

Temporal chirp, temporal lensing, and temporal routing via space-time interfaces

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A time interface (a rapid change of the constitutive parameters of a material in time), applied within an unbounded medium where a wave travels, can enable frequency conversion and is considered the temporal analog of a spatial interface between two materials. Here, we study light-matter interactions in four dimensions, 4D (space, x, y, z , and time, t), by exploring the implications of applying time interfaces, not to the entire space where a wave travels, but to certain regions of space in order to create spatial interfaces in time. Different configurations such as induced perpendicular, parallel, and oblique spatial interfaces via a temporal interface are discussed. It is shown how such four-dimensional combination of temporal and spatial interfaces can enable interesting features such as the 4D *generalized Snell's law* and *temporal chirp*, *temporal lensing*, and *temporal routing* of electromagnetic waves. Such exotic possibilities may provide new ways to manipulate light-matter interactions via a combination of temporal and spatial interfaces.

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It is well known that wave-matter interactions can be tailored by engineering three-dimensional spatial (x, y, z) inhomogeneities of materials along the path of wave propagation [1]. In seeking alternative ways to manipulate waves, the scientific community has reported great progress in fields such as metamaterials, plasmonics, and photonics [2–4], enabling groundbreaking applications in diverse areas such as radars [5–7], imaging [8,9], and computing [10–13], to name a few. While all these applications have been developed using materials with time-invariant parameters such as relative permittivity, ϵ , and relative permeability, μ , the focus has now been shifting towards materials with such parameters being time dependent. In so doing, a higher degree of freedom is enabled by introducing *time* as an additional variable in material parameters, opening new paradigms for a full control of electromagnetic (EM) wave propagation in space and time [14–17].

An interesting type of time-modulated media is time interfaces where ϵ and/or μ of the *whole* medium are rapidly changed in time (with time scales smaller than the period T of the propagating wave) while the wave is in it [18,19]. These time interfaces have been recently proposed for interesting applications such as filters [20,21], temporal inverse and space-time Fresnel prisms [22,23], temporal aim-

ing and Brewster angle [24,25], meta-atoms in space and time [26,27], and temporal and spatiotemporal gratings [28,29], with groundbreaking experimental demonstrations being reported for water waves [30], microwaves [31], and optical frequencies [32,33]. When applying a time interface at a time $t = t_0$, provided that the impedance is changed from (Z_1 , $t < t_0$) to (Z_2 , $t > t_0$) with $Z_1 \neq Z_2$, time-refracted *forward* (FW) and time-reflected *backward* (BW) waves are generated [18,34]. While time interfaces are applied within an unbounded medium, a combination of spatial and temporal interfaces (space-time interfaces) have recently been explored in the realm of surface waves on time-dependent metasurfaces [35] and effective medium theory [36–38], where both spatial and temporal interfaces have dimensions smaller than the incident wavelength, λ , and T of the propagating wave, respectively.

Inspired by the exciting possibilities that time-dependent media can offer, here we explore wave propagation in a medium where spatial interfaces are created in time via an induced time interface at a time $t = t_0$. We first discuss the physical phenomena that arise when inducing in time a single perpendicular, parallel, or oblique spatial interface within an unbounded medium, providing physical insights in terms of field distributions and space-time mapping of wave propagation. Then more complex configurations are presented, showing how the combination of multiple space-time interfaces can lead to phenomena such as the temporal equivalent of generalization of the Snell law in four dimensions (4D) (x, y, z, t) and temporal chirp, temporal lensing, and temporal routing, demonstrating the potential of space-time interfaces.

Consider the scenarios shown in Fig. 1. For times $t < t_0$, a monochromatic EM wave is traveling within an unbounded (homogeneous and isotropic) medium having a relative permittivity value of ϵ_1 . The incident wave is polarized along

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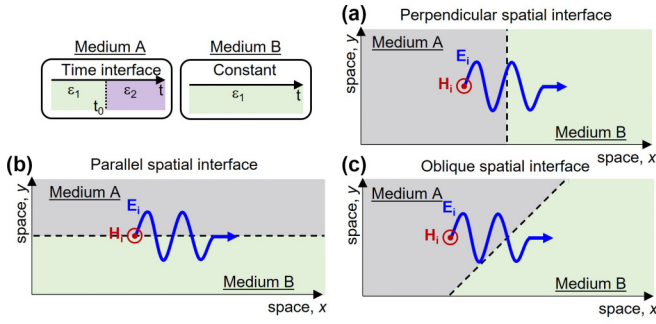


FIG. 1. Spatial interfaces induced by temporal boundaries, (a) perpendicular, (b) parallel, or (c) oblique spatial interface. Gray region: spatial region filled with a time-dependent $\varepsilon(t)$. Green region: ε is constant in time. The schematic representation of $\varepsilon(t)$ for each spatial region is shown in the top-left panel.

y (E_y) and propagates along x . At $t = t_0$, the ε of just one half-space is changed to ε_2 (all values real and larger than 1, with $\mu_1 = \mu_2 = 1$) while the other half-space remains with the same initial value ε_1 (constant ε). The boundary of the spatial region filled with a time-dependent $\varepsilon(t)$ (gray) can be chosen to be perpendicular, parallel, or oblique with respect to the direction of propagation of the wave such that different spatial interfaces can be created (see Fig. 1). To begin with, let us study the case where a perpendicular spatial interface is created using a time interface. This configuration is shown in Fig. 1(a). To evaluate it, we make use of the same configurations used in [38] but now considering that not the *whole* but just half-space is time-modulated via $\varepsilon(t)$. Moreover, the incident signal is switched off when inducing a time interface in order to better appreciate the effects of the temporal and spatial interfaces on the wave that is already in the region. (The numerical setup is available in the Supplemental Materials, SM [39].)

The numerical results of the electric field distribution (E_y) as a function of space and time are shown in Fig. 2(a), where

an EM wave is traveling within an unbounded medium for $t < t_0$ with $\varepsilon_1 = 1$ (see the SM [39] for an animation of this case). At $t \approx 28.35T$ (T being the period of the monochromatic wave for $t < t_0$) a temporal interface is applied to the left half-space in Fig. 1(a) [medium A shown in Fig. 2(b)], where ε is changed to $\varepsilon_2 = 10$, while the right half-space (medium B) remains unchanged. Note that a large change of ε is considered to better appreciate the physical phenomena that occur when inducing a spatial interface via a temporal interface. To better analyze these results, we provide in Fig. 2(b) a space-time map of all the temporal and spatial waves produced by the temporal and induced spatial interfaces. After the time interface is applied, there is a *remnant* signal that continues its propagation in medium B with the same frequency as for times $t < t_0$, i.e., f_1 , since ε is constant in this region (no change of frequency or wavelength λ_0). For medium A, however, the generated FW and BW waves have a new frequency $f_2 = f_1 \sqrt{\varepsilon_1/\varepsilon_2}$ (λ_0 and wave number k do not change). The BW wave simply travels along the negative x axis, i.e., $x/\lambda_0 < 0$, within medium A with a lower speed (reduced T) since $\varepsilon_2 > \varepsilon_1$. The FW wave travels towards the positive x direction, and when it reaches the induced spatial boundary at $x/\lambda_0 = 0$, spatial transmission (T_x) and reflection (R_x) occurs. From Fig. 2(a) this can be seen as a change in wavelength in medium B compared to the wavelength in medium A, which remains unchanged for all times.

A natural question arises: Could we exploit such time-induced perpendicular spatial interfaces for more complex wave-matter interactions? Imagine the spatial scenario shown in Fig. 2(c) [note that for this example the axes have been changed compared to Fig. 1(a) to better compare it with its space-time equivalent]. As is known, if an oblique incident wave with wave number k_1 (k_{x1}, k_{z1}) reaches a spatial boundary between two materials, a reflected (not shown) and a refracted wave will travel within the first and second medium, respectively. The direction of the latter wave is defined by Snell's law, which requires k_{x2} in medium 2 to be the same as k_{x1} in medium 1, i.e., $k_{x2}(x) = k_{x1}$, leading to the fact that the

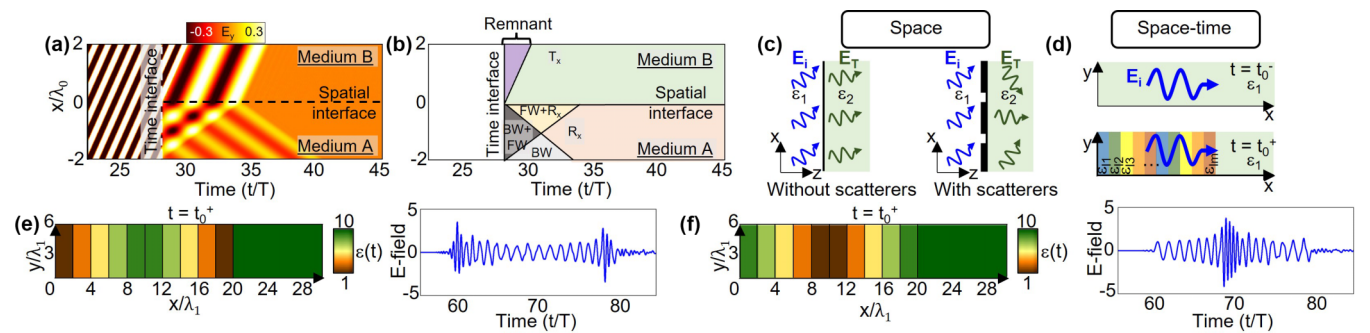


FIG. 2. (a), (b) Space-time diagrams for E_y (calculated at $y = 0$) and waves generated when a perpendicular spatial interface is induced using a temporal interface via a change of ε . μ is time independent, with $\mu_1 = \mu_2 = 1$. (c) Generalized Snell's law in space (three dimensions, x, y, z). (d) 4D generalized Snell's law (space-time, x, y, z, t). The colors for time $t = t_0^+$ represent different values for different spatial regions. (e), (f) E_y for the FW wave (right panels, recorded in the spatial region where ε is time independent, right green region from Fig. 2(d), bottom) after applying different temporal interfaces of different values simultaneously along the spatial x axis. A total of ten layers are produced with a thickness of $2\lambda_1$ (λ_1 is the wavelength inside the whole medium having an ε value as that from $t < t_0$). The values of $\varepsilon(t)$ for each induced spatial layer are shown on the left panels at a time instant $t = t_0^+$, considering $\mu_1 = \mu_{lm} = 1$. For the results from panel (e), ε is changed from $\varepsilon_1 = 10$ to $\varepsilon_{11,10} = 1$ for layers $m = 1$ and $m = 10$, and $\varepsilon_{12,9} = 3$, $\varepsilon_{13,8} = 5$, $\varepsilon_{14,7} = 7$, and $\varepsilon_{15,6} = 9$ for the other layers. For panel (f), ε is changed to $\varepsilon_{11,10} = 9$, $\varepsilon_{12,9} = 7$, $\varepsilon_{13,8} = 5$, $\varepsilon_{14,7} = 3$, and $\varepsilon_{15,6} = 1$.

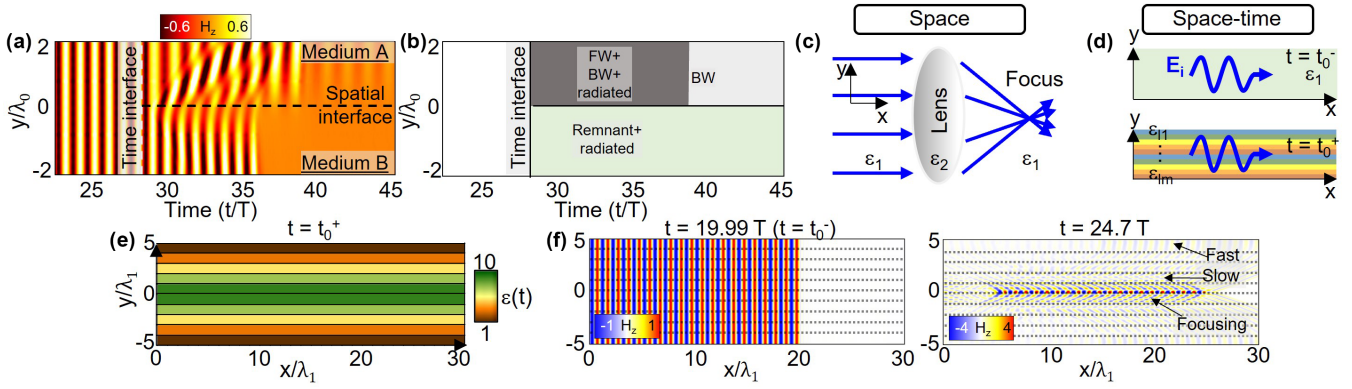


FIG. 3. (a), (b) Space-time diagrams showing the H_z field distribution (at $x = 0$) and waves for a parallel boundary induced with a time interface. The values of ϵ_1 , t_0 , and μ are the same as Fig. 2(a). ϵ is changed from $\epsilon_1 = 1$ to $\epsilon_2 = 2$ at $t = t_0$. (c) Conventional (spatial) lens. (d) *Temporal lens* concept in four dimensions. (e) ϵ values at a time right after the time interface is applied, along with (f) the H_z field distribution at different times showing how a “line” focus is produced after inducing the spatial interfaces via temporal interfaces [right panel from (f)]. A total of ten layers are used with a thickness of λ_1 for each layer (with λ_1 as the wavelength inside the whole medium having ϵ and μ values as those from $t < t_0$). The values of $\epsilon(t)$ for each layer are shown on (e) with $\mu_1 = \mu_{lm} = 1$.

entire refracted wave propagates along a single direction [1]. Interestingly, as shown in Ref. [40], one can generalize Snell’s law in space by placing properly engineered subwavelength scatterers of different geometries/dimensions along the x axis on the spatial interface. These scatterers cause an extra phase shift $\varphi(x)$ between the two sides of the interface. In so doing, a variation of phase $\varphi(x)$ can be introduced such that the wave number can be locally changed to $k_{x2}(x) = k_{x1} + \partial\varphi(x)/\partial x$. This generalized Snell’s law in space has opened new avenues in the field of metasurfaces and metamaterials to arbitrarily manipulate fields and waves [40]. Here we propose a way to further generalize such concepts to four dimensions—space and time (x, y, z, t)—by creating spatial interfaces via time interfaces. Our approach is schematically shown in Fig. 2(d), where multiple perpendicular spatial interfaces are induced at the same time using time interfaces with different values of ϵ , all applied at $t = t_0$. In so doing, an array of spatial layers is created in time with the frequency of the signal inside them being different, as expected ($f_{2,lm} = f_1\sqrt{\epsilon_1/\epsilon_{lm}}$, $m = 1, 2, 3, \dots$, where m is the spatial layer number). This means that for $t < t_0$, the angular frequency of the wave in the entire space is ω_1 , and for $t > t_0$, the converted frequency just due to the temporal interfaces is a function of position $\omega_2(x)$, which can be considered as the spatiotemporal analog of the generalized Snell’s law in space, $k_{x2}(x)$. Note that our concept is different from what has been explored in terms of Snell’s law and its connection with time refraction only [41], as here we create spatial interfaces in time via simultaneously induced time interfaces, i.e., both interfaces are combined, 4D scenario. Examples of this concept are shown in Figs. 2(e) and 2(f), where ten perpendicular spatial layers of thickness $2\lambda_1$ are simultaneously induced at $t = t_0$ (see caption for further details). These two examples consider the scenarios where ϵ is time dependent, $\epsilon(t)$, while μ is constant ($\mu_1 = \mu_{lm} = 1$). The values of $\epsilon(t)$ are shown on the left panels of Figs. 2(e) and 2(f) [similar results can be obtained when fixing ϵ while using time-dependent $\mu(t)$ (not shown)]. The numerically simulated E_y as a function of time, recorded at a

point in the spatial region where ϵ does not change, is shown in Figs. 2(e) and 2(f). The resulting wave traveling towards the region of constant ϵ has oscillations with different amplitudes and time durations, with the latter feature effectively being a *temporal chirped* signal. Such a *temporal chirp* can be tailored by modifying the values of ϵ_{lm} or μ_{lm} (not shown) of each induced spatial layer. This can be explained via the FW and BW waves created in each induced spatial layer. The amplitudes of the FW and BW waves as well as the converted frequency, $f_{2,lm}$, are larger in spatial layers where $\epsilon(t)$ is changed to smaller values ($\epsilon_{lm} < \epsilon_1$) [18]. This can be seen in the recorded final *temporal chirped* signals from the two examples in Figs. 2(e) and 2(f). Larger amplitudes and shorter time oscillations (larger $f_{2,lm}$) appear at the extreme ends (left and right) or at the center of the time window from Fig. 2(e) and 2(f), respectively (generated by layers with $m = 1, 10$ and $m = 5, 6$ respectively), demonstrating the potential to achieve a tailored *temporal chirped* signal. See also the SM video [39] for the case shown in Fig. 2(f).

Let us now study the case where a parallel spatial interface is induced via a temporal interface. This scenario is shown in Fig. 1(b). As in Fig. 2, the half-space of an unbounded medium is filled with a time-dependent $\epsilon(t)$ (medium A, top) while the other half is kept constant (ϵ does not change, medium B, bottom). The numerical results (see caption for constitutive parameters) of the H_z distribution as a function of space and time along with a schematic of the generated space-time waves are shown in Figs. 3(a) and 3(b), respectively. As observed, for $t > t_0$, a FW and a BW wave are created within medium A traveling with a different frequency f_2 (as defined above), while a remnant wave continues its propagation within medium B with the same initial frequency f_1 , as expected. Moreover, a radiated signal is created along the full spatial interface ($y = 0$). Intriguingly, this radiated field is more evident in medium A ($y/\lambda_0 > 0$), where some ripples can be seen for times between $30T < t < 35T$. Remarkably, this is in fact the temporal version of the interfacial antenna scenario explored in [42], where the radiation of a line source placed

at the spatial interface between two dielectrics was studied. In that work it was shown that the radiated field is stronger within the denser material (larger ϵ), and this radiation has its peak along a specific angle defined by the values of ϵ from both media. This, indeed, is analogous to what is happening in the space-time scenario shown in Figs. 3(a) and 3(b), where the radiated field produced along the induced spatial interface is stronger in medium A as $\epsilon_2 = 2$ in this medium, while it is $\epsilon_1 = 1$ in medium B. Such radiated fields shown in Figs. 3(a) and 3(b) can also be explained by considering Snell's law in space where the refracted angle of the *grazing* signal traveling from medium B to A at $t > t_0$ is $\theta_A = 45^\circ$ (in agreement with numerical calculations, see SM [39] for further details along with an animation). (It is worth noting that while it is well known that light indeed tends to travel towards higher-refractive-index media, the analogy with interfacial antennas is made here in order to determine the direction of the scattered signal produced by the temporally induced spatial interfaces, as shown below via an example.)

As an application of such an induced parallel spatial interface, we propose the concept of a *temporal lens* (which is different from time lenses for dilation of optical pulses where engineered dispersion and optimized phase shifters are implemented [43]). Consider the case shown in Fig. 3(c). As is well known, the shape and materials of a conventional (spatial) lens should be designed to manipulate the phase of the output signal such that an incoming collimated beam upon passing through the lens converges to the focal point. Similarly, for our proposed *temporal lens*, as shown in Fig. 3(d), one can induce in time an array of different but simultaneously created parallel spatial layers in which the frequencies and velocities of the waves are different. In this scenario, different from the temporal chirp shown in Figs. 2(d)–2(f), the frequency will change as a function of y , $\omega_2(y)$, for all x following the same expression as that of a time interface as discussed above, $\omega_2(y) = \omega_1 \sqrt{\epsilon_1/\epsilon_2(y)}$. Importantly, for our *temporal lens*, the ϵ and/or μ of each induced parallel layer created at $t = t_0$ should be chosen such that the new frequency and phase velocity in the central layers should be smaller than those of the side layers, i.e., ϵ and/or μ for the central parallel spatial layers should be larger than those in the side layers for $t > t_0$. In so doing, as described in Fig. 3(a), each spatial interface between parallel layers will create radiated fields which will travel towards the denser parallel layers until all converge at $y = 0$, along the entire spatial x axis. Importantly, as multiple spatial layers are created, multiple scattering may occur between each layer. This could be minimized by introducing time interfaces such that an adiabatic value of ϵ along the y axis is introduced (not shown). An example is shown in Figs. 3(e) and 3(f) (and an animation is also available in the SM [39]), where ϵ is modified at $t = 20T$ from $\epsilon_1 = 10$ (for the whole medium where the wave is initially traveling) to different values along the y axis to create ten induced parallel spatial layers each of height λ_1 . Faster/slower FW waves are created within each induced parallel spatial layer depending on the values of ϵ_{lm} used, as expected, with slower FW waves for the central layers. As is shown, a “line” focus (a focal line) is produced along the spatial x axis at $y = 0$. This signal will then diverge in time (see SM video [39]), similar to the divergence of waves after focusing in space using a conventional

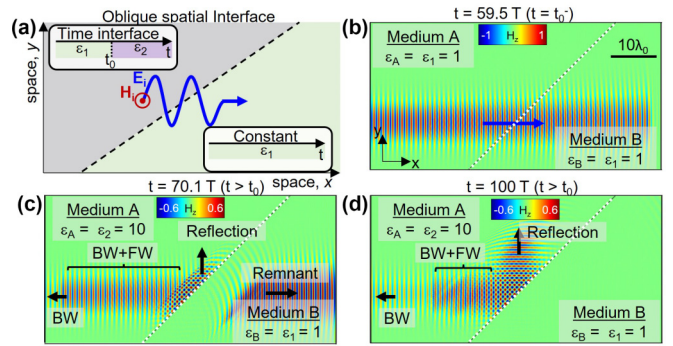


FIG. 4. (a) Inducing an oblique spatial interface using a temporal interface. (b)–(d) H_z field distribution at times before (b) and after (c), (d) applying a temporal interface to the left medium of (a). μ is time independent.

convex lens [Fig. 3(c)], demonstrating the performance of the proposed *temporal lens* concept. The width of the focal line along the transversal y axis will change for different times t , as this is a temporal focusing phenomenon. This response is the time equivalent of a spatial lens as discussed in Figs. 3(c) and 3(d). In a spatial lens, the transversal width of the converging beam changes as a function of position for all times. In the proposed temporal lens, the transversal width of the focal line changes as a function of time within the entire space (propagation x axis). The latter response can also be observed in Fig. 3(f) (right panel) where the produced focal line is $20\lambda_1$ long along the propagation x axis. This is an expected result because the incident signal is covering a length of $x = 20\lambda_1$ at a time right before the time interface is applied [see left panel from Fig. 3(f)]. Hence, the signals created within this spatial region due to the multiple induced space-time boundaries contribute to create the final focal line shown in the right panel of Fig. 3(f).

As a final scenario, consider the case shown in Figs. 1(c) and 4(a), where an oblique spatial interface is induced in time. For an example of an application of such a space-time configuration, we propose the *temporal router* shown in Figs. 4(b)–4(d). A wave is propagating within the whole medium (a Gaussian signal in this case) with a frequency f_1 and $\epsilon_1 = \mu_1 = 1$ [Fig. 4(b)]. At $t = t_0 = 59.7T$, a temporal interface is induced to the left half-space (medium A) to create an oblique spatial boundary of angle $\theta = 45^\circ$. To do this, ϵ of medium A is changed to $\epsilon_2 = 10$ while ϵ for medium B (right half-space) remains unchanged, ϵ_1 . As shown in Fig. 4(c), for a time $t > t_0^+$, a remnant wave travels within medium B with the same frequency and wavelength as the initial frequency and wavelength, as expected. In medium A, a FW and BW waves with the converted frequency are created. When the FW wave reaches the oblique spatial boundary, nothing is transmitted into medium B, but it is perfectly reflected with propagation along the y axis, completely changing its direction of propagation due to the incident angle ($\theta = 45^\circ$) being larger than the critical angle $\theta_c = \text{asin}(\sqrt{\epsilon_B/\epsilon_A}) = \text{asin}(\sqrt{\epsilon_1/\epsilon_2})$ (with ϵ_B and ϵ_A as the relative permittivity values for mediums B and A, respectively, after inducing the time interface in medium A). See also Fig. 4(d) and the SM video [39]. The scientific community has

recently reported groundbreaking experimental work demonstrating time interfaces from water to EM waves. In the latter scenario, different ranges of time-modulated permittivity values have been demonstrated depending on the spectral range (from microwaves up to the optical regime) [31–33]. We hope that our proposed concepts will be demonstrated in the future by leveraging the knowledge, efforts, and methods that many groups are currently developing in the field of time interfaces.

In conclusion, we have discussed how temporal interfaces can be exploited to create different types of spatial interfaces. Different wave characteristics involving such temporally induced spatial interfaces have been discussed, including the 4D generalization of Snell's law for temporal chirp, temporal lensing, as well as temporal routing of EM waves. These results may offer new opportunities and directions for controlling EM waves in multiple dimensions via the im-

plementation of temporal interfaces and the combinations of multiple types of induced spatial interfaces (perpendicular, parallel, and oblique).

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Data Availability. The datasets generated and analyzed during the current study are available from the corresponding author upon reasonable request.

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