

Quantum transport in Hubbard-Rashba mesoscopic rings: Particle-hole symmetry and localization

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We investigate equilibrium charge transport in a finite mesoscopic ring with nearest-neighbor Rashba spin-orbit coupling (SOC) and on-site Hubbard interactions. Using a self-consistent Hartree-Fock (HF) Green's-function approach, we compute persistent currents and the Drude weight while varying the Aharonov-Bohm flux (ϕ/ϕ_0), Rashba strength λ_R , Hubbard interaction U , and electron filling. Within this finite-ring HF description, the transport response is governed by the competition between flux-sensitive single-particle dispersion, SOC-induced level rearrangements, and interaction-induced suppression of charge motion. Increasing U monotonically reduces the Drude weight over the parameter range explored, consistent with a progressive reduction of flux sensitivity in the occupied HF spectrum. By contrast, Rashba SOC does not produce a simple monotonic enhancement. Instead, $D(\lambda_R)$ is strongly nonmonotonic: at selected values of λ_R , SOC brings levels near the chemical potential into near-degeneracy or avoided crossings, producing pronounced resonant enhancements of the flux curvature, whereas at other values the response can be lower than in the $\lambda_R = 0$ case. For the even- N nearest-neighbor ring considered here, particle-hole symmetry relates fillings N_e and $2N - N_e$. In the symmetry-preserving HF solution, the corresponding Drude weights and persistent-current profiles agree within numerical tolerance. In the finite $N = 8$ ring, the largest Drude weight in the explored parameter window occurs at half filling. This result should be understood as a finite-size HF charge-stiffness effect, not as a statement about the exact thermodynamic Mott limit of the one-dimensional Hubbard model. Overall, the results provide a controlled mean-field picture of how Rashba SOC and Hubbard interactions compete in flux-sensitive mesoscopic transport and they identify tunable SOC regimes where the charge stiffness can be strongly enhanced.

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I. INTRODUCTION

One of the most remarkable signatures of quantum coherence in mesoscopic systems is the persistent current, a dissipationless equilibrium current sustained in rings threaded by a magnetic flux. Since the theoretical predictions of Büttiker, Imry, and Landauer [1,2] and subsequent experimental observations [3,4], persistent currents have provided a central probe of phase-coherent transport. Two ingredients are especially relevant for present mesoscopic devices: Rashba spin-orbit coupling (SOC), generated by structural inversion asymmetry and tunable by electric fields [5–9], and electron-electron interactions, which are commonly modeled by the Hubbard Hamiltonian [10].

Persistent currents have been studied in rings containing either Rashba SOC [11–20] or Hubbard interactions [21–26]. The combined influence of these two mechanisms

is less settled, although it is directly relevant for spintronic interferometers where electric-field control of SOC and density-dependent interaction effects coexist. We therefore focus on how Rashba-induced spin-dependent interference competes with correlation-induced localization in a finite Hubbard ring.

The motivation for the present work is not a claim that a single recent experiment has already realized this precise lattice Hamiltonian. Rather, it is the continuing experimental and theoretical activity in gate-tunable spin-orbit mesoscopic systems, quantum rings, and Aharonov-Bohm interferometers. These platforms make it timely to establish a clean mean-field baseline for how Rashba-induced spin-dependent interference competes with Hubbard-like interaction effects in a finite ring. The HF treatment used below is not intended to reproduce the exact strongly correlated Mott regime. Instead, it provides a self-consistent finite-ring description for weak and moderate interactions and allows us to scan systematically over flux, filling, Rashba coupling, and interaction strength.

Throughout the paper, a “bipartite ring” means an even- N ring with only nearest-neighbor (NN) links, including the NN Rashba terms. Such a graph can be divided into alternating sublattices A and B , so that hopping occurs only between A and B . The quantity H_0 introduced below denotes the quadratic NN part of Eq. (5), after subtracting any uniform

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on-site energy. In the (A, B) basis it has the block off-diagonal form

$$H_0(\phi, \lambda_R) = \begin{pmatrix} 0 & T \\ T^\dagger & 0 \end{pmatrix}, \quad \Gamma = \begin{pmatrix} \mathbb{1}_A & 0 \\ 0 & -\mathbb{1}_B \end{pmatrix}, \quad (1)$$

$$\{\Gamma, H_0\} = 0, \quad (2)$$

where Γ acts trivially on spin. Thus, if $H_0 \mathbf{v} = E \mathbf{v}$, then $H_0(\Gamma \mathbf{v}) = -E(\Gamma \mathbf{v})$. The single-particle spectrum is therefore symmetric under $E \leftrightarrow -E$, up to an overall shift produced by uniform on-site or Hartree terms. At half filling this symmetry constrains the transport response. Away from half filling, the responses at electron numbers N_e and $2N - N_e$ are related by particle-hole transformation. In the numerical HF calculation this relation is exact for a symmetry-preserving self-consistent solution; the small residual differences reported below come from finite numerical tolerance, finite broadening η , and finite-difference evaluation of derivatives. Same-sublattice hopping, an odd number of sites, staggered on-site terms, or a symmetry-broken HF solution would in general remove the exact particle-hole relation.

The paper is organized as follows. Section II defines the Hubbard-Rashba ring and the HF self-consistency scheme. Section III derives the persistent-current expression in the Green's-function formulation. Section IV presents the numerical spectra, currents, and Drude weights. The Appendixes provide the self-consistent density calculation, the current-density derivation, and the relation between retarded, advanced, and lesser Green's functions.

II. RASHBA-HUBBARD RING MODEL

We consider a mesoscopic ring of N sites, with periodic boundary conditions, at $T = 0$ [see Fig. 1(a)]. The ring includes NN hopping, NN Rashba SOC, and on-site Hubbard repulsion, and is pierced by an Aharonov-Bohm flux ϕ/ϕ_0 . We measure the flux in units of $\phi_0 = hc/e$ and set $e = h = c = 1$ in the numerical calculations.

To avoid ambiguity in the operator notation, we define the local spinor

$$\Psi_n = \begin{pmatrix} c_{n\uparrow} \\ c_{n\downarrow} \end{pmatrix}, \quad \Psi_n^\dagger = (c_{n\uparrow}^\dagger, c_{n\downarrow}^\dagger), \quad (3)$$

where the spin basis is the local $\{\uparrow, \downarrow\}$ basis along the fixed z axis. The on-site energy is the 2×2 spin-space matrix

$$\epsilon_n = \text{diag}(E_{n\uparrow}, E_{n\downarrow}). \quad (4)$$

Unless otherwise stated, we use a clean ring, $E_{n\uparrow} = E_{n\downarrow} = 0$, so $E_{n\sigma}$ is a bare local on-site energy and should not be confused with the HF eigenvalues $\epsilon_\ell^{\text{HF}}$ discussed below.

With this notation the full Hamiltonian is

$$H = \sum_n \Psi_n^\dagger \epsilon_n \Psi_n + \sum_n [\Psi_{n+1}^\dagger e^{i\theta} \mathcal{T}_{n,n+1} \Psi_n + \text{H.c.}] + U \sum_n n_{n\uparrow} n_{n\downarrow}, \quad (5)$$

where

$$\mathcal{T}_{n,n+1} = \gamma \mathbb{1}_2 - i\lambda_R(\sigma_x \cos \varphi_{n,n+1} + \sigma_y \sin \varphi_{n,n+1}), \quad (6)$$

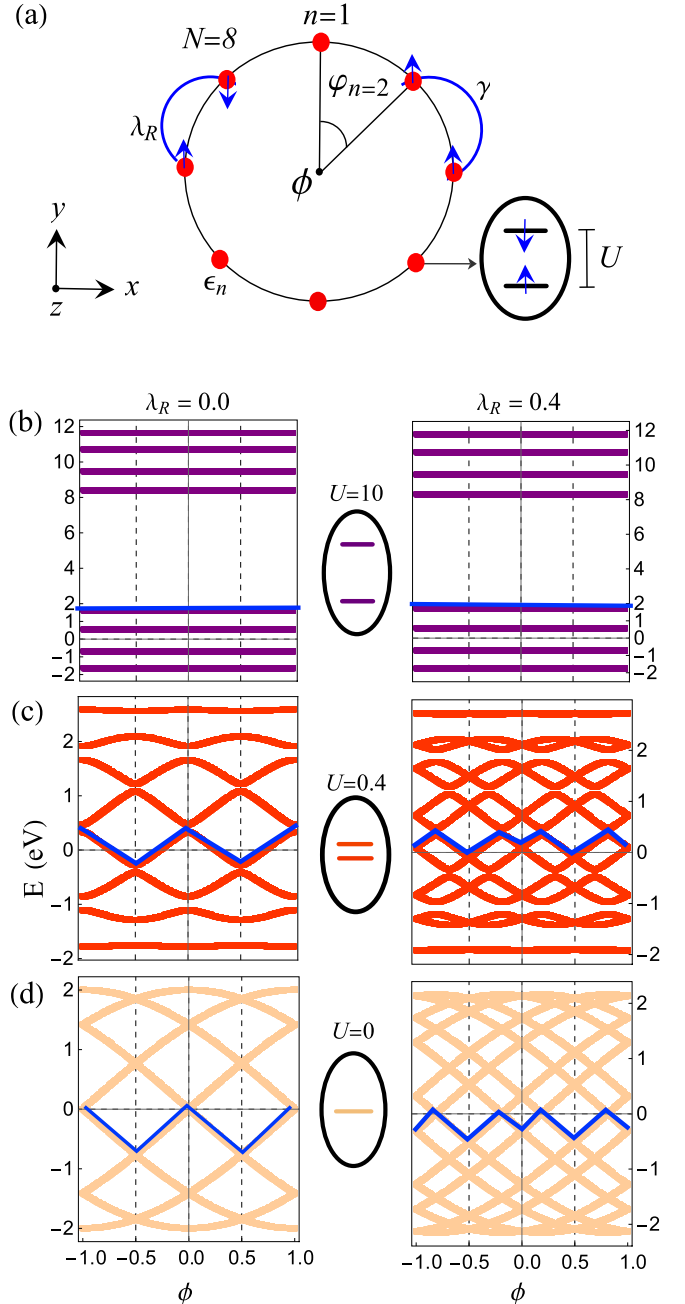


FIG. 1. (a) Schematic of the mesoscopic ring. For even N , alternating sites may be assigned to the two sublattices A and B discussed in the text; the schematic does not explicitly color them. The magnetic flux ϕ pierces the ring along the z direction. The on-site energy is ϵ_n , the NN hopping is γ , the Rashba interaction is λ_R , and the on-site Hubbard interaction is U . Panels (b), (c), and (d) show spectra as a function of ϕ/ϕ_0 for $U = 10$, $U = 0.4$, and $U = 0$, respectively. Each panel compares $\lambda_R = 0$ with $\lambda_R = 0.4$. The Fermi level is highlighted in blue.

$n_{n\sigma} = c_{n\sigma}^\dagger c_{n\sigma}$, $\theta = (2\pi/N)(\phi/\phi_0)$, $\varphi_{n,n+1} = (\varphi_n + \varphi_{n+1})/2$, and $\varphi_n = 2\pi(n-1)/N$. The hopping amplitude γ sets the energy scale. The parameter λ_R is the lattice Rashba coupling and U is the on-site Hubbard repulsion. The quadratic part of Eq. (5) is the H_0 used in Eq. (2) when the uniform on-site shift is removed.

The interaction term in Eq. (5) is treated in a self-consistent Hartree-Fock approximation,

$$U n_{n\uparrow} n_{n\downarrow} \rightarrow -U \langle n_{n\uparrow} \rangle \langle n_{n\downarrow} \rangle + U \langle n_{n\uparrow} \rangle n_{n\downarrow} + U n_{n\uparrow} \langle n_{n\downarrow} \rangle. \quad (7)$$

The HF decoupling reduces the interacting many-body problem to a quadratic one-particle problem, but the calculation remains self-consistent and numerical because the renormalized site energies depend on the occupations:

$$E'_{n\sigma} = E_{n\sigma} + U \langle n_{n,-\sigma} \rangle. \quad (8)$$

The corresponding HF Hamiltonian is obtained from Eq. (5) by replacing ϵ_n with

$$\epsilon'_n = \text{diag}(E'_{n\uparrow}, E'_{n\downarrow}) \quad (9)$$

and omitting the constant double-counting term when computing eigenvectors and Green's functions. For each set of parameters (ϕ , λ_R , U , N_e), we iterate the densities $\langle n_{n\sigma} \rangle$ until convergence and then diagonalize the converged HF Hamiltonian or, equivalently, compute the corresponding Green's functions. Thus the spectra shown in Fig. 1 are not exact many-body spectra; they are the self-consistent HF single-particle spectra used to interpret the current and Drude-weight calculations.

For the smallest rings, including the $N = 8$ half filled case, exact diagonalization is in principle feasible and would provide a useful benchmark. We use HF here because our objective is a systematic scan over flux, filling, interaction strength, and Rashba coupling within the same Green's-function framework used for the current density. This framework also extends naturally to larger rings, nonuniform systems, disorder, and lead-coupled geometries, where exact diagonalization becomes rapidly more expensive and the finite-ring eigenstate picture is no longer sufficient. The approximation neglects dynamical correlations, but it captures static interaction-induced level shifts and provides a transparent finite-ring baseline for the weak-to-moderate interaction window studied here.

All energies in the numerical results are reported in units of γ . For a continuum discretization with lattice spacing a ,

$$\gamma = \frac{\hbar^2}{2m^*a^2}, \quad \lambda_R \simeq \frac{\alpha_R}{2a},$$

so that

$$\frac{\lambda_R}{\gamma} \simeq \frac{\alpha_R m^* a}{\hbar^2}.$$

This relation gives the connection between the dimensionless lattice parameter used here and the experimentally quoted Rashba coefficient α_R . For orientation, experimentally reported Rashba parameters in III–V semiconductor quantum wells and nanowires are commonly of order 10^{-12} – 10^{-11} eV m. In InAs- and InGaAs-based structures, gate control of spin-orbit interaction has been demonstrated in inverted InGaAs/InAlAs heterostructures [5], in InAlAs/InGaAs/InAlAs quantum wells using weak-antilocalization analysis [27], and in InAs quantum wells where the Rashba coupling is tunable by roughly a factor of 2 [8]. In single InAs nanowires, electrolyte or surrounding gates can tune the Rashba coefficient by large factors [9]; gate-all-around InAs nanowire devices report Rashba parameters in

the range $\alpha_R \sim (0.3 \times 10^{-11})$ – (1×10^{-11}) eV m and tunability reaching approximately 1.2×10^{-11} eV m/V [28]. Thus the values used in the transport analysis, especially $\lambda_R/\gamma \lesssim 1$, should be read as dimensionless moderate-to-strong SOC parameters rather than as a fit to a single material. Similarly, U/γ represents an effective screened on-site repulsion; $U/\gamma \lesssim 1$ corresponds to the moderate-interaction regime in which the HF baseline is expected to be most meaningful.

For each value of ϕ , U , and λ_R , the spectra in Figs. 1(b)–1(d) are obtained numerically by diagonalizing the converged HF Hamiltonian. The filling fraction determines the chemical potential; at half filling it lies at the particle-hole symmetric point after subtracting the uniform Hartree shift. The blue level marks the closest occupied level to the chemical potential. If we first look at the case without Rashba coupling, we see two primary effects from increasing the Hubbard interaction. (i) Gap opening: interaction-induced gaps appear at half-integer values of the flux ($\phi/\phi_0 = \pm 0.5$), which is where level crossings would occur in the noninteracting case. (ii) Energy level shifting: the entire spectrum shifts by an amount proportional to U . This reflects the modified on-site energies that result from electron-electron repulsion. As U increases, the HF spectrum becomes progressively less dispersive with flux. In the present closed-ring geometry this should be interpreted as an interaction-induced suppression of charge stiffness and flux sensitivity, rather than as Coulomb blockade in the transport-through-a-dot sense. The relevant observation for the present work is that the occupied HF levels become less sensitive to the Aharonov-Bohm phase and this reduced curvature is reflected directly in the suppression of the persistent current and Drude weight. When we introduce Rashba spin-orbit coupling, it adds even more complexity to the energy spectrum. The most immediate effect is the lifting of spin degeneracy. This results in a doubling of energy levels compared to the case without SOC. This spin splitting is characterized by the appearance of additional gap openings at the newly formed avoided crossings. A particularly interesting feature emerges when we examine the competition between the Hubbard and Rashba interactions. As U increases, we see that the nondegenerate pairs of levels begin to approach each other. This demonstrates the counteracting effect of the Coulomb interaction on the Rashba-induced spin splitting. At large U values [Fig. 1(b)], the Hubbard interaction essentially overwhelms the Rashba effect. This results in what appears to be nearly spin-degenerate levels at the scale shown, though a small splitting does persist. These spectral features have direct implications for the system's transport properties, since the persistent current is determined by the flux dependence of the occupied energy levels. The interplay between level repulsion (from U) and enhanced dispersion (from λ_R) will be reflected in the behavior of the persistent currents, which we will analyze in Sec. III.

III. PERSISTENT CHARGE CURRENTS

At $T = 0$, for a fixed electron filling, the persistent charge current may be obtained from the flux derivative of the occupied spectrum, $I_c = -\sum_{\ell \in \text{occ}} \partial \epsilon_\ell / \partial \phi$ [29]. For the clean closed ring studied here, this eigenvalue-derivative route is

sufficient for the total persistent current and the corresponding HF eigenvectors could also be used to evaluate bond-resolved current densities. We nevertheless use the Green's-function formulation because it gives a unified language for the HF self-consistency, the spectral response, and the local current-density calculation. The same formulation extends naturally

to disordered rings, open geometries with attached leads, and treatments in which interactions enter through self-energies rather than only through static mean-field shifts. We obtain the current operator from the commutator of the HF Hamiltonian with the position operator $x = \sum_j j \Psi_j^\dagger \Psi_j$ (see Appendix B). This gives

$$J_c = \frac{e}{N(e\beta\omega + 1)} \sum_n \left\{ \gamma [e^{i\theta}(G_{n\uparrow,n+1\uparrow}^{(r)} - G_{n\uparrow,n+1\uparrow}^{(a)}) + e^{i\theta}(G_{n\downarrow,n+1\downarrow}^{(r)} - G_{n\downarrow,n+1\downarrow}^{(a)}) - e^{-i\theta}(G_{n+1\uparrow,n\uparrow}^{(r)} - G_{n+1\uparrow,n\uparrow}^{(a)}) - e^{-i\theta}(G_{n+1\downarrow,n\downarrow}^{(r)} - G_{n+1\downarrow,n\downarrow}^{(a)})] - i\lambda_R [e^{i\theta}(G_{n\downarrow,n+1\uparrow}^{(r)} - G_{n\downarrow,n+1\uparrow}^{(a)})e^{-i\varphi_{n,n+1}} + e^{i\theta}(G_{n\uparrow,n+1\downarrow}^{(r)} - G_{n\uparrow,n+1\downarrow}^{(a)})e^{i\varphi_{n,n+1}} + e^{-i\theta}(G_{n+1\downarrow,n\uparrow}^{(r)} - G_{n+1\downarrow,n\uparrow}^{(a)})e^{-i\varphi_{n,n+1}} + e^{-i\theta}(G_{n+1\uparrow,n\downarrow}^{(r)} - G_{n+1\uparrow,n\downarrow}^{(a)})e^{i\varphi_{n,n+1}}] \right\}, \quad (10)$$

where $G^{(a)}$ is the retarded (advanced) Green's function. This expression is useful because it captures both the conventional hopping contribution (the γ terms) and the spin-orbit mediated transport (the λ_R terms). The latter introduces spin-dependent phase factors that modify the current flow.

To find the total persistent charge current for a given electron filling, we integrate over all occupied energy states:

$$I_c(\phi) = \int_{-\infty}^{\mu} J_c(\omega) d\omega, \quad (11)$$

where μ denotes the chemical potential, which is determined by the electron filling of the ring.

To calculate the retarded and advanced Green's functions entering Eq. (10), we construct the converged HF matrix $H_{\text{HF}}(\phi, \lambda_R, U)$ in the site-spin basis

$$(\Psi_1, \Psi_2, \dots, \Psi_N).$$

The Green's functions are then obtained from the compact matrix relation

$$G^r(\omega) = [(\omega + i\eta)\mathbb{1} - H_{\text{HF}}(\phi, \lambda_R, U)]^{-1}, \\ G^a(\omega) = [G^r(\omega)]^\dagger, \quad (12)$$

where $\eta \rightarrow 0^+$. The matrix elements $G_{n\sigma,m\sigma'}^r(\omega)$ and $G_{n\sigma,m\sigma'}^a(\omega)$ appearing in Eq. (10) are read directly from Eq. (12). This notation is equivalent to the equation-of-motion formulation, but it is more compact and avoids unnecessary index clutter in the main text. The explicit equation-of-motion form is given in Appendix B.

IV. NUMERICAL RESULTS

A. Persistent charge currents and the Hubbard-Rashba interactions

Persistent charge currents obtained by integrating contributions from all energy levels up to the Fermi level Eq. (7) are shown in Figs. 2(a)–2(c) for Hubbard interaction strengths $U = 0, 0.2, 0.4$, and 0.8 . Left panels show currents without Rashba coupling; right panels include Rashba spin-orbit coupling with $\lambda_R = 0.4$.

1. Direct Green's-function calculation of persistent currents

The persistent current is computed directly from Eq. (10) by evaluating the Green's functions at different spatial sites and integrating over all occupied energies up to the chemical potential μ . The flux dependence enters through the phase factors $e^{\pm i\theta}$ multiplying the Green's functions $G_{n,m}^r(\omega, \phi)$, which are computed from Eq. (12) with flux-dependent hopping amplitudes $\gamma e^{\pm i\theta}$ and Rashba couplings.

Different electron numbers N_e lead to different chemical potentials μ , which serve as the upper integration limit in Eq. (11). The single-particle spectral function

$$\rho_{n\sigma}(\omega, \phi) = -\frac{1}{\pi} \text{Im} G_{n\sigma,n\sigma}^r(\omega, \phi) \quad (13)$$

exhibits sharp peaks at discrete energies $\varepsilon_n^{\text{HF}}(\phi)$, which correspond to the poles of the Green's function obtained from the self-consistent matrix Green's function in Eq. (12). As ϕ varies, these spectral peaks shift in energy—this flux dispersion of the spectrum determines how strongly the off-diagonal Green's functions $G_{n,n+1}^r(\omega, \phi)$ are modulated by flux. Energies where the spectrum disperses strongly with ϕ contribute more to the integrated current than energies where the spectrum is flat.

2. Filling-dependent hierarchy and physical mechanisms

A striking hierarchy emerges across the three filling regimes: persistent currents at half filling [Fig. 2(b)] exhibit amplitudes 2–3 times larger than those away from half filling [Figs. 2(a) and 2(c)].

At half filling for the $N = 8$ ring ($N_e = 8$ electrons occupying 8 of the 16 spin orbitals), the chemical potential sits at the particle-hole symmetric point and the integration in Eq. (11) runs over eight occupied spectral peaks. Examining the energy spectra in Fig. 1, these eight levels show pronounced flux dispersion. When we evaluate $G_{n,n+1}^r(\omega, \phi)$ at these energies for different values of ϕ , the Green's functions change substantially, producing large flux modulations in $J_c(\omega, \phi)$. Moreover, at half filling these contributions add coherently: the phase factors $e^{i\theta}$ and the Green's functions conspire such that current densities from different occupied levels reinforce each other—a consequence of the particle-hole symmetric point in

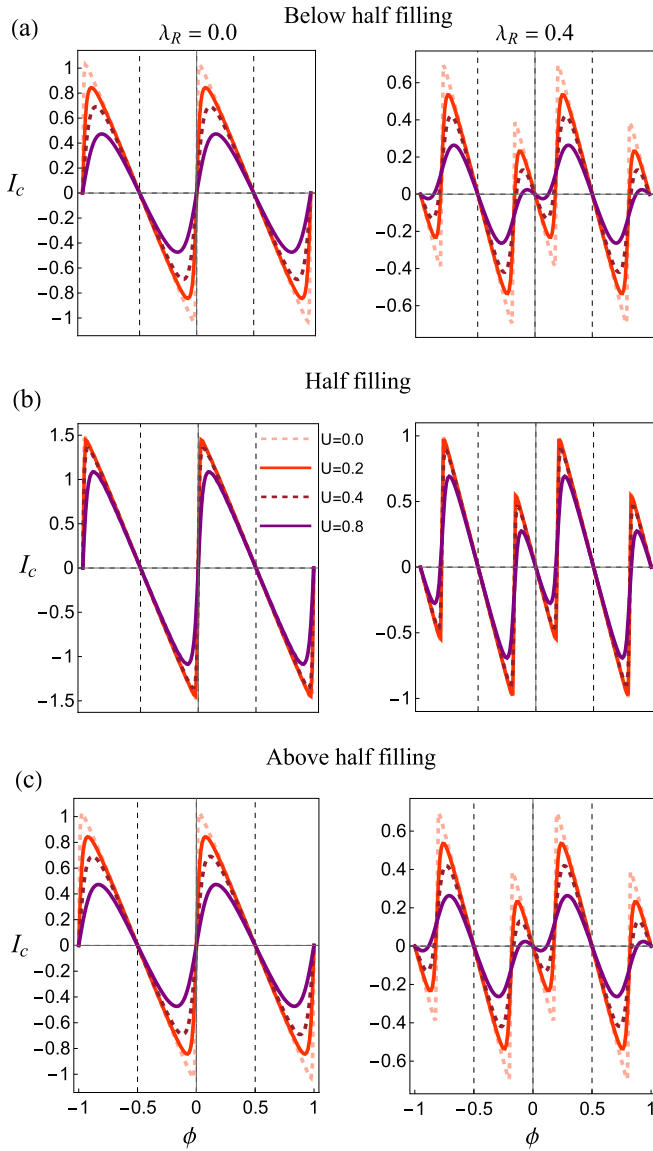


FIG. 2. Persistent current for (a) below half filling, (b) half filling, and (c) above half filling. *Left:* $\lambda_R = 0$. *Right:* $\lambda_R = 0.4$. Lines: $U = 0$ (dashed, light), $U = 0.2$ (solid, light), $U = 0.4$ (dashed, dark), and $U = 0.8$ (solid, dark).

a bipartite lattice. With one electron per site on average, all NN links participate in the coherent response.

By contrast, at quarter filling ($N_e = 4$) the integral runs only over the four lowest energy states. While the Green's functions at these energies may be reasonable, the limited integration range reduces the net current. Additionally, with fewer electrons many bonds are empty, reducing the magnitude of $G_{n,n+1}^r(\omega)$. At three-quarter filling ($N_e = 12$), 12 peaks contribute to the integral, but at higher energies the Green's functions exhibit weaker flux response due to correlation effects. More importantly, contributions from the extra four peaks can interfere destructively with lower-energy contributions.

This mechanism explains the observed hierarchy: at half filling, integrating $J_c(\omega, \phi)$ over the specific range of the eight lowest peaks maximizes the persistent current because (i) the

Green's functions in this energy range are large, (ii) they exhibit strong flux modulation, and (iii) their contributions add constructively.

3. Effects of Hubbard interaction

As U increases from 0 to 0.8, persistent current amplitudes decrease by factors of 2–3 across all three filling regimes. This universal suppression reflects how electron-electron repulsion modifies the Green's functions: the interaction-induced self-energy corrections in Eq. (8) shift the spectral peaks and reduce the magnitude of the off-diagonal elements $G_{n,n+1}^r(\omega)$. Physically, the Hubbard U penalizes double occupancy, suppressing the hopping processes that allow electrons to propagate around the ring. Additionally, the interactions smooth the flux dispersion of the spectral peaks: as U increases, evaluating $G_{n,n+1}^r(\omega, \phi)$ at different flux values yields functions that vary less dramatically, reducing the flux modulation of $J_c(\omega, \phi)$.

Even at the largest U explored ($U = 0.8$), the hierarchy remains intact: half filling continues to exhibit the largest currents. This persistence indicates that the Green's function structure—determined primarily by band structure and filling—dominates over correlation-induced modifications within this parameter regime.

4. Particle-hole symmetry

The close agreement between $N_e = 4$ electrons [panel (a)] and $N_e = 12$ electrons [panel (c)] reflects particle-hole symmetry of the even- N NN ring. For the symmetry-preserving HF solution the mapping $c_{i\sigma} \rightarrow (-1)^i c_{i\sigma}^\dagger$ relates the two fillings exactly, up to the uniform Hartree energy shift. The small residual differences visible in the plotted curves are numerical rather than physical; they arise from finite convergence tolerance, the finite broadening η used in the Green's functions, and finite-difference evaluation of flux derivatives. We have therefore replaced the previous wording “nearly perfect” with “within numerical tolerance” where appropriate.

5. Effects of Rashba spin-orbit coupling

Rashba SOC modifies the persistent current by lifting spin degeneracies, mixing spin channels, and introducing spin-dependent phase structure into the off-diagonal Green's functions that enter Eq. (10). The resulting effect is not purely monotonic. Depending on filling, interaction strength, and the value of λ_R , SOC can either enhance or reduce the current amplitude at a fixed flux. What is robust in the present calculation is that Rashba coupling reorganizes the flux-dependent spectrum and produces additional structures in the current profiles, especially near crossings or avoided crossings of HF levels.

At values of λ_R for which levels close to the chemical potential become nearly degenerate or undergo avoided crossings, the Green's functions $G_{n,n+1}^r(\omega, \phi)$ become especially sensitive to small changes in flux. After integration over the occupied energies in Eq. (11), this enhanced local flux sensitivity produces sharper features in $I_c(\phi)$ and, correspondingly, resonant structures in the Drude weight. Thus the role of Rashba SOC is best described as producing a nonmonotonic, resonance-like modulation of the flux response, rather than as a uniform enhancement for all values of λ_R .

B. Drude weight and the Hubbard-Rashba interactions

The Drude weight D quantifies the flux curvature of the ground-state energy and is a standard measure of charge stiffness. In the present HF calculation we evaluate it from the flux derivative of the persistent current, equivalently

$$D = C_N \left. \frac{\partial^2 E_0(\phi)}{\partial \phi^2} \right|_{\phi=\phi_*} = -C_N \left. \frac{\partial I_c(\phi)}{\partial \phi} \right|_{\phi=\phi_*}, \quad (14)$$

where C_N is the geometry- and units-dependent normalization factor and $\phi_* = \phi_0$ is the flux point used in the numerical comparison. Numerically, we approximate the derivative by a small finite difference of $I_c(\phi)$ computed from Eq. (11). A larger D therefore indicates a stronger flux sensitivity of the occupied HF spectrum and of the off-diagonal Green's functions entering the current.

Figures 3 and 4 summarize two distinct trends. First, increasing U suppresses D monotonically in all three fillings considered. Second, Rashba SOC produces a nonmonotonic response: it can strongly enhance D at resonant values of λ_R , but it does not enhance the charge stiffness uniformly for every fixed value of λ_R . This distinction is important because Fig. 3 compares only two Rashba values, $\lambda_R = 0$ and 0.4 , whereas Fig. 4 displays the full resonance structure as λ_R is varied.

1. Finite-ring half filling hierarchy

Within the finite $N = 8$, self-consistent HF calculation, the half filled case exhibits the largest Drude weight throughout the explored parameter window. This is a statement about the flux curvature of the occupied HF spectrum in the finite ring, not about the exact thermodynamic Hubbard limit. In the present calculation, at half filling the chemical potential lies at the particle-hole symmetric point, and the occupied HF levels contribute strongly to the flux-dependent current. Consequently, evaluating $I_c(\phi)$ at nearby flux values produces a larger finite-difference response than in the below- and above-half filled cases.

This hierarchy should therefore be interpreted as a finite-size HF charge-stiffness effect. The Hubbard interaction still suppresses the response: as U increases, D decreases monotonically at all fillings. However, within the explored range, this suppression does not invert the filling hierarchy. The result is thus not that interactions enhance half filled transport, but rather that the half filled finite ring retains the largest HF flux curvature while all fillings become less flux sensitive as U grows.

2. Physical origin of the resonances in $D(\lambda_R)$

The sharp structures in $D(\lambda_R)$ arise when changing λ_R reorganizes the HF spectrum near the chemical potential. As Rashba SOC is increased, spin-degenerate levels split and levels with different spin character can approach one another. Because the Rashba term mixes spin channels, these approaches generally produce avoided crossings rather than exact crossings. Near such avoided crossings, the corresponding spectral peaks are highly sensitive to the Aharonov-Bohm flux.

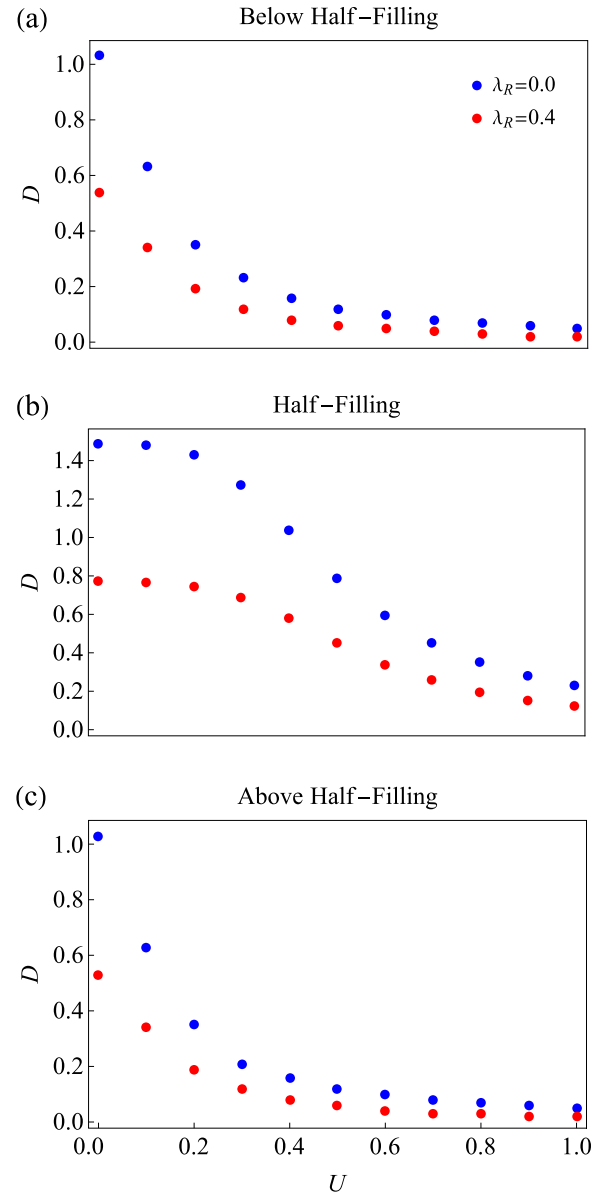


FIG. 3. Drude weight for the finite $N = 8$ ring within self-consistent HF. For the finite $N = 8$ ring within self-consistent HF, the half filled case ($N_e = 8$) has the largest D over the explored U range. In all fillings shown, D decreases monotonically as U increases, indicating a reduction of flux sensitivity and charge stiffness. The comparison between $\lambda_R = 0$ and $\lambda_R = 0.4$ should not be read as a universal SOC enhancement.

This enhanced sensitivity is reflected in the Green's functions $G_{n,n+1}^r(\omega, \phi)$. At resonant values of λ_R , a small change in ϕ produces a comparatively large change in the phase relation and spectral weight of the states close to μ . The finite-difference estimate of $-\partial I_c / \partial \phi$ is then large, producing a peak in $D(\lambda_R)$. Once λ_R moves the relevant levels away from μ , or once the avoided crossing no longer controls the occupied spectrum, the response decreases again. This explains the nonmonotonic Rashba dependence observed in Fig. 4.

We therefore avoid assigning a universal resonance position to λ_R . The relevant SOC values depend on filling, U , flux point, and the finite-size level structure. The robust conclusion

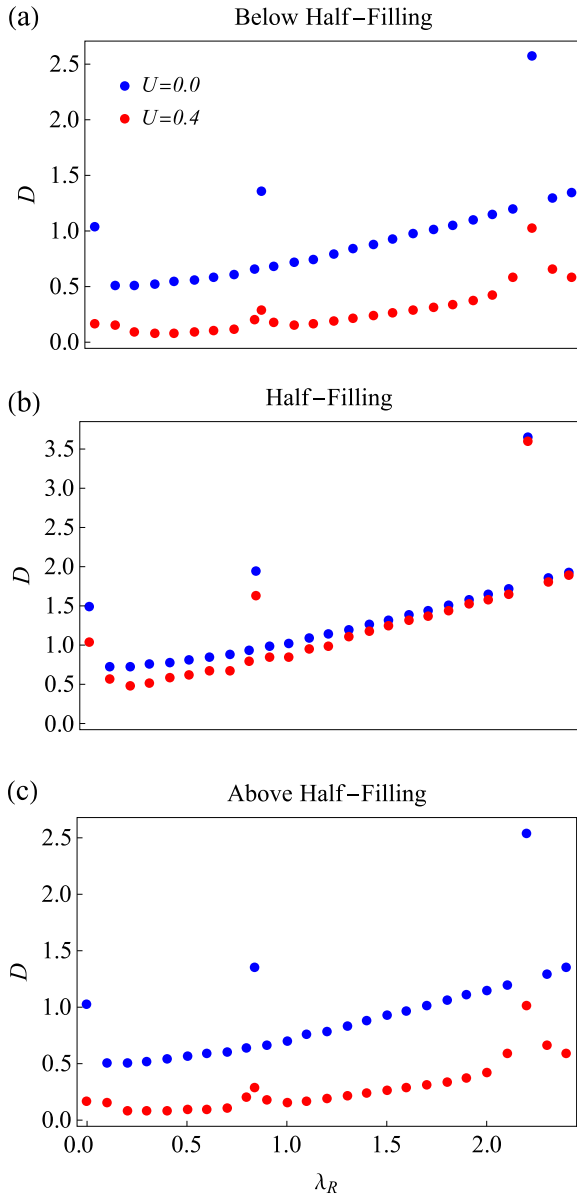


FIG. 4. Drude weight vs λ_R . The Rashba dependence is strongly nonmonotonic. Pronounced maxima occur at selected values of λ_R , where SOC brings HF levels near the chemical potential into near degeneracy or avoided crossings. Away from resonant values, D can decrease and may lie below the $\lambda_R = 0$ reference. The half filled finite ring shows the largest D in the explored range, while $N_e = 4$ and $N_e = 12$ are related by particle-hole symmetry within numerical tolerance.

is the presence of Rashba-tuned resonant enhancements, not a monotonic increase of D with λ_R .

V. SUMMARY AND CONCLUSIONS

We have investigated equilibrium charge transport in mesoscopic Hubbard-Rashba rings using a self-consistent Hartree-Fock Green's-function approach, computing persistent currents and Drude weights as functions of interaction strength U , Rashba coupling λ_R , and electron filling. The revised interpretation emphasizes finite-ring HF charge

stiffness, monotonic suppression by Hubbard interactions, nonmonotonic Rashba-tuned resonances, and particle-hole symmetry within numerical tolerance.

A. Main findings

Finite-ring filling hierarchy. In the finite $N = 8$ ring treated within self-consistent HF, half filling ($N_e = 8$ electrons in 16 spin orbitals) exhibits the largest Drude weight in the explored parameter range. This reflects the large flux curvature of the occupied HF spectrum at the particle-hole symmetric point. The statement is specific to the finite-size HF calculation and should not be interpreted as a claim about the exact thermodynamic Mott limit of the one-dimensional Hubbard model.

Suppression by Hubbard interactions. Increasing U monotonically reduces D for all fillings considered. In the Green's-function language, the interaction-induced HF shifts reduce the flux sensitivity of the occupied spectral peaks and of the off-diagonal propagators entering the current. The filling hierarchy remains intact over the explored interaction range, but the overall charge stiffness is suppressed.

Nonmonotonic Rashba response. Rashba SOC reorganizes the finite-ring spectrum and produces a strongly nonmonotonic $D(\lambda_R)$. At selected values of λ_R , avoided crossings or near degeneracies close to the chemical potential enhance the flux curvature and generate pronounced peaks in D . At other values, including fixed values such as $\lambda_R = 0.4$ in Fig. 3, the Drude weight can be smaller than for $\lambda_R = 0$. Thus Rashba coupling should be described as a tunable source of resonant enhancement, not as a uniform delocalizing enhancement for all λ_R .

Particle-hole symmetry. The responses at $N_e = 4$ and $N_e = 12$ are related by particle-hole symmetry for the even- N , nearest-neighbor ring with Rashba SOC. In the symmetry-preserving HF solution, the corresponding curves agree within numerical tolerance. Any small residual differences in the plotted data arise from convergence tolerance, finite broadening, and numerical differentiation, rather than from physical breaking of particle-hole symmetry.

B. Implications, limitations, and outlook

The trends found here suggest possible operating regimes for mesoscopic interferometers with gate-tunable SOC. In a finite coherent ring, tuning the carrier density toward the particle-hole symmetric point can maximize the HF flux curvature, while tuning λ_R can move levels near avoided crossings and thereby enhance the charge stiffness resonantly. The practical message is therefore not that Rashba SOC always increases the current, but that it provides a gate-controllable way to move between weak- and strong-response regimes.

Our Hartree-Fock treatment captures static interaction effects but neglects dynamical spin and charge fluctuations, which become important as U/γ increases. Exact diagonalization is feasible for the smallest rings, including the $N = 8$ half-filled case, and would provide a useful benchmark. We use HF here because the objective is a systematic scan over flux, filling, U , and λ_R , and because the same Green's-function framework can be extended to larger rings, nonuniform systems, disordered rings, and lead-coupled geometries where

exact diagonalization becomes rapidly expensive or the closed-system eigenstate picture is no longer adequate.

In the large- N clean limit at fixed density, the persistent-current amplitude is expected to decrease with ring length, while the Drude weight remains finite in a metallic mean-field regime and is suppressed by interactions or disorder. The Rashba-induced resonances should become denser with increasing N , turning the sharp few-level features of small rings into a smoother envelope. Quantitative device predictions will require ingredients beyond the present minimal model, including larger rings, disorder, finite temperature, coupling to leads, and beyond-HF benchmarks. The present calculation should therefore be viewed as a transparent finite-ring HF baseline for the interplay of Rashba-induced level rearrangements, Hubbard-induced suppression of charge stiffness, and particle-hole symmetry in mesoscopic rings.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: CALCULATION OF THE AVERAGE NUMBER OF PARTICLES

The average number of particles $\langle n(t) \rangle$ is

$$\langle n(t) \rangle = \langle d^\dagger(t) d(t) \rangle. \quad (\text{A1})$$

We first introduce the Green's function:

$$G(t, t') = -i\Theta(t, t') \langle d(t) d^\dagger(t') \rangle + i\Theta(t', t) \langle d^\dagger(t') d(t) \rangle, \quad (\text{A2})$$

so that

$$\begin{aligned} \langle n(t) \rangle &= \langle d^\dagger d \rangle = -iG^<(t, t') \\ &= \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi i} e^{-i(\omega-\omega')t} G^<(\omega, \omega'). \end{aligned} \quad (\text{A3})$$

Since our ring is a closed system in equilibrium, the average number of particles is time independent, so $\langle n(t) \rangle \equiv \langle n \rangle$. This is given by

$$\langle n_\sigma \rangle = \frac{1}{\pi} \int_{-\infty}^{\infty} d\omega \text{Im}(G_\sigma^<(\omega)). \quad (\text{A4})$$

This integral can be related to the retarded and advanced Green's functions through the fluctuation-dissipation theorem. This relationship is particularly useful for numerical calculations (see Appendix C). The flow chart in Fig. 5 summarizes the numerical loop used in the main text: starting from an initial density, we build $E'_{n\sigma}$, solve the HF Green's functions, update $\langle n_{n\sigma} \rangle$, and repeat until the stated tolerance is reached. This point is emphasized here because the HF approximation makes the Hamiltonian quadratic but does not make the self-consistency problem analytic. This self-consistent approach has been widely used in similar contexts [26,30].

APPENDIX B: CHARGE AND CURRENT DENSITY CALCULATIONS

We start with the position operator:

$$\mathbf{x} = \sum_j \mathbf{c}_j^\dagger j \mathbf{c}_j, \quad (\text{B1})$$

which means the charge current density operator is

$$J_c = \frac{1}{N} e \langle \dot{\mathbf{x}} \rangle = \frac{ie}{N} \langle [\mathbf{H}, \mathbf{x}] \rangle. \quad (\text{B2})$$

Evaluating the commutator gives

$$\begin{aligned} [\mathbf{H}, \mathbf{x}] &= \gamma \sum_{n\sigma} (e^{i\theta} \mathbf{c}_{n\sigma}^\dagger \mathbf{c}_{n+1\sigma} - e^{-i\theta} \mathbf{c}_{n+1\sigma}^\dagger \mathbf{c}_{n\sigma}) - i\lambda_R \sum_{n\sigma} e^{-i\sigma\varphi_{n,n+1}} \\ &\quad \times (e^{i\theta} \mathbf{c}_{n\sigma}^\dagger \mathbf{c}_{n+1(-\sigma)} - e^{-i\theta} \mathbf{c}_{n+1\sigma}^\dagger \mathbf{c}_{n(-\sigma)}). \end{aligned} \quad (\text{B3})$$

Substituting (B3) into (B2) and then using Green's functions gives

$$\begin{aligned} &(\omega - E'_{n\sigma} + i\eta) \sum_{n\sigma} G_{n,\sigma;n,\sigma}^r(\omega) \\ &= \delta_{n,\sigma;n,\sigma} + \gamma \sum_{n\sigma} e^{i\theta} G_{n+N+1(\sigma),n\sigma}^r(\omega) \\ &\quad + \gamma \sum_{n\sigma} e^{-i\theta} G_{n+N-1(\sigma),n\sigma}^r(\omega) \\ &\quad - i\frac{\lambda_R}{2} e^{-i\sigma\varphi_{n,n+1}} \sum_{n\sigma} e^{i\theta} G_{n+N+1(\sigma'),n\sigma}^r(\omega) \\ &\quad - i\frac{\lambda_R}{2} e^{-i\sigma\varphi_{n,n+1}} \sum_{n\sigma} e^{-i\theta} G_{n+N-1(\sigma'),n\sigma}^r(\omega), \end{aligned} \quad (\text{B4})$$

with $\eta \rightarrow 0^+$.

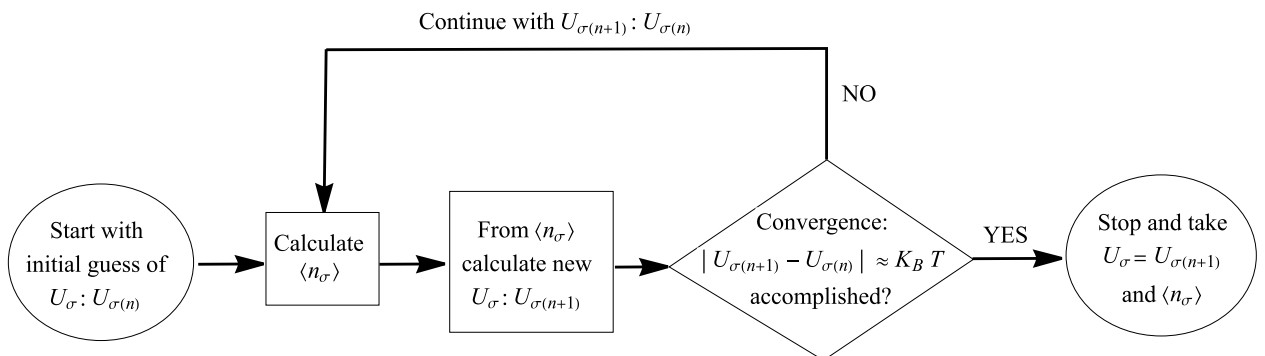


FIG. 5. Flux diagram illustrates the iterative procedure we use to calculate $\langle n_\sigma \rangle$ and U_σ in a self-consistent loop. In practice, we iterate this HF loop until a strict convergence criterion is met, $\Delta U = U_\sigma(n-1) - U_\sigma(n) < 10^{-6}$.

APPENDIX C: RELATIONSHIP BETWEEN THE RETARDED AND ADVANCED GREEN'S FUNCTIONS AND THE LESSER GREEN'S FUNCTION

We can decompose the contour-ordered Green's function as follows:

$$\begin{aligned} G(t, t') &= -i\langle T_C c_i(t) c_j^\dagger(t') \rangle \\ &= -i\Theta(t - t')\langle c_i(t) c_j^\dagger(t') \rangle \\ &\quad \mp i\Theta(t' - t)\langle c_i^\dagger(t) c_j(t') \rangle \\ &= \Theta(t - t')G^>(t, t') \mp \Theta(t' - t)G^<(t, t'), \end{aligned}$$

where the upper (lower) sign is for fermions (bosons). In an equilibrium state, $G^\Lambda(t, t') = G^\Lambda(t - t')$, and

$$G^\Lambda(t - t') = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega(t-t')} G^\Lambda(\omega), \quad \Lambda = <, >, r, a. \quad (C1)$$

Thermal equilibrium implies the KMS relation:

$$G^>(t - t') = \pm G^<(t - t' - i\beta), \quad \beta = (k_B T)^{-1}, \quad (C2)$$

which, in frequency space, becomes

$$G^>(\omega) = \pm e^{\beta\omega} G^<(\omega). \quad (C3)$$

By definition, we have

$$\begin{aligned} G^a(t, t') &= -\Theta(t' - t)[G^>(t, t') - G^<(t, t')], \\ G^r(t, t') &= \Theta(t - t')[G^>(t, t') - G^<(t, t')], \end{aligned} \quad (C4)$$

so that

$$G^r(t, t') - G^a(t, t') = G^>(t, t') - G^<(t, t'). \quad (C5)$$

If we Fourier transform this and use (C3), we arrive at the fluctuation-dissipation form:

$$G^<(\omega) = \pm \frac{G^r(\omega) - G^a(\omega)}{e^{\beta\omega} \mp 1}, \quad (C6)$$

with the upper sign for fermions. Equation (C6) completes the expressions for the current density in terms of $G^{r,a}$ that we used in the main text.

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