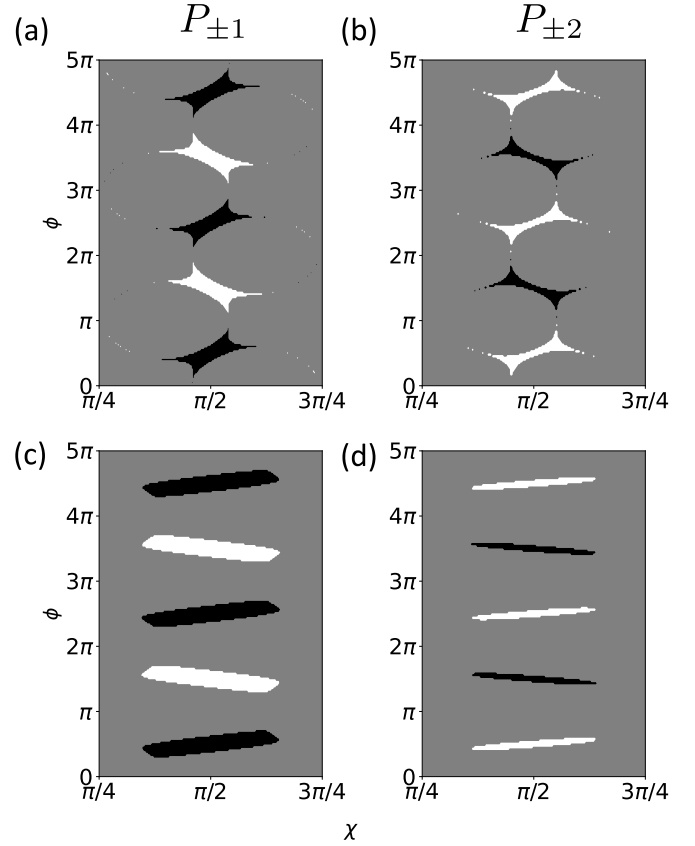


FIG. 3. The starfish setup. Convergence regions to QTC points of opposite chirality $\pm$ are shown in black and white, respectively, with the gray background indicating the flow to the insulating point. For (a) and (c), the environment is $\mathcal{E}_1$ favoring $P_{\pm 1}$, while in (b) and (d) the environment is $\mathcal{E}_2$ favoring $P_{\pm 2}$. For (a) and (b), all dynamical phases are chosen to be equal $\chi_i \equiv \chi$, whereas for (c) and (d) the choice is $\chi_0 \equiv \chi$, with the rest $\chi_{i \neq 0} = \pi/2$.

nals generally is larger than that to the next-nearest-neighbor terminals. Figures 3(c) and 3(d) show the convergence regions as a function of the dynamical phase between splitters 0 and 1, $\chi_0 \equiv \chi$, and flux. All other dynamical phases are set to the optimal value of $\pi/2$. This is why the regions in Fig. 3(c) are bigger than in Fig. 3(a). We observe peculiar shapes of the convergence regions for the symmetric configurations in Figs. 3(a) and 3(b). The shapes make cusps from where thin, quickly tapering filaments extend over some curves in the plane. Since the filaments are very thin, we had difficulty to resolve them with proper precision. These features are not seen for the less symmetric configurations in Figs. 3(c) and 3(d), so we regard those as artifacts of the high symmetry and do not pay special attention to them.

An obvious drawback of the starfish setup is the escape of an electron to the terminals while traversing the loop, so the chance to complete the loop is relatively small. We partially circumvent this in the football setup composed of five 4×4 reflectionless beam splitters connected in a loop and to a central 5×5 splitter. The phase diagram becomes more complex, with QTC convergence regions taking a bigger part of the phase space. We present the concrete results in the Supplemental Material [16].

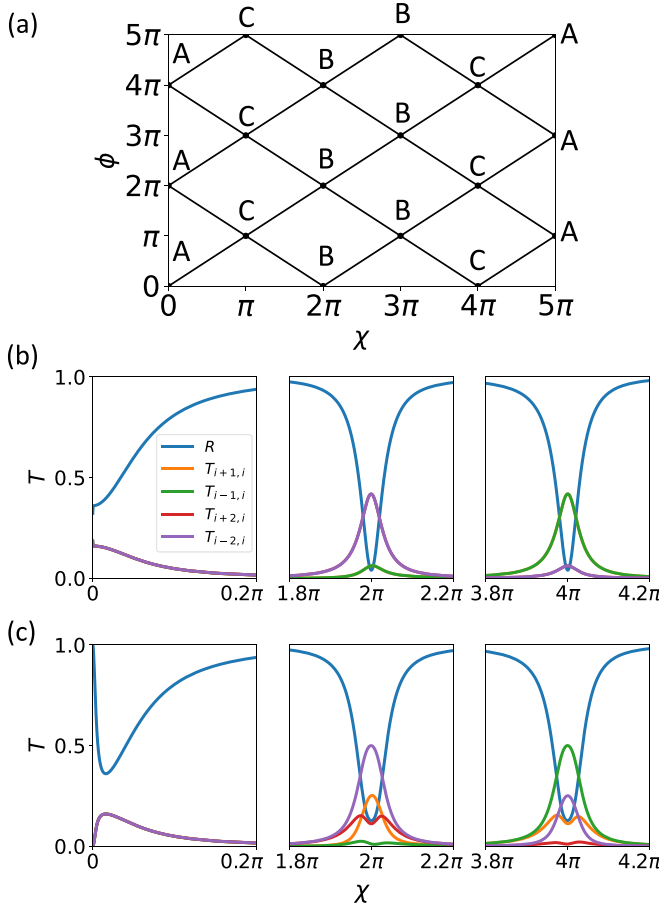


FIG. 4. The resonator setup. (a) The positions of the resonance levels vs dynamical, χ , and magnetic, ϕ , phases. There are three types of level crossings, A–C, as labeled. (b) Initial transmission coefficients in the vicinity of the resonance crossings for $\phi = 0$. (c) Initial transmission coefficients for $\phi = 0.01$ of the order of the small entrance transmission coefficient. Crossings A–C, are characterized by equal, next-nearest-neighbor, and nearest-neighbor transmission, respectively.

In the starfish and football setups electrons leave the contact relatively fast, which limits flux sensitivity and the number of terminals explored. This can be circumvented with the resonator setup, where the beam splitters have small entrance/exit amplitudes and are almost reflectionless within the channels that connect them to each other. Once an electron enters the loop, it spends a long time there, allowing it to feel the flux and explore all terminals. This only occurs at a resonant condition: the Fermi energy coinciding with one of the resonant energy levels formed in the loop. The resonance condition is fulfilled at $\phi \pm \chi = 2\pi N$, with N an integer and ϕ, χ the magnetic and dynamical phase acquired upon traversing the loop once. The reflection coefficient is smallest at the crossings between two resonances. The resonance positions as a function of phases are shown in Fig. 4(a). There are three types of crossings labeled A–C. The difference between them is best illustrated in Fig. 4(b), where we plot the transmission coefficients $T_{i,j}$ as a function of χ at $\phi = 0$. At an A crossing, the transmission coefficients are all the same, while at a B crossing, $T_{i\pm 1, i} < T_{i\pm 2, i}$, and at a C crossing, $T_{i\pm 1, i} > T_{i\pm 2, i}$.

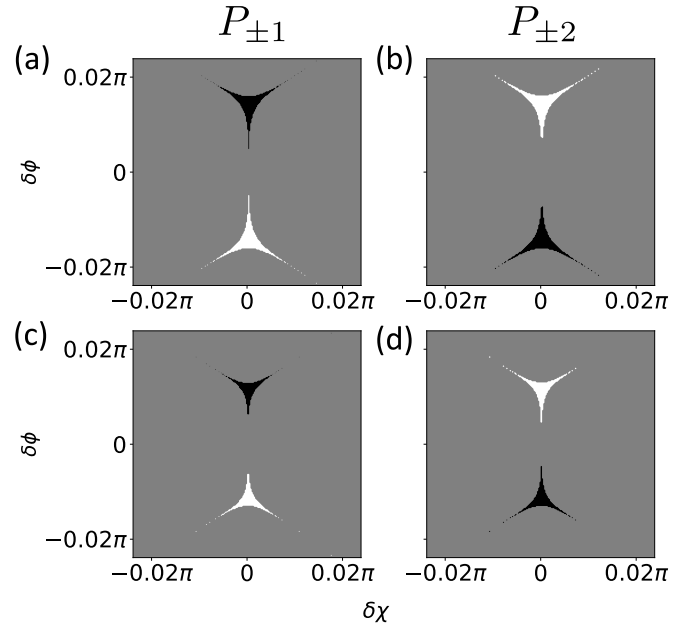


FIG. 5. The resonator setup. Convergence diagram. (a) and (c) show the convergence to $P_{\pm 1}$ in the vicinity of a C crossing, while (b) and (d) show the convergence to $P_{\pm 2}$ near a B crossing. The flux and dynamical phase are counted from the crossing positions. The scale of the plots is the small transmission coefficient T_s . Concretely, $T_s = 0.04$ for (a) and (b) and $T_s = 0.03$ for (c) and (d).

Figure 4(c) illustrates the transmission coefficients near the crossings at finite small flux $\phi = 0.01$ (of the order of the small entrance transmission coefficient). Near an A crossing, the flux simply shifts the resonance introducing no asymmetry. The flow to the insulating fixed point is therefore expected near these crossings. The flux does introduce asymmetry near B and C crossings, enabling the flow to $P_{\pm 2}$ and $P_{\pm 1}$, respectively, as confirmed by numerical calculations.

We explore the convergence to the QTC points in the vicinity of a C crossing, $\chi, \phi = \pi$, and a B crossing, $\chi = 2\pi, \phi = 0$, choosing the environments $\mathcal{E}_1, \mathcal{E}_2$, respectively. In computations, tunneling to and from the leads is modeled by the 3×3 scattering matrix,

$$\hat{s}_r = \begin{pmatrix} \cos(2\delta) & \frac{i}{\sqrt{2}} \sin(2\delta) & \frac{i}{\sqrt{2}} \sin(2\delta) \\ \frac{i}{\sqrt{2}} \sin(2\delta) & -\sin^2 \delta & \cos^2 \delta \\ \frac{i}{\sqrt{2}} \sin(2\delta) & \cos^2 \delta & -\sin^2 \delta \end{pmatrix}, \quad (6)$$

and the small entrance transmission coefficient $T_s = 2\delta^2$.

The convergence regions are shown in Fig. 5. Figures 5(a) and 5(c) correspond to the vicinity of a C crossing (where $P_{\pm 1}$ are stable), while Figs. 5(b) and 5(d) correspond to the vicinity of a B crossing (where $P_{\pm 2}$ are stable), $\delta\phi, \delta\chi$ being the deviations of the magnetic and dynamical phase from the crossing position. In all plots, the scale of the convergence regions in both $\delta\phi$ and $\delta\chi$ is determined by T_s . The centers of the black and white regions are separated by $\Delta\phi \approx 2T_s$, and their width is correspondingly $\Delta\phi, \Delta\chi \approx T_s/4$. We also notice the shape peculiarities of the region: Similar to the shapes in Figs. 3(a) and 3(b), there are filaments extending and tapering along the lines $\delta\chi = 0$, and two slanted lines

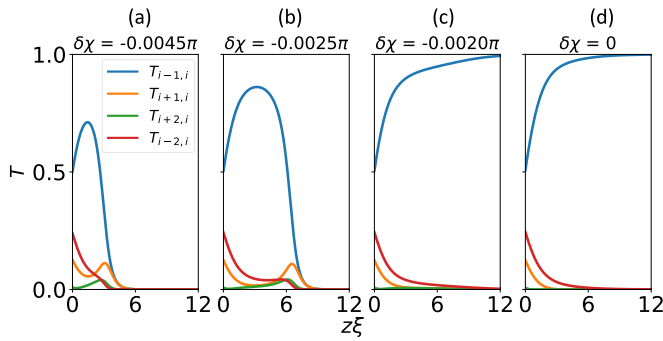


FIG. 6. The resonator setup. The renormalization flow for a selection of parameters in Fig. 5(a). We plot the transmission coefficients vs the renormalization parameter $z\xi$.

(not parallel to the resonant level positions). We regard it as an artifact of the high symmetry and do not investigate these in detail. The plots on the left (for $P_{\pm 1}$) and on the right (for $P_{\pm 2}$) have the same shape in the limit of small T_s , the relative difference being of the order of T_s . This is because of almost reflectionless propagation along the loop. To make the scaling with T_s visible, we use slightly different T_s in Figs. 5(a) and 5(b) ($T_s = 0.04$) and in Figs. 5(c) and 5(d) ($T_s = 0.03$).

In Fig. 6 we present the renormalization flow near a C crossing with the parameters corresponding to Fig. 5(a). The flux is chosen in the middle of the white region $\delta\phi = -0.015\pi$, while dynamical phase settings are deep in the gray region [Fig. 6(a)], very close to the boundary from the gray side [Fig. 6(b)], very close to the boundary from the white side [Fig. 6(c)], and in the middle of the white region [Fig. 6(d)]. For all settings, we plot the transmission coefficients as a function of $z\xi$. $T_{i\pm n,i}$ are the same for all i , and that does not change with the flow. In Fig. 6(a), all coefficients go to 0 in the low-energy limit, approaching the insulating point. As the dynamical phase approaches the stability threshold, the flow becomes slower. Above the threshold, $T_{i-1,i}$ quickly flows to 1, approaching P_{-1} . The traces in Figs. 6(b) and 6(c) are very similar in an interval $z\xi = [0, 4]$.

The resonator setup looks promising, yet two points need to be clarified. First, the small entrance transmission coefficient results in a Coulomb blockade, which we formally did not account for. We think it is not important, since the Coulomb blockade only shifts the resonant level positions and thus can be compensated with proper gates. Second, the resonance

implies the energy dependence of the scattering matrix on an energy scale of the order of the resonant level width. By virtue of our renormalization approach, this energy scale should be *larger* or of the order of the upper cutoff energy for the renormalization. This condition can always be met by choosing very short contacts.

Finally, we express an important note about the setups presented: They do not have to be realized literally, by carving actual loops and channels. The only important condition is the exploration of the whole nanostructure by the scattered electrons. For instance, the loops can be replaced by continuous propagation regions, such as a disk connected to five terminals. All flux effects except the periodicity should be preserved in these setups.

Conclusion and discussion. In conclusion, we have demonstrated that quantum fluctuations of a time-reversible Ohmic environment may lead to quantized transconductance in the limit of low energies. This is the result of the stability of nontrivial fixed points in the renormalization flow. Rather ironically, one needs at least five-terminal quantum contacts and environments to observe this phenomenon. We concentrated on a cyclically symmetric environment and derived the stability criteria for the QTC fixed points in terms of the characteristic impedances. We have shown how to realize the required environment from resistors and transformers. Importantly, we found examples of quantum contacts that converge to the QTC fixed points upon renormalization. This facilitates the experimental observation of the phenomenon.

We note that this topic motivates further research. Our consideration was limited to the simplest case of one channel per terminal. It is interesting to investigate the case of multiple channels, in particular the semiclassical case of many channels, to extend to bigger nanostructures. The permutation group suggests the existence of even more complex QTC fixed points, and it would be interesting to identify those in such nanostructures. Furthermore, we restricted our analysis to small dimensionless impedances of the environment. Assessing the situation at bigger impedances is a challenging theoretical problem. In principle, the QTC fixed points must remain fixed at any impedance, while the stability criteria can be very different.

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Data availability. The data that support the findings of this article are openly available [26].

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