Stark many-body localization under periodic driving

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We study the stability of localization under periodic driving in Stark many-body systems. We find that localization is stable except near special resonant frequencies, where resonances cause delocalization. We provide approximate analytical arguments and numerical evidence in support of these results. This shows that disorder-free broken ergodicity is stable to driving, opening up the way to studying nonequilibrium driven physics in a different setting.

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I. INTRODUCTION

Generic ergodic quantum systems are expected to thermalize according to the predictions of statistical mechanics. Such ergodic systems are understood to be in agreement with the eigenstate thermalization hypothesis (ETH) [1–3], which states that the eigenstate expectation values of any local observable form a smooth function of energy. Any deviation from this expectation is, by default, interesting, as it leads to the unusual deviations from statistical mechanics. More practically, nonadiabatic manipulations of ergodic systems lead to runaway entropic increase and heat death [4]; therefore, creating nonequilibrium phases such as time crystals [5–7] *requires* breaking ergodicity, as usually these rely on periodic driving.

It is therefore interesting to study systems that exhibit nonergodic behavior. Ergodicity can be broken in various ways including glassiness, prethermalization [3], or kinetic constraints [8–11]. More recently there has been considerable attention devoted to disorder-induced many-body localization (MBL) [12–15]. In Anderson's original work, it was shown that transport in a tight-binding model for a single particle is suppressed by inherent disorder [16]. MBL extends this phenomenon to the case of interacting systems. Similarly, it has been long known that transport can instead be suppressed by the presence of a tilted potential, leading to so-called Wannier-Stark localization [17]. Here, the single-particle eigenstates become localized in space and equidistant in energy by a spacing that corresponds to the gradient of the tilt. Only much more recently was this shown to also persist in the presence of interactions, giving what is known as Stark MBL [18–23].

Periodic driving of a noninteracting Wannier-Stark system has been studied thoroughly. It was found that driving at a resonant frequency determined by the tilt angle results in delocalization for arbitrarily weak driving [24,25]. Here, we investigate the analogous situation in an interacting system, finding that delocalization does still occur near certain resonant frequencies, but only above a finite driving strength. We use approximate analytical arguments to explain how this driven system delocalizes in the noninteracting limit, and extend this to the interacting case using exact numerical methods. This therefore establishes that the situation is analogous to that for disordered MBL systems [4,26]. Previous work on driving Stark systems has focused on the high-frequency regime [27].

II. MODEL AND ANALYTICAL ARGUMENTS

A. Hamiltonian

We will focus on the time-periodic Hamiltonian

$$H(t) = \begin{cases} H_0, & 0 < \mathrm{mod}(t, T) \leqslant T_0, \\ H_1, & T_0 < \mathrm{mod}(t, T) \leqslant T, \end{cases} \tag{1}$$

with period $T = T_0 + T_1$ and

$$H_0 = -\gamma \sum_j^L j n_j + \sum_j^L h_j n_j + V \sum_j^{L-1} n_j n_{j+1}$$

diagonal in the basis of product (Fock) states, while

$$H_1 = -J \sum_j^{L-1} (b_j b_{j+1}^\dagger + \mathrm{H.c.}).$$

Here, the b_j are hard-core bosonic operators and our system lives on a one-dimensional (1D) lattice of size L. The parameters J and V denote the amplitudes of nearest-neighbor hopping and interactions, respectively, while γ denotes the gradient of the tilted potential. The times T_0 and T_1 correspond to the diagonal H_0 and off-diagonal H_1, respectively, such that T_1 controls the strength of the driving and the system is trivially localized in the $T_1 = 0$ limit. We choose this model as it has been well established for studying driven systems [26].

In the limit $h_j = 0$, the static system $H_0 + H_1$ is not localized [28], suggesting that the Floquet system described by Eq. (1) will also be delocalized. We therefore add a small disorder term where the h_j are drawn from a zero-mean normal

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distribution with variance w. To ensure that the disorder itself does not cause MBL, we set $w \ll V$. This is enough for the static system to be localized [28]. Alternatively, one could also achieve this by introducing a curvature term to the tilted potential as a function of system size, however, this brings with it the possible disadvantage of additional finite-size effects.[1]

Throughout this paper, the parameters used will be $J = V = T_1 = 0.5$, $\gamma = 5$, $w = 0.05$ (see Supplemental Material [29] for motivation). Finally, we work at half filling and with open boundary conditions.

The Floquet operator for this Hamiltonian, propagating the system over a period, is then

$$\mathcal{F} = e^{-iH_0T_0}e^{-iH_1T_1}. \tag{2}$$

B. Mapping to RZS

One can obtain analytical insight into the behavior of this system by recasting the problem in the so-called repeated zone scheme (RZS) [30]. To facilitate this, we recast the Hamiltonian of Eq. (1) as

$$H(t) = \bar{H} + \delta H(t), \tag{3}$$

where the time-independent part $\bar{H}$ is the time average of $H(t)$ over a single period while $\delta H(t)$ is the rest. We have

$$\begin{aligned}\bar{H} &= H_0\frac{T_0}{T} + H_1\frac{T_1}{T} \\ &= \sum_j [\tilde{J}(b_j b_{j+1}^\dagger + \text{H.c.}) + \tilde{V} n_j n_{j+1} + (\tilde{\gamma} j + \tilde{h}_j) n_j],\end{aligned} \tag{4}$$

with $\tilde{J} = JT_1/T$, $\tilde{V} = VT_0/T$, $\tilde{\gamma} = \gamma T_0/T$, and $\tilde{h}_j = h_j T_0/T$, and

$$\delta H(t) = \begin{cases} \delta H_1 = \bar{H}\frac{T}{T_0} - H_1\frac{T_1}{T_0}, & \text{mod}(t,T) \leqslant T_0, \\ \delta H_2 = \bar{H}\frac{T}{T_1} - H_0\frac{T_0}{T_1}, & T_0 < \text{mod}(t,T) \leqslant T. \end{cases} \tag{5}$$

One can eliminate the time dependence at the price of working with infinitely many coupled undriven systems [30]. Expanding in Fourier components, one finds that our system corresponds to a time-independent system on an infinite ladder, the nth rung of which hosts a copy of the static part of the Hamiltonian $\overline{H}$ shifted in energy by $n\omega$. The rungs are then coupled by the Fourier components of the driving Hamiltonian $\delta H(t)$ which cause hopping between the rungs. Denoting by $|\alpha\rangle$ a basis state of the Hilbert space of $\overline{H}$, the time-independent system can be written in terms of the states $|\alpha, n\rangle = |\alpha\rangle \otimes |n\rangle$ localized in rung n and the physical basis state α in the form

$$\langle \alpha, n | H_F | \beta, m \rangle = H_{\alpha,\beta}^{(n-m)} - \omega n \delta_{n,m}\delta_{\alpha,\beta}. \tag{6}$$

Here, $H_{\alpha,\beta}^{(p)}$ are the matrix elements $\langle\alpha| H^{(p)} |\beta\rangle$ of the pth Fourier component of the physical Hamiltonian $H^{(p)}$.

Writing $H^{(0)} = \overline{H}$ and $H^{(p)} = \delta H^{(p)}$ for $p \neq 0$ [since these are the Fourier components of δH of Eq. (3)], H_F can be represented as follows:

$$H_F = \left(\begin{array}{c|c|c|c|c|c|c} \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \iddots \\ \hline \cdots & \overline{H} - 2\omega & \delta H^{(1)} & \delta H^{(2)} & \delta H^{(3)} & \delta H^{(4)} & \cdots \\ \hline \cdots & \delta H^{(1)} & \overline{H} - \omega & \delta H^{(1)} & \delta H^{(2)} & \delta H^{(3)} & \cdots \\ \hline \cdots & \delta H^{(2)} & \delta H^{(1)} & \overline{H} & \delta H^{(1)} & \delta H^{(2)} & \cdots \\ \hline \cdots & \delta H^{(3)} & \delta H^{(2)} & \delta H^{(1)} & \overline{H} + \omega & \delta H^{(1)} & \cdots \\ \hline \cdots & \delta H^{(4)} & \delta H^{(3)} & \delta H^{(2)} & \delta H^{(1)} & \overline{H} + 2\omega & \cdots \\ \hline \iddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{array}\right). \tag{7}$$

Physically, the hopping between different rungs corresponds to the system gaining or losing energy in integer multiples of ω, and is mediated by the components

$$\delta H^{(n)} = \frac{\delta H_0}{2\pi i n}(e^{2\pi i n T_0/T} - 1) + \frac{\delta H_1}{2\pi i n}(e^{2\pi i n} - e^{2\pi i n T_1/T}). \tag{8}$$

We note for later reference that the components decay algebraically with n.

Solutions of the time-dependent Schrödinger equation $[i\partial_t - H(t)]|\phi(t)\rangle = 0$ can be written as $|\phi_a(t)\rangle = e^{-i\Omega_a t}\sum_n |\varphi_a^{(n)}\rangle e^{-in\omega t}$, where $|\varphi_a\rangle = \sum_{n=-\infty}^{\infty} |\varphi_a^{(n)}\rangle \otimes |n\rangle$ is an eigenstate of the time-independent Hamiltonian (7) with eigenvalue Ω_a. Here, $|n\rangle$ indexes the rung, so that $|\varphi_a^{(n)}\rangle$ is the part of the state living in a single rung. For stroboscopic dynamics, $|\phi_a(pT)\rangle = e^{-i\Omega_a pT}\sum_n |\varphi_a^{(n)}\rangle$ (for integer p) so that one sums up across all rungs.

What does this picture say about the system of Eq. (3)? In the limit we are working in ($h_j = V = 0$) and $\delta H(t) = 0$, the $|\varphi_a^{(n)}\rangle$ are localized and independent of n; they are the eigenstates $|\alpha\rangle$ of $\overline{H}$, which are many-body Stark localized states with eigenvalues $q_\alpha\tilde{\gamma}$ where the q_α are integers.[2]

[1] Alternatively, in the limit of $T_1 = h_j = V = 0$, conservation of the dipole moment means that all Fock states with the same dipole moment (center of mass) will be degenerate. If then $T_1 > 0$, these states will strongly hybridize, delocalizing the system. Adding weak disorder lifts the degeneracy.

[2] This is because the single-particle states have energies $m\tilde{\gamma}$ with m integer, and a many-body state is simply a Slater determinant of such states thus having an energy that is a sum of these.

To delocalize the system the driving must couple $|\varphi_a^{(n)}\rangle$ with different a in different rungs n, so that the stroboscopic dynamics of the resulting state will be delocalized. Perturbatively in δH, there are two leading-order ways this can happen.

The first is that two eigenstates $|\alpha\rangle$ and $|\beta\rangle$ in neighboring rungs are coupled by the matrix element $\langle\alpha|\,\delta H^{(1)}\,|\beta\rangle$; the energy difference is $\overline{\epsilon}_\alpha - \overline{\epsilon}_\beta = m\tilde{\gamma}$ with integer m; since they are on neighboring rungs, the condition on ω for this is $\overline{\epsilon}_\alpha - \overline{\epsilon}_\beta = m\tilde{\gamma} = \omega$ from which we find $\frac{\gamma T_0}{2\pi} = \frac{1}{m}$.

Now the matrix element $\langle\alpha|\,\delta H^{(1)}\,|\beta\rangle$ is exponentially suppressed whenever $|\alpha\rangle$ and $|\beta\rangle$ differ by moving a particle by more than one site (because $\delta H^{(1)}$ has only nearest-neighbor hopping and the $|\alpha\rangle$ are localized). Thus $m = 1$ is dominant and we expect delocalization to be clearly visible at $\frac{\gamma T_0}{2\pi} = 1$.

The second is that two eigenstates $|\alpha\rangle$ and $|\beta\rangle$ in rungs n and $n+p$, with $p > 1$, respectively, are coupled by $\langle\alpha|\,\delta H^{(p)}\,|\beta\rangle$. The resonance condition is then $\overline{\epsilon}_\alpha - \overline{\epsilon}_\beta = m\tilde{\gamma} = p\omega$ leading to $\frac{\gamma T_0}{2\pi} = \frac{p}{m}$. As before, $m > 1$ is exponentially suppressed, while the matrix element $\langle\alpha|\,\delta H^{(p)}\,|\beta\rangle$ decays linearly with p; since $p > 1$, $p = 2$ dominates and we expect strong signs of delocalization at $\frac{\gamma T_0}{2\pi} = 2$, with weaker delocalization for increasing p.

Thus there are resonances at

$$\gamma T_0/2\pi = p/m \tag{9}$$

for integer p, m, with $p = m = 1$ the dominant one, followed by $p = 2, m = 1$, then $p = 3, m = 1$, and so on. The ones for $m > 1$ are strongly suppressed because of the locality of $\delta H(t)$ and will not be visible in our numerics. These conclusions are in broad agreement with the results of Holthaus and Hone on single-particle systems [24].

Away from the limit $h_j = V = 0$, this picture is no longer exact. We therefore use numerics to study to what extent the conclusions survive away from this limit. To check whether the system is localized we implement two numerical probes of localization, and find that they do indeed confirm our conclusions. Fig. 1 shows a summary of the numerical calculations that we now describe.

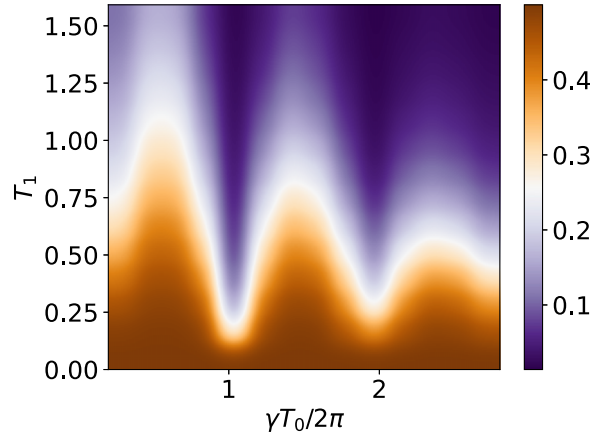


FIG. 1. Phase diagram of a driven Stark MBL system for the unkicked (diagonal) period T_0 vs kicked (off-diagonal) period T_1. The metric used is the standard deviation σ of the probability distribution for the EEVs $\langle n_6\rangle$, where n_6 is the number operator for a particle on site 6 of a 1D lattice. The ergodic regime is represented by the purple region $\sigma \approx 0$, corresponding to all EEVs equaling 0.5. The MBL regime is represented by the orange region $\sigma \approx 0.5$, corresponding to all EEVs equaling 0 or 1. The valleys in the MBL regions at integer multiples of $\gamma T_0/2\pi$ indicate resonant points, where driving occurs at a frequency that eliminates energy separations caused by the tilted potential with gradient γ. Parameters used are $L = 14$, $J = V = 0.5$, $\gamma = 5$, $w = 0.05$, and 14 data points are used for both axes.

III. EIGENSTATE PROPERTIES

A localized system violates the ETH, while delocalization via periodic driving is expected to coincide with its reestablishment [4,26]. We therefore probe directly ETH by checking whether or not the eigenstate expectation values (EEVs) form a smooth function of the eigenenergies. Selecting our local observable to be the number density of a particle on some fixed site j, we calculate the EEV for each eigenstate $|\epsilon_\alpha\rangle$ as $\langle n_i\rangle = \langle\epsilon_\alpha|n_i|\epsilon_\alpha\rangle$, and plot these versus the quasienergies $\epsilon_\alpha = -(\ln\Theta_\alpha)/iT$ where Θ_α are the eigenvalues of the propagator $\mathcal{F}$ corresponding to $|\epsilon_\alpha\rangle$.

In a maximally localized system half of the EEVs will equal 1 and the rest 0, while in an infinite-temperature system, as is expected for a driven, delocalized, ergodic system [31], they will all be equal to each other and to the infinite-temperature result $\mathrm{tr}(n_j) = 1/2$ [4].

In Fig. 2 we plot both the density of states (DOS) as a function of quasienergy (left column) and the probability densities of the EEVs (right column) for various system sizes, both on resonance and off resonance, with the resonant points as determined earlier.

In the off-resonant plots, the EEVs are evenly distributed between both $\langle n_6\rangle = 0$ and $\langle n_6\rangle = 1$ as anticipated for the localized case. On the other hand, for resonant driving the EEVs become sharply peaked at $\langle n_6\rangle = 0.5$, consistent with Floquet ETH. Crucially, the standard deviation decreases with system size, indicating that the system is indeed delocalized in the thermodynamic limit.

To further assess whether our results are reliable, we turn to the DOS plots. In the thermodynamic limit, we expect the DOS to be uniform due to the folding of the quasienergies and the mixing of the states caused by the driving. Indeed, in the off-resonant cases the DOS fluctuates around a constant value, with the amplitude of the fluctuations decreasing with system size. This shows that our driving protocol is able to mix the bands that are present in the undriven $T_1 = 0$ case (discussed in Supplemental Material [29]).

In contrast, in the resonant cases the DOS instead shows strong quasienergy dependence. This suggests that our driving is not strong enough, or not low-frequency enough (compared to the static system's bandwidth), to mix the bands. From this viewpoint, increasing the system size will eventually result in mixing, which suggests that we are far from the thermodynamic limit. Nevertheless, this is not an issue: The EEVs already indicate delocalization, which is favored by better mixing of the bands. Thus, if anything the nonuniform DOS shows that our results *underestimate* delocalization, which is fine given that our result is that delocalization occurs at all. Finally, the DOS becomes more evenly distributed with

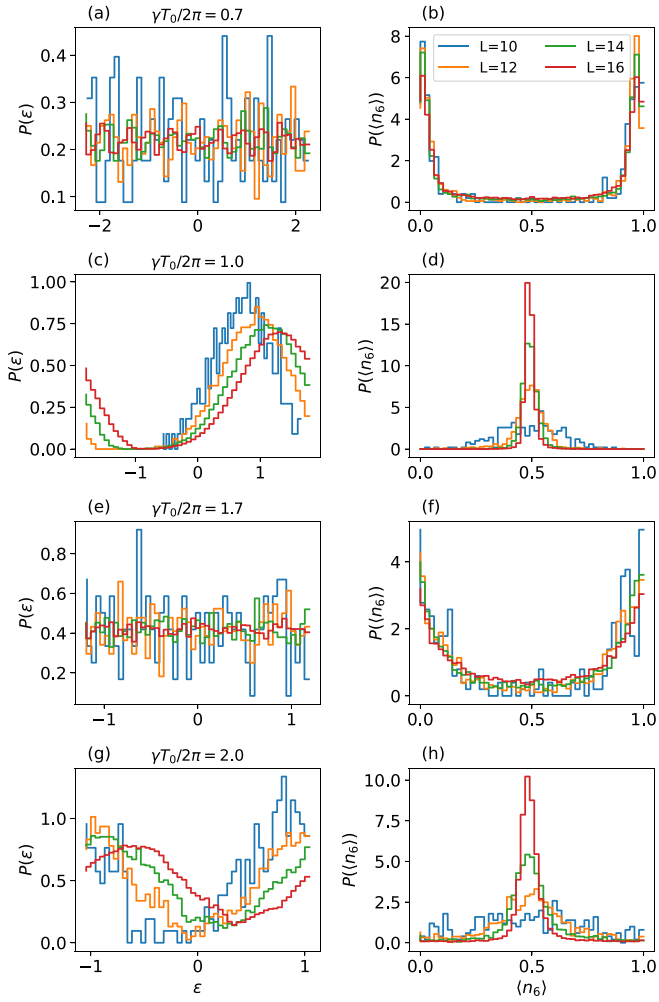


FIG. 2. Left column: Density of states as a function of quasienergy ϵ. Right column: Probability distributions of eigenstate expectation values of the particle number density on site $j = 6$, $\langle n_6 \rangle$ for a periodically driven Stark MBL system, at various system sizes L and unkicked periods T_0. The second and fourth rows correspond to resonant driving at the resonant points $n = 1.0$ and $n = 2.0$, respectively, where the strongly peaked behavior around $\langle n_6 \rangle = 0.5$ in (d) and (h) indicates a strong lack of energy dependence between the EEVs, which corresponds to the infinite temperature ensemble that is associated with systems exhibiting Floquet ETH. The remaining figures show off-resonant driving, where the EEVs are strongly peaked around $\langle n_6 \rangle = 0$ and $\langle n_6 \rangle = 1.0$. This indicates violation of the ETH and thus the presence of MBL. Parameters used are $J = V = T_1 = 0.5$, $\gamma = 5$, $w = 0.05$.

system size, which further supports the claim that this is a finite-size effect.

IV. MULTIFRACTALITY

Another signature of broken ergodicity is accessed by studying the structure of the eigenstates of $\mathcal{F}$. In an ergodic Floquet system, an eigenstate is simply a uniform superposition of all Fock states [31]. Conversely, in MBL systems the eigenstates are multifractal in Fock space [32]. This

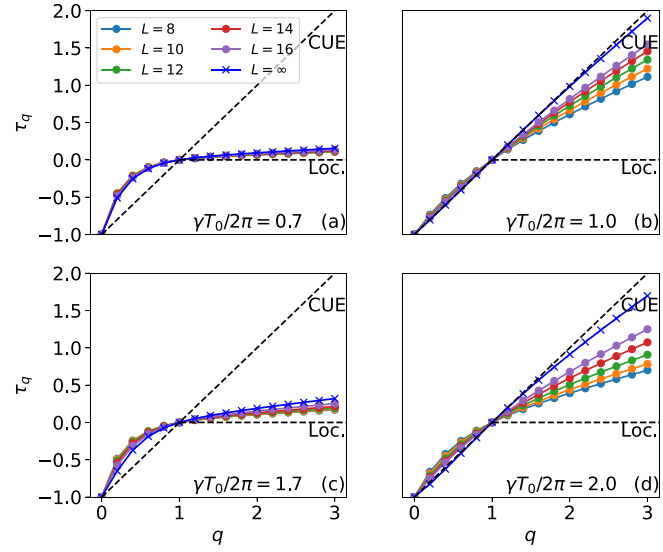


FIG. 3. Averaged scaling exponent τ_q at finite sizes and infinite size for the driven Stark MBL system under periodic driving. Averaging takes place over all eigenvectors and m disorder realizations, where $m = \text{ceil}(10\,000/\mathcal{D})$. (a) and (c) correspond to off-resonant driving, which shows the expected multifractal behavior for MBL eigenstates, specifically that the τ_q sit above the thermal prediction at $0 < q < 1$ and below it at $q > 1$. (b) and (d) show resonant driving at two different resonant points, which shows agreement with the thermal prediction up to values above $q = 1$, indicating that the eigenstates lie in the ETH regime. Parameters used are $J = V = T_1 = 0.5$, $\gamma = 5$, $w = 0.05$.

multifractal property can be studied by invoking a metric called generalized inverse participation ratios (IPRs) [33]. This is defined by

$$I_q = \sum_j |\langle \epsilon | j \rangle|^{2q}. \tag{10}$$

At $q = 2$ this is the usual IPR. This depends on the Hilbert space dimension $\mathcal{D}$ via $I_q \sim \mathcal{D}^{-\tau_q}$ with $\tau_q = -\frac{\ln I_q}{\ln \mathcal{D}}$ the scaling exponent. The τ_q are related to the multifractal dimension D_q, via $D_q = \frac{\tau_q}{q-1}$.

In a maximally localized system (in Fock space), $D_q = 0$ for $q > 0$, which translates to $\tau_q = 0$. In a completely delocalized system, $D_q = 1$, which leads to the random matrix theory (RMT) prediction $\tau_q = q - 1$. The general behavior for disordered interacting systems is intermediate [33–35]: Close to the transition point between MBL and the ergodic phase, the eigenstates are multifractal in both regimes.

In Fig. 3 we show the τ_q for a given q at several sizes, as well as our thermodynamic limit prediction. This was obtained by plotting two finite-size values against $1/\ln \mathcal{D}$ and extrapolating to $1/\ln \mathcal{D} = 0$ [33]. In these plots we average over all eigenvectors, as well as a number m of disorder realizations; we select $m = \text{ceil}(10\,000/\mathcal{D})$.

On resonance [Figs. 3(b) and 3(d)], our extrapolated result for the thermodynamic limit agrees with the thermal prediction up to moments $q^* > 1$. This indicates that the system is in the ETH phase, with a residual multifractality confined

to high-order moments. Off resonance, the τ_q sit above the thermal prediction in the interval $0 < q < 1$, and below it for all $q > 1$, thus exhibiting behavior similar to that observed in MBL systems.

V. CONCLUSIONS

We have showed that Stark-MBL systems may remain localized under periodic driving. Such systems are clean and translationally invariant, in contrast to the usual disordered MBL systems, which also have this property. We expect that phenomena usually studied in disordered cases should have analogs for Stark systems [36]. There also remain several open questions: How quickly does delocalization set in at resonances, and how does it depend on the detuning? Similarly, how slow is heating in the absence of a nonlinear component to the potential, when the static system is known to be delocalized, but correlations spread very slowly [28]?

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