

Nonreciprocal Goos-Hänchen and photonic spin Hall shifts driven by incident angle and polarization at a planar interface

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Most studies on Goos-Hänchen and photonic spin Hall beam shifts are focused on reciprocal, or direction-independent, beam shifts. In this work, we present a strategy to induce nonreciprocal beam shifts in the reflected field from a planar heterostructure by exploiting direction- and polarization-dependent coupling between the incident field and the structure. We show that the heterostructure's resonant modes lead to polarization-dependent, nonreciprocal reflection at specific incident angles. This behavior enables controlled tuning of the direction-dependent transformation of optical momentum. Consequently, we observe nonreciprocal Goos-Hänchen and photonic spin Hall shifts in the reflection plane, manifesting as lateral and transverse displacements, respectively. These shifts arise from the conversion of linear momentum and orbital angular momentum into spin angular momentum. The results are verified using three independent approaches—comsol multiphysics simulations, the stationary phase method, and Gaussian beam scattering calculations—with excellent agreement among them.

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I. INTRODUCTION

When an electromagnetic field is scattered from a planar interface, the scattered plane may undergo a lateral shift, namely, a Goos-Hänchen (GH) shift [1,2], and a transverse spin-dependent shift, known as an Imbert-Fedorov or photonic spin Hall (SH) shift [3]. These lateral and transverse shifts in the scattered plane occur due to the transformation of optical linear and angular momentum [4–6], respectively. Because of momentum exchange between incident and scattered fields [3,7], the GH shift occurs parallel to the plane of incidence, while the SH shift occurs perpendicular to the plane of incidence [8]. As a result, the scattered field appears to deviate from the original positions [9,10]. These photonic shifts have been extensively investigated, both theoretically and experimentally, in various media such as epsilon near zero (ENZ) metamaterials [11–13], absorptive and lossy dielectric materials [14–16], photonic crystals [17–19], bianisotropic structures [20–26], quasibound states in the continuum [27,28], and nonreciprocal (NR) photonic crystals [29,30]. These shifts have potential applications in diverse fields, including measurement of the rate of chemical reaction

[31], chiral resolution [10], slab thickness measurement [32], optical conductivity measurement [33], and biosensing [34], to name a few.

To understand the origin of beam shifts, different methods were employed, such as the transformation of the optical momentum [10,35]. When a linearly polarized field scatters from a generic interface, an exchange of linear momentum between the incident field and the reflected field takes place. This interaction leads to positive or negative GH shifts depending on whether the reflected signal loses or gains energy, respectively [14,36]. As an example, in Fig. 1(a), we schematically illustrate a positive GH shift in the reflected field for the NR multilayered structure. In contrast, the SH shift appears due to the splitting of a linearly polarized incident field into the right-handed circularly polarized (RCP) and left-handed circularly polarized (LCP) components of the reflected field, as schematically illustrated in Fig. 1(b). The SH shift is directly related to the spin-orbit interaction of light; a stronger spin-to-orbital angular momentum conversion leads to a larger SH shift [35,37].

Generally, GH and SH shifts are small compared to the incident wavelength, making their experimental measurement challenging. Tremendous efforts have been made to enhance these shifts to enrich their applications [25,38–40]. For instance, large shifts are measured near Brewster angle [41], total internal reflection [42,43], ENZ materials [44,45], PT-symmetric materials [20,21], grating structure [46], and anisotropic materials [26,45,47,48]. The investigation of the beam shift discussed in these works is direction independent due to reciprocal scattering behavior, where the scattering response does not depend on the propagation direction of the incident beam. However, there are limited studies that explore

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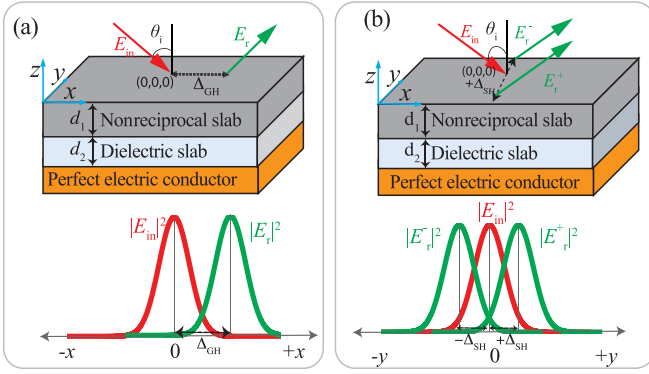


FIG. 1. (a) Schematic illustration of NR planar structures, where an incident field impinges obliquely at an incident angle θ_i onto a NR slab of thickness d_1 , which is placed over a dielectric slab of thickness d_2 . The bottom slab is a perfect reflector. The reflected field is subjected to the GH shift represented by Δ_{GH} . The bottom figure illustrates the lateral distance of the reflected field intensity from the incident field intensity. (b) With the same planar structure, the reflected field is subjected to the SH shift represented by Δ_{SH} . The bottom figure shows the shift of each spin component of the scattered field intensity along the transverse direction with respect to the incident field intensity.

the NR GH shift, particularly those arising from asymmetric reflection in photonic crystals [29,30].

Recently, it has been shown that NR scattering by a planar structure can be achieved by applying the external magnetic field parallel to the incident plane and tuned by adjusting the geometrical parameters [49–54]. For the experimental realization of such structures, Weyl semimetals, such as WP_2 , $Y_2Ir_2O_7$, $HgCr_2Se_4$, TaAs, and $Co_3Sn_2S_2$, exhibit magneto-optical properties [55] and are potential candidates for observing the nonreciprocity in the visible frequency region. They are widely used for different applications, such as bidirectional phase modulation [51,56,57] and magnetic Hall effect [50,58]. In this work, by considering an NR planar heterostructure, we aim to explore direction- (incident angle) and polarization-dependent NR reflection. We numerically (using the commercial software comsol multiphysics) show that the NR reflection is enabled by the selective coupling of the incident field with the structure at specific combinations of incident angle and polarization. This NR reflection provides a promising platform for achieving strong spin-orbit interaction [8,10,34], which leads to both linear and angular momentum transformation. By tailoring the incident angle, we optimize GH and SH beam shifts in the reflected field through coherent control of the nonreciprocity. In addition, we investigate these beam shifts by considering experimentally viable Gaussian beams and demonstrate the real-space displacement of the scattered fields.

This paper is organized as follows. In Sec. II, we describe the theoretical framework, including the transfer matrix method and the expressions of beam shifts. In Sec. III, we present the main calculated results and compare the stationary phase approach with comsol simulations. Additionally, we test the experimental viability of our approach by comparing the analytical results obtained via the stationary phase method with those derived from Gaussian beam scattering. Finally, in Sec. IV, we conclude our findings.

II. METHODS

We consider a planar structure composed of three layers: an NR slab of thickness d_1 (top layer), a dielectric slab of thickness d_2 (middle layer), and a perfect reflecting medium (bottom layer) as shown in Fig. 1. To characterize the system, the relative permittivity tensor of the NR slab is defined as

$$\bar{\epsilon}_{NR} = \begin{pmatrix} \epsilon_{xx} & 0 & \epsilon_{xz} \\ 0 & \epsilon_{yy} & 0 \\ \epsilon_{zx} & 0 & \epsilon_{zz} \end{pmatrix}, \quad (1)$$

where $\epsilon_{xx} = \epsilon_{yy} = \epsilon_{zz} = \epsilon_d$, and $\epsilon_{xz} = -\epsilon_{zx} = i\epsilon_a$ is a purely imaginary number. The nonzero ϵ_{xz} can be obtained by applying a magnetic field on Weyl semimetals along the y direction [49]. The dielectric slab has $\epsilon_2 = 2.25$. We consider an electric field $\mathbf{E} = (E_x, E_y, E_z) \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)]$ incident onto the top of the structure at $(0,0,0)$, i.e., the NR layer, with an incident angle θ_i ranging from -90° to 90° , as illustrated in Fig. 1. Here, $\mathbf{k}$ is the wave vector and ω is the angular frequency, and E_x , E_y , and E_z are the electric field components along the Cartesian coordinate axes. To find the electromagnetic field in the NR medium, we solve the Maxwell equations $\nabla \times \mathbf{H} = \epsilon_0 \bar{\epsilon}_{NR} \cdot \partial_t \mathbf{E}$ and $\nabla \times \mathbf{E} = -\mu_0 \partial_t \mathbf{H}$ in the Cartesian coordinate system, where ϵ_0 and μ_0 are the permittivity and permeability of vacuum, respectively. The incident wave is propagating in the xz plane with wave-vector components $k_x = n_1 k_0 \sin \theta_i$ and $k_z = n_1 k_0 \cos \theta_i$, where n_1 is the relative refractive index of the background medium, $k_0 = 2\pi/\lambda_0$ and λ_0 is the incident wavelength in vacuum. Our structure exhibits medium discontinuity along the z axis; the electromagnetic fields across the interface can be determined by applying the continuity conditions. This allows determining the fields E_x , E_y , H_x , and H_y in the NR medium (see details in Appendix A) as

$$\partial_z \begin{pmatrix} E_x \\ E_y \\ H_x \\ H_y \end{pmatrix} = i\mathbf{M} \cdot \begin{pmatrix} E_x \\ E_y \\ H_x \\ H_y \end{pmatrix}, \quad (2)$$

with

$$\mathbf{M} = \begin{pmatrix} -\epsilon_{zx}k_x/\epsilon_d & 0 & 0 & \omega\mu_0 - k_x^2/\epsilon_0\omega\epsilon_d \\ 0 & 0 & -\mu_0\omega & 0 \\ 0 & k_x^2/\omega\mu_0 - \omega\epsilon_0\epsilon_d & 0 & 0 \\ \omega\epsilon_0\epsilon_v & 0 & 0 & -\epsilon_{xz}k_x/\epsilon_d \end{pmatrix},$$

where $\epsilon_v = \epsilon_d - \epsilon_{zx}\epsilon_{xz}/\epsilon_d$. Since the incident field impinges on the NR slab at $z = 0$, by integrating the above equation over z , we get the field across the slab interfaces as

$$\begin{pmatrix} E_x(\Delta z) \\ E_y(\Delta z) \\ H_x(\Delta z) \\ H_y(\Delta z) \end{pmatrix} = \exp(i\mathbf{M}\Delta z) \cdot \begin{pmatrix} E_x(0) \\ E_y(0) \\ H_x(0) \\ H_y(0) \end{pmatrix}, \quad (3)$$

where Δz is the path of the electric field that passes through the NR medium along the z axis. In our configuration shown in Fig. 1, and the incident wave carries a k_z along the $-z$ direction. Thus, we have $\Delta z = -d_1$, where d_1 is the thickness of the NR slab. The propagation matrix for the NR slab $\mathbf{P}_1 = \exp(-i\mathbf{M}d_1)$ can be expressed as (see details in Appendix B)

$$\mathbf{P}_1 = \begin{pmatrix} \cos(d_1 k_{zp1}) - \frac{i\epsilon_{xz}k_x \sin(d_1 k_{zp1})}{\epsilon_d k_{zp1}} & 0 & 0 & -\frac{ik_{zs1}^2 \sin(d_1 k_{zp1})}{\omega\epsilon_0\epsilon_d k_{zp1}} \\ 0 & \cos(d_1 k_{kzs1}) & \frac{i\mu_0\omega \sin(d_1 k_{zp1})}{k_{zs1}} & 0 \\ 0 & \frac{ik_{zs1} \sin(d_1 k_{zs1})}{\mu_0\omega} & \cos(d_1 k_{kzs1}) & 0 \\ -\frac{i\omega\epsilon_0\epsilon_v \sin(d_1 k_{zp1})}{k_{zp1}} & 0 & 0 & \cos(d_1 k_{zp1}) + \frac{i\epsilon_{xz}k_x \sin(d_1 k_{zp1})}{\epsilon_d k_{zp1}} \end{pmatrix},$$

where $k_{zs1} = \sqrt{\epsilon_d k_0^2 - k_x^2}$ and $k_{zp1} = \sqrt{\epsilon_v k_0^2 - k_x^2}$ are wave numbers for s - and p -polarized fields, respectively. Note that the k_{zs1} does not involve off-diagonal elements of the permittivity tensor. Thus, the s -polarized field is rotationally symmetric and does not exhibit NR scattering. On the other hand, k_{zp1} depends on ϵ_{xz} , and the scattering response will depend on the direction of incidence.

The propagation matrix carries all polarization information (such as s and p polarization) passing through the NR slab. In each layer of our multilayered system, the propagation matrix is defined similarly by accommodating the appropriate relative permittivity. The propagation matrix $\mathbf{P}_2$ for the dielectric slab of thickness d_2 can be obtained by taking an isotropic permittivity $\epsilon_2 \mathbf{I}$, where $\mathbf{I}$ is the identity matrix. Since the bottom layer of the structures is a perfect reflector and no transmitted wave in this system, our focus lies on the reflected field. To find the s - and p -polarized reflection coefficients along a generic direction θ_r ($\theta_r = \theta_i$ as required by Snell's law), we rotate the incident fields E_x and E_y to define the s - and p -polarized fields as follows:

$$\begin{pmatrix} E_x \\ E_y \\ H_x \\ H_y \end{pmatrix} = \mathbf{R}(n, \theta) \cdot \begin{pmatrix} a_s \\ a_p \\ r_s \\ r_p \end{pmatrix}, \quad (4)$$

where

$$\mathbf{R}(n, \theta) = \begin{pmatrix} 0 & \cos \theta & 0 & -\cos \theta \\ 1 & 0 & 1 & 0 \\ n \cos \theta \sqrt{\frac{\epsilon_0}{\mu_0}} & 0 & -n \cos \theta \sqrt{\frac{\epsilon_0}{\mu_0}} & 0 \\ 0 & -n \sqrt{\frac{\epsilon_0}{\mu_0}} & 0 & -n \sqrt{\frac{\epsilon_0}{\mu_0}} \end{pmatrix},$$

and a_σ and r_σ ($\sigma = s, p$) are the incident field and reflected field coefficients. We can then define the transmission t_σ and reflection r_σ coefficients of the entire structure (excluding the

perfect reflector) as

$$\begin{pmatrix} t_s \\ t_p \\ 0 \\ 0 \end{pmatrix} = \mathbf{R}^{-1}(n_3, \theta_3) \cdot \mathbf{P}_2 \cdot \mathbf{P}_1 \cdot \mathbf{R}(n_1, \theta_1) \cdot \begin{pmatrix} a_s \\ a_p \\ r_s \\ r_p \end{pmatrix}, \quad (5)$$

where $n_3 = 10^3 i$ is the refractive index used to model the perfect electric conductor, θ_3 is the transmission angle at the interface between the dielectric slab and the perfect reflector, and $\mathbf{T} = \mathbf{R}^{-1}(n_3, \theta_3) \cdot \mathbf{P}_2 \cdot \mathbf{P}_1 \cdot \mathbf{R}(n_1, \theta_1)$ is the transfer matrix. Finally, the normalized reflection coefficients for the s - ($a_s = 1, a_p = 0$) and p - ($a_s = 0, a_p = 1$) polarized waves can be defined as $r_{ss} = T_{31}/T_{33}$ and $r_{pp} = T_{42}/T_{44}$, respectively. It is important to note that due to the particular configuration of the permittivity tensor, the cross-polarized reflection coefficients (i.e., r_{sp}, r_{ps}) are zero. Using r_{ss} and r_{pp} , we can define the reflection amplitudes of the copolarized reflected fields as

$$R_{pp} = |r_{pp}|^2, \quad (6)$$

$$R_{ss} = |r_{ss}|^2. \quad (7)$$

Using these reflection coefficients, we can investigate the NR beam shifts driven by anisotropic permittivity. Without loss of generality, we define the beam shifts by considering the incident field of a large beam waist $w_0 \gg \lambda_0$ with a narrow angular spectrum. Under this assumption, we use Artmann's formalism [2] or the stationary phase method to define the maximum possible shifts that are independent of w_0 (see details in Appendix C). According to the stationary phase theory, the lateral shift in the reflected fields can be defined as [1,2]

$$\Delta_{\text{GH}}^\sigma = -\frac{1}{k_0} \frac{\partial \phi_\sigma}{\partial \theta_i}, \quad (8)$$

where $\phi_\sigma = \arg(r_{\sigma\sigma})$ and $\sigma = s, p$. Using the above equation, it is convenient to define the Goos-Hänchen shift as

$$\Delta_{\text{GH}}^\sigma = -\frac{1}{k_0} \frac{1}{R_{\sigma\sigma}} \left\{ \text{Re}[r_{\sigma\sigma}] \frac{d \text{Im}[r_{\sigma\sigma}]}{d(\theta_i)} - \text{Im}[r_{\sigma\sigma}] \frac{d \text{Re}[r_{\sigma\sigma}]}{d(\theta_i)} \right\}, \quad (9)$$

where $\text{Re}[\cdot]$ and $\text{Im}[\cdot]$ denote the real and imaginary parts of the complex function $[\cdot]$, respectively.

On the other hand, the SH shift appears due to the splitting of the incident field into LCP and RCP fields along the transversal direction (along the y axis) as shown in Fig. 1(b) (bottom figure), where the centroids of the scattered field intensity distribution (green) are shifted away from the centroid of the incident field intensity (red). This splitting is a direct consequence of spin-orbit interaction, where the orbital angular momentum transforms into nonzero spin angular momentum. This splitting, or photonic SH shift of light, compensates for the conservation of total angular momentum in the normal direction. In accordance with the geometric prediction, a spin-dependent splitting of the reflected field can be defined as

$$\Delta_{\text{SH}}^{H,V} = \frac{\iint y E_{r,\pm}^{H,V} dy_r dz_r}{\iint E_{r,\pm}^{H,V} dy_r dz_r}, \quad (10)$$

where the positive (+) and negative (−) signs represent the right- and left-circularly polarized reflected fields, respectively. Accommodating horizontally H -polarized and vertically V -polarized fields, one can get the expressions of the SH shifts [38–40,59] as

$$\Delta_{\text{SH}}^H = \pm \frac{\lambda_0}{2\pi} \left[1 + \frac{|r_{ss}|}{|r_{pp}|} \cos(\phi_s - \phi_p) \right] \cot \theta_i, \quad (11)$$

$$\Delta_{\text{SH}}^V = \pm \frac{\lambda_0}{2\pi} \left[1 + \frac{|r_{pp}|}{|r_{ss}|} \cos(\phi_p - \phi_s) \right] \cot \theta_i. \quad (12)$$

These expressions are only valid for a large beam waist and give the maximum possible SH shifts (see details in Appendix C). Note that the strong coupling of the incident light of polarization p - and s -polarized with the structure leads to large SH shifts in the H - and V -polarized fields, respectively.

III. RESULTS AND DISCUSSION

We consider an incident field of vacuum wavelength $\lambda_0 = 1 \mu\text{m}$ impinging on the planar structure at angle θ_i as shown in Fig. 1, and the reflected field is subjected to the GH and SH shift, as illustrated in Figs. 1(a) and 1(b), respectively. We fix other parameters as $n_1 = 1$, $\epsilon_d = 9 + 0.03i$, $d_1 = 0.1 \mu\text{m}$, and $d_2 = 0.32 \mu\text{m}$. Figures 2(a) and 2(b) show the analytically calculated, using Eqs. (6) and (7), respectively, copolarized reflection amplitudes as functions of the incident angle and off-diagonal terms of the permittivity tensor ϵ_a . Figure 2(a) exhibits an asymmetric behavior indicating that the p -copolarized reflected amplitude is not symmetric for the incident angle, i.e., $R_{pp}(\theta_i) \neq R_{pp}(-\theta_i)$, where $R_{pp}(\theta_i)$ and $R_{pp}(-\theta_i)$ represent the reflection amplitudes when the incident field is coming from the left and right, respectively. Our analytical formula, Eq. (B4), predicts such asymmetric reflection because k_{zp1} depends on the anisotropic parameter ϵ_{xz} . In contrast, the s -copolarized reflected amplitude is symmetric and does not depend on the incident direction, i.e., $R_{ss}(\theta_i) = R_{ss}(-\theta_i)$, because k_{zs1} does not depend on ϵ_{xz} . Thus, only the p -polarized field is subjected to NR reflection. Note that Fig. 2(a) shows that the reflected amplitude vanishes at $\{\theta_i = -41.7^\circ, \epsilon_a = 12\}$, meaning all incident field is coupled with the structure. On the other hand, the reflection amplitudes

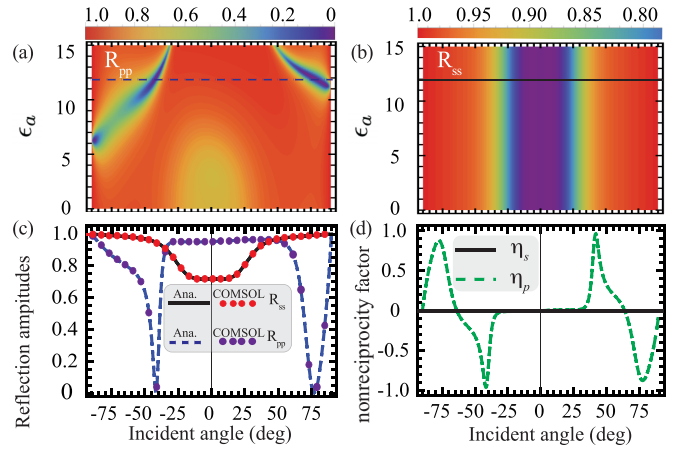


FIG. 2. Normalized reflection amplitudes for the NR planar structure: phase plots of (a) R_{pp} and (b) R_{ss} as functions of the incident angle and the off-diagonal component ϵ_a of the NR permittivity tensor. The reflection amplitudes R_{pp} along the horizontal dashed line in (a) and R_{ss} along the horizontal black line in (b) are plotted in (c) as a function of the incident angle, where $\epsilon_a = 12$. Here, the solid and dashed lines are obtained from Eqs. (6) and (7), while the circular markers represent numerical results calculated using COMSOL multiphysics. (d) The nonreciprocity factors $\eta_p(\theta_i) = R_{pp}(\theta_i) - R_{pp}(-\theta_i)$ and $\eta_s(\theta_i) = R_{ss}(\theta_i) - R_{ss}(-\theta_i)$ are shown as a function of the incident angle.

for the left incident field at $\{\theta_i = +41.7^\circ, \epsilon_a = 12\}$ are close to 1. Thus, the large nonreciprocity factor $\eta_p(\theta_i) = R_{pp}(\theta_i) - R_{pp}(-\theta_i)$ is achieved.

In Fig. 2(c), we calculate the reflection amplitudes as a function of incident angle for p -polarized (dashed) and s -polarized (solid black) incident fields with fixed $\epsilon_a = 12$, as marked by the horizontal lines in Figs. 2(a) and 2(b). To verify our analytical formula, we numerically calculate the reflection amplitudes as a function of incident angle using COMSOL multiphysics, a finite-element package, and plot in Fig. 2(c) in solid dots. Good agreement is seen between the analytical and numerical approaches. The reflection amplitude of the p -polarized field exhibits an asymmetric behavior, especially at $\theta_i = \pm 41.7^\circ$, where $R_{pp}(\theta_i = -41.7^\circ) \approx 0$ and $R_{pp}(\theta_i = 41.7^\circ) \approx 1$ and the nonreciprocity factor is nearly unity $|\eta_p(\theta_i = \pm 41.7^\circ)| \approx 1$. In Fig. 2(d), we plot $\eta_o(\theta_i)$ as a function of the incident angle for the s -polarized field (black curve) and p -polarized field (dashed curve). It clearly shows that while the s -polarized wave has symmetric reflection, the p -polarized wave exhibits strong asymmetric reflection at several incident angles, such as $\theta_i = -41.7^\circ$ and $\theta_i = 77^\circ$. To get deeper insight into the direction-dependent coupling of the incident field, in Fig. 3 we plot the numerically calculated field distributions by setting the incident angle at $\theta_i = \pm 41.7^\circ$. Figures 3(a) and 3(b) show the real part of the magnetic field for a p -polarized wave incident from the left and from the right, respectively. Figure 3(b) shows that the p -polarized field incident from the right fully couples to the slab, and this direction-dependent coupling leads to a large value of $\eta_p(\theta_i)$. However, for the s -polarized incident beam, the reflection behavior remains reciprocal, i.e., independent of the incident direction, as shown in Figs. 3(c) and 3(d). Therefore, only

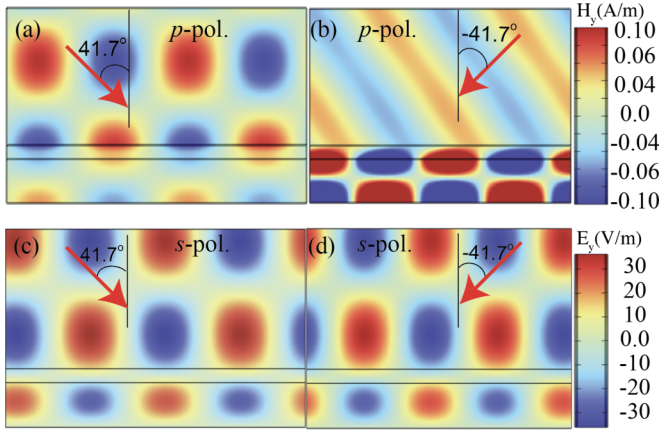


FIG. 3. The magnetic field distributions when the p -polarized incident field incidence forms (a) left and (b) right. The electric field distributions when the s -polarized incident field is impinging from (c) left and (d) right. All other parameters are the same as those in Fig. 2(c).

the p -copolarized reflected field experiences both enhanced lateral and transverse beam shifts, which are predicted by Eqs. (9) and (11), respectively.

We calculate the GH shift using Eq. (9) for the NR structure with $\epsilon_a = 12$ and for an isotropic structure with $\epsilon_a = 0$, and plot the results in Fig. 4. When the p -polarized wave impinges from the right (negative angle), Fig. 4(a) shows that Δ_{GH}^p is small [$\Delta_{\text{GH}}^p(\theta_i = -41.7^\circ) \approx \lambda_0$] for the isotropic case. Whereas for the anisotropic case, when $|\eta(\theta_i = -41.7^\circ)| \approx 1$, the GH shift $\Delta_{\text{GH}}^p(\theta_i = -41.7^\circ) \approx -52\lambda_0$. When the p -polarized wave impinges from the left (positive angle), Fig. 4(b) shows that Δ_{GH}^p is small at $\theta_i = 41.7^\circ$; this is because the incident field is not coupled with the structure as shown in Fig. 3(a), i.e., $R_{pp}(\theta_i = 41.7^\circ) \approx 1$. However, a large GH shift also appears at $\theta_i \approx 77^\circ$ [$\Delta_{\text{GH}}^p(\theta_i = 77^\circ) \approx 16\lambda_0$] where the nonreciprocity factor $|\eta(\theta_i = 77^\circ)| \approx 0.89$. To reveal the effect of the reflection phase (ϕ_p) on the GH shift, we plot ϕ_p in the insets of Figs. 4(a) and 4(b). The results show that near the maximum coupling angles, where the reflectance approaches zero, the phase varies continuously with a large slope, as marked by vertical dashed lines. Thus, a large shift is found. In contrast, the smooth phase variation for the isotropic case ($\epsilon_a = 0$), results in a weaker GH shift. Since the incident angle is positive, the resulting large GH shift is also positive. The sign of the GH shift is determined by the direction of illumination. It is important to mention that the fundamental relation $\Delta_{\text{GH}}^\sigma(-\theta_i) = -\Delta_{\text{GH}}^\sigma(\theta_i)$ is only valid for the reciprocal system, as shown by the dashed lines in Figs. 4(a) and 4(b). For an NR system, such a relation is not necessarily valid. Figure 4(c) shows the GH shifts for s -polarized waves as a function of positive incident angle. The s -polarized reflected wave is independent of ϵ_a and exhibits weak coupling with the structure, as shown in Figs. 3(c) and 3(d). Consequently, the phase ϕ_s varies smoothly [inset of Fig. 4(c)] and is independent of ϵ_a , leading to a weak reciprocal GH shift. Altogether, we have shown that the reflected field from our proposed structure exhibits a large NR GH shift for p -polarized light and a reciprocal GH shift for s -polarized light. This behavior

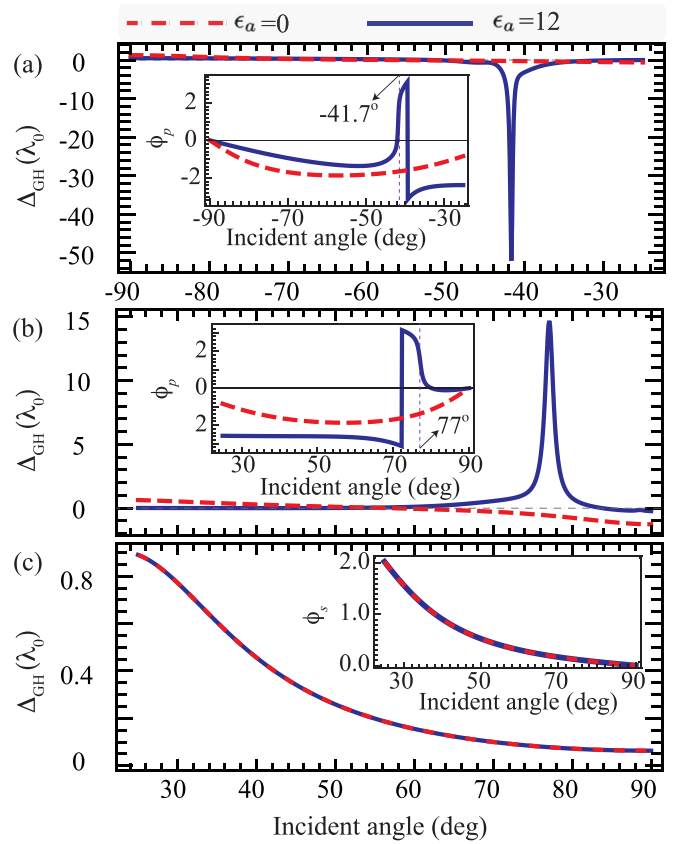


FIG. 4. GH shift (in units of λ_0) for a p -polarized incident field illuminating the slab from (a) the right side (with a negative incident angle) and (b) the left side (with a positive incident angle), plotted as a function of the incident angle. (c) GH shift for an s -polarized field incident from the left side. The off-diagonal elements of the permittivity tensor are fixed at $\epsilon_a = 0$ (dashed) and $\epsilon_a = 12$ (blue), and all other parameters are the same as those in Fig. 2(c). The insets show the reflection phase variation as a function of the incident angle.

arises from the polarization- and angle-dependent reflection caused by the anisotropic nature of the permittivity tensor.

We conduct a similar study for the SH shift. Figure 5(a) shows the SH shift as a function of the incident angle for the V - and H -polarized fields for fixed $\epsilon_a = 12$. It is seen that the SH shift for the H -polarized field is subjected to a large shift $|\Delta_{\text{SH}}^H(\theta_i = -41.7^\circ)| \approx 3.5\lambda_0$ due to full optical momentum conversion to the slab, as shown in Figs. 2(a) and 2(c). In this case, the p -polarized field is completely coupled with the NR structure as shown in Fig. 3(b) such that $R_{pp}/R_{ss} \ll R_{ss}/R_{pp}$ at $\theta_i = -41.7^\circ$ as shown in the inset of Fig. 5(a). Thus, a large shift is found as suggested by Eq. (11). Figure 5(b) shows that the SH shift for left-side incidence is very small $|\Delta_{\text{SH}}^H(\theta_i = 41.7^\circ)| \approx 0.04\lambda_0$ because the p -polarized field does not couple with the structure, as shown in Fig. 3(a), which leads to $R_{pp}/R_{ss} \approx 1$ at $\theta_i = 41.7^\circ$ as shown in the inset of Fig. 5(b). On the other hand, the SH shift for the V -polarized field coming from the left $|\Delta_{\text{SH}}^V(\theta_i = 41.7^\circ)| \approx 0.05\lambda_0$ and right $|\Delta_{\text{SH}}^V(\theta_i = -41.7^\circ)| \approx 0.19\lambda_0$ is small, due to weak coupling of the s -polarized field with the structure, as shown in Figs. 3(c) and 3(d), thus $R_{pp}/R_{ss} \approx 1$. In contrast to the GH shift, it is important to mention that in

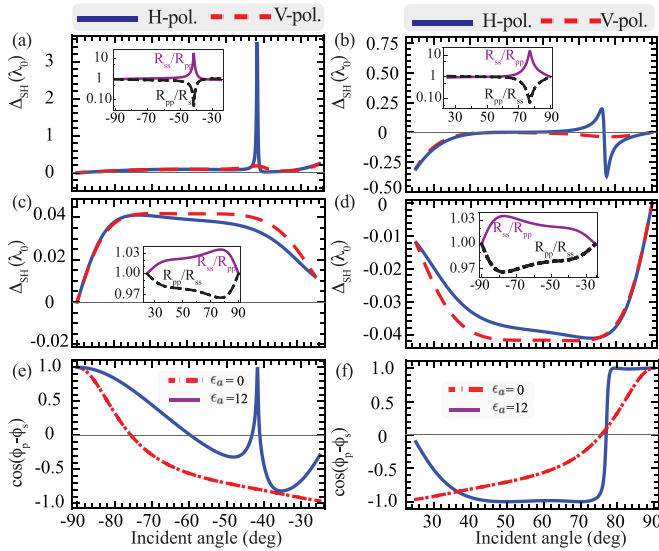


FIG. 5. Δ_{SH} in units of vacuum wavelength for H polarized (V polarized) blue (dashed) as a function of incident angle for (a) right illumination (with negative θ_i) (b) for left illumination (with positive θ_i) with $\epsilon_a = 12$. Similarly, Δ_{SH} for (c) right illumination and (d) for left illumination with $\epsilon_a = 0$. Here, the insets show the ratio of reflection amplitudes as a function of incident angle. Plot of $\cos(\phi_s - \phi_p)$ as a function of incident angle for (e) right illumination and for (f) left illumination. All other parameters are the same as those in Fig. 4.

our structure, the SH shift for the left incident wave $|\Delta_{\text{SH}}^{H,V}(\theta_i)|$ and the right incident wave $|\Delta_{\text{SH}}^{H,V}(-\theta_i)|$ is not symmetric, as illustrated in Figs. 5(a) and 5(b) due to the involvement of R_{pp} (which is nonreciprocal) [see Eqs. (11) and (12)]. Therefore, both SH shifts Δ_{SH}^V and Δ_{SH}^H are NR.

To explore the effect of nonreciprocity, we plot the SH shift for the isotropic slab in Figs. 5(c) and 5(d) for negative and positive incidences, respectively. The results show that the reflected field does not exhibit a large beam shift due to the smooth variation of the reflection amplitude versus the incident angle. In this context, $R_{pp}/R_{ss} \approx R_{ss}/R_{pp} \approx 1$, as shown by the insets of Figs. 5(c) and 5(d). Therefore, Eqs. (11) and (12) predict a small shift. In addition, for the isotropic structure, the SH shift is reciprocal, i.e., $|\Delta_{\text{SH}}^{H,V}(\theta_i)| = |\Delta_{\text{SH}}^{H,V}(-\theta_i)|$, because reflection amplitudes for both polarizations are independent of the incident direction, such that $\eta_\sigma(\pm\theta_i) = 0$.

To gain a deeper insight of the physics of the SH shift, we analyze the relative phases of the reflected field defined as $\cos(\phi_p - \phi_s)$, where $\phi_\sigma = \arg(r_{\sigma\sigma})$, and $\sigma = s$ or p . In Figs. 5(e) and 5(f), the relative reflected phase is plotted as a function of negative and positive incident angles, respectively, which shows an abrupt phase variation at $\theta_i = -41.7^\circ$ in the field reflected from the NR structure. At this angle, the optical linear momentum of the reflected field is significantly reduced, corresponding to a near-zero reflected field amplitude $R_{pp} \approx 0$. This abrupt variation in the phase suggests a large beam shift as shown in Fig. 5(a). On the other hand, for an isotropic structure, $\cos(\phi_p - \phi_s)$ is smoothly varying as a function of the angles, and no sharp optical momentum transformation occurs. Thus, the shift is negligible. A similar

trend is observed in Fig. 5(f), where the phases vary smoothly and the resultant SH shift is weak.

Gaussian beam incidence

The GH and SH shifts obtained using the stationary-phase method, as discussed earlier, correspond to the maximum possible beam shifts. In this formalism, we assume that the beam waist is very large, such that the shifts become independent of the beam waist (see details in Appendix C). To verify the conclusions we drew, we calculate the GH and SH shifts by illuminating the entire structure with a Gaussian beam whose waist $w_0 \gg \lambda_0$ and compare them with stationary phase calculations. We consider an incident Gaussian beam with electric field magnitude E_i in the input co-ordinates $(x', y', z' = 0)$ as follows [60]:

$$E_{\text{in}}(x', y', z' = 0) = E_0 \exp\left(-\frac{(x'^2 + y'^2)}{2w_0^2}\right), \quad (13)$$

and set the amplitude of the incident wave as 1 at the origin, i.e., $E_0 = 1$. The input field can be expressed by the two-dimensional inverse Fourier transform as [60]

$$E_{\text{in}}(x', y', z') = \frac{w_0^2}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp\left(-\frac{k_x'^2 + k_y'^2}{2w_0^{-2}}\right) \times \exp(-i(k_x'x' + k_y'y')) \exp(-i\alpha) dk_x' dk_y', \quad (14)$$

where $\alpha = k_z'z' = (n_1^2k_0^2 - k_x'^2 - k_y'^2)^{1/2}z_1$, with z_1 the distance between the incident source and the NR slab. The reflected wave is expressed in local coordinates (x'', y'', z'') as illustrated in Fig. 6(a). Using the reflection coefficients, we can define the reflected field at $z'' = 0$:

$$E_r^\sigma(x'', y'', z'' = 0) = \frac{w_0^2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} r_\sigma(k_x, k_y) \exp\left(-\frac{k_x^2 + k_y^2}{2w_0^{-2}}\right) \times \exp[-i(k_x'x' + k_y'y')] \exp(-i\beta) dk_x' dk_y', \quad (15)$$

where $\beta = k_z'(z_1 + z_2) = (n_1^2k_0^2 - k_x'^2 - k_y'^2)^{1/2}(z_1 + z_2)$, with z_2 the distance between the slab and the reflected wave detector and $\sigma = s, p$. We plot the intensity of the p -polarized incident waves, $E_{\text{in}}(x, y = 0, z = 0)$, at a fixed incident angle $\theta_i = -41.7^\circ$ along the x axis in Fig. 6(b) in dashed curves, where the peak intensity occurs at $x = 0$. Similar behavior is found in Fig. 6(c) for the same incident wave, but opposite incident angle, i.e., $\theta_i = +41.7^\circ$. To demonstrate the GH shift in the reflected field, we plot the normalized reflected field intensity along the x axis ($y = 0$) in the same figures for their respective incident angles. The result shows that for the right incident Gaussian beam ($\theta_i = -41.7^\circ$), the reflected field is shifted from the origin by a distance $\Delta x = -52\lambda_0$, which is referred to as a GH shift. Noticeably, the simulated GH shift is the same as we have shown in Fig. 4(a), near $\theta_i = -41.7^\circ$. For clear illustration, we zoom in the peak intensity distribution around the origin as shown in the inset of Fig. 6(b). In contrast, when the Gaussian beam is incident from the left ($\theta_i = 41.7^\circ$), the shift in the reflected field is negligible as shown in Fig. 6(c)

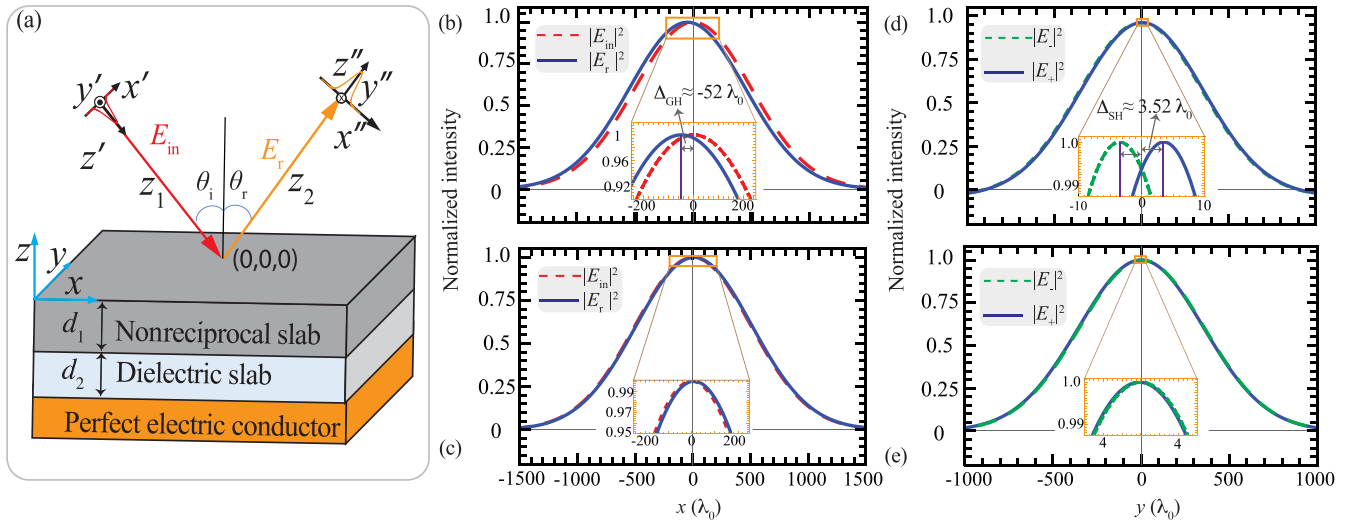


FIG. 6. (a) A schematic illustration shows a Gaussian source positioned at a distance z_1 from the multilayer slab, illuminating the structure at an incident angle θ_i . The reflected field is collected by a detector located at a distance z_2 from the slab. The incident plane (x, z) is defined in the laboratory coordinate system (x, y, z). The input plane (x', z') and output plane (x'', z'') are described using local coordinate systems (x', y', z') and (x'', y'', z''), respectively, with the y axes aligned parallel, i.e., $y \parallel y' \parallel y''$. The normalized incident field intensity (dashed curve) and reflected field intensity (solid curve) along the x axis ($y = 0$) for fixed incident angle (b) $\theta_i = -41.7^\circ$ and (c) $\theta_i = +41.7^\circ$. The lateral displacement Δx of the solid line from the origin represents the GH shift as shown in the insets. (d) The normalized reflected field intensity distribution along the y axis ($x = 0$) for fixed incident angle (d) $\theta_i = -41.7^\circ$ and (e) $\theta_i = 41.7^\circ$. Here, $\pm \Delta y$ represents the spin-dependent splitting of the LCP and RCP reflected field intensity as clearly shown in the inset of each figure. Here, we set $w_0 = 500\lambda_0$, $z_1 = z_2 = 200w_0$, and the rest of the parameters are the same as we used in Fig. 4(a).

as predicted in Fig. 4(b). Thus, our stationary method calculation agrees with the Gaussian beam analysis.

To compare the SH shift, we plot the normalized reflected intensity along the y axis ($x = 0$) for fixed incident angle $\theta_i = -41.7^\circ$ in Fig. 6(d) and $\theta_i = 41.7^\circ$ in Fig. 6(e). The incident field has a Gaussian profile with maximum intensity centered around $y = 0$ (not shown here), and the reflected field undergoes a spin-dependent splitting of $E_r^\pm$ along the positive and negative y axis, as we schematically illustrated in Fig. 1(b). It can be seen in the inset of Fig. 6(d), where the reflected field is split into left- and right-polarized fields. The splitting is symmetric along the y axis [as suggested by Eq. (12)] and shifted through a distance of $\Delta y \approx 3.5\lambda_0$, which is the same as shown in Fig. 5(a) (blue lines at $\theta_i = -41.7^\circ$). On the other hand, for left-side incidence ($\theta_i = 41.7^\circ$), the spin-dependent splitting is negligible as shown by the inset of Fig. 6(e) and predicted in Fig. 5(b). Thus, it also agrees with our stationary phase method calculation.

IV. CONCLUSION

In this work, we have investigated the NR Goos-Hänchen shift and photonic SH shift by a planar heterostructure made of an NR slab and a dielectric slab placed over a perfect reflector. The results demonstrate that by tuning the off-diagonal elements of the permittivity tensor of the NR medium, the structure exhibits NR reflection due to polarization- and direction-dependent coupling of the incident field with the structure. Our numerical results obtained using COMSOL multiphysics- show that the engineered coupling enables direction-selective optical momentum transfer to the slab. As

a result, the reflected beam exhibits both a large NR GH shift and SH shift due to the conversion of linear and orbital angular optical momentum, respectively. To validate our findings, we employed two independent analytical approaches: the stationary phase method for plane-wave incidence and the scattering analysis of a Gaussian beam. The results reveal that when a Gaussian beam is reflected by the structure, it experiences both GH and SH shifts. These observed shifts are in excellent agreement with those predicted by the stationary phase method. Thus, the presented results are reliable and experimentally achievable. We believe our findings pave the way to control the flow of light, enabling the development of efficient optical devices based on directional-dependent beam manipulation and polarization control, with potential applications in the field of sensing.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: ELECTROMAGNETIC FIELD IN THE NONRECIPROCAL MEDIUM

We consider time-harmonic electromagnetic fields propagating in an NR medium of anisotropic permittivity tensor $\bar{\epsilon}_{\text{NR}}$, as given in Eq. (1). Under the time convention $\exp(-i\omega t)$, Maxwell's equations are written as follows:

$$\nabla \times \mathbf{E} = i\omega\mu_0\mathbf{H}, \quad (\text{A1})$$

$$\nabla \times \mathbf{H} = -i\omega\epsilon_0\bar{\epsilon}_{\text{NR}} \cdot \mathbf{E}. \quad (\text{A2})$$

We assume that the incident field propagates in the xz plane, and the medium has discontinuities along the z axis. To apply the boundary condition at each interface, we expand Eq. (A1) as

$$-\partial_z E_y = i\omega\mu_0 H_x, \quad (\text{A3})$$

$$\partial_z E_x = i\omega\mu_0 H_y + ik_x E_z, \quad (\text{A4})$$

$$k_x E_y = \omega\mu_0 H_z, \quad (\text{A5})$$

and Eq. (A2) as

$$-\partial_z H_y = -i\omega\epsilon_0(\epsilon_d E_x + \epsilon_{xz} E_z), \quad (\text{A6})$$

$$\partial_z H_x = -i\omega\epsilon_0\epsilon_d E_y + ik_x H_z, \quad (\text{A7})$$

$$k_x H_y = -\omega\epsilon_0(\epsilon_{zx} E_x + \epsilon_d E_z). \quad (\text{A8})$$

Since the NR medium has discontinuities along the z axis, we will apply the boundary conditions at $z = 0$ and $z = -d_1$. Therefore, we eliminate E_z and H_z from Eqs. (A4) and (A7), respectively. We get $H_z = \frac{k_x}{\omega\mu_0} E_y$ and $E_z = -\frac{1}{\omega\epsilon_0\epsilon_d}(k_x H_y + \omega\epsilon_0\epsilon_{zx} E_x)$. By inserting into the equations. (A3), (A5), (A6), and (A8), we obtain the coupled system of differential equations as

$$\partial_z E_x = -\frac{ik_x\epsilon_{zx}}{\epsilon_d} E_x + i\left(\mu_0\omega - \frac{k_x^2}{\omega\epsilon_0\epsilon_d}\right) H_y, \quad (\text{A9})$$

$$\partial_z E_y = -i\omega\mu_0 H_x, \quad (\text{A10})$$

$$\partial_z H_x = i\left(\frac{k_x^2}{\mu_0\omega} - \omega\epsilon_0\epsilon_d\right) E_y, \quad (\text{A11})$$

$$\partial_z H_y = i\omega\epsilon_0\left(\epsilon_d - \frac{\epsilon_{xz}\epsilon_{zx}}{\epsilon_d}\right) E_x - \frac{ik_x\epsilon_{xz}}{\epsilon_d} H_y. \quad (\text{A12})$$

The above coupled equations can be compactly expressed in a matrix form as

$$\partial_z \begin{pmatrix} E_x \\ E_y \\ H_x \\ H_y \end{pmatrix} = i\mathbf{M} \cdot \begin{pmatrix} E_x \\ E_y \\ H_x \\ H_y \end{pmatrix},$$

with the matrix $\mathbf{M}$ given as

$$\mathbf{M} = \begin{pmatrix} -\frac{\epsilon_{zx}k_x}{\epsilon_d} & 0 & 0 & \mu_0\omega - \frac{k_x^2}{\omega\epsilon_0\epsilon_d} \\ 0 & 0 & -\mu_0\omega & 0 \\ 0 & \frac{k_x^2}{\mu_0\omega} - \omega\epsilon_0\epsilon_d & 0 & 0 \\ \omega\epsilon_0\epsilon_v & 0 & 0 & -\frac{\epsilon_{xz}k_x}{\epsilon_d} \end{pmatrix},$$

$\epsilon_v = (\epsilon_{xx} - \frac{\epsilon_{xz}\epsilon_{zx}}{\epsilon_d})$. This matrix $\mathbf{M}$ is used in the main text.

APPENDIX B: EXPLICIT EXPRESSION OF THE PROPAGATION MATRIX $\mathbf{P}_1$

We start with the given matrix $\mathbf{M}$, which describes the field coupling in an NR slab. The matrix $\mathbf{M}$ is Block-diagonal and can be decomposed into an s -polarization (E_y, H_x) Block matrix as

$$\mathbf{M}_s = \begin{pmatrix} 0 & -\mu_0\omega \\ \frac{k_x^2 - \omega^2\mu_0\epsilon_0\epsilon_d}{\omega\mu_0} & 0 \end{pmatrix} \quad (\text{B1})$$

with eigenvalues $\lambda_{\pm} = \pm i\sqrt{\epsilon_d k_0^2 - k_x^2} = \pm ik_{zs1}$. By using the identity $\exp(-i\mathbf{M}_s d_1) = \cos(k_{zs1} d_1) \mathbf{I} - \frac{i \sin(k_{zs1} d_1)}{k_{zs1}} \mathbf{M}_s$, we express the solution as

$$\exp(-i\mathbf{M}_s d_1) = \begin{pmatrix} \cos(k_{zs1} d_1) & \frac{i\mu_0\omega \sin(k_{zs1} d_1)}{k_{zs1}} \\ \frac{ik_{zs1} \sin(k_{zs1} d_1)}{\mu_0\omega} & \cos(k_{zs1} d_1) \end{pmatrix}. \quad (\text{B2})$$

Similarly, the p -polarization (E_x, H_y) component of the Block matrix can be expressed as

$$\mathbf{M}_p = \begin{pmatrix} -\frac{\epsilon_{zx}k_x}{\epsilon_d} & \mu_0\omega - \frac{k_x^2}{\omega\epsilon_0\epsilon_d} \\ \omega\epsilon_0\epsilon_v & -\frac{\epsilon_{xz}k_x}{\epsilon_d} \end{pmatrix}. \quad (\text{B3})$$

The eigenvalues of the matrix can be written as

$$\det(\mathbf{M}_p - \lambda \mathbf{I}) = \lambda^2 + \left(-\frac{\epsilon_{xz}^2 k_x^2}{\epsilon_d^2} - \omega^2 \mu_0 \epsilon_0 \epsilon_v + \frac{k_x^2 \epsilon_v}{\epsilon_d} \right) = 0,$$

$$\lambda_{\pm} = \pm i\sqrt{\epsilon_v k_0^2 - k_x^2} = \pm ik_{zp1}.$$

By using the identity $\exp(-i\mathbf{M}_p d_1) = \cos(k_{zp1} d_1) \mathbf{I} - \frac{i \sin(k_{zp1} d_1)}{k_{zp1}} \mathbf{M}_p$ we express the solution as

$$\exp(-i\mathbf{M}_p d_1) = \begin{pmatrix} \cos(k_{zp1} d_1) - \frac{i\epsilon_{xz}k_x \sin(k_{zp1} d_1)}{\epsilon_d k_{zp1}} & -\frac{ik_{zs1}^2 \sin(k_{zp1} d_1)}{\omega\epsilon_0\epsilon_d k_{zp1}} \\ -\frac{i\omega\epsilon_0\epsilon_v \sin(k_{zp1} d_1)}{k_{zp1}} & \cos(k_{zp1} d_1) - \frac{i\epsilon_{xz}k_x \sin(k_{zp1} d_1)}{\epsilon_d k_{zp1}} \end{pmatrix}. \quad (\text{B4})$$

By combining Eqs. (B3) and (B4) we write the complete solution as

$$\mathbf{P}_1 = \begin{pmatrix} \cos(k_{zp1}d_1) - \frac{i\epsilon_{xz}k_x \sin(k_{zp1}d_1)}{\epsilon_d k_{zp1}} & 0 & 0 & -\frac{ik_{zs1}^2 \sin(k_{zp1}d_1)}{\omega\epsilon_0\epsilon_d k_{zp1}} \\ 0 & \cos(k_{zs1}d_1) & \frac{i\mu_0\omega \sin(k_{zs1}d_1)}{k_{zs1}} & 0 \\ 0 & \frac{ik_{zs1} \sin(k_{zs1}d_1)}{\mu_0\omega} & \cos(k_{zs1}d_1) & 0 \\ -\frac{i\omega\epsilon_0\epsilon_v \sin(k_{zp1}d_1)}{k_{zp1}} & 0 & 0 & \cos(k_{zp1}d_1) - \frac{i\epsilon_{xz}k_x \sin(k_{zp1}d_1)}{\epsilon_d k_{zp1}} \end{pmatrix},$$

which is the propagation matrix used in the main text.

APPENDIX C: EFFECT OF BEAM WAIST ON THE BEAM SHIFTS

1. Effect of beam waist on the Goos-Hänchen shift

In the main text, we use the stationary-phase method to define the GH shift, which is valid for a large beam waist. However, to explore the effect of the beam waist on the GH shift and provide the limitations of the our work, we compute the GH shift using a Gaussian beam in Fig. 7, by plotting incident and reflected fields for several beam waists: (a) $w_0 = 10\lambda_0$, (b) $w_0 = 50\lambda_0$, (c) $w_0 = 100\lambda_0$, (d) $w_0 = 200\lambda_0$, (e) $w_0 = 500\lambda_0$, and (f) $w_0 = 600\lambda_0$. The lateral displacement (GH shift) in the reflected field is illustrated by arrows. The results show that, as the beam waist increases, the GH shift also increases and eventually converges to the value of $-52\lambda_0$, consistent with our analytical prediction obtained using the stationary-phase method. This confirms that our analytical formulation accurately describes the GH shift in the large beam waist limit, i.e., $w_0 \geq 500\lambda_0$. Therefore, we choose $w_0 = 500\lambda_0$ in Fig. 6.

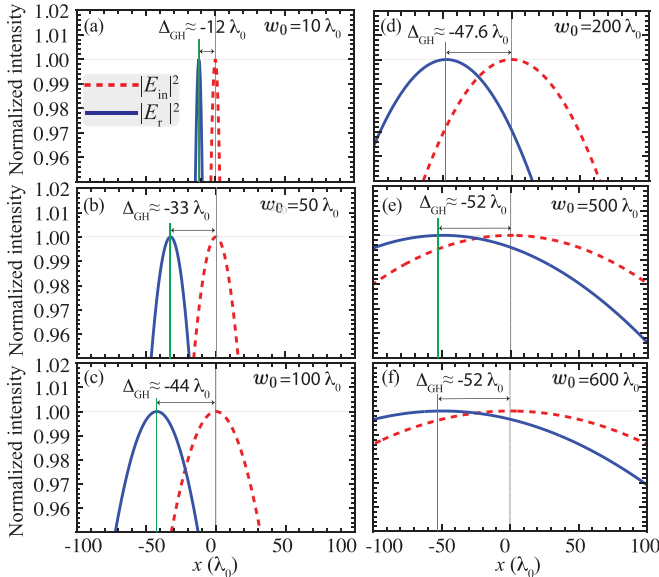


FIG. 7. Effect of beam waist on the GH shift. The normalized incident field intensity (dashed curve) and reflected field intensity (solid curve) for different beam waists, (a) $w_0 = 10\lambda_0$, (b) $w_0 = 50\lambda_0$, (c) $w_0 = 100\lambda_0$, (d) $w_0 = 200\lambda_0$, (e) $w_0 = 500\lambda_0$, and (f) $w_0 = 600\lambda_0$, are plotted along the x axis (for $y = 0$) for a fixed incident angle $\theta_i = -41.7^\circ$. The remaining parameters are the same as those used in Fig. 6(b).

2. Effect of beam waist on the photonic spin Hall shift

In the main text, we employ the stationary-phase method to define the SH shift, which provides the maximum possible shift and is valid only when the incident field has a large beam waist and a narrow angular spectrum. To explore the limitations of our work, we calculate the beam waist dependent SH shift by considering an incident Gaussian beam with the angular spectrum as

$$\tilde{E}_{in} = \frac{w_0}{\sqrt{2\pi}} \exp\left[-\frac{w_0^2(k_{x'}^2 + k_{y'}^2)}{4}\right]. \quad (C1)$$

Using the reflected coefficients, we can calculate the angular spectrum $\tilde{E}_{r+}$ or $\tilde{E}_{r-}$ of the LCP and RCP components of the

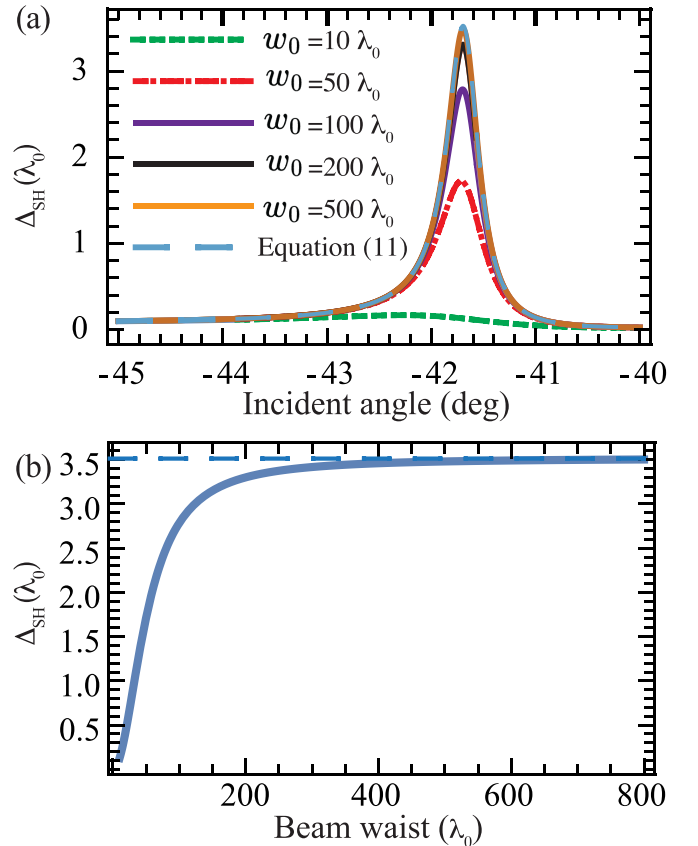


FIG. 8. (a) SH shift as a function of the incident angle for different beam waists, computed using Eqs. (C2) and (11) based on the stationary phase method. (b) SH shift as a function of beam waist for fixed incident angle $\theta_i = -41.7^\circ$. The remaining parameters are the same as those used in Fig. 5.

reflected field. By substituting them into

$$\Delta_{\text{SH}}^H = \frac{\langle \tilde{E}_{r\pm} | i \partial_{k_y} | \tilde{E}_{r\pm} \rangle}{\langle \tilde{E}_{r\pm} | \tilde{E}_{r\pm} \rangle},$$

and a straightforward calculation leads to a simplified w_0 -dependent expression of the SH shift given as [19]

$$\Delta_{\text{SH}}^H = \frac{\pm k_0 w_0^2 |r_{pp}| [|r_{pp}| + |r_{ss}| \Delta \Phi] \cot \theta_i}{k_0^2 w_0^2 |r_{pp}|^2 + \chi^2 + [|r_{pp}|^2 + |r_{ss}|^2 + 2|r_{pp}||r_{ss}| \Delta \Phi] \cot^2 \theta_i}, \quad (\text{C2})$$

where $\chi = |\partial r_{pp} / \partial \theta_i|$ and $\Delta \Phi = \cos(\varphi_s - \varphi_p)$. This equation shows that the SH shift is proportional to w_0 , and in the asymptotic limit ($w_0 \rightarrow \infty$), reaches its maximum value. Under this limit, the analytical expression for the SH shift can be expressed as

$$\Delta_{\text{SH}}^H = \pm \frac{\lambda_0}{2\pi} \left[1 + \frac{|r_{ss}|}{|r_{pp}|} \cos(\phi_s - \phi_p) \right] \cot \theta_i, \quad (\text{C3})$$

which corresponds to Eq. (11) (likewise for V polarized) in the main text. In this case, the shifts are proportional to the

ratio of reflection amplitudes. A large shift occurs when the $|r_{ss}|/|r_{pp}|$ becomes significantly large.

To demonstrate the effect of w_0 on the SH shift, we plot it as a function of the incident angle for different values of w_0 in Fig. 8(a). The results show that as w_0 increases, the SH shift approaches the value calculated using Eq. (11). For better illustration, we also plot the SH shift as a function of the w_0 in Fig. 8(b). The results indicate that for $w_0 \geq 500 \lambda_0$, the SH shift converges to the value predicted by Eq. (11), as marked by the dashed line. This confirms that our approach is applicable in the large beam waist limit.

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