

Light-controlled terahertz plasmonic time-varying media: Momentum gaps, entangled plasmon pairs, and pulse-induced time reversal

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This Letter establishes a Floquet engineering framework in which coherent high frequency light with a time dependent amplitude can be used to parametrically excite and amplify THz plasmons, mirror plasmonic wave packets in time, and generate momentum-gapped plasmonic band structures, entangled plasmon pairs, and THz radiation in two dimensional Dirac systems. Our results show how low frequency plasmons can be coherently excited and manipulated without the need for THz light.

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Introduction. Time varying media [1], discussed by Morghenthaler as early as 1958 [2], and by Holberg and Kunz in 1966 [3], have recently attracted a great deal of attention due to a variety of exotic effects and potential applications. Notable examples are time-varying mirrors [4], space-time crystals [5,6], time-varying metasurfaces [7–9], temporal switching [10–12], amplified emission and lasing [13], photon pair generation [13,14], nontrivial topological [15] and statistical properties [16], non-Hermiticity [17,18], momentum-gapped (k -gapped) states, and unusual solitonic behavior [19]. Beyond photonics, time varying media have been realized in classical liquids [20] and acoustic media [21,22].

Floquet engineering of quantum materials, on the other hand, is a powerful tool for the manipulation of band structures and for creating nonequilibrium correlated states in atomic, optical, and condensed matter physics [23–51]. A recent work suggested that Floquet engineering using high frequency drives enables control of low-energy collective modes of many body systems through modulated Floquet parametric driving (MFPD) and leads to new correlated states [52]. Here, we demonstrate that an amplitude modulated optical Floquet drive can be used to create momentum gapped plasmonic time varying media—terahertz (THz) analogues of so called photonic time crystals (PTCs) [15]. Plasmonic time varying media can be used to create entangled plasmon pairs, amplify plasmons, as well as reverse their propagation in time. MFPD surpasses experimental difficulties associated with the excitation and manipulation of low frequency plasmons: A high frequency (eV-range) drive is used to engineer an effective band structure. Varying the drive's amplitude in time then creates a medium with time dependent properties.

Compared to many other solid state Floquet engineering schemes, MFPD offers two major advantages: First, the Floquet drive couples to a resonant mode (the plasmon), which stores its energy over many oscillation cycles and thus amplifies its effects. Second, it operates in a regime with strongly suppressed photon absorption and heating (see Fig. 1).

While we focus on THz plasmons in this manuscript, we note that the MFPD principle is very general, and could be employed to create time varying media with other kinds of collective modes and at different frequencies. The main limitation with respect to the frequency range is that the amplitude modulation, which is responsible for the time-varying properties, is significantly slower than the carrier frequency of the Floquet drive. Potentially, MFPD can be applied to magnons in ferro- and antiferromagnets, which span a range of frequencies between GHz and THz [53,54]. Here similar effects could be mediated by the renormalization of the effective exchange constant by coherent light [55–57].

Our starting point are electrons in a two dimensional material. An effective quasienergy band structure is induced by driving the electrons coherently with light of frequency Ω_F . The induced band structure depends on the amplitude of the driving signal. A subsequent periodic modulation of the drive amplitude with a frequency $2\omega_1 \ll \Omega_F$ results in a periodically changing Fermi velocity. The oscillation of the Fermi velocity parametrically couples to the soft plasmon modes of the two dimensional electron gas. This is the principle of MFPD, which is illustrated in Fig. 1. In this Letter, we will consider plasmons with low frequencies of a few THz [58]. The principle, however, can be applied more broadly, depending on material and driving parameters.

In the linear approximation, the Floquet-engineered dynamics of the plane wave plasmon modes $\delta\rho_{\mathbf{q}}$ is governed by the equation

$$\partial_t(1 + h \cos 2\omega_1 t) \partial_t \delta\rho_{\mathbf{q}} + \omega_{\text{pl}}^2(q) \delta\rho_{\mathbf{q}} = 0, \quad (1)$$

which is derived below. Here h is an amplitude describing the effect of a slow modulation of the driving field [see Eq. (3)],

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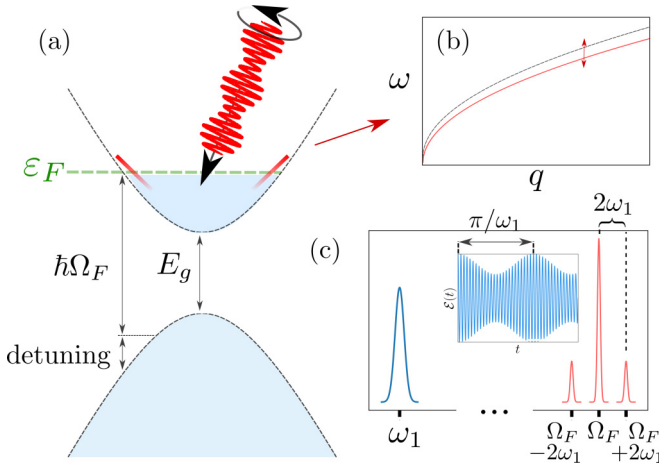


FIG. 1. Floquet engineering and MFPD. (a) The band structure (gray) of a gapped Dirac material, e.g., a transition metal dichalcogenide, with gap E_g driven with circularly polarized infrared light. The driving frequency Ω_F and the Fermi energy ε_F are chosen detuned, such that single photon absorption and particle-hole creation with energy transfer $\hbar\Omega_F$ is blocked by the Fermi sea. The driving leads to a change of the group velocity at the Fermi surface (red). (b) This change results in a shift of the plasmon dispersion $\omega_{\text{pl}}(q)$. (c) Modulating the amplitude of the Floquet drive with a frequency $\omega_1 \ll \Omega_F$ will lead to an oscillating Fermi velocity. This oscillation induces a parametric resonance of the plasmon modes. The spectrum of the modulated signal consists of a strong central peak at Ω_F and two side bands at $\Omega_F \pm 2\omega_1$ (red peaks). The parametric generation of plasmons at half the difference frequency (blue peak) corresponds to frequency mixing.

and $\omega_{\text{pl}}(q)$ is the plasmon dispersion defined in Eq. (5). Equation (1) is a plasmonic version of the equation describing the evolution of electromagnetic waves in PTCs [13], where the driving enters through a modulation of the dielectric constant $\varepsilon(t)$.

In essence, Eq. (1) is a parametric oscillator equation describing parametrically excited plasmonic plane-wave modes. We note that momentum conservation requires that modes with wave vectors $\mathbf{q}$ and $-\mathbf{q}$ are equally excited, such that the net momentum is zero [52]. Parametric resonance occurs for $\omega_{\text{pl}}(q) \approx n\omega_1$, with $n = 1, 2, 3, \dots$ (see Sec. 27 in Ref. [59]). In the presence of damping, resonance occurs above a critical amplitude h_c , which grows with increasing n [60]. We will focus on the $n = 1$ case, which requires the lowest h_c . It can be useful to exploit resonances with $n > 1$ to lower the required modulation frequency [61,62], however, modulation frequencies of up to 10 THz can be reached by interfering laser beams with slightly detuned frequencies [63,64].

This reveals another basic functionality of MFPD. It can be used as a frequency mixer, which creates standing wave plasmons at frequencies corresponding to the beating of the high frequency signal [see Fig. 1(c)]. These plasmons, which will have μm wavelengths and create strong electric fields in the THz range (see the discussion of experimental parameters at the end of this Letter), can be detected by near field optical microscopy [65,66].

Interesting effects on plasmon propagation can also be achieved with pulsed signals. In analogy to recent experiments

with surface waves in classical liquids [20], we propose that pulses can be used to induce time reversal of propagating plasmon wave packets via a temporary change of the electrons' effective mass. A high frequency pulse will split a propagating plasmon wave packet into two parts. While one part will continue to propagate in the initial direction, the other will evolve backwards in time and propagate toward the origin. We suggest that pulse induced time reversal is a promising way to control the propagation of THz plasmons with off-resonant, high frequency light, and discuss this functionality in more detail toward the end of this Letter.

Modulated Floquet parametric driving: Floquet engineering of time varying materials. We now show, in more detail, how MFPD can be used to create a time varying plasmonic medium. Consider a gapped Dirac Hamiltonian which describes electrons near the Fermi level of a two dimensional material, e.g., a transition metal dichalcogenide or black phosphorus [67–69],

$$H = \sum_{\mathbf{k}} \mathbf{c}_{\mathbf{k}}^\dagger [H_0(\mathbf{k}) + H_d(t)] \mathbf{c}_{\mathbf{k}} + \sum_{\mathbf{q}} V(\mathbf{q}) \rho_{\mathbf{q}} \rho_{-\mathbf{q}}. \quad (2)$$

We focus on a single valley and write $H_0 = \mathbf{d} \cdot \boldsymbol{\sigma}$ with $\mathbf{d} = [\lambda k_x, \lambda k_y, E_g/2]$, where, E_g is the energy gap between the two bands, $\boldsymbol{\sigma}$ is a Pauli vector describing pseudospin-orbit coupling, $\mathbf{c}_{\mathbf{k}}^\dagger, \mathbf{c}_{\mathbf{k}}$ are electron creation and annihilation operators, $\rho_{\mathbf{q}} = \sum_{\mathbf{k}} \mathbf{c}_{\mathbf{k}+\mathbf{q}}^\dagger \mathbf{c}_{\mathbf{k}}$ is the density operator, and $V(\mathbf{q}) = 2\pi/q$ is the 2D Fourier transform of the Coulomb potential. The driving part of the Hamiltonian is derived from minimal coupling: $H_d(\mathbf{k}) = e\mathbf{A} \cdot \nabla_{\mathbf{k}} H_0$. We assume driving by circularly polarized light with an amplitude $\mathcal{E}$ described by the vector potential $\mathbf{A} = (\mathcal{E}/\Omega_F)[- \sin \Omega_F t, \cos \Omega_F t, 0]$. Spin-valley degeneracy is included in the final results.

To avoid direct single photon absorptions, which are a dominant source of heating in Floquet engineered systems [36,38,70], we suggest to operate in an off-resonant regime, where the Fermi surface lies close to, but above, the single photon resonance (see Fig. 1). In this off-resonant regime, all states supporting excitations of a single electron by photons with an energy of $\hbar\Omega_F$ and near-zero momentum transfer are blocked. Processes involving the absorption of multiple photons are suppressed to second order in the small ratio of driving amplitude over driving frequency $e\mathcal{E}\lambda/\Omega_F^2 \hbar^2$ [38,70]. Interaction and disorder assisted single photon absorption is possible, but can be shown to not cause an overheating of the system (see Sec. G in the Supplemental Material [71]).

Let us now carry out the MFPD program and subject the Floquet drive amplitude $\mathcal{E}$ to a slow modulation. Expanding the dispersion relation around the Fermi momentum [72], we find $\varepsilon_k \approx \hbar v_F(\mathcal{E}, \Omega_F)(k - k_F) + \varepsilon_{k_F}(\mathcal{E}, \Omega_F)$ [73]. Here, the Fermi velocity $v_F(\mathcal{E}, \Omega_F)$ depends on the amplitude and frequency of the Floquet drive. A slow, adiabatic, time periodic modulation of $\mathcal{E}$ according to $\mathcal{E}(t) = \bar{\mathcal{E}} + \delta\mathcal{E} \cos(2\omega_1 t)$, where $\omega_1 \ll \Omega_F$, will result in an oscillating Fermi velocity. For our purposes, it will be convenient to parametrize this time dependence as an oscillation of the effective mass $m^* = \hbar k_F/v_F(\mathcal{E}, \Omega_F)$:

$$m^*(t) = \bar{m}^*(1 + h \cos(2\omega_1 t)). \quad (3)$$

Here, h is a small dimensionless number quantifying the amplitude of the oscillatory component of the effective mass. Below, we estimate that for reasonable driving strengths h is of the order of 10^{-2} . Explicit formulas for $\varepsilon_k(\mathcal{E}, \Omega_F)$, $v_F(\mathcal{E}, \Omega_F)$, and $\bar{m}^*(\mathcal{E}, \Omega_F)$ are given in Sec. A in the Supplemental Material [71].

Momentum-gapped states. We now investigate the influence of the oscillating effective mass of Eq. (3) on the dispersion relation of plasmons in a Coulomb interacting 2D electron gas. The plasmon dynamics can be inferred from charge and momentum conservation [74–77]. An inhomogeneous charge distribution $\rho(\mathbf{x}, t)$ will induce an electrostatic potential $\phi(\mathbf{x}, t)$ in the system, which will accelerate the electrons according to Newton's law: $\partial_t \mathbf{p}(\mathbf{x}, t) = -e\rho(\mathbf{x}, t)\nabla\phi(\mathbf{x}, t)$. Here, $\mathbf{p}$ is the momentum density of the electrons. Combining the continuity equation $\partial_t \rho(\mathbf{x}, t) = -\nabla \cdot \mathbf{j}(\mathbf{x}, t)$ and the relationship between momentum and current densities $e\mathbf{p}(\mathbf{x}, t) = m^*(t)\mathbf{j}(\mathbf{x}, t)$ (see Sec. B in the Supplemental Material [71]) with Newton's law, we find

$$\partial_t m^*(t) \partial_t \rho(\mathbf{x}, t) = \frac{e}{4\pi\varepsilon} \nabla \cdot \rho(\mathbf{x}, t) \nabla \int d^2x' \frac{\rho(\mathbf{x}', t)}{|\mathbf{x} - \mathbf{x}'|}, \quad (4)$$

where the integral extends over the sample. We separate the oscillating part of ρ from the homogeneous background $\bar{\rho}$ and write $\rho = \bar{\rho} + \delta\rho$. Linearizing Eq. (4) in $\delta\rho$, and assuming plane-wave solutions $\delta\rho(\mathbf{x}) = \delta\rho_{\mathbf{q}} \exp(i\mathbf{q} \cdot \mathbf{x})$, we obtain Eq. (1).

The plasmon dispersion $\omega_{\text{pl}}(q)$ is given by

$$\omega_{\text{pl}}(q) = \sqrt{\frac{e\bar{\rho}}{4\pi\bar{m}^*\varepsilon}} q^2 V(q). \quad (5)$$

$V(q)$ is the Fourier transform of the Coulomb potential.

Consider a quasi one-dimensional, waveguidelike setup of infinite length and width l , oriented parallel to the x axis. For this strip geometry we find

$$V(q) \approx \int_0^l dy \int_{-\infty}^{\infty} dx \frac{e^{iq \cdot \mathbf{x}}}{|\mathbf{x}|} \approx 2l \left| \ln \frac{|q|l}{4} \right| \quad (6)$$

for $q \ll 1/l$, where $\mathbf{q} = q\hat{\mathbf{x}}$. The precise geometry of the device is not very important for the physics presented here. The calculations would be very similar for an infinite 2D sample with $V(q) = 2\pi/q$, but we would need to keep track of the vector nature of $\mathbf{q}$.

Let us study the behavior of $\delta\rho_q$ in the vicinity of the parametric resonance, where $|\omega_{\text{pl}}(q) - \omega_1| \approx h\omega_1$. Using the ansatz $\delta\rho_q = a_q(t) \cos(\omega_1 t) + b_q(t) \sin(\omega_1 t)$, where the slowly varying coefficients are given by $a_q(t) = \tilde{a}e^{st}$ and $b_q(t) = \tilde{b}e^{st}$, we find (see Sec. C in the Supplemental Material [71])

$$s_{\pm}(q) = \pm \frac{\omega_1}{2} \sqrt{\frac{1}{4}h^2 - \left(\frac{\omega_{\text{pl}}^2(q) - \omega_1^2}{\omega_1^2} \right)^2}. \quad (7)$$

For $|\omega_{\text{pl}}^2(q) - \omega_1^2| > \omega_1^2 h/2$ the exponent is imaginary, corresponding to dispersive plane-wave solutions, whereas for $|\omega_{\text{pl}}^2(q) - \omega_1^2| < \omega_1^2 h/2$, we find exponentially growing (or decaying) unstable nondispersive solutions oscillating at the frequency ω_1 . This results in a momentum gapped dispersion,

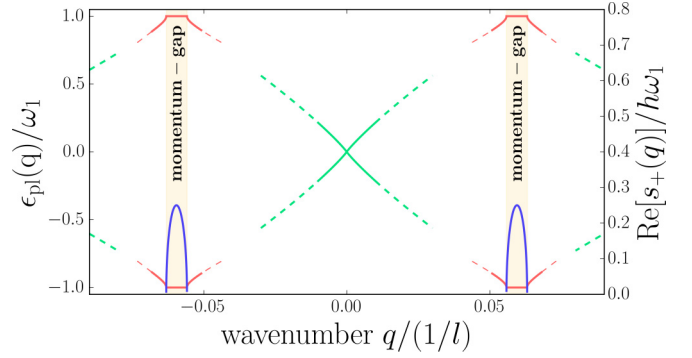


FIG. 2. Dispersion of the plasmonic time varying medium. The modulated Floquet drive couples to plasmon modes and induces gaps (yellow segments) near the resonance $\omega_{\text{pl}}(q) = \pm\omega_1$. In the vicinity of the gaps, the modes are given by Eq. (7) (red curves). Away from the gap, the drive-induced changes to the dispersion are small (green curves) and Eqs. (5) and (6) are approximately valid. The growth rate of the unstable modes $\text{Re}[s_{\pm}(q)]$ is shown in blue.

similar to the one appearing in so called photonic time crystals [1,15]. The gap is centered at the wave vectors $\pm q^*$, where q^* is determined by the resonance condition $\omega_{\text{pl}}(q^*) = \omega_1$.

Notice that Eq. (1) is invariant with respect to time translations by $T = \pi/\omega_1$. This symmetry is broken by the system's response—a feature that is known from discrete time crystals [78–84] and generic to parametric resonances. The momentum gap and the dispersion in the vicinity of the gap are shown in Fig. 2.

So far we did not consider the finite lifetime of plasmons. For small q , the plasmon dispersion lies outside the particle-hole continuum and Landau damping can be neglected (see Sec. G in the Supplemental Material). In clean materials the main source of damping is momentum relaxing phonon scattering [85]. To model this effect, we add momentum decay at a rate γ to Newton's law $(\partial_t + \gamma)\mathbf{p}(\mathbf{x}, t) = -\rho(\mathbf{x}, t)\nabla\phi(\mathbf{x}, t)$. To first order in γ , $s_{\pm}(q)$ in Eq. (7) obtains a negative real part of $-\gamma/2$ (see Sec. C in the Supplemental Material [71]). The instability condition $\text{Re}(s_{\pm}) > 0$ is realized in a narrow region around q^* for

$$h > 2\gamma/\omega_1. \quad (8)$$

Interestingly, even if $h < 2\gamma/\omega_1$ holds, the momentum-gap remains intact. The gap then hosts nondispersive modes with decay rates $-\gamma/2 \pm \omega_1 h^2/4$ at $q = q^*$.

Equation (7) suggests that the opening of the momentum gap is associated with the joining of two branches of the plasmon dispersion described by the plus and minus signs. To interpret the two branches, it is useful to write the solution of Eq. (7) in the form known from Floquet's theorem [86]:

$$\delta\rho_q = e^{-i\epsilon_{\text{pl}}(q)t} u_{\epsilon_{\text{pl}}}(t), \quad (9)$$

where $\epsilon_{\text{pl}}(q)$ is the plasmon quasidispersion that determines wave propagation once the periodic modulation is switched on [87], and $u_{\epsilon_{\text{pl}}}(t) = e^{-i\epsilon_{\text{pl}}t} [a(e^{2i(\epsilon_{\text{pl},\pm} + i s_{\pm})t} + 1) \pm ib(e^{2i(\epsilon_{\text{pl},\pm} + i s_{\pm})t} - 1)]/2$, where the plus is chosen for $\epsilon_{\text{pl}} < 0$ and vice versa. It is easy to verify that $u_{\epsilon_{\text{pl}}}(t + \pi/\omega_1) = u_{\epsilon_{\text{pl}}}(t)$, in accordance with Floquet's theorem. The quasidispersion $\epsilon_{\text{pl}}(q)$ can be confined to a Brillouin zone of width

$2\omega_1$. We choose $\pm\omega_1$ as the boundaries of the Brillouin zone. Away from the gap, for small q , the dispersion is given by $\epsilon_{\text{pl}}(q) \approx \pm i\omega_{\text{pl}}(q)/\sqrt{1+h^2/4}$, showing that the impact of the drive is small. The plasmonic band structure in terms of ϵ_{pl} is shown in Fig. 2.

Production of entangled plasmon pairs. Since the momentum supplied by the optical drive is negligible, plasmons can only be produced in pairs with opposite momenta $\pm\hbar q$. This results in generation of entangled plasmon pairs. To see this, we find the Hamiltonian leading to Eq. (1) and then apply a second quantization procedure following Refs. [13,14]. Details are given in Sec. D in the Supplemental Material [71]. For the driven, resonant plasmon modes we find the Hamiltonian in the interaction representation

$$H_{\text{pl,int}} \approx \frac{\hbar}{8}\omega_1(a_{q^*}^\dagger a_{-q^*}^\dagger + a_{-q^*} a_{q^*}), \quad (10)$$

where $a_{q^*}^\dagger$ and a_{q^*} are plasmon creation and annihilation operators for the resonant wavenumber q^* .

Applied to the vacuum, the time evolution operator $U_{\text{pl,int}}(t, 0) = \exp(-tH_{\text{pl,int}}/\hbar)$ generates a nonfactorizable two mode squeezed state $|\psi_{\text{pl}}(t)\rangle = U_{\text{pl,int}}(t, 0)|0, 0\rangle$ in the basis $|n_q, n_{-q}\rangle$ [88], where n_q gives the number of plasmons in the state q . The generation of entangled plasmon pairs has been discussed previously in Ref. [89]. The authors suggested to excite the longitudinal plasmon modes of a graphene ribbon via their coupling to a resonantly pumped transverse mode and discussed modes of detection.

Pulse induced time reversal. So far we have considered the action of a periodic oscillation of the effective mass $m^*(t)$ resulting from an amplitude modulated high frequency Floquet driving. We now discuss how a pulse induced, abrupt change of $m^*(t)$ can time reverse a propagating plasmon wave packet. The effect is shown in Fig. 3. We consider a pulse with peak amplitude $\mathcal{E}_0$, center frequency Ω_F , and duration Δt . As shown in Sec. E in the Supplemental Material [71], the effect of such a pulse at time $t = t_0$ on the effective mass can be described by the formula $m^*(t) = \bar{m}^*(1 + \bar{h}\delta(t - t_0))$, where $\bar{h}$ is an amplitude characterizing the impact of the pulse. This assumes that the pulse is very short compared to a plasmon oscillation cycle. For an incident wave packet $\delta\rho_i(\mathbf{x}, t)$, the time reversed part, which appears after the pulse, is given by (Sec. E in the Supplemental Material [71])

$$\delta\rho_{\text{rev}}(\mathbf{x}, t) \approx A(q_0)\delta\rho_i(\mathbf{x}, -t). \quad (11)$$

Here, $A(q_0) = \frac{1}{2}\omega_{\text{pl}}(q_0)\bar{h}e^{-i2\omega_{\text{pl}}(q_0)t_0}$ is a complex amplitude and q_0 is the central wavenumber of the packet.

The assumption of a delta function, like abrupt change of m^* , while convenient for analytic calculations (see Sec. E in the Supplemental Material [71]), is only valid if the pulse duration is much shorter than the time scale of the plasmon oscillations. To achieve a strong reshaping of the electron dispersion, typically pulses with field strengths of $10^7 - 10^8$ V/m and durations of around 100 fs are employed [27,30]. Being interested in the pulse induced time reversal of plasmons with frequencies of a few THz, we address the question of whether pulses with durations of up to half the plasmon oscillation cycle T_{pl} can be used. Using the Dedalus library [90], we carry out simulations (see Sec. E in the Supplemental Material [71]) for pulse durations of $\Delta t = 0.1T_{\text{pl}}$ (see Fig. 3),

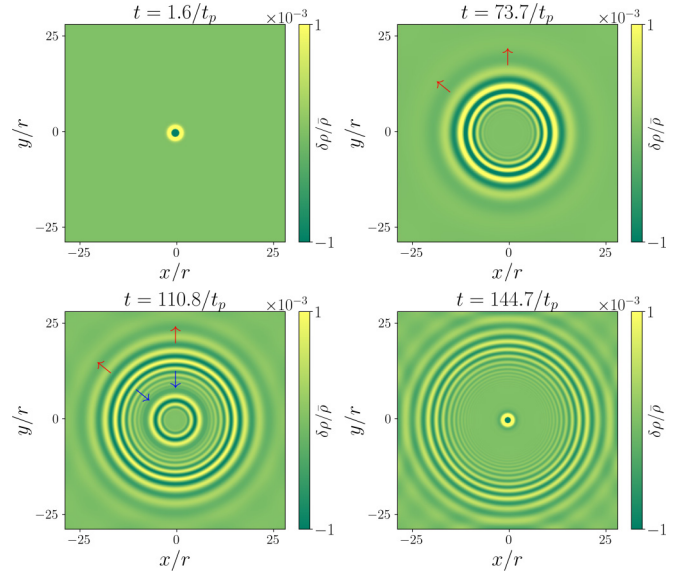


FIG. 3. Numerical simulation of pulse induced time reversal of a propagating cylindrical plasmon wave packet. The plasmon is triggered by an electromagnetic pulse with duration t_p localized in a region with radius r around the origin. A high frequency gaussian pulse, resulting in a change of the electron's Fermi velocity and effective mass, is applied at time $t_0 = 75t_p$. The wavefront is split into parts, with the time-reversed part propagating back toward the origin (blue arrows). At $2t_0$, the time reversed front is focused back at the origin. For this simulation, Eq. (4) was solved using the Dedalus package [90].

$\Delta t = 0.22T_{\text{pl}}$, and $\Delta t = 0.5T_{\text{pl}}$. We find that pulse induced time reversal is observable for all three values.

Discussion. The reshaping of bandstructure by light in the off-resonant regime considered here has recently been demonstrated in doped black phosphorus [30], showing that the regime of operation for MFPD controlled plasmonic time varying materials is within experimental reach. In Sec. F in the Supplemental Material [71], we estimate the critical driving strength to be $\bar{\mathcal{E}} \approx 4 \cdot 10^5$ V/m using Eq. (8) and the parameters $\bar{\rho} = 1.18 \cdot 10^{11}/\text{cm}^{-2}$, $E_g = 0.3$ eV, $\hbar\Omega_F = 0.35$ eV, $\lambda = 15$ eVÅ, $\epsilon = 6\epsilon_0$. The corresponding laser intensity is orders of magnitude smaller than in current solid state Floquet experiments [27–30]. The choice of material parameters was inspired by black phosphorus, and can be adjusted for other materials. The estimate assumes a plasmon quality factor of $Q = \omega_1/\gamma \approx 10^2$, reachable in clean materials [24,28,85]. In theory, Q factors of $10^3 - 10^4$ are believed to be achievable [85,91], which would further reduce the necessary driving power. Moreover we demonstrate in Sec. G in the Supplemental Material [71] that the heating caused by photon absorption from MFPD can be compensated for by the cooling power of the lattice. We stress that the realization of plasmonic time reversal requires only pulsed signals and could be demonstrated with setups similar to the ones used in most Floquet experiments [27–30]. Near-field optical microscopy can be used for detection.

To estimate the experimental parameters, we consider plasmons with frequencies of 1–5 THz, and use the above values for material and driving parameters. Table I shows

TABLE I. Plasmon frequencies, wavelengths, and field strengths of MFPD driven plasmons for two dimensional and striplike samples.

Sample	Frequency	Wavelength	Field strength
2D	1 THz	8.2 μm	$2.3 \cdot 10^5$ V/m
2D	5 THz	0.3 μm	$2.3 \cdot 10^5$ V/m
Strip, $l = 0.2 \mu\text{m}$	1 THz	1.7 μm	$1.6 \cdot 10^5$ V/m
Strip, $l = 0.05 \mu\text{m}$	3 THz	0.2 μm	$2.8 \cdot 10^5$ V/m

our estimates for plasmon wavelengths and field strengths. The momentum gap has typically a width of 1–2% of q^* , e.g., 0.05/ μm for the strip and 1 THz. To estimate the field strengths, we draw on Ref. [52], which predicts that, due to nonlinear effects, the exponential plasmon growth saturates at amplitudes of $\sim \sqrt{8\hbar c \rho} \epsilon^{1/4}$, where ϵ is the relative distance

of the drive from the critical driving strength. We assume a driving that is 10% above threshold, giving $\epsilon = 0.1$. Summing up, we proposed a method to induce low frequency plasmons by amplitude modulated high frequency signals, and to create time varying materials for plasmons with effects such as parametric amplification, the opening of momentum gaps, time reversal mirroring, and creation of entangled plasmon pairs. The proposed effects can be measured with near field microscopy [58,65,66] and used in device applications, e.g., as plasmon sources or to control the propagation of plasmons.

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