

Geometry-controlled directional wave-packet transmission in graphene with photonic-crystal-like electrostatic patterns

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We investigate electron wave-packet transport in monolayer graphene subjected to photonic-crystal-inspired electrostatic superlattices composed of periodically arranged circular barriers and wells with zero net potential. Using the time-dependent Dirac equation combined with a split-operator propagation method, we systematically examine how lattice geometry, barrier-well ordering, propagation direction, carrier energy, and scatterer radius govern transient dynamics and long-time transmission behavior. Three representative superlattice configurations are analyzed: square, staggered, and triangular superlattices. While all systems exhibit a universal temporal evolution characterized by an initial transient regime followed by transmission saturation, the saturated transmission strongly depends on structural geometry and propagation direction (i.e., forward and backward). In particular, square and triangular lattices display pronounced directional asymmetry, whereas the staggered configuration serves as an isotropic system in both the forward and backward directions. By introducing the normalized asymmetry coefficient η , we demonstrate that both the magnitude and sign of directional preference can be controllably tuned—and even reversed—through variations in electrostatic potential strength and incident wave-packet energy. These findings establish circular electrostatic superlattices as a versatile platform for geometry-programmable Dirac-fermion transport, opening pathways toward graphene-based directional filters, tunable energy-selective transport elements, and advanced electron-optical devices.

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I. INTRODUCTION

Graphene, a two-dimensional monolayer of carbon atoms arranged in a honeycomb lattice, has remained at the forefront of condensed-matter and nanomaterials research since its experimental isolation because of its exceptional electrical, mechanical, and thermal properties [1–3]. A defining characteristic of this material is that its charge carriers, in the vicinity of the Dirac points, obey a linear dispersion relation and behave as massless Dirac fermions [4]. This unusual electronic structure gives rise to ultrahigh carrier mobility and long mean free paths, which collectively make graphene a

highly attractive platform for nanoelectronics, ballistic transport devices, and high-frequency applications [5,6]. At the same time, the relativistic dynamics of charge carriers in graphene leads to Klein tunneling, namely, the anomalously high transmission of normally incident electrons across electrostatic barriers [7–9]. Although this effect is one of the most remarkable manifestations of Dirac physics in solids, it also creates a major obstacle for electrostatic confinement and switching, thereby motivating sustained efforts to identify potential landscapes capable of controlling, suppressing, or redirecting carrier transport in graphene [10–12].

Among the most promising strategies for tailoring transport in graphene is the use of periodic electrostatic modulations, commonly described as graphene superlattices [13–16]. These structures, which can be realized through patterned top gates, substrate engineering, or moiré-induced modulation, introduce new spatial length scales that reconstruct the Dirac spectrum, generate secondary Dirac points, and induce pronounced anisotropy in carrier propagation [17,18]. From a transport perspective, periodic potentials lead to rich interference effects, including resonant transmission

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and Fabry-Pérot-type oscillations, whose character depends strongly on the barrier geometry, incidence conditions, and spatial symmetry of the pattern [11,19]. More broadly, the close analogy between electron propagation in modulated graphene and light propagation in photonic crystals has opened a conceptually powerful route toward geometry-driven electron-optical functionality [20]. Within this framework, appropriately designed electrostatic patterns can produce velocity renormalization, directional selectivity, and valley-sensitive transmission, all of which are controlled by the arrangement, periodicity, and amplitude of the imposed potentials [16,18].

To capture such phenomena with sufficient realism, wave-packet approaches based on the time-dependent Dirac equation have become especially valuable, particularly when combined with the split-operator technique [21,22]. In contrast to stationary plane-wave descriptions, wave-packet simulations naturally incorporate finite spatial extent, angular spread, transient dwell dynamics, and multiple-scattering processes, thereby providing a more physically transparent picture of carrier transport in structured graphene systems [23,24]. This framework has been successfully used to analyze Klein tunneling, *Zitterbewegung*, and scattering by electrostatic and magnetic barriers [21,25], as well as wave-packet dynamics in strained graphene and related valley-selective transport regimes [26]. More recently, attention has turned to transport through engineered arrays of electrostatic scatterers. Earlier studies demonstrated that even in the presence of Klein tunneling, random and periodic scatterer configurations can substantially reduce transmission through interference and collective scattering effects [27–30]. Subsequent works further showed that ordered arrays of alternating positive and negative electrostatic regions can induce strong transmission suppression, a nonmonotonic response, and propagation-direction asymmetry, depending sensitively on the geometry and polarity sequence of the potential arrangement [31,32]. Complementary investigations of square- and triangular-type scatterer configurations likewise revealed that geometric organization plays a decisive role in controlling transmission and confinement in graphene-based nanodevices [33].

These developments suggest that photonic-crystal-like electrostatic patterns offer a particularly rich platform for controlling Dirac-fermion transport in graphene. In direct analogy with optical photonic crystals, periodic potential landscapes in graphene may generate stop-band-like behavior, directional filtering, and tunable confinement through repeated scattering and interference [20]. This perspective is additionally supported by the observation of Fabry-Pérot-like transport signatures in graphene heterostructures and superlattice-type systems [11,19], as well as by experimental progress in graphene ballistic rectifiers and related high-frequency electron-optical devices [6,34]. Nevertheless, despite substantial progress in understanding transport across isolated barriers, random scatterers, and several classes of periodic electrostatic arrays, the role of lattice symmetry and potential ordering in controlling direction-dependent wave-packet transport remains insufficiently understood.

Previous wave-packet studies, including our recent work [28], demonstrated that transmission can be strongly affected by the size, spacing, polarity, and arrangement of periodic

electrostatic scatterers. However, these studies primarily addressed changes in the overall transmission and did not systematically examine how lattice symmetry and barrier-well ordering affect transport under reversal of the propagation direction. In particular, the possibility of reversing the preferred transmission direction by tuning the electrostatic potential strength and carrier energy remains insufficiently explored within this framework.

In this work, we address this question by investigating electron wave-packet propagation in monolayer graphene subjected to photonic-crystal-like periodic electrostatic patterns composed of circular regions with alternating positive and negative potentials. Three representative geometries are considered: a regular square superlattice, its staggered configuration with laterally shifted rows, and a triangular superlattice. By explicitly comparing forward and backward propagation, we determine how lattice symmetry and barrier-well ordering govern direction-dependent transport. Using the Dirac continuum model and the split-operator scheme, we systematically examine the influence of geometric arrangement, propagation direction, scatterer radius, potential strength, and carrier energy on both the transient evolution and saturated transmission probability. In particular, we show that the directional asymmetry can change sign with potential strength and carrier energy, corresponding to a tunable reversal of the preferred propagation direction. These results establish geometry and electrostatic tuning as complementary means of controlling directional Dirac-fermion transport in graphene.

II. MODEL SYSTEM AND THEORETICAL FRAMEWORK

We consider a monolayer graphene sheet of lateral dimensions $L = 4096$ nm and $W = 128$ nm, subjected to a spatially modulated scalar electrostatic potential. The potential landscape is engineered as a periodic array of circular regions with alternating positive ($+V$, potential barriers) and negative ($-V$, potential wells) amplitudes, forming a photonic-crystal-like structure (Fig. 1). The positive and negative circular regions are arranged so that the spatial average of the imposed scalar potential over the patterned region vanishes. This zero-net-potential condition eliminates a global electrostatic bias and allows the observed transmission asymmetry to be attributed primarily to lattice geometry, barrier-well ordering, and interference effects rather than to a net potential drop across the scattering region.

As shown in Fig. 1, to systematically explore geometry-induced transport effects, three distinct configurations are examined. Sample 1 represents a regular square superlattice of circular scatterers with horizontal and vertical periodicities δ and d_1 , respectively. Although the scatterer positions in sample 1 form a square lattice, the alternating barrier-well sequence is not equivalent under reversal of the propagation direction. Therefore, forward and backward propagations correspond to different potential sequences encountered by the wave packet. Sample 2 corresponds to a staggered arrangement of sample 1, where adjacent rows are laterally shifted; the lateral shift of adjacent rows restores an effective inversion symmetry of the barrier-well sequence along the propagation direction, making forward and backward propagation equivalent. Sample 3 is a triangular superlattice, in

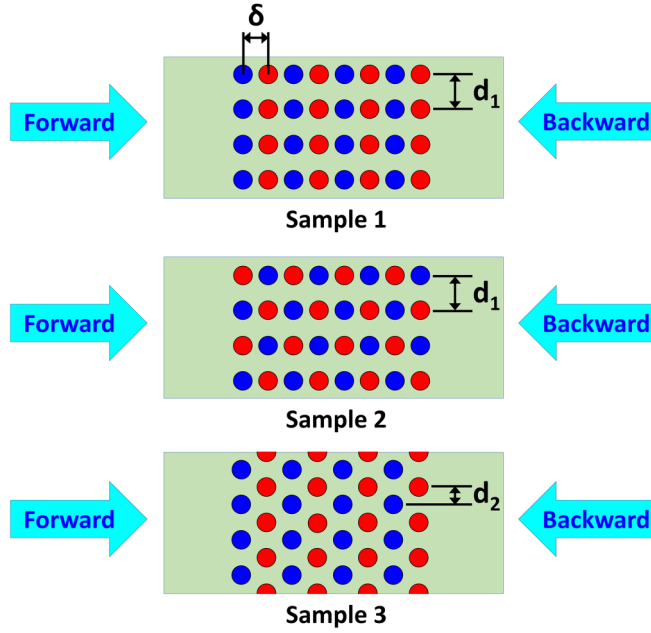


FIG. 1. Schematic illustration of wave-packet propagation in graphene through photonic-crystal-like electrostatic potential patterns. Circular regions represent periodically arranged potentials, with red and blue circles corresponding to positive ($+V$, potential barriers) and negative ($-V$, potential wells) values, respectively. Three geometrical configurations are considered: Sample 1 is a regular square superlattice with spacings δ and d_1 , sample 2 is its staggered arrangement, and sample 3 is a triangular superlattice with modified vertical spacing d_2 . Arrows indicate forward (left-to-right) and backward (right-to-left) propagation, enabling the analysis of geometry- and direction-dependent transport.

which the effective vertical spacing is modified to d_2 , leading to a different packing density of scatterers within the same simulation domain. The triangular superlattice not only increases the scatterer density but also changes the topology of available propagation pathways. Compared with the square and staggered configurations, this geometry provides a richer multipath interference network in which longitudinal and transverse scattering channels compete. As a result, sample 3 can either suppress or facilitate transmission depending on energy and potential strength, leading to sign-changing directional asymmetry and switchable transmission preference.

In all configurations, each circular region of radius R acts as a localized electrostatic scatterer. As mentioned above, the alternating $\pm V$ distribution ensures that barrier and well regions are balanced, resulting in a zero net potential across the system. This condition eliminates any global electrostatic bias while enhancing interference and multiple-scattering effects, which are crucial for understanding geometry-controlled transport.

The center-to-center spacing between neighboring scatterers in the transverse direction (perpendicular to wave-packet propagation) is defined by d_1 and d_2 , with $d_1 = 32$ nm for samples 1 and 2 and $d_2 = 28$ nm for sample 3 to accommodate the triangular arrangement. The scatterer radius is varied as $R = 7, 10$, and 15 nm to examine size-dependent

effects. Along the propagation direction, the spacing between consecutive rows is fixed at $\delta = 32$ nm for all configurations.

The wave packet is modeled as a Gaussian wave front characterized by its energy $E = 100, 150$, and 200 meV and width $a_x = 10$ nm. Specifically, the initial wave packet is a Gaussian function constant in the y direction, with a finite width in the x direction, and is expressed as

$$\Psi(x, y, 0) = N \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{ikx - \frac{x^2}{2a_x^2}}, \quad (1)$$

where N is the normalization constant and $k = E/\hbar v_F$, with v_F being the Fermi velocity in graphene. The pseudospinor $(1 \ 1)^T$ is selected to ensure propagation along the x direction, with $\langle \sigma_x \rangle = 1$ and $\langle \sigma_y \rangle = \langle \sigma_z \rangle = 0$. This choice avoids *Zitterbewegung*, a trembling motion of the wave packet, along the x direction [26]. Wave-packet propagation is governed by the time-evolution operator applied to the initial wave packet:

$$\Psi(x, y, t + \Delta t) = e^{-\frac{i}{\hbar} H \Delta t} \Psi(x, y, t), \quad (2)$$

where H is the Hamiltonian for low-energy electrons in graphene,

$$H = v_F (\vec{\sigma} \cdot \vec{p}) + V(x, y) \mathbf{I}, \quad (3)$$

with $\vec{\sigma}$ representing the Pauli matrices, $\mathbf{I}$ being the 2×2 identity matrix, and $V(x, y)$ denoting the scalar potential. The wave functions are written as pseudospinors $\Psi = (\Psi_A, \Psi_B)^T$, where Ψ_A and Ψ_B correspond to the probabilities of the electron being in sublattices A and B , respectively. To simplify calculations, the split-operator technique [35,36] is employed:

$$\begin{aligned} \exp\left[-\frac{i}{\hbar} H \Delta t\right] &= \exp\left[-\frac{i}{2\hbar} V(x, y) \mathbf{I} \Delta t\right] \exp\left[-\frac{i}{\hbar} v_F \vec{p} \cdot \vec{\sigma} \Delta t\right] \\ &\times \exp\left[-\frac{i}{2\hbar} V(x, y) \mathbf{I} \Delta t\right], \end{aligned} \quad (4)$$

where terms of order $O(\Delta t^3)$ and higher are neglected. This approach enables efficient multiplication in real and reciprocal spaces, avoiding the explicit differentiation of the momentum operator by utilizing the Fourier transform and expressing $\vec{p} = \hbar \vec{k}$. Furthermore, the exponentials involving Pauli matrices can be exactly reformulated as matrices. Simulations are performed with a time step of $\Delta t = 0.1$ fs, and the probabilities of finding the electron before, inside, and after the scattering region are computed. The latter is interpreted as the transmission probability P through the scattering region. To avoid numerical artifacts associated with periodic boundary conditions, we employ a sufficiently large computational grid of 4096×128 nm², while the scattering centers are confined to a smaller region of 320×128 nm² around the $x = 0$ axis. This ensures accurate evaluation of the transmission and reflection probabilities. All simulations are conducted at a wave-packet energy of $E = 100, 150$, and 200 meV to ensure that the results remain within the range of applicability of the Dirac continuum model. Note that the longitudinal length of the computational domain was chosen to be much larger than the patterned scattering region so that reflected and transmitted components remain well separated during the simulated time window. The transverse width was also selected to minimize artificial interactions between scattered wave-packet components and their periodic images. This setup

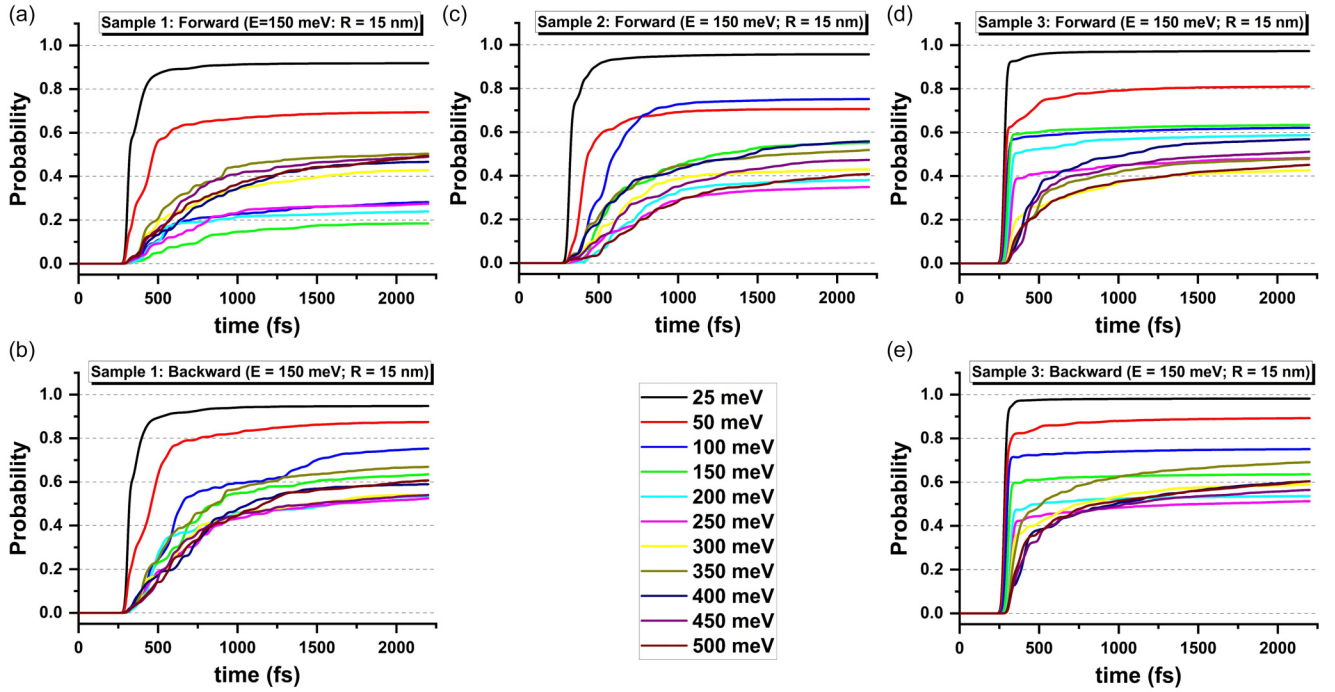


FIG. 2. The transmission probability P for a wave packet with energy $E = 150$ meV and scatterer radius $R = 15$ nm, shown for different electrostatic potential magnitudes ($|V| = 25\text{--}500$ meV) with alternating $\pm V$. (a) and (b) correspond to sample 1 under forward and backward propagation, respectively, revealing a clear direction-dependent behavior that is consistent with the reversed sequence of potential wells and barriers. (c) shows the results for sample 2, where only one direction is shown because forward and backward propagation are equivalent for this configuration. (d) and (e) present sample 3 under forward and backward propagation, respectively, highlighting the role of the triangular superlattice, which enhances multiple scattering and leads to pronounced asymmetry between propagation directions. In all configurations, P exhibits a characteristic temporal behavior, consisting of an initial rapid increase followed by a slower evolution toward a geometry-dependent saturated regime at long times ($t \gtrsim 1500$ fs).

ensures that the extracted transmission and reflection probabilities are not dominated by boundary artifacts.

To quantify the direction-dependent response, we introduced the normalized asymmetry coefficient $\eta = (P_F - P_B)/(P_F + P_B)$, where P_F and P_B are the saturated transmission probabilities for forward and backward propagation, respectively. The sign η identifies the preferred propagation direction, with positive (negative) values indicating preferential forward (backward) transmission, while its magnitude quantifies the strength of the directional contrast.

III. RESULTS AND DISCUSSION

To elucidate the role of geometric configuration in wave-packet transport in graphene, we analyzed the transmission probability P across a range of wave-packet energies and scatterer radii (see previous section). As a representative case, Fig. 2 presents results for wave-packet energy $E = 150$ meV and scatterer radius $R = 15$ nm, evaluated for varying electrostatic potential strengths ($|V| = 25\text{--}500$ meV). Complementary results for other energies and scatterer sizes are provided in the Supplemental Material [37].

To systematically investigate the role of the geometry and propagation direction, we considered five configurations derived from the three model systems (samples 1–3) introduced in Fig. 1. Accounting for both forward (left-to-right) and backward (right-to-left) propagation, samples 1 and 3 each yield two distinct transport cases, while sample 2 remains direction

independent. Configurations 1 and 2 [Figs. 2(a) and 2(b)] correspond to sample 1 under forward and backward propagation, respectively, where the wave packet encounters reversed sequences of potential wells and barriers. Configuration 3 [Fig. 2(c)] represents sample 2, whose staggered geometry preserves effective symmetry and leads to identical transport in both directions. Configurations 4 and 5 [Figs. 2(d) and 2(e)] correspond to sample 3, where the triangular superlattice enhances scattering and modifies the interference landscape. For all configurations, the wave-packet dynamics were analyzed over a sufficiently long time interval ($t = 2500$ fs), ensuring convergence to a well-defined saturation regime. As shown in Fig. 2, the transmission probability exhibits a universal temporal evolution: a rapid initial increase associated with the arrival of the wave packet in the scattering region, followed by slower growth governed by multiple scattering and interference processes. For $t \gtrsim 1500$ fs, P approaches a constant value, indicating balance between transmission, reflection, and partial confinement.

Figure 2(a) shows the results for sample 1 under forward propagation. As is clear, the transmission exhibits a pronounced nonmonotonic dependence on the potential magnitude V , decreasing rapidly with increasing V and reaching a minimum of approximately $P = 0.18$ at $V = 150$ meV. For larger V , the transmission partially recovers and stabilizes below $P \approx 0.5$. This behavior reflects enhanced scattering and destructive interference consistent with the periodic sequence of alternating wells and barriers, leading to an

optimal suppression at intermediate potentials. In contrast, Fig. 2(b) (sample 1, backward propagation) shows a more gradual and nearly monotonic decrease of P with increasing V . The transmission saturates at approximately $P \approx 0.5$ around $V \approx 250$ meV and remains weakly dependent on further increases in V , approaching values near $P \approx 0.6$. The reduced suppression compared to the forward case arises from the reversed sequence of potentials, where the initial interaction with a barrier is partially compensated by subsequent well regions. This comparison highlights a clear direction-dependent transport asymmetry.

Figure 2(c) (sample 2) exhibits a qualitatively different behavior. The transmission decreases with increasing V , reaching a minimum of approximately $P \approx 0.35$ at $V \approx 250$ meV, and subsequently stabilizes within the range $P \approx 0.4$ – 0.5 . This relatively smooth and almost monotonic behavior indicates that the staggered geometry creates a more uniform scattering landscape, resulting in a more even distribution of scattering events, reflecting the inherent symmetry of the configuration for which forward and backward propagations are equivalent.

For sample 3 under forward propagation [Fig. 2(d)], the transmission decreases with increasing V , reaching values of approximately $P \approx 0.42$ – 0.45 for intermediate and large potential magnitudes. Compared with sample 1, the triangular arrangement leads to a faster approach to the saturated regime, indicating stronger and more rapid multiple scattering within the potential landscape. However, the increased scatterer density does not necessarily produce a stronger suppression of transmission. In this geometry, the triangular packing suggests the presence of interference-mediated transmission pathways, allowing part of the wave packet to propagate efficiently through the structure even at large $|V|$. Thus, sample 3 mainly modifies the transient dynamics and saturation time, rather than simply reducing the final transmission probability. Figure 2(e) shows the corresponding backward-propagation case for sample 3. Here, P decreases more systematically with increasing $|V|$, reaching a minimum of approximately $P \approx 0.5$ at intermediate potentials and remaining weakly dependent on further increases in $|V|$. The reversed sequence of barriers and wells modifies the interference conditions and leads to a different saturation level compared with forward propagation. This confirms that, in the triangular configuration, transport is controlled not only by scatterer density but also by the ordering of positive and negative potential regions encountered by the wave packet.

Overall, the results in Fig. 2 reveal that wave-packet transmission in graphene is not determined solely by scatterer density, but by the interplay between geometric arrangement, potential sequencing, and interference effects. While all configurations exhibit a common temporal evolution toward a saturated regime, their transmission shows qualitatively different responses to the potential strength. In sample 1, a pronounced nonmonotonic dependence emerges, with strong suppression at intermediate potentials. This behavior can be understood in terms of potential-dependent phase accumulation and coherent multiple scattering within the periodic barrier-well landscape. At small $|V|$, individual scattering events are relatively weak, and the transmission remains high. As $|V|$ increases, both the scattering strength and the relative

phases accumulating along different propagation pathways change. At intermediate potentials, multiply scattered wave-packet components can interfere predominantly destructively in the transmitted region, while repeated scattering increases the dwell time within the patterned region, producing a pronounced transmission minimum. A further increase in $|V|$ changes the relative phases again, moving the system away from this strongly suppressive interference condition and allowing partial recovery through constructive or less destructive interference pathways. The position and depth of the transmission minimum therefore depend on lattice geometry, carrier energy, and propagation direction. In contrast, the staggered geometry (sample 2) produces a smoother, nearly monotonic behavior, consistent with a more homogeneous scattering landscape and reduced sensitivity to potential ordering. Notably, the triangular superlattice (sample 3) does not simply lead to stronger suppression; instead, it promotes faster saturation while maintaining relatively higher transmission levels, indicating the presence of efficient transmission pathways supported by its packing geometry. These results demonstrate that transmission is governed by a competition between multiple scattering, destructive interference, and interference-assisted transport rather than by geometric density or scattering strength alone. The relative importance of these mechanisms also depends on the scatterer size and carrier energy, as discussed below.

Although Fig. 2 presents representative results for $R = 15$ nm, the complete set of time-dependent transmission curves for $R = 7, 10$, and 15 nm and wave-packet energies $E = 100, 150$, and 200 meV is shown in Figs. S1–S5 in the Supplemental Material [37]. These results confirm that the characteristic transport behavior persists across the investigated scatterer sizes and carrier energies, with all systems exhibiting the same general temporal evolution toward a saturated regime, while the detailed transmission values and interference features depend on R , E , and the geometric configuration. At the same time, the scatterer radius and wave-packet energy introduce important quantitative modifications. Increasing the radius R generally enhances the scattering, leading to a systematic reduction of the saturated transmission and a more pronounced separation between curves corresponding to different potential magnitudes. In contrast, smaller radii result in weaker scattering and higher transmission levels, indicating that the degree of confinement and interference can be tuned through the scatterer size. The wave-packet energy E also plays an important role. In the present parameter range, higher-energy wave packets show stronger nonmonotonicity and deeper minima, indicating that their phase accumulation and interference with the periodic potential are more sensitive to the barrier-well sequence. This behavior reflects the increased contribution of multiple scattering and interference processes as the wave packet interacts with the spatially periodic potential landscape. Conversely, at lower energies, the transmission exhibits a more monotonic dependence on V , consistent with weaker effective backscattering and the persistence of Klein-tunneling-assisted transmission. In general, these results demonstrate that while geometry sets the dominant transport regime, the scatterer radius and carrier energy provide effective tuning parameters that control the strength of scattering, the

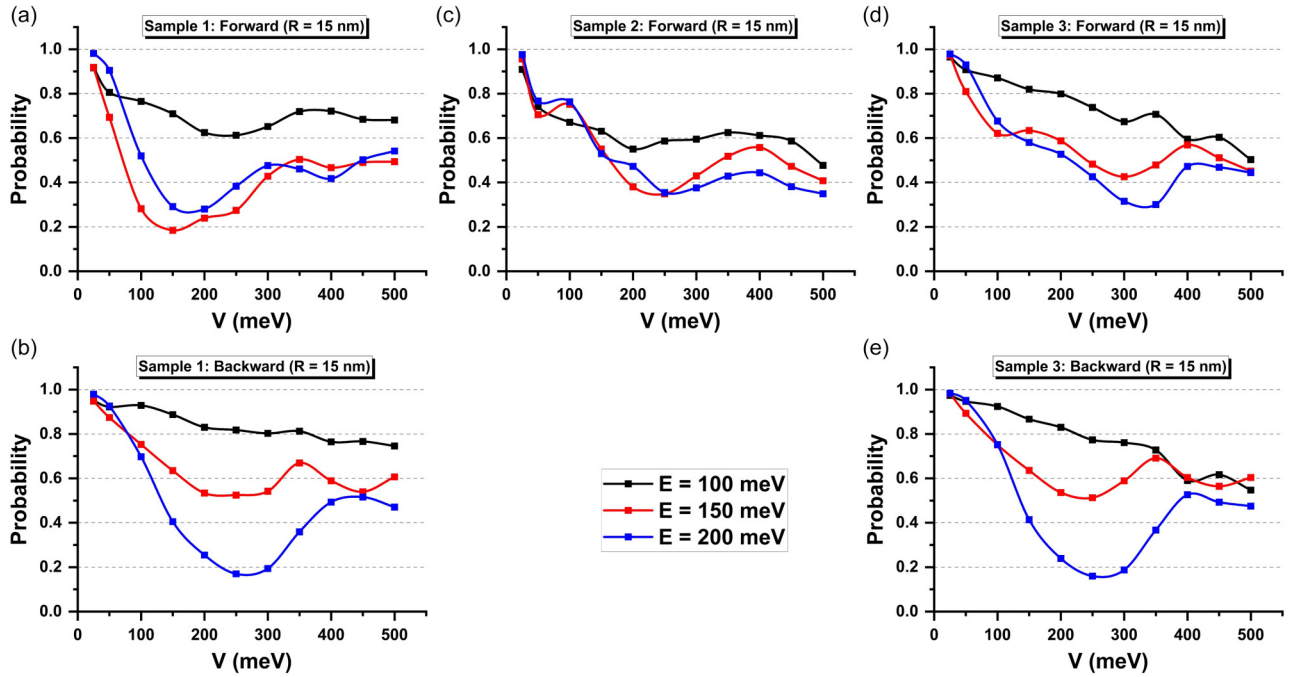


FIG. 3. Saturated transmission probability P as a function of electrostatic potential V for the five configurations: (a) sample 1 forward, (b) sample 1 backward, (c) sample 2, (d) sample 3 forward, and (e) sample 3 backward. Results are shown for wave-packet energies $E = 100, 150$, and 200 meV at fixed scatterer radius $R = 15$ nm. Although increasing V generally enhances scattering, the resulting dependence of P on V is strongly controlled by geometry and propagation direction, leading to monotonic suppression in some cases and pronounced nonmonotonic minima in others.

onset of interference effects, and the resulting transmission characteristics.

Figure 3 represents the saturated transmission probability P as a function of the potential V for all five configurations. Figure 3(a) shows the results for sample 1 under forward propagation. A clear energy dependence is observed. For $E = 100$ meV, the transmission decreases gradually with increasing V , exhibiting a monotonic behavior. In contrast, for higher energies ($E = 150$ and 200 meV), a pronounced nonmonotonic dependence emerges: P initially decreases sharply, reaching a minimum of approximately $P \approx 0.16$ at $V \approx 150$ meV (for $E = 150$ meV), and then partially recovers, stabilizing around $P \approx 0.5$ at larger V . This behavior indicates the formation of an optimal scattering regime at intermediate potentials, where interference and multiple-scattering effects are maximized. The stronger nonmonotonicity at higher energies suggests an increased sensitivity of energetic wave packets to the periodic potential landscape. Figure 3(b) presents the corresponding results for sample 1 under backward propagation. While the overall trends remain energy dependent, the behavior differs due to the reversed sequence of potentials. For $E = 100$ meV, the transmission decreases smoothly with increasing V . For higher energies, however, P drops sharply and reaches a minimum of approximately $P \approx 0.17$ at $V \approx 250$ meV (for $E = 200$ meV), followed by a partial recovery. At large V , the transmission remains around $P \approx 0.5$, indicating a saturation regime with limited sensitivity to further increases in potential strength. This behavior reflects the combined influence of wave-packet energy and propagation direction, where the initial interaction with

barrier regions enhances reflection and strengthens suppression at intermediate potentials.

Figure 3(c) shows the results for sample 2. In this case, all three energies exhibit a similar response: The transmission decreases rapidly with increasing V , reaching a minimum of approximately $P \approx 0.35$ at $V \approx 250$ meV, and then stabilizes near $P \approx 0.4$ for larger V . The weak dependence on energy reflects the effective symmetry of the staggered geometry, which leads to a more uniform distribution of scattering events and suppresses direction-dependent effects.

For sample 3 under forward propagation [Fig. 3(d)], the transmission behavior again shows a strong energy dependence. For $E = 100$ meV, P decreases monotonically with increasing V . For higher energies, a nonmonotonic trend appears, with a minimum at approximately $V \approx 300$ meV. The strongest suppression is observed for $E = 200$ meV, where $P \approx 0.3$. At larger potentials, the transmission partially recovers and saturates near $P \approx 0.45$. This behavior arises from the higher density of scatterers in the triangular superlattice, which enhances multiple scattering and interference at intermediate potentials while still allowing finite transmission channels at larger V . Figure 3(e) presents the backward propagation case for sample 3. For $E = 100$ meV, the transmission decreases monotonically with increasing V . For higher energies, strong suppression is observed, with a minimum of approximately $P \approx 0.17$ at $V \approx 250$ meV (for $E = 200$ meV), followed by a slight recovery. However, even at large V , the transmission remains near $P \approx 0.5$, indicating persistent suppression. The combined effect of the triangular geometry and the reversed potential sequence modifies the

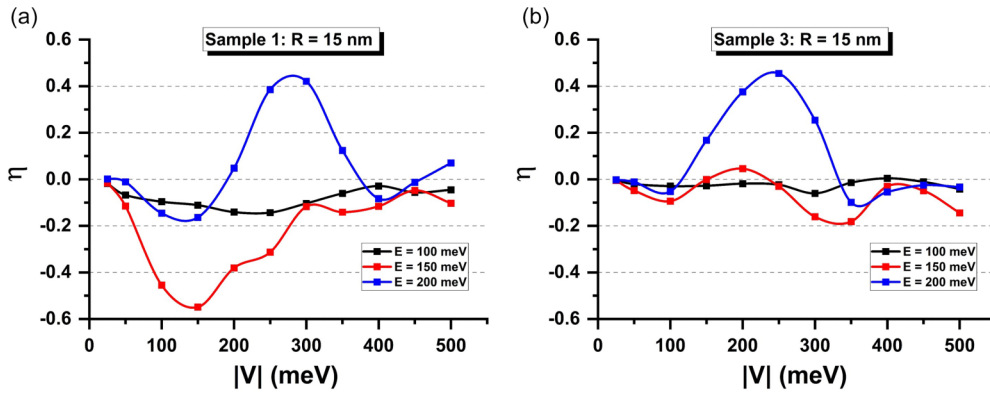


FIG. 4. Normalized directional asymmetry coefficient η for samples 1 and 3 at $R = 15$ nm as a function of the electrostatic potential magnitude. Positive values indicate preferential forward transmission, whereas negative values indicate preferential backward transmission. The sign changes of η demonstrate that the preferred propagation direction can be reversed by tuning both the potential strength and the wave-packet energy.

interference conditions, enhancing reflection in selected parameter regimes and making this configuration particularly effective for transmission suppression.

The saturated transmission probabilities for other scatterer radii ($R = 7$ nm and 10 nm) are provided in Fig. S6 in the Supplemental Material [37]. Consistent with the overall trends, P exhibits a pronounced and systematic dependence on the scatterer size, which serves as an additional control parameter alongside wave-packet energy, the scatterer's potential, and the propagation direction. Increasing the radius (from 7 to 15 nm) enhances scattering, deepens transmission minima, and amplifies nonmonotonic features at higher energies, reflecting stronger interference and multiple-scattering effects. While low-energy wave packets retain a relatively smooth, monotonic suppression, higher-energy wave packets develop pronounced minima at intermediate potentials that become increasingly sharp with increasing radius. The staggered geometry (sample 2) remains largely insensitive to energy, preserving uniform transport, whereas samples 1 and 3 display progressively stronger directional asymmetry with increasing radius. Notably, the triangular configuration (sample 3), particularly under backward propagation at large radius ($R = 15$ nm), produces the strongest transmission suppression, identifying it as the most efficient regime for tunable electron filtering. These results establish scatterer size as a key tuning parameter for interference-controlled transport in graphene superlattices.

Figure 4 shows the directional transmission contrast (measured as the normalized asymmetry coefficient η) for samples 1 and 3 at $R = 15$ nm. For sample 1, the asymmetry is strongly energy dependent and changes sign as the electrostatic potential varies. This behavior indicates that the preferred propagation direction is not determined solely by the geometry but can be tuned by adjusting both the carrier energy and potential strength. Specifically, for wave-packet energies $E = 100$ meV and $E = 150$ meV, backward propagation is generally favored. In particular, at $E = 150$ meV, the strongest backward preference occurs around $V = 150$ meV. In contrast, for $E = 200$ meV, both propagation directions can dominate depending on the potential magnitude, with a pronounced forward preference emerging near $V \approx 300$ meV.

The sensitivity of the directional response is largest near the zero crossings of η , where the forward and backward transmission probabilities become comparable and relatively small changes in the electrostatic potential may reverse the preferred propagation direction. In contrast, parameter regions with larger $|\eta|$ provide a more stable directional preference. As further shown in Fig. S7 in the Supplemental Material [37], the oscillatory and sign-changing behavior persists for $R = 7$, 10 , and 15 nm, indicating that the effect is not restricted to a single geometric scale, although the magnitude of η depends appreciably on the scatterer size. Since the present calculations employ discrete values of V and the geometric parameters, a quantitative tolerance to small electrostatic or geometric variations would require a finer parameter-resolution analysis.

Similar behavior is observed for sample 3, which exhibits an oscillatory directional response reflecting the more complex interference pattern induced by the triangular arrangement, especially at $E = 150$ meV. In this case, the asymmetry likewise changes sign as a function of $|V|$, indicating that the triangular superlattice not only enhances scattering but also enables geometry-controlled, switchable directional filtering.

The results for η for other scatterer radii ($R = 7$ nm and 10 nm) are provided in Fig. S7 in the Supplemental Material [37]. Consistent with the trends observed at larger radii, the directional asymmetry remains strongly dependent on both energy and potential strength, exhibiting multiple sign reversals as a function of $|V|$. However, the magnitude of η is generally reduced at smaller radii, indicating weaker directional contrast due to diminished scattering and interference effects. As the radius increases from 7 to 10 nm, the amplitude of the oscillations becomes more pronounced, and the regions of strong positive and negative η expand, reflecting enhanced multiple scattering and more effective interference control. In sample 3, the triangular geometry continues to produce a more complex and tunable directional response compared to sample 1, with sharper transitions between forward- and backward-dominated regimes, particularly at intermediate potentials. These results confirm that scatterer size acts as a key parameter for amplifying and

stabilizing geometry-induced directional transport in graphene superlattices.

More generally, although the present calculations demonstrate sign reversal explicitly for the square and triangular arrangements, the underlying mechanism is not expected to be unique to these particular lattice types. Rather, directional reversal can, in principle, arise whenever the electrostatic pattern produces inequivalent forward and backward scattering pathways whose relative interference can be tuned by carrier energy or potential strength. The absence of directional asymmetry in the effectively reversal-symmetric staggered configuration supports this interpretation. Other periodic and, potentially, quasiperiodic electrostatic patterns satisfying these conditions may therefore exhibit analogous behavior, although verification for specific lattices requires separate calculations.

Overall, the saturated transmission probability P is governed by a nontrivial interplay of energy, potential strength, geometry, and propagation direction. While low-energy wave packets exhibit a nearly monotonic decay with increasing V , higher energies produce pronounced nonmonotonic behavior with well-defined minima at intermediate potentials, signaling enhanced multiple scattering and interference. The staggered geometry yields direction-independent transport, whereas the square and triangular configurations produce pronounced directional asymmetry. Notably, the triangular superlattice under backward propagation exhibits the strongest transmission suppression among the geometries considered here, making it particularly favorable for directional filtering. These results demonstrate that electrostatic patterning enables geometry-controlled modulation of Dirac-fermion transport in graphene, functioning not only as a transmission suppressor but also as a tunable directional selector for Dirac wave packets.

From an experimental perspective, the proposed potential landscapes may, in principle, be approached using gate-defined graphene superlattices, for which periodic electrostatic modulation of carrier transport was already experimentally demonstrated [18]. More complex patterned top- or bottom-gate architectures could potentially generate alternating electron- and hole-doped regions analogous to the $+V$ and $-V$ regions considered here. The characteristic dimensions used in our calculations, with circular potential regions of radius $R = 7\text{--}15\text{ nm}$ and lattice spacings of approximately $28\text{--}32\text{ nm}$, correspond to nanoscale electrostatic patterning. In an actual device, the gate-induced potential profiles would be smoother than the ideal steplike potentials assumed in the model. Although the present calculations do not provide a quantitative fabrication tolerance, the persistence of the qualitative directional behavior over the investigated range of scatterer radii indicates that the effect is not restricted to a single finely tuned geometric size.

The present simulations nevertheless assume ideal, phase-coherent electrostatic patterns, whereas realistic devices will exhibit positional and dimensional variations, gate-voltage fluctuations, charge inhomogeneity, and finite-temperature effects. Since the directional response originates from coherent multiple scattering and interference, such imperfections are expected to reduce the magnitude of the asymmetry and to broaden or shift the transmission minima and

sign-reversal points. Nevertheless, the persistence of sign-changing directional asymmetry over the investigated ranges of carrier energy and scatterer radius indicates that the effect is not confined to a single finely tuned parameter set. The most pronounced directional contrast is therefore expected in ballistic or quasiballistic samples with sufficiently long phase-coherence lengths. A quantitative assessment of disorder, thermal averaging, and decoherence would require a dedicated analysis beyond the scope of the present work.

IV. CONCLUSIONS

In this work, we investigated the propagation of electron wave packets in graphene subjected to photonic-crystal-like electrostatic potential patterns. By employing a time-dependent Dirac framework and the split-operator method, we analyzed how the geometric configuration, propagation direction, potential magnitude, scatterer radius, and wave-packet energy influence transmission dynamics.

The central result is that transmission in these patterned graphene systems is not governed solely by the magnitude or density of electrostatic scatterers. Instead, it is controlled by the combined effect of lattice geometry, barrier-well ordering, and wave-packet energy. This interplay gives rise to a well-defined temporal saturation of the transmission probability and to both monotonic suppression and pronounced nonmonotonic minima at intermediate potentials, where destructive interference and multiple scattering are most effective.

A key outcome of this study is the strong geometry- and direction-dependent control of transport. While the square and staggered configurations provide moderate and relatively symmetric transmission characteristics, the triangular superlattice enhances multiple scattering and produces strong direction-dependent responses. Depending on propagation direction and energy, this triangular geometry can either support efficient transmission pathways or yield strong suppression. The comparison between forward and backward propagation further revealed an intrinsic directional asymmetry governed by the sequence of potential barriers and wells encountered by the wave packet.

By quantifying the directional contrast through η , we showed that the preferred propagation direction is not fixed but can be tuned by adjusting the carrier energy and electrostatic potential strength. This sign-changing directional asymmetry is especially pronounced in the square and triangular arrangements, where the barrier-well ordering and pathway topology generate strong interference-mediated transport contrast. These findings show that photonic-crystal-like electrostatic patterning can be used not only to suppress or enhance graphene wave-packet transmission but also to program the directionality of Dirac-fermion transport. The results therefore point to a route toward switchable graphene electron-optical elements, including energy-selective directional filters and reconfigurable transmission devices.

Overall, our findings highlight that engineered electrostatic patterns inspired by photonic crystals offer a versatile platform for tuning Dirac-fermion transport in graphene. This approach opens new possibilities for the design of

graphene-based electron-optical devices with controllable transmission properties, including directional filters, collimators, and rectifying elements.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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