

# Light-induced topological phase transitions and anomalous thermal transport in $d$ -wave altermagnets

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(Received 24 April 2026; revised 25 June 2026; accepted 31 July 2026; published 1 September 2026)

We study intrinsic thermal transport and Floquet-engineered topology in a two-dimensional  $d$ -wave altermagnetic topological insulator powered by linearly polarized light. We analyze the anomalous Hall, Nernst, and thermal Hall conductivities, as well as their spin-resolved equivalents, and develop closed-form formulas for the Berry curvature using an analytically calculated high-frequency effective Hamiltonian. We demonstrate that linearly polarized light, in contrast to conventional antiferromagnets, breaks the symmetry connecting spin sectors in altermagnets, allowing a series of spin-selective topological phase transitions from a quantum spin Hall state to a spin-polarized Chern insulator and finally to a trivial phase. The Nernst response shows substantial thermal activation and significant sensitivity to the gap size in the Chern domain, but both the electrical and thermal Hall responses become quantized and meet the anomalous Wiedemann-Franz law. Every anomalous transport coefficient exhibits a distinctive  $d$ -wave dependence on the polarization angle, reversing sign under orthogonal rotation and vanishing at symmetry-restoring directions. Our findings show a path to all-optical regulation of topological and caloritronic responses beyond traditional magnetic systems and establish thermal transport as a sensitive probe of altermagnetic order.

DOI: [10.1103/8x4v-gf3y](https://doi.org/10.1103/8x4v-gf3y)

## I. INTRODUCTION

Altermagnetism (AM) has recently emerged as a distinct magnetic phase characterized by zero net magnetization coexisting with strongly anisotropic, momentum-dependent spin splitting [1–5], and has been identified in a growing number of materials [6–10]. Unlike conventional antiferromagnets (AFMs), which preserve combined parity-time ( $\mathcal{PT}$ ) symmetry and exhibit spin-degenerate bands, altermagnets break both time-reversal ( $\mathcal{T}$ ) and  $\mathcal{PT}$  symmetries while preserving their combination with specific crystal rotations [11]. This distinctive symmetry structure [12] enables a wide range of spintronic and topological phenomena, including giant magnetoresistance [13], anomalous Hall effects [14], topological insulating phases [15–17], and unconventional superconducting proximity effects [18].

Light is a particularly powerful external stimulus for controlling quantum materials, as it couples directly to electronic degrees of freedom and can dynamically modify band structure and symmetry. Beyond enabling control of intrinsic

electronic and magnetic properties, time-periodic driving can induce nonequilibrium phases that have no static counterparts. Within the framework of Floquet theory [19–21], such periodic driving provides a versatile route to engineer effective Hamiltonians and realize dynamically generated topological states [22,23].

In conventional nonmagnetic and antiferromagnetic systems, Floquet engineering has enabled a wide range of phenomena, including light-induced Hall effects [24,25], Floquet topological insulators [2,15,19], light-induced Weyl semimetals [26], and Floquet time crystals [2]. It has also opened a route to ultrafast control of spin and magnetic textures on picosecond and sub-picosecond timescales, forming the basis of ultrafast spintronics [3,21]. In topological systems, light-driven Floquet states have enabled novel phenomena such as Majorana modes, light-controlled edge transport, and dynamical probes of topological phase transitions.

Recently, the effects of light on unconventional magnetic systems, particularly altermagnets, have begun to attract significant attention [11,27,28]. However, existing studies have largely focused on charge transport and high-frequency regimes. While Berry-curvature-driven thermal transport is well established in topological materials [29], its Floquet engineering in altermagnetic systems remains essentially unexplored. This is especially important because symmetry considerations allow linearly polarized light (LPL) to induce Hall responses in altermagnets, in contrast to conventional antiferromagnets.

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More broadly, recent work has demonstrated that Berry-curvature-related responses in low-dimensional and compensated magnetic systems can be tuned by a variety of external control parameters beyond optical driving, including strain, electric fields, and heterostructure engineering. Notable recent advances include the demonstration of externally controlled anomalous Hall and spin-dependent transport responses in low-dimensional materials [30] and compensated magnetic systems [31]. Our work contributes to this broader direction by establishing linearly polarized light as an all-optical control knob for anomalous thermal and spin-caloritronic responses in altermagnetic topological insulators, without requiring magnetic field reversal or structural modification.

In nonmagnetic and conventional antiferromagnetic systems, LPL generally cannot generate a Hall response because  $\mathcal{PT}$  symmetry is preserved. Altermagnets circumvent this restriction: their intrinsically broken  $\mathcal{PT}$  symmetry allows LPL, by breaking the spin-rotation symmetry  $C_{4z}\mathcal{T}$  that relates the two spin sublattices, to induce a finite anomalous Hall effect [11]. This mechanism is part of a wider effort to identify external control parameters that can tune Berry-curvature-driven responses in systems with compensated magnetic order, where conventional Hall probes are ineffective [30,31]. Thermal and spin-caloritronic channels, as studied here, offer particularly sensitive probes of such responses precisely because they weight the Berry curvature differently from charge transport. Recent works have demonstrated LPL-induced anomalous Hall responses and topological phase transitions in altermagnetic topological insulators [11], as well as circularly polarized light-induced quantum anomalous Hall phases [32], light-induced odd-parity magnetism [27,32], and higher-order spin-orbit coupling effects [28].

Despite this progress, a comprehensive understanding of thermal transport under LPL irradiation in altermagnetics remains lacking. Thermal transport coefficients, namely, the anomalous Nernst conductivity (ANC) and anomalous thermal Hall conductivity (ATHC), probe the Berry curvature through distinct thermal weightings that differ fundamentally from the anomalous Hall conductivity (AHC). As a result, they provide complementary and often more sensitive insights into the underlying topology and magnetic order. The ANC, being thermally activated, serves as a sensitive probe of band topology at finite temperature, while the ATHC, together with its relation to  $\sigma_{xy}$  via the Wiedemann-Franz (WF) law, offers a direct test of topological quantization. Furthermore, the spin-resolved counterparts; the spin Nernst and spin thermal Hall conductivities enable access to spin caloritronic effects of direct relevance for device applications [33].

In this work we present a comprehensive theoretical study of thermal transport in a  $d$ -wave altermagnetic under LPL irradiation. Starting from an analytically derived Floquet effective Hamiltonian in the high-frequency limit, we obtain explicit expressions for the Berry curvature in each spin sector and compute six intrinsic transport coefficients: the AHC  $\sigma_{xy}$ , ANC  $\alpha_{xy}$ , and ATHC  $\kappa_{xy}^{(0)}$ , as well as their spin-resolved counterparts  $\sigma_{xy}^s$ ,  $\alpha_{xy}^s$ , and  $\kappa_{xy}^{(0),s}$ . We analyze their dependence on topological phase, polarization angle, and temperature, and establish their interrelations via the Mott and Wiedemann-Franz laws. Finally, we contrast the altermagnetic response

with that of conventional antiferromagnets, identifying clear experimental signatures accessible through thermal Hall and Nernst measurements.

## II. MODEL AND FLOQUET EFFECTIVE HAMILTONIAN

We employ the four-band two-dimensional out-of-plane  $d$ -wave altermagnetic model on a square lattice [11,15,16] in which the orbitals are  $p_{z\uparrow}$ ,  $\frac{1}{\sqrt{2}}(d_{xz\uparrow} + id_{yz\uparrow})$ ,  $p_{z\downarrow}$ , and  $\frac{1}{\sqrt{2}}(d_{xz\downarrow} - id_{yz\downarrow})$ . The low-energy Hamiltonian has the block-diagonal form

$$H(\mathbf{k}) = \begin{pmatrix} H_{\uparrow}(\mathbf{k}) & 0 \\ 0 & H_{\downarrow}(\mathbf{k}) \end{pmatrix}, \quad (1)$$

where each  $2 \times 2$  spin block is written in terms of a  $d$ -vector as  $H_{\sigma}(\mathbf{k}) = \mathbf{d}^{\sigma}(\mathbf{k}) \cdot \boldsymbol{\tau}$ , with  $\boldsymbol{\tau} = (\tau_x, \tau_y, \tau_z)$  the Pauli matrices acting on the orbital subspace. The explicit  $d$ -vector components are

$$\begin{aligned} d_x^{\uparrow} &= -v \sin k_x, & d_y^{\uparrow} &= -vt_a \sin k_y, \\ d_z^{\uparrow} &= m + b(\cos k_x + t_a^2 \cos k_y), \end{aligned} \quad (2)$$

and  $H_{\downarrow}(k_x, k_y) = H_{\uparrow}(k_y, k_x)$ , i.e., the spin-down block is obtained by the exchange  $k_x \leftrightarrow k_y$ . Here  $m$  is the on-site potential,  $v$  the hopping amplitude,  $b$  the inversion-breaking hopping, and  $t_a$  the asymmetry parameter. The two spin channels are related by  $C_{4z}\mathcal{T}$  or mirror symmetry  $\mathcal{M}_{xy}$ . The lattice constant is assumed to be unity. Setting  $t_a = 1$  restores  $\mathcal{PT}$  symmetry, reducing the system to a conventional AFM;  $t_a \neq 1$  breaks  $\mathcal{PT}$  and yields the  $d$ -wave AM. The equilibrium system is a QSH insulator when

$$|m| < |b|(1 + t_a^2), \quad (3)$$

and topologically trivial otherwise [15]. The parameters used throughout this work,

$$m = 3.8v, \quad b = -v, \quad t_a = \sqrt{3}, \quad (4)$$

satisfy this condition since

$$|b|(1 + t_a^2) = v(1 + 3) = 4v > 3.8v = m, \quad (5)$$

placing the system in the QSH regime but sufficiently close to the topological phase boundary at  $m = 4v$  that light-induced renormalization of the hopping parameters can drive sequential gap closings in the two spin sectors. The hopping parameter  $v$  sets the overall energy scale of the model, the ratio  $m/b$  controls the degree of band inversion, and  $t_a \neq 1$  breaks the combined parity-time ( $\mathcal{PT}$ ) symmetry that would otherwise reduce the system to a conventional antiferromagnet.

We drive the system with LPL whose vector potential is given as

$$\mathbf{A}(t) = A_0[\cos \theta \cos(\omega t), \sin \theta \cos(\omega t)], \quad (6)$$

where  $A_0$  is the amplitude (it is dimensionless in our units),  $\theta$  the polarization angle relative to the  $x$  axis, and  $\omega$  the frequency. The time-dependent Hamiltonian is obtained via the Peierls substitution  $\mathbf{k} \rightarrow \mathbf{k} + e\mathbf{A}(t)/\hbar$ . In the high-frequency limit and off-resonant regime, the leading order commutator correction vanishes because  $\mathbf{A}(t) = \mathbf{A}(-t + \tau)$  for a

fractional time translation  $\tau$ , and the effective Hamiltonian reduces to the time-averaged part [22] in Floquet high-frequency expansion regime.

To derive the effective hamiltonian explicitly we substitute  $\mathbf{k} \rightarrow \mathbf{k} + e\mathbf{A}(t)/\hbar$  into the static  $d$ -vector of Eq. (2) For the spin-up block this gives, for example,

$$d_x^\uparrow(t) = -v \sin\left(k_x + \frac{eA_0 \cos \theta}{\hbar} \cos \omega t\right), \quad (7)$$

and similarly for  $d_y^\uparrow(t)$  and  $d_z^\uparrow(t)$ , with the driving entering along  $k_x$  through  $A_0 \cos \theta$  and along  $k_y$  through  $A_0 \sin \theta$ . In the high-frequency off-resonant regime, the leading correction to the effective Hamiltonian is given by the time average over one driving period,

$$H_{\text{eff}}^\sigma(\mathbf{k}) = \frac{1}{T_0} \int_0^{T_0} H^\sigma(\mathbf{k}, t) dt, \quad T_0 = \frac{2\pi}{\omega}. \quad (8)$$

Expanding  $\sin[k_x + \phi(t)]$  and  $\cos[k_x + \phi(t)]$  using standard trigonometric identities and performing the time average using the Jacobi-Anger expansion,

$$\frac{1}{T_0} \int_0^{T_0} \cos\left(\frac{eA_0 \cos \theta}{\hbar} \cos \omega t\right) dt = J_0(A_0 \cos \theta) \equiv j_1 \quad (9)$$

[and analogously for the  $k_y$ -dependent terms, giving  $j_2 = J_0(A_0 \sin \theta)$ ], one obtains directly the renormalized  $d$ -vector components in Eqs. (11) and (12): every term that originally depended on  $k_x$  acquires a factor  $j_1$ , and every term depending on  $k_y$  acquires a factor  $j_2$ . The terms odd in the driving field (generating Bessel functions  $J_{n \geq 1}$ ) average to zero in this leading-order expansion, leaving only the  $J_0$  renormalization factors  $j_1$  and  $j_2$ .

Physically,  $j_1$  and  $j_2$  represent the LPL-induced renormalization of the effective hopping amplitudes along the  $x$  and  $y$  directions, respectively. Because the spin-down block is related to the spin-up block by the coordinate exchange  $k_x \leftrightarrow k_y$  [Eq. (2)], the same substitution shows that  $H_{\downarrow, \text{eff}}$  is obtained from  $H_{\uparrow, \text{eff}}$  by simultaneously exchanging  $k_x \leftrightarrow k_y$  and  $j_1 \leftrightarrow j_2$ , which is precisely the origin of Eq. (11). When  $\theta = \pm\pi/4$ ,  $j_1 = j_2$  and the two spin blocks remain related by the original symmetry operation; for any other  $\theta$ ,  $j_1 \neq j_2$  and this symmetry is broken, which is the microscopic origin of the spin-selective topological phase transitions discussed throughout this work.

In the off-resonant limit, the heating rate is significantly reduced, allowing the system to reach a long-lived Floquet prethermal [34,35] state where the effective Hamiltonian description remains valid or the situation where a finite pulse duration in order of ps or ns is used.

Finite temperature primarily broadens the Fermi distribution but does not destroy the topological band structure, provided  $k_B T \ll \hbar\omega$ . The high-frequency expansion underlying Eqs. (11) and (12) is controlled by the parameter  $W/\hbar\omega$ , where  $W \sim |v| + |b|(1 + t_a^2)$  is the electronic bandwidth of the static model. Truncation at leading order in this expansion, as performed here, is justified when

$$\hbar\omega \gg W. \quad (10)$$

For typical hopping parameters  $v, |b| \sim 0.1$ – $1$  eV, the bandwidth is  $W \sim 0.5$ – $2$  eV in 2D semiconductor materials.

Driving frequencies in the midinfrared to terahertz range,  $\omega/2\pi \sim 10$ – $100$  THz (i.e.,  $\hbar\omega \sim 0.04$ – $0.4$  eV), therefore satisfy Eq. (10) only marginally for the larger end of  $W$ , but comfortably for systems with smaller bandwidths or for higher driving frequencies. We assume throughout that the system is operated in this regime, so that higher-order Floquet corrections do not qualitatively modify the topological phases discussed below.

Applying the Jacobi-Anger expansion gives renormalized  $d$ -vectors:

$$d_{x, \text{eff}}^\uparrow = -vj_1 \sin k_x, \quad d_{y, \text{eff}}^\uparrow = -vt_a j_2 \sin k_y, \\ d_{z, \text{eff}}^\uparrow = m + b[j_1 \cos k_x + j_2 t_a^2 \cos k_y], \quad (11)$$

$$d_{x, \text{eff}}^\downarrow = -vj_2 \sin k_y, \quad d_{y, \text{eff}}^\downarrow = -vt_a j_1 \sin k_x, \\ d_{z, \text{eff}}^\downarrow = m + b[j_2 \cos k_y + j_1 t_a^2 \cos k_x], \quad (12)$$

where

$$j_1 = J_0(A_0 \cos \theta), \quad j_2 = J_0(A_0 \sin \theta), \quad (13)$$

and  $J_0$  is the zeroth-order Bessel function. The key physical content of Eqs. (11)–(13) is transparent: when  $\theta \neq \pm\pi/4$ , we have  $j_1 \neq j_2$ , which means the spin-up and spin-down effective Hamiltonians are no longer related by any symmetry operation, breaking  $C_{4z}\mathcal{T}$  and  $\mathcal{M}_{xy}$ . In contrast, for the conventional AFM ( $t_a = 1$ ), the  $\mathcal{PT}$  symmetry is preserved for any  $\theta$ , locking  $j_1 = j_2$  in its effect on the combined system and hinders any Hall response.

The eigenvalues of  $H_{\text{eff}}^\sigma$  are  $\varepsilon_\pm^\sigma(\mathbf{k}) = \pm|\mathbf{d}_{\text{eff}}^\sigma(\mathbf{k})|$ . A topological phase transition occurs whenever a spin-resolved gap  $\Delta_\sigma = \min_{\mathbf{k}}[2|\mathbf{d}_{\text{eff}}^\sigma(\mathbf{k})|]$  closes. Since  $j_1 \neq j_2$  under LPL in the AM, the two gaps  $\Delta_\uparrow$  and  $\Delta_\downarrow$  evolve independently with  $A_0$ , enabling sequential closings that are forbidden in the AFM.

We emphasize that the parameter set in Eq. (4) defines a symmetry-allowed minimal-model regime and should not be interpreted as a quantitative fit to any specific material. To our knowledge, an intrinsic QSH phase in a monolayer altermagnetic compound has not yet been unambiguously confirmed experimentally. Our model instead provides a minimal theoretical platform that isolates the role of altermagnetic symmetry and linearly polarized light in producing spin-selective topological phase transitions.

Several candidate material directions may nevertheless allow the required conditions to be realized:

**MnTe-based heterostructures.** MnTe is a well-established altermagnetic candidate [13] with strong spin-momentum locking. Thin-film or heterostructure engineering could introduce the band inversion needed for a QSH phase while preserving the altermagnetic symmetry.

**RuO<sub>2</sub>-based thin films.** RuO<sub>2</sub> has been identified as a  $d$ -wave altermagnet [6], and its electronic structure can be tuned significantly by epitaxial strain and substrate choice, potentially bringing the system into a band-inverted regime.

**Artificial lattice platforms.** Moiré superlattices and cold-atom optical lattices with engineered square-lattice geometry offer full tunability of the hopping parameters  $v$ ,  $b$ , and  $t_a$ , making it possible to place the system directly into the QSH altermagnetic regime defined by Eq. (3).

Quantitative predictions for these specific systems would require material-specific first-principles input and are left for future work.

### III. BERRY CURVATURE

Throughout this work we adopt the following conventions for the spin-resolved and total Chern numbers. The spin-sector Chern numbers are defined as

$$C_{\uparrow} = \frac{1}{2\pi} \int_{\text{BZ}} d^2k \Omega_{-}^{z,\uparrow}(\mathbf{k}), \quad C_{\downarrow} = \frac{1}{2\pi} \int_{\text{BZ}} d^2k \Omega_{-}^{z,\downarrow}(\mathbf{k}), \quad (14)$$

the total Chern number is

$$C = C_{\uparrow} + C_{\downarrow}, \quad (15)$$

and the spin Chern number is

$$C_s = \frac{C_{\uparrow} - C_{\downarrow}}{2}. \quad (16)$$

The three topological phases realized under LPL irradiation are then characterized as follows:

$$\text{QSH phase: } C_{\uparrow} = -1, \quad C_{\downarrow} = +1, \quad C = 0, \quad C_s = -1, \quad (17)$$

$$\text{Chern phase: } C_{\uparrow} = -1, \quad C_{\downarrow} = 0, \quad C = -1, \quad C_s = -\frac{1}{2}, \quad (18)$$

$$\text{Trivial phase: } C_{\uparrow} = 0, \quad C_{\downarrow} = 0, \quad C = 0, \quad C_s = 0. \quad (19)$$

The spin-up and spin-down components in the Eq. (1) are decoupled, their Berry connections and Chern numbers can be evaluated independently. For the lower band  $\varepsilon_{-}^{\sigma} = -|\mathbf{d}^{\sigma}|$  of each spin block, the normalized eigenstate in terms of the spherical coordinates  $\hat{d}^{\sigma} = (\sin \Theta \cos \Phi, \sin \Theta \sin \Phi, \cos \Theta)$  of the effective  $\mathbf{d}$ -vector is

$$|u_{-}^{\sigma}(\mathbf{k})\rangle = \begin{pmatrix} -\sin(\Theta/2) e^{-i\Phi} \\ \cos(\Theta/2) \end{pmatrix}. \quad (20)$$

Computing the Berry connection  $\mathcal{A}_{\mu}^{\sigma} = i\langle u_{-}^{\sigma} | \partial_{k_{\mu}} | u_{-}^{\sigma} \rangle$  from Eq. (20) gives

$$\mathcal{A}_{\mu}^{\sigma}(\mathbf{k}) = \sin^2\left(\frac{\Theta}{2}\right) \partial_{k_{\mu}} \Phi. \quad (21)$$

Taking the antisymmetric curl of Eq. (21) and converting to the solid angle form, the  $z$  component of the Berry curvature for the lower band of spin  $\sigma$  can be expressed as

$$\Omega_{-}^{z,\sigma}(\mathbf{k}) = \frac{1}{2} \hat{d}^{\sigma} \cdot (\partial_{k_x} \hat{d}^{\sigma} \times \partial_{k_y} \hat{d}^{\sigma}) = \frac{\mathbf{d}^{\sigma} \cdot (\partial_{k_x} \mathbf{d}^{\sigma} \times \partial_{k_y} \mathbf{d}^{\sigma})}{2|\mathbf{d}^{\sigma}|^3}. \quad (22)$$

Evaluating the triple product explicitly with the LPL renormalized  $\mathbf{d}$ -vectors (11) and (12), we obtain the closed form expression for the spin-up Berry curvature as

$$\Omega_{-}^{z,\uparrow}(\mathbf{k}) = \frac{bv^2 t_a j_1^2 j_2 \sin^2 k_x \cos k_y + bv^2 t_a^3 j_1^2 j_2 \sin^2 k_y \cos k_x}{2|\mathbf{d}_{\text{eff}}^{\uparrow}|^3} + \frac{d_{z,\text{eff}}^{\uparrow} v^2 t_a j_1 j_2 \cos k_x \cos k_y}{2|\mathbf{d}_{\text{eff}}^{\uparrow}|^3}, \quad (23)$$

and for spin-down

$$\Omega_{-}^{z,\downarrow}(\mathbf{k}) = \frac{-bv^2 t_a j_1 j_2^2 \sin^2 k_y \cos k_x - bv^2 t_a^3 j_1^2 j_2 \sin^2 k_x \cos k_y}{2|\mathbf{d}_{\text{eff}}^{\downarrow}|^3} - \frac{d_{z,\text{eff}}^{\downarrow} v^2 t_a j_1 j_2 \cos k_x \cos k_y}{2|\mathbf{d}_{\text{eff}}^{\downarrow}|^3}. \quad (24)$$

Equations (23) and (24) encode several important symmetry properties. First, when  $j_1 = j_2$  (i.e.,  $A_0 = 0$  or  $\theta = \pm\pi/4$ ), we can verify directly that  $\Omega_{-}^{z,\uparrow}(\mathbf{k}) = -\Omega_{-}^{z,\downarrow}(\mathbf{k})$ , so the total Berry curvature  $\Omega_{-}^{z,\uparrow} + \Omega_{-}^{z,\downarrow} = 0$  at every  $\mathbf{k}$ , consistent with the preserved  $C_{4z}\mathcal{T}$  symmetry. Second, under  $\mathcal{PT}$  symmetry (AFM,  $t_a = 1$ , any  $j_1, j_2$ ):  $\mathcal{PT}$  maps  $\Omega_n^z(\mathbf{k}) \rightarrow -\Omega_n^z(\mathbf{k})$ , forcing  $\Omega_n^z(\mathbf{k}) = 0$  for all  $\mathbf{k}$ , which prevents any Hall response. Third, the symmetry  $\sigma_{xy}(\pi/2 - \theta) = -\sigma_{xy}(\theta)$  is encoded in Eqs. (23)–(24) through the exchange  $j_1 \leftrightarrow j_2$  under  $\theta \rightarrow \pi/2 - \theta$ .

### IV. THERMAL TRANSPORT

Having calculated the Berry phase, we can calculate some thermal transport quantities like anomalous Hall conductivity, Nernst conductivity, and thermal Hall conductivity.

#### A. Anomalous Hall conductivity

The low-energy and effective Hamiltonian is block diagonal in spin space, thus the spin  $S_z$  is a good quantum number, and the valence bands split into spin-up and spin-down

components. Therefore, we can define the spin current in the system. Within the semiclassical Boltzmann framework, the anomalous (intrinsic) contribution to the transverse conductivity is [36]

$$\sigma_{xy} = \frac{e^2}{\hbar} \sum_{\sigma=\uparrow,\downarrow} \sum_{n=\pm} \int_{\text{BZ}} \frac{d^2k}{(2\pi)^2} \Omega_n^{z,\sigma}(\mathbf{k}) f_n^{\sigma}, \quad (25)$$

where  $f(\varepsilon) = (e^{\varepsilon/k_B T} + 1)^{-1}$  is the Fermi-Dirac distribution and  $\mu$  is the chemical potential. At  $T = 0$  in a gapped insulating phase, this reduces to  $\sigma_{xy} = (e^2/h)C$ . Where  $C = C_{\uparrow} + C_{\downarrow}$  is the total Chern number [37].

For the two-band model considered here, the Berry curvatures of the upper and lower bands satisfy  $\Omega_{+}^{z,\sigma}(\mathbf{k}) = -\Omega_{-}^{z,\sigma}(\mathbf{k})$ , so when the chemical potential lies in the gap at  $T = 0$ , only the lower band contributes and one recovers the familiar Chern-number formula. For a general chemical potential or at finite temperature, however, both bands must be retained to correctly capture the decay of  $\sigma_{xy}$  as  $E_F$  moves into the conduction band.

### B. Anomalous Nernst conductivity

The intrinsic anomalous Nernst conductivity (ANC), relating the transverse electric current to a longitudinal temperature gradient via  $J_y = \alpha_{xy}(-\partial_x T)$ , is given by the Berry curvature weighted by the fermionic entropy kernel [29,38]:

$$\alpha_{xy} = \frac{ek_B}{\hbar} \sum_{\sigma=\uparrow,\downarrow} \sum_{n=\pm} \int_{\text{BZ}} \frac{d^2k}{(2\pi)^2} \Omega_n^{z,\sigma}(\mathbf{k}) s(f_n^\sigma), \quad (26)$$

where

$$s(f) = -f \ln f - (1-f) \ln(1-f) \quad (27)$$

is the entropy carried by a single fermionic state, and  $f_n^\sigma = (\exp\{[\varepsilon_n^\sigma(\mathbf{k}) - \mu]/k_B T\} + 1)^{-1}$  is the Fermi-Dirac distribution.

The vanishing of  $\alpha_{xy}$  at zero temperature follows immediately from this form. As  $T \rightarrow 0$ , the Fermi function approaches a step function, so every state is either fully occupied ( $f = 1$ ) or fully empty ( $f = 0$ ). Since

$$s(1) = 0, \quad s(0) = 0, \quad (28)$$

the integrand vanishes identically at every  $\mathbf{k}$  point when  $\mu$  lies inside the bulk gap, and therefore

$$\alpha_{xy}(T = 0) = 0. \quad (29)$$

This result is a direct consequence of the third law of thermodynamics: a completely filled or completely empty state carries no entropy, and the Nernst response is therefore purely thermally activated. No cancellation between upper and lower bands is required; the vanishing follows directly from  $s(0) = s(1) = 0$ .

We note that the entropy-kernel form in Eq. (26) is equivalent to the logarithmic representation obtained via integration by parts,

$$s(f_n^\sigma) \longleftrightarrow \frac{1}{k_B T} \int_{\varepsilon_n^\sigma}^{\infty} f(\varepsilon' - \mu) d\varepsilon' = \ln(1 + e^{(\mu - \varepsilon_n^\sigma)/k_B T}), \quad (30)$$

up to an overall factor of  $k_B T$ . The logarithmic form is convenient for numerical evaluation, but the apparent divergence of  $\ln(1 + e^{(\mu - \varepsilon)/k_B T}) \approx (\mu - \varepsilon)/k_B T$  for occupied bands at low temperature is canceled exactly by the vanishing Fermi occupation factor in the full thermoelectric kernel. The entropy-kernel form makes this cancellation manifest and is therefore the physically transparent starting point. In practice, the entropy kernel  $s(f)$  and the logarithmic form  $\ln(1 + e^{(\mu - \varepsilon)/k_B T})$  are numerically equivalent up to the factor  $k_B T$ , and both are well behaved for  $k_B T > 0$ . Our numerical calculations use the entropy-kernel form of Eq. (26) directly, which avoids any ambiguity about the low-temperature limit and is manifestly consistent with  $\alpha_{xy}(T = 0) = 0$ .

### C. Anomalous thermal Hall conductivity

The ATHC, defined by  $J_y^Q = -\kappa_{xy}^{(0)} \partial_x T$ , is given by the  $c_2$  function formula derived via Luttinger's gravitational potential approach [39,40]:

$$\kappa_{xy}^{(0)} = -\frac{k_B^2 T}{\hbar} \sum_{\sigma=\uparrow,\downarrow} \sum_{n=\pm} \int_{\text{BZ}} \frac{d^2k}{(2\pi)^2} \Omega_n^{z,\sigma}(\mathbf{k}) c_2(f_n^\sigma), \quad (31)$$

where the function  $c_2$  is defined as

$$c_2(f) = \int_0^f \left( \ln \frac{1-t}{t} \right)^2 dt, \quad c_2(0) = 0, \quad c_2(1) = \frac{\pi^2}{3}. \quad (32)$$

The value  $c_2(1) = \pi^2/3$  can be established precisely using the Dirichlet eta function:  $c_2(1) = 2 \int_0^\infty u^2 e^u / (e^u + 1)^2 du = 4\eta(2) = \pi^2/3$ . In the low-temperature limit with the chemical potential sitting inside the gap, the upper band is completely empty so  $f_+^\sigma \rightarrow 0$  and  $c_2(0) = 0$ , while the lower band is completely filled so  $f_-^\sigma \rightarrow 1$  and  $c_2(1) = \pi^2/3$ . The upper band therefore contributes nothing in this limit, and Eq. (31) reduces to

$$\left. \frac{\kappa_{xy}^{(0)}}{T} \right|_{T \rightarrow 0} = -\frac{\pi^2 k_B^2}{3\hbar} \cdot \frac{C}{2\pi} = -\frac{\pi^2 k_B^2}{3h} C. \quad (33)$$

### D. Wiedemann-Franz law for anomalous transport

Comparing Eq. (33) with  $\sigma_{xy} = e^2 C/h$  at  $T = 0$ , we obtain the anomalous Wiedemann-Franz law: [41]

$$\frac{\kappa_{xy}^{(0)}}{\sigma_{xy}} = -\frac{\pi^2 k_B^2}{3e^2} T \equiv -L_0 T, \quad (34)$$

where  $L_0 = \pi^2 k_B^2 / 3e^2 \approx 2.44 \times 10^{-8} \text{ W } \Omega \text{ K}^{-2}$  is the Lorenz number. This relation holds exactly in any topological phase at  $T \rightarrow 0$  and serves as an unambiguous experimental test of the Berry phase origin of the thermal Hall response.

### E. Mott formula for the Nernst conductivity

At low temperature  $k_B T \ll \mu$ , the Sommerfeld expansion gives [41]

$$\alpha_{xy} \approx -\frac{\pi^2 k_B^2 T}{3e} \left. \frac{\partial \sigma_{xy}(\varepsilon)}{\partial \varepsilon} \right|_{\varepsilon=\mu}, \quad (35)$$

the anomalous analog of the Mott formula. This low-temperature expression is fully consistent with Eq. (26): expanding  $s(f)$  to leading order in  $k_B T/(\varepsilon - \mu)$  via the Sommerfeld expansion and integrating by parts recovers Eq. (35) directly [41]. The Mott relation therefore provides an independent check that the entropy-kernel form correctly reduces to zero at  $T = 0$  and grows linearly in  $T$  at low temperatures, consistent with the thermally activated picture established in Eq. (29). Equation (35) shows that  $\alpha_{xy}$  is proportional to the energy derivative of the AHC, making it sensitive to the sharpness of Berry curvature features near the Fermi level.

### F. Spin-resolved transport coefficients

Since the altermagnetic effective Hamiltonian is block diagonal in spin, we define spin-resolved versions of all three transport coefficients. In doing so, we apply the conventional charge-to-spin conversion factor  $\hbar/(2e)$  only where physically appropriate, namely, for coefficients derived from charge currents. For thermal responses, which measure transverse heat currents independent of the elementary charge  $e$ , the prefactor remains unchanged from the charge sector. The spin Hall

conductivity

$$\sigma_{xy}^s = \frac{e}{2} \sum_{n=\pm} \int_{\text{BZ}} \frac{d^2k}{(2\pi)^2} [\Omega_n^{z,\uparrow}(\mathbf{k}) f_n^\uparrow - \Omega_n^{z,\downarrow}(\mathbf{k}) f_n^\downarrow]; \quad (36)$$

the spin Nernst conductivity

$$\alpha_{xy}^s = \frac{k_B}{2} \int \frac{d^2k}{(2\pi)^2} [\Omega_-^{z,\uparrow} \ln(1 + e^{(\mu - \varepsilon_-^\uparrow)/k_B T}) - \Omega_-^{z,\downarrow} \ln(1 + e^{(\mu - \varepsilon_-^\downarrow)/k_B T})], \quad (37)$$

which is equivalent to

$$\alpha_{xy}^s = \frac{k_B}{2} \sum_{n=\pm} \int_{\text{BZ}} \frac{d^2k}{(2\pi)^2} [\Omega_n^{z,\uparrow}(\mathbf{k}) s(f_n^\uparrow) - \Omega_n^{z,\downarrow}(\mathbf{k}) s(f_n^\downarrow)],$$

where

$$s(f) = -f \ln f - (1 - f) \ln(1 - f) \quad (38)$$

is the fermionic entropy kernel, which is related to the logarithmic form  $\ln(1 + e^{(\mu - \varepsilon)/k_B T})$  through the same integration-by-parts identity. The prefactor  $k_B/2$  that is corrected follows from applying the conventional charge-to-spin conversion factor  $\hbar/(2e)$  to the charge Nernst prefactor  $ek_B/\hbar$  in Eq. (26):

$$\frac{ek_B}{\hbar} \times \frac{\hbar}{2e} = \frac{k_B}{2}, \quad (39)$$

and the spin thermal Hall conductivity

$$\kappa_{xy}^{(0),s} = -\frac{k_B^2 T}{\hbar} \sum_{n=\pm} \int_{\text{BZ}} \frac{d^2k}{(2\pi)^2} [\Omega_n^{z,\uparrow}(\mathbf{k}) c_2(f_n^\uparrow) - \Omega_n^{z,\downarrow}(\mathbf{k}) c_2(f_n^\downarrow)]. \quad (40)$$

There is the same thermal prefactor  $k_B^2 T/\hbar$  for both  $\kappa_{xy}^{(0),s}$  and the anomalous thermal Hall conductivity in Eq. (31). This is due to the reason that the spin thermal Hall coefficient represents the difference in transverse heat currents carried by the two spin channels,  $\kappa_{xy}^{(0),s} = \kappa_{xy}^{(0),\uparrow} - \kappa_{xy}^{(0),\downarrow}$ . Since heat transport does not involve the elementary charge  $e$ , applying the charge-to-spin conversion factor  $\hbar/(2e)$  here would be physically meaningless. In the QSH phase where  $\Omega_-^{z,\uparrow} = -\Omega_-^{z,\downarrow}$  and  $\varepsilon_-^\uparrow = \varepsilon_-^\downarrow$ , the total  $\sigma_{xy} = \alpha_{xy} = \kappa_{xy}^{(0)} = 0$  but both  $\sigma_{xy}^s$  and  $\kappa_{xy}^{(0),s}$  are nonzero, providing a route to pure spin caloritronic effects.

### G. Symmetry of transport coefficients under $\theta$

The polarization angle dependence of all three anomalous coefficients follows from the symmetry  $j_1(-\theta) = j_1(\theta)$  and  $j_2(-\theta) = j_2(\theta)$  (even in  $\theta$ ) combined with the exchange symmetry  $j_1 \leftrightarrow j_2$  under  $\theta \rightarrow \pi/2 - \theta$ . We can show that

$$X(\pi/2 - \theta) = -X(\theta), \quad X \in \{\sigma_{xy}, \alpha_{xy}, \kappa_{xy}^{(0)}\}, \quad (41)$$

which requires  $X(\pi/4) = 0$ , i.e., all anomalous transport coefficients vanish at the diagonal polarization angle where  $\mathcal{M}_{xy}$  symmetry is preserved. Additionally,  $|X|$  is maximized when  $\theta = 0$  or  $\theta = \pi/2$  (LPL along a crystal axis), and all three coefficients share the same  $d$ -wave nodal pattern in the  $(A_0, \theta)$  plane.

## V. RESULTS AND DISCUSSION

We consider a pristine system, and thus our results describe intrinsic contributions; extrinsic effects may modify magnitudes but not symmetry or signatures of studied phases. In this study we explain the Floquet-engineered altermagnetic system using a noninteracting band framework. There are various arguments to support this approximation. First, the anomalous Hall, Nernst, and thermal Hall responses taken into consideration here are intrinsic transport coefficients that are mostly governed by the underlying band structure's Berry curvature, which is well represented in an effective single-particle picture. Second, as long as the driving frequency is higher than the characteristic interaction and bandwidth scales, electron-electron interactions in the high-frequency (off-resonant) Floquet regime primarily result in weak renormalizations of the band parameters and do not qualitatively change the topology of the effective Hamiltonian. Third, the main phenomena discussed here, the symmetry-controlled angular dependence, spin, selective gap closings, and Chern number changes, are resistant against weak interaction effects because they are determined by symmetry and band topology. Lastly, investigations and first-principles research show that a quasiparticle description is still viable for candidate altermagnetic materials like RuO<sub>2</sub> and MnTe-based systems, supporting the use of an effective noninteracting model.

### A. Band structures

The spin-resolved band structures of the QSH altermagnetic (AM) and QSH antiferromagnetic (AFM) insulators are examined for four values of the LPL amplitude, and results are shown in Fig. 1. In the QSH AM case, the spin-up and spin-down bands evolve differently with increasing  $A_0$ : the spin-down gap closes at  $A_0^{c1} \approx 0.521$  while the spin-up gap remains open, leading to the emergence of an intermediate Chern phase. Upon further increasing the amplitude to  $A_0^{c2} \approx 0.918$ , the spin-up gap also closes, and the system transitions into a trivial phase. In contrast, for the QSH AFM case, the spin-up and spin-down bands remain perfectly degenerate for all values of  $A_0$ , and both gaps close simultaneously at a single critical amplitude, resulting in the absence of any intermediate Chern phase. This contrast is made explicit in phase diagram which shows the topological phase transitions from QSH topological insulator to the Chern insulator and then to the trivial altermagnetic insulator. The spin-resolved Chern numbers evolve continuously with the amplitude  $A_0$ , revealing two critical values. At  $A_0^{c1} \approx 0.521$  the spin-down gap closes:  $C_\downarrow$  changes from  $+1$  to  $0$  while  $C_\uparrow$  remains  $-1$ , so the total Chern number changes from  $C = 0$  to  $C = -1$ , signaling the entry into the Chern insulating phase. At  $A_0^{c2} \approx 0.918$  the spin-up gap closes:  $C_\uparrow$  changes from  $-1$  to  $0$ , so  $C$  returns to  $0$  and the system enters the trivial insulating phase. The correct phase sequence is therefore

$$C : 0 \longrightarrow -1 \longrightarrow 0, \quad (42)$$

which corresponds to

$$\begin{aligned} \text{QSH} &\longrightarrow \text{spin-polarized Chern insulator} \\ &\longrightarrow \text{trivial insulator.} \end{aligned} \quad (43)$$

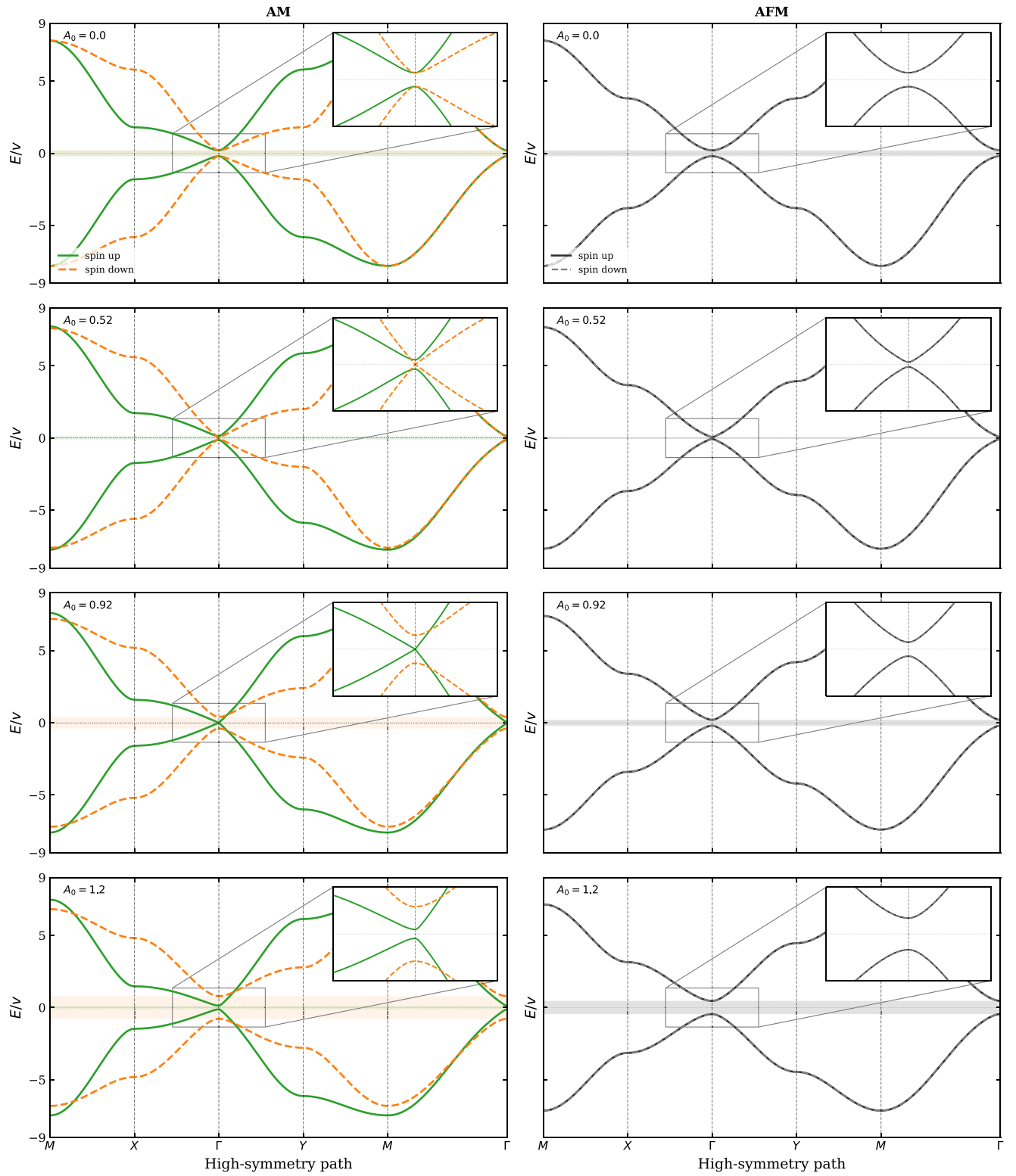


FIG. 1. Energy dispersions and topological phase diagram of the QSH AM under LPL. Spin-resolved band structure of the QSH AM ( $m = 3.8v$ ,  $b = -v$ ,  $t_a = \sqrt{3}$ ) at  $A_0 = 0.0, 0.52, 0.92, 1.2$  ( $\theta = 0$ ). Spin-up (solid teal) and spin-down (dashed orange) bands evolve differently. At  $A_0^{c1} \approx 0.521$ , the spin-down gap closes, changing  $C_\downarrow$  from  $+1$  to  $0$  and driving the system into the Chern insulating phase with  $C = -1$ . At  $A_0^{c2} \approx 0.918$ , the spin-up gap closes, changing  $C_\uparrow$  from  $-1$  to  $0$  and driving the system into the trivial phase with  $C = 0$ .

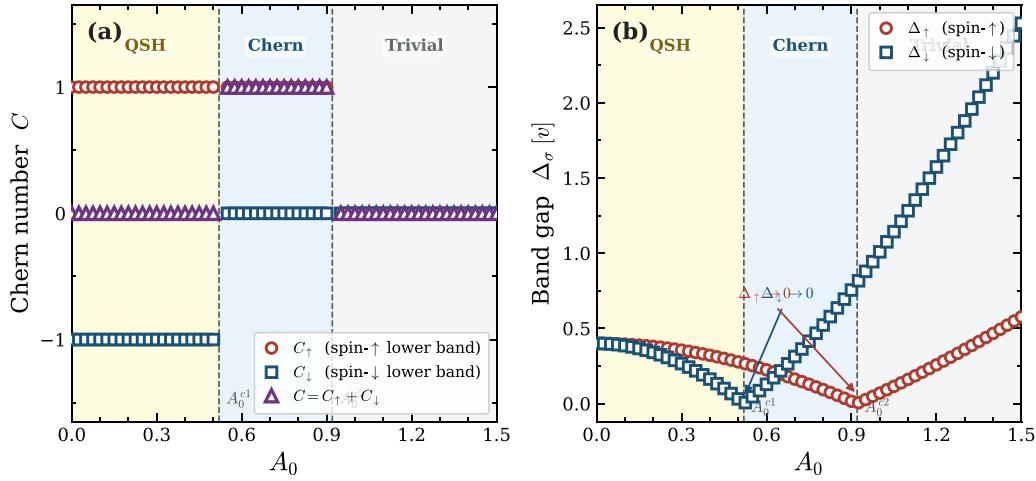


FIG. 2. Topological phase transition (a) Spin Chern numbers  $C_\uparrow$ ,  $C_\downarrow$ , total  $C$  as a function of the  $A_0$ . The Chern number  $C$  in each panel is calculated by integrating the total Berry curvature. (b) Band gap as a function of the  $A_0$ . At two different values of amplitude the band gap vanishes separately for spin-up and spin-down that shows two topological phase transitions. For the trivial AM the Chern number remains  $C = 0$  throughout. For the QSH AM, the total Chern number  $C = C_\uparrow + C_\downarrow$  evolves as  $0 \rightarrow -1 \rightarrow 0$ , reflecting the sequential topological transitions driven by the differential spin-sector renormalization  $j_1 \neq j_2$ . In the QSH phase ( $A_0 = 0.0, 0.3$ ):  $C_\uparrow = -1$ ,  $C_\downarrow = +1$ ,  $C = 0$ . In the intermediate Chern phase ( $A_0 = 0.6$ ):  $C_\uparrow = -1$ ,  $C_\downarrow = 0$ ,  $C = -1$ . In the trivial phase ( $A_0 = 0.9$ ):  $C_\uparrow = 0$ ,  $C_\downarrow = 0$ ,  $C = 0$ .

This sequential gap-closing mechanism arises because the linearly polarized light (LPL) renormalizes the two spin blocks differently due to the condition  $j_1 \neq j_2$ , causing the spin-down gap to vanish first [Fig. 2(b)]. Such a sequence is entirely absent in the antiferromagnetic (AFM) case, where the preserved  $\mathcal{PT}$  symmetry enforces  $\Delta_\uparrow = \Delta_\downarrow$  for all  $A_0$ .

### B. Berry curvature maps and topological phase transitions

For the trivial AM, the Berry curvature distribution shifts and concentrates as  $A_0$  increases, but the Chern number remains  $C = 0$  throughout since no gap closing occurs. The gradual transfer of Berry curvature weight between the  $\Gamma$  and  $X/Y$  regions reflects the LPL induced renormalization of the spin dependent hopping amplitudes,  $j_1 = J_0(A_0) \rightarrow 0$  while  $j_2 = J_0(0) = 1$  is unchanged for  $\theta = 0$ .

For the QSH AM, the initial  $A_0 = 0$  configuration has  $C = 0$  (QSH phase with  $C_\uparrow = -1$ ,  $C_\downarrow = +1$ ). At  $A_0 = 0.6$  the Berry curvature develops a noticeable asymmetry between spin sectors, the Chern number jumps to  $C = +1$ , making the entry into the Chern insulating phase. The occupied Berry curvature, shown in Fig. 3, underlies these behaviors and reflects the sequential topological transitions driven by the differential renormalization of the spin sectors ( $j_1 \neq j_2$ ).

### C. Anomalous Hall conductivity

At zero temperature, the anomalous Hall conductivity shows distinct behavior depending on the altermagnetic phase as shown in Figs. 4(a)–4(c). In the trivial altermagnetic phase [Fig. 4(a)], no topological transition occurs, yet increasing the staggered potential strength  $A_0$  leads to a nontrivial energy dependence as the broken  $C_{4z}\mathcal{T}$  symmetry allows Berry curvature to accumulate. In contrast, the quantum spin Hall altermagnetic phase in Fig. 4(b) shows that as  $A_0$  increases through the first critical value  $A_0^{c1}$ , a quantized plateau

$\sigma_{xy} = +e^2/h$  emerges within the gap region, signaling a Chern insulating phase. Notice that the Berry curvature sharpens near gap closing, and thus the physical quantities show a peak at this location. The conductivity satisfies the exact even symmetry  $\sigma_{xy}(E_F) = \sigma_{xy}(-E_F)$ , as shown in Fig. 4(c) confirmed by a residual root-mean-square error of  $7.1 \times 10^{-17} e^2/h$ , which reflects a particle-hole-like symmetry of the Berry curvature. Finally, tracking the spin-resolved Chern numbers and total  $\sigma_{xy}$  as functions of  $A_0$  unambiguously identifies three phases: quantum spin Hall ( $C = 0$ ), Chern ( $C = -1$ ), and trivial ( $C = 0$ ), with critical amplitudes  $A_0^{c1} = 0.521$  and  $A_0^{c2} = 0.918$  as shown in Fig. 4(d). This unambiguously confirms the phase sequence  $C : 0 \rightarrow -1 \rightarrow 0$  (QSH  $\rightarrow$  Chern  $\rightarrow$  Trivial), consistent with the spin-sector analysis in Sec. III and the convention defined in Eq. (42).

### D. Anomalous nernst conductivity: Thermal activation and Mott scaling

Figure 5 shows the temperature evolution of the anomalous Nernst coefficient (ANC). For the trivial altermagnet [Fig. 5(a)], the ANC vanishes as  $T \rightarrow 0$ , confirming the thermally activated character established analytically in Eq. (29). This vanishing is a direct consequence of  $s(0) = s(1) = 0$ : at zero temperature every state is either completely filled or completely empty, carrying no entropy, so the Berry-curvature-weighted integral in Eq. (26) evaluates to zero identically. No cancellation between Berry curvature contributions of opposite signs is required. At low temperatures, plotting  $\alpha_{xy}/(k_B T)$  in Fig. 5(b) versus Fermi energy reveals convergence to the Mott prediction  $-\frac{\pi^2 k_B}{3e} \frac{\partial \sigma_{xy}}{\partial \varepsilon} \big|_{\varepsilon=\mu}$ , with the root-mean-square deviation decreasing as temperature lowers. Increasing the light amplitude  $A_0$  both enhances the magnitude of the ANC and shifts its peak position, because linearly polarized light modifies the Berry curvature distribution and hence the energy derivative of the anomalous Hall

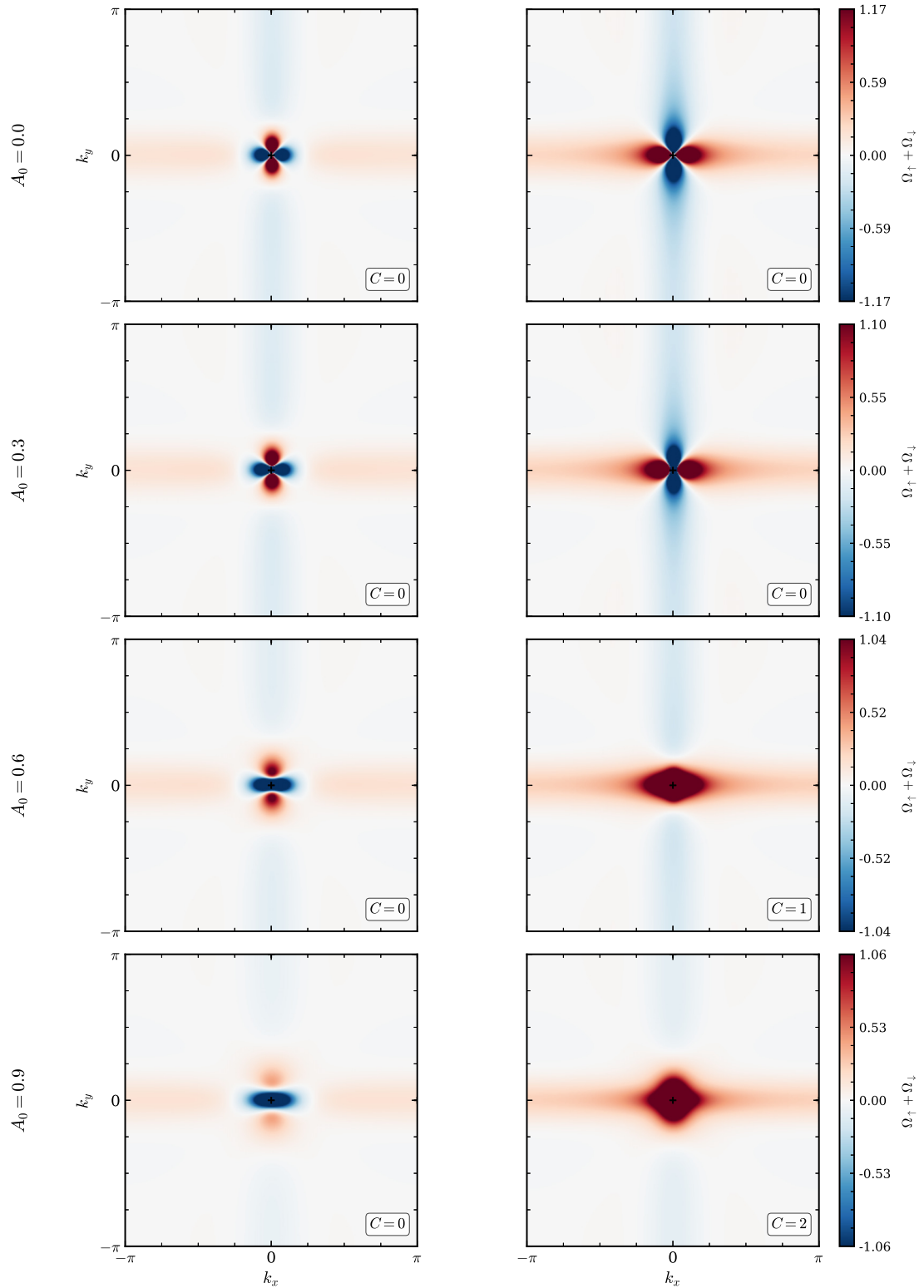


FIG. 3. Total occupied Berry curvature  $\Omega_{\uparrow}^{\dagger}(\mathbf{k}) + \Omega_{\downarrow}^{\dagger}(\mathbf{k})$  across the Brillouin zone. (Left panels) Trivial AM ( $m = 4.2v, b = -v, t_a = \sqrt{3}$ ). (Right panels) QSH AM ( $m = 3.8v, b = -v, t_a = \sqrt{3}$ ). Rows from top to bottom:  $A_0 = 0.0, 0.3, 0.6, 0.9$  at  $\theta = 0$ . The Chern number  $C$  in each panel is calculated by integrating the total Berry curvature. For the trivial AM the Chern number remains  $C = 0$  throughout. For the QSH AM, the total Chern number  $C$  evolves as  $0 \rightarrow -1 \rightarrow 0$ , reflecting the sequential topological transitions driven by the differential spin-sector renormalization  $j_1 \neq j_2$ . The redistribution of Berry curvature weight between the  $\Gamma$  and  $X/Y$  points is visible at each transition.

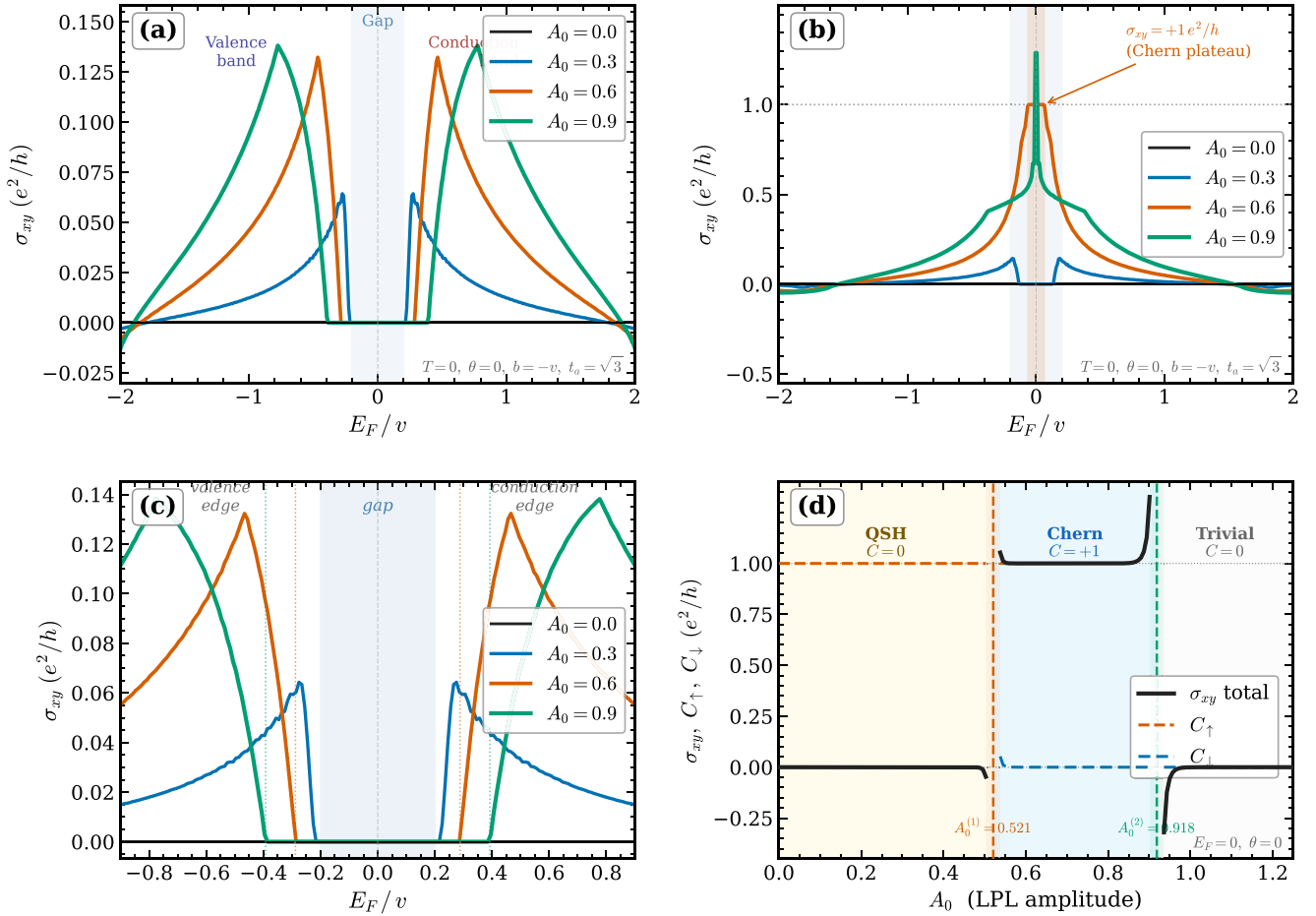


FIG. 4. Anomalous Hall conductivity  $\sigma_{xy}$  at  $T = 0$  as a function of Fermi energy  $E_F/v$  for four LPL amplitudes. Panel (a) shows  $\sigma_{xy}$  for the trivial AM. Although no topological transition occurs,  $\sigma_{xy}$  develops a nontrivial energy dependence with increasing  $A_0$ , because the broken  $C_{4z}T$  symmetry allows Berry curvature to accumulate. Panel (b) shows  $\sigma_{xy}$  for the QSH AM. As  $A_0$  increases through  $A_0^c$ , a quantized plateau  $\sigma_{xy} = +e^2/h$  emerges in the gap region, characteristic of the Chern insulating phase. The dashed line marks the transition value. Panel (c) verifies the exact even symmetry  $\sigma_{xy}(E_F) = \sigma_{xy}(-E_F)$ , with a residual RMS error of  $7.1 \times 10^{-17} e^2/h$ , confirming the particle-hole-like symmetry  $\varepsilon \rightarrow -\varepsilon$  of the Berry curvature in this model. Panel (d) tracks the spin-resolved Chern numbers  $C_\uparrow, C_\downarrow$  and the total  $\sigma_{xy}$  as functions of  $A_0$ , unambiguously identifying the QSH ( $C = 0$ ), Chern ( $C = -1$ ), and trivial ( $C = 0$ ) phases with critical amplitudes  $A_0^{c1} = 0.521$  and  $A_0^{c2} = 0.918$ .

conductivity as shown in Fig. 5(c). The ANC satisfies the exact odd symmetry  $\alpha_{xy}(E_F) = -\alpha_{xy}(-E_F)$  in Fig. 5(d). The temperature dependence of the ANC at fixed Fermi energies for both trivial and quantum spin Hall altermagnets shows a characteristic nonmonotonic behavior: it rises from zero at  $T = 0$ , peaks at a characteristic temperature  $T^*$ , and then decreases at high temperature as thermal broadening averages over Berry curvature features of opposite signs. The peak temperature  $T^*$  is tunable by light intensity; for the trivial altermagnet at a given Fermi energy,  $k_B T^*$  increases with  $A_0$ , while for the quantum spin Hall altermagnet  $T^*$  is even more strongly  $A_0$ -dependent. This tunability provides a direct experimental knob for thermoelectric optimization.

To connect this tunability to physical units, we note that the peak temperature satisfies  $k_B T^* \sim \Delta$  up to an order-unity prefactor, where  $\Delta$  is the relevant spin-resolved gap. For a representative hopping scale  $v \sim 0.1\text{--}0.5$  eV, gaps of order  $\Delta \sim 0.01\text{--}0.1v$  correspond to  $T^* \sim 10\text{--}60$  K, placing the predicted Nernst peak well within the range of standard cryogenic transport measurements. Closer to the critical amplitudes  $A_0^{c1}$

and  $A_0^{c2}$ , where  $\Delta$  becomes very small (see discussion below),  $T^*$  correspondingly decreases, and the Nernst peak shifts to sub-Kelvin or few-Kelvin temperatures.

The spin Nernst conductivity  $\alpha_{xy}^s$  and spin thermal Hall conductivity  $\kappa_{xy}^{(0),s}$  are computed with the corrected prefactors  $k_B/2$  and  $k_B^2 T/\hbar$ , respectively, as detailed in Sec. IV F. The qualitative features, including the nodal structure, sign reversal under polarization-angle rotation, and the nonzero spin response in the QSH phase, remain unchanged. Only the quantitative magnitudes of the spin-resolved coefficients are affected.

### E. Temperature and chemical potential dependence

Figure 6 shows all three physical conductivities which share an identical nodal structure with zeros at  $\theta = \pm\pi/4$  (where  $\mathcal{M}_{xy}$  symmetry is restored) and extrema near  $\theta = 0, \pm\pi/2$ . The sign reversal under  $\theta \rightarrow -\theta$  is visible, and the magnitude scales with  $A_0$ . Furthermore, the sign of all three coefficients can be controlled by rotating the polarization

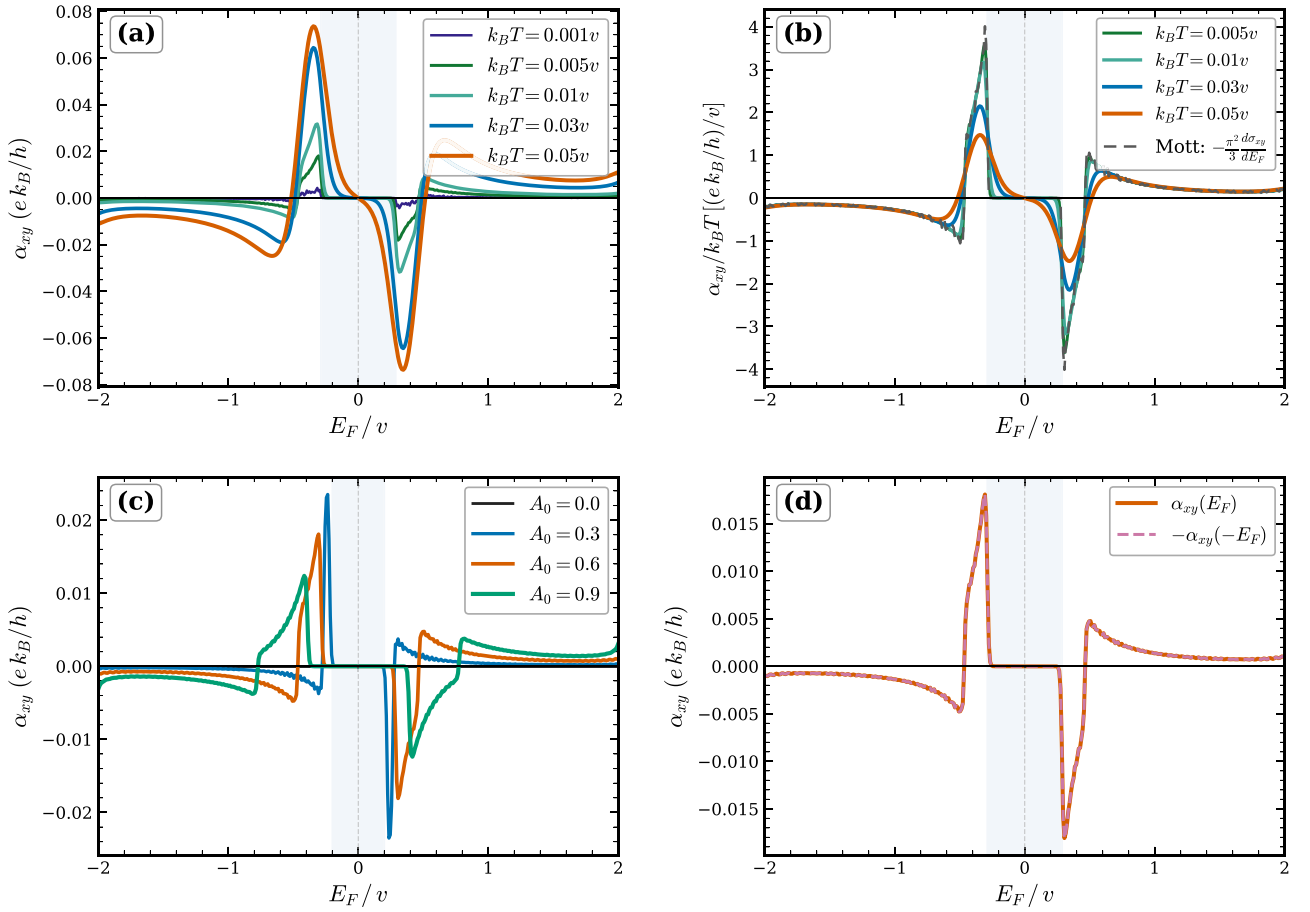


FIG. 5. Anomalous Nernst conductivity  $\alpha_{xy}$  as a function of Fermi energy. In all panels the trivial AM is considered by given  $m = 4.2v$ ,  $b = -v$ ,  $t_a = \sqrt{3}$ ,  $\theta = 0$ . (a) at  $A_0 = 0.6$  for six temperatures, demonstrating the thermally activated nature ( $\alpha_{xy} \rightarrow 0$  as  $T \rightarrow 0$ ). (b) Scaled quantity  $\alpha_{xy}/(k_B T)$  as a function of  $E_F$  at  $A_0 = 0.6$ ; the Mott prediction  $-(\pi^2 k_B/3e) \partial \sigma_{xy}/\partial \varepsilon|_{\mu}$  (dashed) is approached as  $T \rightarrow 0$ . (c) at  $k_B T = 0.005v$  for four LPL amplitudes, showing the evolution of the Nernst response with light intensity. (d) Odd symmetry  $\alpha_{xy}(E_F) = -\alpha_{xy}(-E_F)$  verified for  $A_0 = 0.6$ ; Notice that a RMS error is  $5.8 \times 10^{-17} e k_B/h$ . All spin-resolved quantities shown here have been recomputed using the dimensionally corrected prefactors  $k_B/2$  for  $\alpha_{xy}^s$  and  $k_B^2 T/\hbar$  for  $\kappa_{xy}^{(0),s}$ , following the corrections described in Sec. IV F.

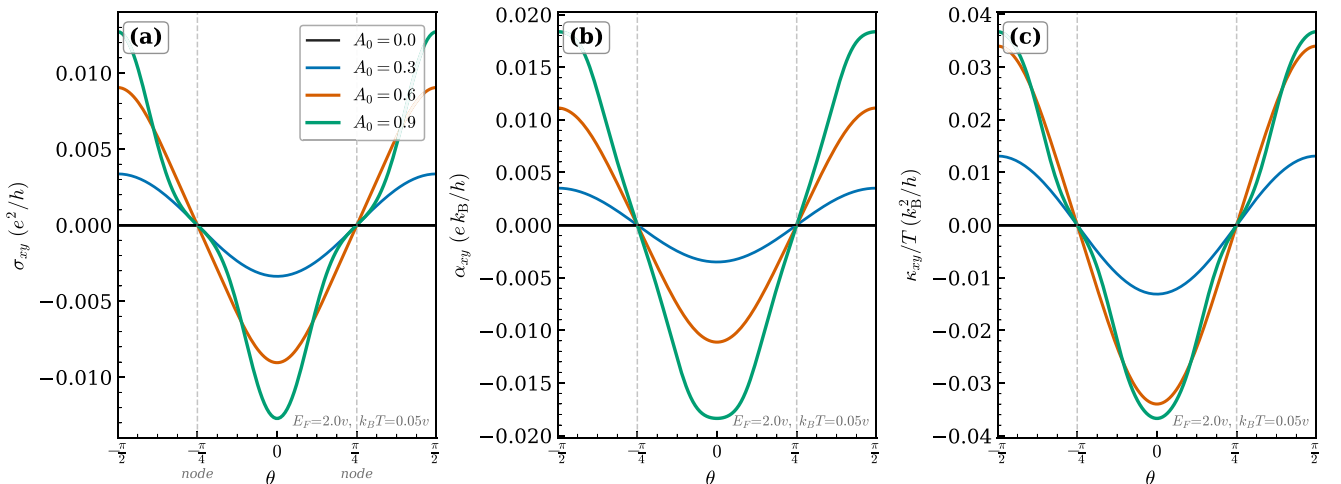


FIG. 6. Physical conductivities as a function of the Polarization angle  $\theta$ . All three conductivities (a)–(c) show a  $d$ -wave nature under LPL that is a remarkable result and a fingerprint for AM.

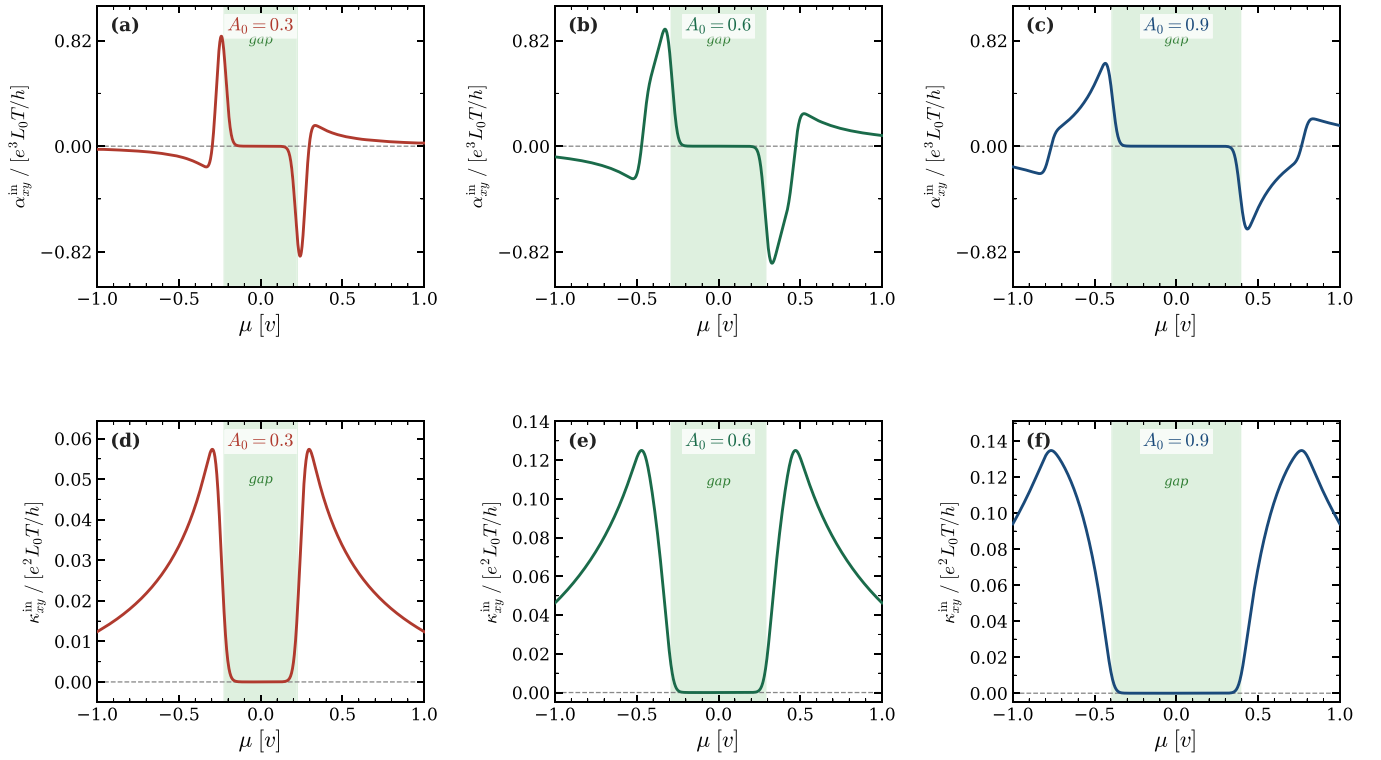


FIG. 7. For trivial phase intrinsic Nernst conductivity  $\alpha_{xy}^{\text{in}}$  (top row, panels (a)–(c)) and thermal Hall conductivity  $\kappa_{xy}^{\text{in}}$  (bottom row, panels (d)–(f)) as functions of chemical potential  $\mu$  for the trivial altermagnetic insulator ( $m = 4.2v$ ,  $b = -v$ ,  $t_a = \sqrt{3}$ ,  $\theta = 0$ ) at three LPL amplitudes:  $A_0 = 0.3$  (red),  $A_0 = 0.6$  (dark green), and  $A_0 = 0.9$  (blue). The green shaded region marks the insulating gap in each panel. All quantities are normalized by  $e^3 L_0 T / h$  for the Nernst conductivity and  $e^2 L_0 T / h$  for the thermal Hall conductivity, where  $L_0 = \pi^2 k_B^2 / 3e^2$  is the Lorenz number.

direction, providing a purely optical route to reversing the thermal Hall response a significant advantage over approaches requiring magnetic field reversal. Importantly, this entire polarization pattern is absent in the conventional AFM, where  $\sigma_{xy} = \alpha_{xy} = \kappa_{xy}^{(0)} = 0$  for all  $\theta$ , making the angle-dependent thermal transport a definitive experimental fingerprint of altermagnetism.

#### F. Intrinsic Nernst and thermal Hall response in the trivial and QSH AM insulators

Figure 7 shows the intrinsic Nernst  $\alpha_{xy}^{\text{in}}$  and thermal Hall  $\kappa_{xy}^{\text{in}}$  coefficients as functions of chemical potential for the trivial AM insulator at three LPL amplitudes. The Nernst coefficient [Figs. 7(a)–7(c)] exhibits a characteristic double-peak structure with opposite signs across the gap, whose magnitude grows with  $A_0$ . The thermal Hall coefficient [Figs. 7(d)–7(f)] shows a single negative peak inside the gap that deepens as  $A_0$  increases. At  $A_0 = 0.9$  the peak value reaches approximately  $-0.14 e^2 k_B T / h$ , about 2.3 times larger than at  $A_0 = 0.3$ . The insulating gap is highlighted in green; within it,  $\alpha_{xy}^{\text{in}} \rightarrow 0$  as  $T \rightarrow 0$  (consistent with our analytical prediction) while  $\kappa_{xy}^{\text{in}}$  remains finite, tracking the WF prediction.

Figure 8 shows the same two coefficients for the QSH altermagnet ( $m = 3.8v$ ,  $b = -v$ ,  $t_a = \sqrt{3}$ ,  $\theta = 0$ ) at the same three amplitudes. Because this parameter set is closer to the phase boundary, the gaps are smaller and the Berry curvature is more sharply peaked near the gap-closing momenta. The Nernst

coefficient in panels 8(a–c) keeps the bipolar lineshape as in Fig. 7, but its scale grows from  $\approx 0.82$  to  $\approx 3.97 e^3 L_0 T / h$ . The shape also changes with amplitude: at  $A_0 = 0.3$  the two lobes are narrow and nearly antisymmetric about  $\mu = 0$ , whereas at  $A_0 = 0.6$ , once the system has entered the Chern regime, a deep negative trough opens below the gap and a sharp positive edge above it.

The thermal Hall coefficient in panels Figs. 8(d)–8(f) departs more strongly from the trivial case. In Figs. 7(d)–7(f) it is suppressed inside the gap and peaks only at the band edges. In QSH phase, at  $A_0 = 0.6$  and  $0.9$ , it peaks at the centre of the gap and is an order of magnitude larger than at  $A_0 = 0.3$ . This is the thermal signature of the light-induced Chern phase. Once one spin sector has passed through its gap closing, the surviving inverted sector leaves an uncompensated Berry curvature, and the thermal Hall response follows the resulting nonzero  $\sigma_{xy}$ . Together, Figs. 7 and 8 show that the thermal channel separates the trivial from the topological altermagnetic insulator even when the chemical potential lies deep inside the gap, where charge transport alone is not providing complete information.

#### G. Transport signatures across all topological phases

We commence by examining the band structure induced by LPL as a foundation for understanding the transport signatures. Figure 9 shows the spin-resolved band structures and the corresponding constant energy contours of the LPL

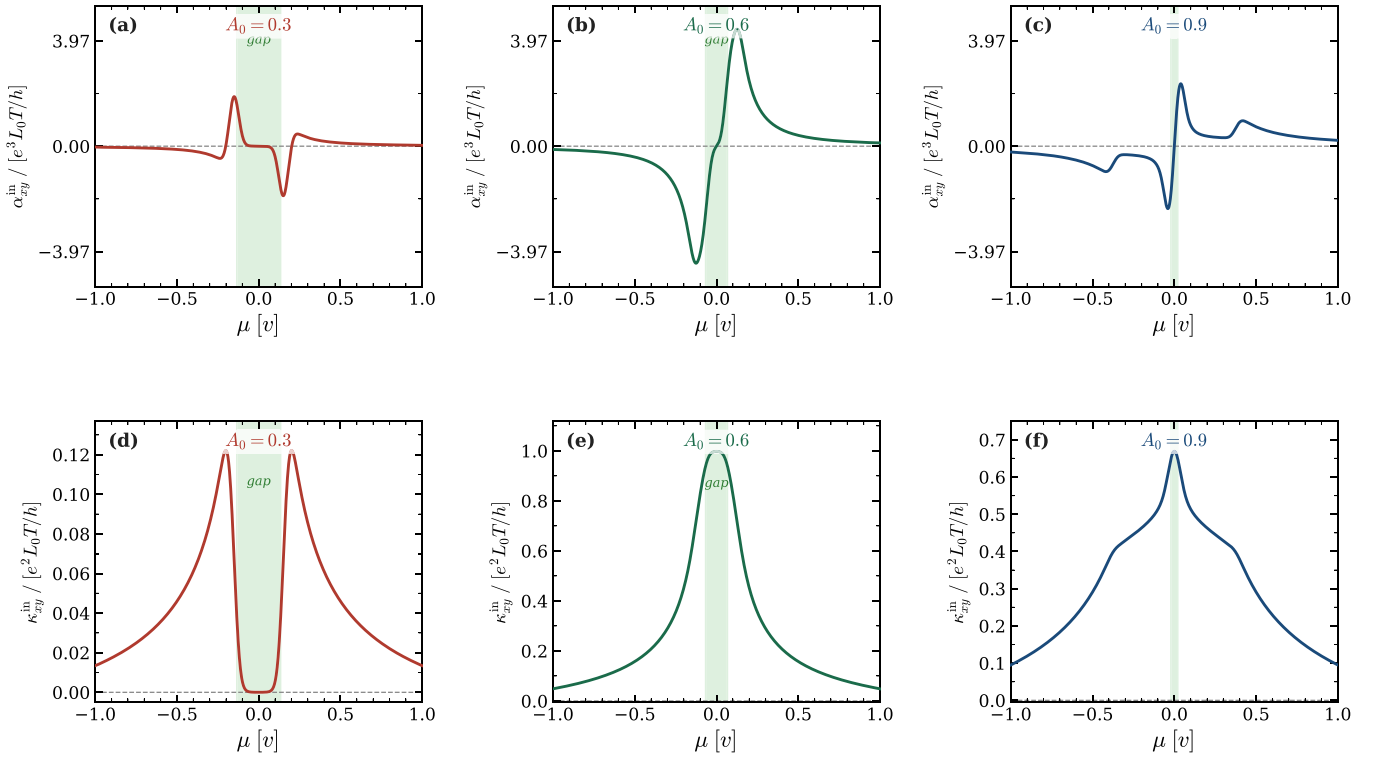


FIG. 8. For QSH phase Intrinsic Nernst conductivity  $\alpha_{xy}^{\text{in}}$  (top row, panels (a)–(c)) and thermal Hall conductivity  $\kappa_{xy}^{\text{in}}$  (bottom row, panels (d)–(f)) as functions of chemical potential  $\mu$  for the QSH altermagnetic insulator ( $m = 3.8v$ ,  $b = -v$ ,  $t_a = \sqrt{3}$ ,  $\theta = 0$ ) at three LPL amplitudes:  $A_0 = 0.3$  (red),  $A_0 = 0.6$  (dark green), and  $A_0 = 0.9$  (blue). The green shaded region marks the insulating gap in each panel. All quantities are normalized by  $e^3 L_0 T / h$  for the Nernst conductivity and  $e^2 L_0 T / h$  for the thermal Hall conductivity, where  $L_0 = \pi^2 k_B^2 / 3e^2$  is the Lorenz number.

driven  $d$ -wave altermagnetic quantum spin Hall system. Solid curves represent the spin-up sector, while dashed curves correspond to the spin-down sector. The parameters used are altermagnetic parameters and these are fixed in all phases,  $m = 3.8v$ ,  $b = -v$ ,  $t_a = \sqrt{3}$  for these specified values of AM parameters the system is in equilibrium and satisfies  $|m| < |b|(1 + t_a^2)$ . Since  $|b|(1 + t_a^2) = 4v$ , the undriven or weakly driven system sits comfortably within the quantum spin Hall regime.

Figure 9(a) presents the band structure in the quantum spin Hall phase. The bulk gap remains open throughout this regime, as the inset makes clear, yet the valence and conduction bands are inverted relative to the trivial case. The spin-up and spin-down sectors carry opposite Chern numbers,  $C_\uparrow = -1$  and  $C_\downarrow = +1$  so the total Chern number vanishes, while the spin Chern number remains finite,  $C_s \sim C_\uparrow - C_\downarrow \neq 0$ . This confirms that the system is a QSH insulator rather than a trivial one. The open gap (visible in the inset) signals

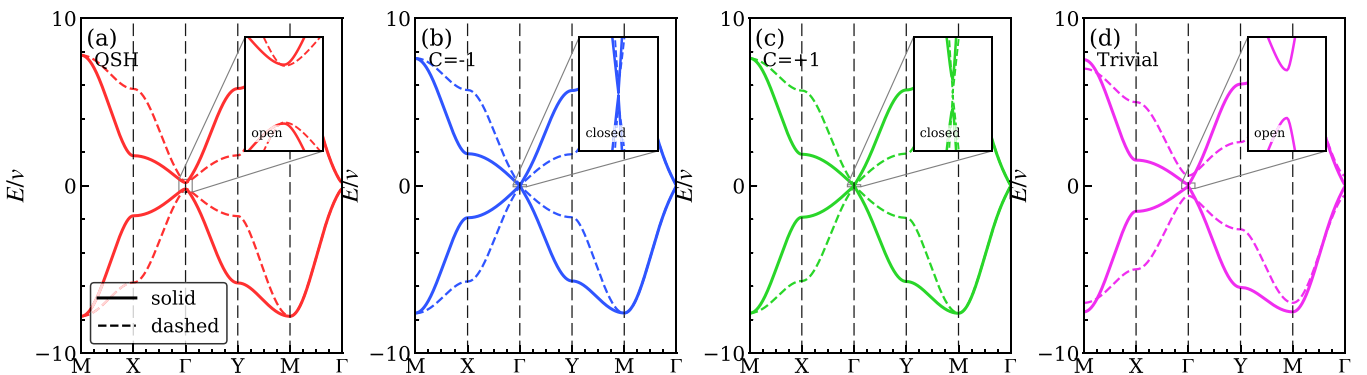


FIG. 9. Band structures and topological phase diagram of the QSH AM under LPL. (a)–(d) Spin-resolved band structure of the QSH AM ( $m = 3.8v$ ,  $b = -v$ ,  $t_a = \sqrt{3}$ ) at  $A_0 = 0.0, 0.3, 0.7, 1.0$  ( $\theta = 0$ ). Spin-up (solid teal) and spin-down (dashed orange) bands evolve differently; the spin-down gap closes at  $A_0^{c1}$  producing the Chern phase, while the spin-up gap closes at  $A_0^{c2}$ . Insets show  $\Delta_\uparrow$  and  $\Delta_\downarrow$ . Lower panels: Spin Chern numbers  $C_\uparrow$ ,  $C_\downarrow$ , total  $C$ , and spin Chern number  $C_s = (C_\uparrow - C_\downarrow)/2$  vs  $A_0$ .

insulating behavior, and the band inversion signals the underlying nontrivial topology.

Figure 9(b) shows the first light-induced Chern insulating phase, which carries a total Chern number  $C = -1$ . With increase in light amplitude  $A_0$ , the Bessel-renormalized hopping terms redirect the effective masses of the two spin sectors in different directions. Consequently, one spin gap closes and reopens before the other. Here the spin-down sector turns topologically trivial first, while the spin-up sector remains inverted,  $C_\uparrow = -1$  and  $C_\downarrow = 0$  that results in total Chern number is  $-1$ .

The inset captures the local gap closing associated with this topological phase transition that is a necessary feature, since a change in Chern number can occur only when the bulk gap closes and then reopens. The result is a spin-polarized Chern insulator whose nonzero Chern number originates predominantly from a single spin channel.

Figure 9(c) illustrates the complementary spin-polarized Chern phase with total Chern number  $C = +1$ . Now it is the other spin sector that remains topologically active, while the first has already become trivial,  $C_\uparrow = 0$  and  $C_\downarrow = +1$  so that the total Chern number is  $+1$ .

The sign of the light-induced Chern number can be controlled through the polarization direction combined with the anisotropic Floquet renormalization. By adjusting  $\theta$ , one can access either  $C = -1$  or  $C = +1$  intermediate Chern phases depending on which spin sector closes its gap first. This tunability is a distinguishing feature of the altermagnetic system.

Figure 9(d) shows the trivial insulating phase reached at larger light amplitude. Once the remaining inverted spin sector also undergoes its own gap closing and reopening, both sectors become topologically trivial,  $C_\uparrow = 0$ ,  $C_\downarrow = 0$

Although the inset confirms that the bulk gap is still open, the phase has lost its topological character entirely because the band inversion is gone. This panel therefore represents an ordinary trivial insulator, and the full sequence of phase transitions reads

$$\text{QSH } (C = 0) \longrightarrow \text{Chern } (C = -1) \longrightarrow \text{Trivial } (C = 0). \quad (44)$$

A particularly illuminating way to present our results is to compare the three anomalous transport coefficients; the AHC  $\sigma_{xy}$ , ANC  $\alpha_{xy}$ , and ATHC  $\kappa_{xy}^{(0)}/T$  across all four distinct topological phases that the altermagnetic system can have under LPL irradiation: the QSH phase ( $C = 0$ ), the two Chern insulating phases ( $C = -1$  and  $C = +1$ ), and the trivial phase ( $C = 0$ ). This phase comparison reveals how each transport coefficient encodes a distinct topological fingerprint and how the Berry curvature redistribution in the Brillouin zone manifests differently in charge, thermoelectric, and thermal channels.

Figure 10 presents  $\sigma_{xy}$  as a function of Fermi energy  $E_F$  for representative parameter choices in each of the four topological phases. The top panel overlays all four phases on a common axis, making the contrast immediately apparent. The QSH phase [Fig. 10(a)] displays  $\sigma_{xy} \approx 0$  throughout the gap region, as expected for a  $\mathcal{PT}$ -broken but  $C_{4z}\mathcal{T}$ -symmetric state where  $C_\uparrow = -C_\downarrow$  and the Berry

curvatures of the two spin sectors cancel. Outside the gap,  $\sigma_{xy}$  acquires small but nonzero values as the Fermi level probes bands with unequal spin resolved Berry curvature densities.

The two Chern insulating phases shows different behavior. For the  $C = -1$  phase [Fig. 10(b)] ( $A_0 = 0.625$ ,  $\theta = 0.829$  rad,  $\Delta = 0.002v$ ),  $\sigma_{xy}$  develops a sharp negative dip near  $E_F = 0$ , approaching  $-e^2/h$  as the Fermi level sits in the tiny gap. For the  $C = +1$  phase [Fig. 10(c)] ( $A_0 = 0.625$ ,  $\theta = 0.742$  rad,  $\Delta = 0.002v$ ), the sign is reversed and  $\sigma_{xy}$  rises sharply toward  $+e^2/h$  within the gap. The near quantized values in Figs. 10(b) and 10(c) reflect the fact that  $\sigma_{xy} = Ce^2/h$  in the low-temperature insulating limit, with the sign controlled entirely by the polarization angle  $\theta$ . The ability to switch between  $C = -1$  and  $C = +1$  by rotating the polarization direction, without any change in material or magnetic field is one of the most striking features of Floquet engineered altermagnetism.

The trivial phase Fig. 10(d)] ( $A_0 = 1.075$ ,  $\theta = 0$ ,  $\Delta = 0.137v$ ) shows a broad, positive  $\sigma_{xy}$  that peaks near the band edges and decays away from the gap. This finite but non quantized response arises from the residual Berry curvature of the trivially gapped bands and has no topological protection. The contrast between the sharp, sign changing  $\sigma_{xy}$  in the Chern phases and the broad, featureless response in the trivial phase provides a clear experimental discriminator between topological and nontopological regimes.

Figure 11 shows  $\alpha_{xy}$  at  $k_B T = 0.01v$  for the same four phases. Since  $\alpha_{xy}(T = 0) \equiv 0$  in any gapped phase, all curves are shown at a small but finite temperature to reveal the thermally activated structure.

The QSH phase [Fig. 11(a)] displays a very small  $\alpha_{xy}$  with a peak value of only  $+0.015 ek_B/h$ , concentrated near the gap edges. This small magnitude reflects the nearly complete cancellation of Berry curvature contributions from the two spin sectors in the QSH state: the Nernst response is thermally activated by band edge excitations across a relatively large gap  $\Delta = 0.377v$ , and the partial cancellation suppresses the signal further.

The Chern phases exhibit dramatically enhanced Nernst responses. For  $C = -1$  [Fig. 11(b)]  $\alpha_{xy}$  develops a pronounced bipolar structure near  $E_F = 0$ , with a positive peak of  $+1.882 ek_B/h$  and a negative trough of comparable magnitude, separated by the tiny gap  $\Delta = 0.002v$ . The  $C = +1$  phase, Fig. 11(c) shows the mirror image of this structure, with a peak of  $+1.883 ek_B/h$  of opposite internal asymmetry. These large values, more than 100 times the QSH response arise because the extremely small gaps in the Chern phases ( $\Delta \approx 0.002v$ ) allow thermal excitation across the gap even at  $k_B T = 0.01v$ , and the Berry curvature is sharply concentrated near the gap closing  $\mathbf{k}$  points. The Nernst response is therefore a far more sensitive probe of topological gap size than the Hall conductivity: while  $\sigma_{xy}$  approaches  $\pm e^2/h$  in both Chern phases,  $\alpha_{xy}$  distinguishes them by its magnitude and internal line shape.

The trivial phase, Fig. 11(d) gives a small, broad Nernst response with peak  $+0.102 ek_B/h$ , confined to the band edges and showing no sign change within the gap. This is qualitatively different from both Chern phases and provides a clear thermal fingerprint of the trivial insulating state.

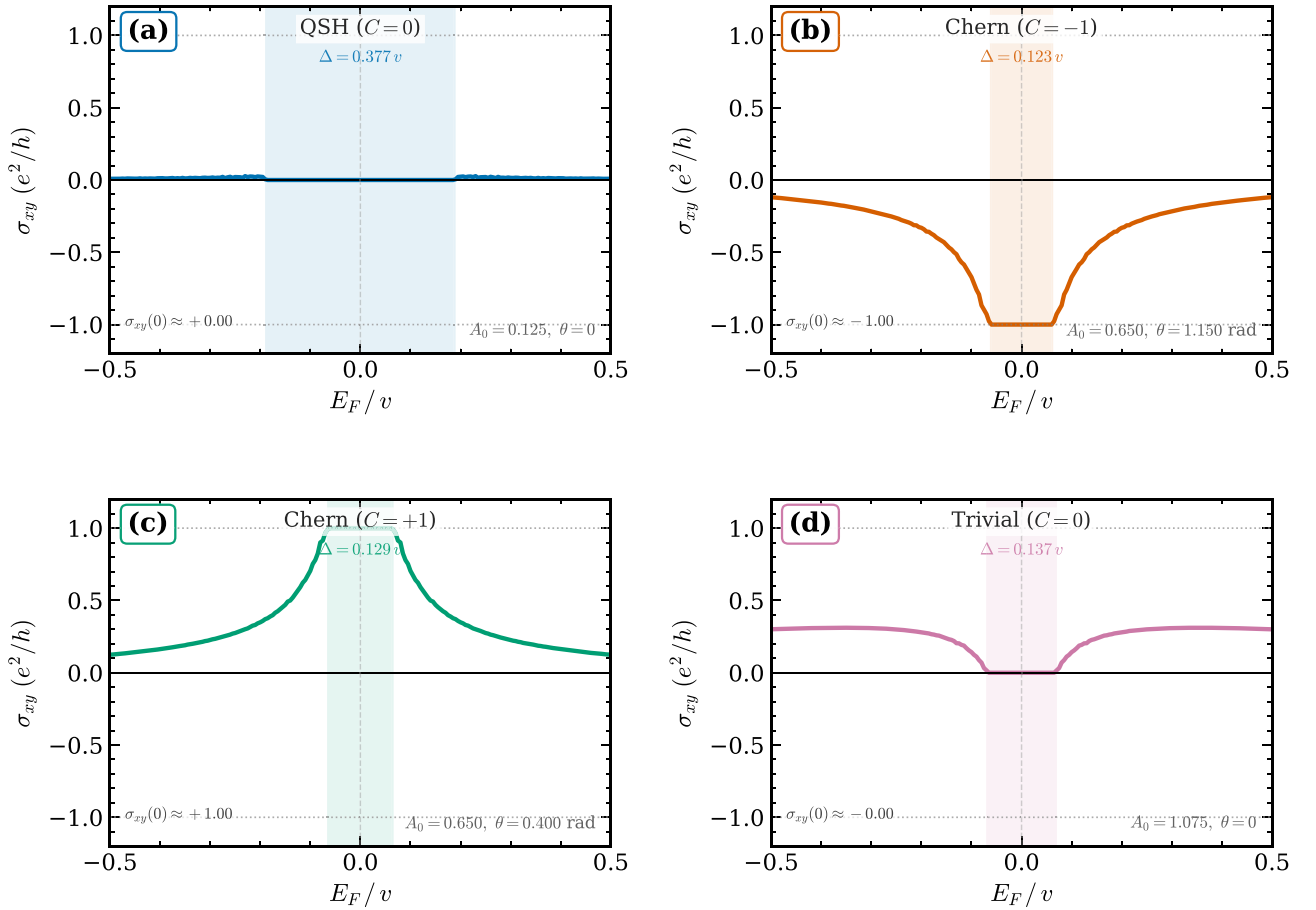


FIG. 10. Anomalous Hall conductivity  $\sigma_{xy}$  across all four topological phases. Top panel: Overlay of all four phases as a function of Fermi energy  $E_F/v$ . Dotted horizontal lines mark  $\pm e^2/h$ . Individual panels: (a) QSH phase ( $C = 0$ ,  $A_0 = 0.125$ ,  $\theta = 0$ ,  $\Delta = 0.377v$ ); (b) Chern phase ( $C = -1$ ,  $A_0 = 0.625$ ,  $\theta = 0.829$  rad,  $\Delta = 0.002v$ ); (c) Chern phase ( $C = +1$ ,  $A_0 = 0.625$ ,  $\theta = 0.742$  rad,  $\Delta = 0.002v$ ); (d) trivial phase ( $C = 0$ ,  $A_0 = 1.075$ ,  $\theta = 0$ ,  $\Delta = 0.137v$ ). Blue shading marks the insulating gap in each panel. The Chern phases develop near-quantized values  $\sigma_{xy} \rightarrow Ce^2/h$  in the gap, while the QSH and trivial phases show vanishing and nonquantized responses, respectively.

The parameter choices in Figs. 10(b) and 10(c) to 12(b) and 12(c) correspond to a gap  $\Delta \approx 0.002v$ , which was selected to demonstrate the near-quantization of  $\kappa_{xy}^{(0)}/T$  and the maximal enhancement of  $\alpha_{xy}$  in the Chern phase. For  $v \sim 0.1$ – $0.5$  eV, this gap corresponds to  $\Delta \sim 0.2$ – $1$  meV, or  $T \sim 2$ – $12$  K, which is comparable to or smaller than typical thermal broadening and disorder scales in real materials, and may be difficult to resolve in practice.

However, the qualitative features highlighted in this work, the enhancement of  $\alpha_{xy}$  in the Chern phase relative to the QSH phase by more than two orders of magnitude, the sign reversal of all three coefficients under polarization rotation, and the approach of  $\kappa_{xy}^{(0)}/T$  toward  $\pi^2 k_B^2/3h$ , persist for any gap within the Chern phase, not only at the fine-tuned point  $\Delta \approx 0.002v$ . Moving away from the immediate vicinity of  $A_0^1$  and  $A_0^2$  while remaining in the Chern phase increases  $\Delta$  to  $\sim 0.01$ – $0.1v$  (i.e., 1–50 meV, or  $T \sim 10$ – $60$  K), at which point thermal broadening and disorder are not expected to wash out the enhanced Nernst signal or the near-quantized thermal Hall plateau, since these features reflect the overall magnitude of the gap and the associated Berry curvature concentration rather than a fine-tuned cancellation at  $\Delta \rightarrow 0$ . We have added this discussion to clarify that the dramatic

enhancements reported here are not an artifact of an experimentally inaccessible fine-tuned point, though the sharpest, most quantized features are expected closer to the critical amplitudes and correspondingly lower temperatures.

Figure 12 presents  $\kappa_{xy}^{(0)}/T$  at  $T = 0$  for all four phases. The top overlay panel and the four individual panels confirm the Wiedemann Franz relation  $\kappa_{xy}^{(0)}/T|_{T=0} = (\pi^2/3)\sigma_{xy}$  throughout.

In the QSH phase [Fig. 12(a)]  $\kappa_{xy}^{(0)}/T$  is nearly zero in the gap and shows a small positive peak of  $+0.080 k_B^2/h$  near the band edge. The flatness in the gap is consistent with  $\sigma_{xy} \approx 0$  there and the WF law.

In the  $C = -1$  phase [Fig. 12(b)]  $\kappa_{xy}^{(0)}/T$  develops a negative peak of  $-0.025 k_B^2/h$  at  $E_F = 0$ , tracking the negative  $\sigma_{xy}$  of this phase. By contrast, the  $C = +1$  phase has a sharp positive peak of  $+1.913 k_B^2/h$ , which is close to the quantized value  $\pi^2 k_B^2/3h \approx \pi^2/3 k_B^2/h$  expected for a  $C = +1$  Chern insulator, panel Fig. 12(c), at  $T \rightarrow 0$ . The near quantization here is particularly significant: it demonstrates that the LPL induced Chern insulating phase carries a genuine topological thermal Hall response satisfying the anomalous WF law, placing it on the same place as magnetically ordered quantum anomalous Hall insulators.

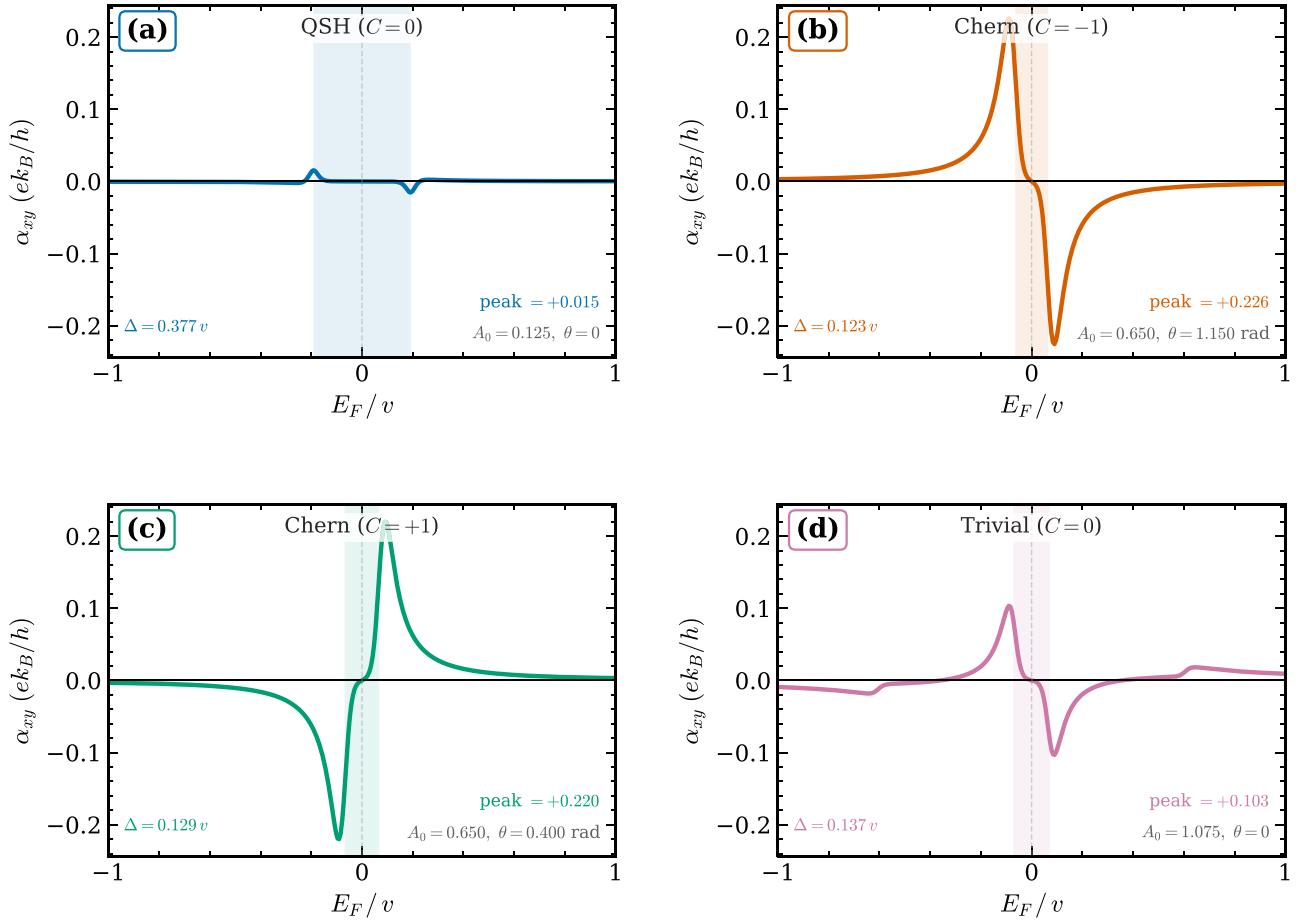


FIG. 11. Anomalous Nernst conductivity  $\alpha_{xy}$  across all four topological phases at  $k_B T = 0.01v$ . Top panel: Overlay of all four phases. Individual panels with the same parameter choices as Fig. 10. Notice that the dramatic enhancement in the Chern phases (b), (c): peak values of  $\approx 1.88 ek_B/h$  compared to  $0.015 ek_B/h$  in the QSH phase (a), arising from the exponential sensitivity of  $\alpha_{xy}$  to the small gap  $\Delta \approx 0.002v$ . The Nernst response provides a far more sensitive probe of topological gap size than the Hall conductivity.

The trivial phase [Fig. 12(d)] shows a broad positive  $\kappa_{xy}^{(0)}/T$  with peak  $+1.023 k_B^2/h$ , again showing its non quantized  $\sigma_{xy}$  through the WF law. Comparing all, the most striking result is the sign structure:  $\kappa_{xy}^{(0)}/T$  is positive for the QSH,  $C = +1$ , and trivial phases, but negative for the  $C = -1$  phase. This sign change is a direct consequence of the sign of the Chern number and provides an unambiguous thermal Hall signature of the different topological phases. The results reveal a clear hierarchy of sensitivity among the three transport coefficients. The AHC  $\sigma_{xy}$  is the most direct topological probe, approaching quantized values in the Chern phases and vanishing in the QSH gap. The ATHC  $\kappa_{xy}^{(0)}/T$  mirrors  $\sigma_{xy}$  through the WF law, offering a thermal channel to the same topological information. The ANC  $\alpha_{xy}$ , however, goes beyond this: it is exponentially sensitive to the gap size and provides a quantitative measure of how close the system is to a phase boundary. The enhancement of  $\alpha_{xy}$  in the near critical Chern phases compared to the QSH phase makes the Nernst effect a natural experimental tool for detecting topological phase transitions driven by LPL intensity or polarization angle.

From an experimental perspective, the four transport signatures described here are all accessible in a single device geometry. The Hall conductivity  $\sigma_{xy}$  can be measured by

transverse voltage in a Hall bar geometry under LPL irradiation. The Nernst coefficient  $\alpha_{xy}$  requires an additional temperature gradient, achievable through laser heating of one contact. The thermal Hall conductivity  $\kappa_{xy}^{(0)}$  is accessed by the transverse temperature difference under longitudinal heat current. The fact that all three quantities share the same polarization angle dependence vanishing at  $\theta = \pm\pi/4$  and reversing sign under  $\theta \rightarrow \pi/2 - \theta$  means that a single angle rotation measurement can confirm the altermagnetic origin of all observed responses simultaneously.

Table I summarizes the contrasting behavior of the AM and AFM under LPL irradiation across all transport and topological observables. The central distinction is that LPL breaks  $C_4\mathcal{T}$  in the AM but cannot break  $\mathcal{PT}$  in the AFM. This propagates to every observable: the AFM has  $\sigma_{xy} = \alpha_{xy} = \kappa_{xy}^{(0)} = \sigma_{xy}^s = \alpha_{xy}^s = \kappa_{xy}^{(0),s} = 0$  for all  $A_0$ ,  $\theta$ , and  $T$ . Only the QSH  $\rightarrow$  Trivial transition (direct, no intermediate Chern phase) is possible in the AFM.

## VI. MEASUREMENT OF THE ANOMALOUS TRANSPORT COEFFICIENTS IN EXPERIMENTS

Altermagnetic order and tunable band topology, the two essential ingredients of our proposal, are currently being

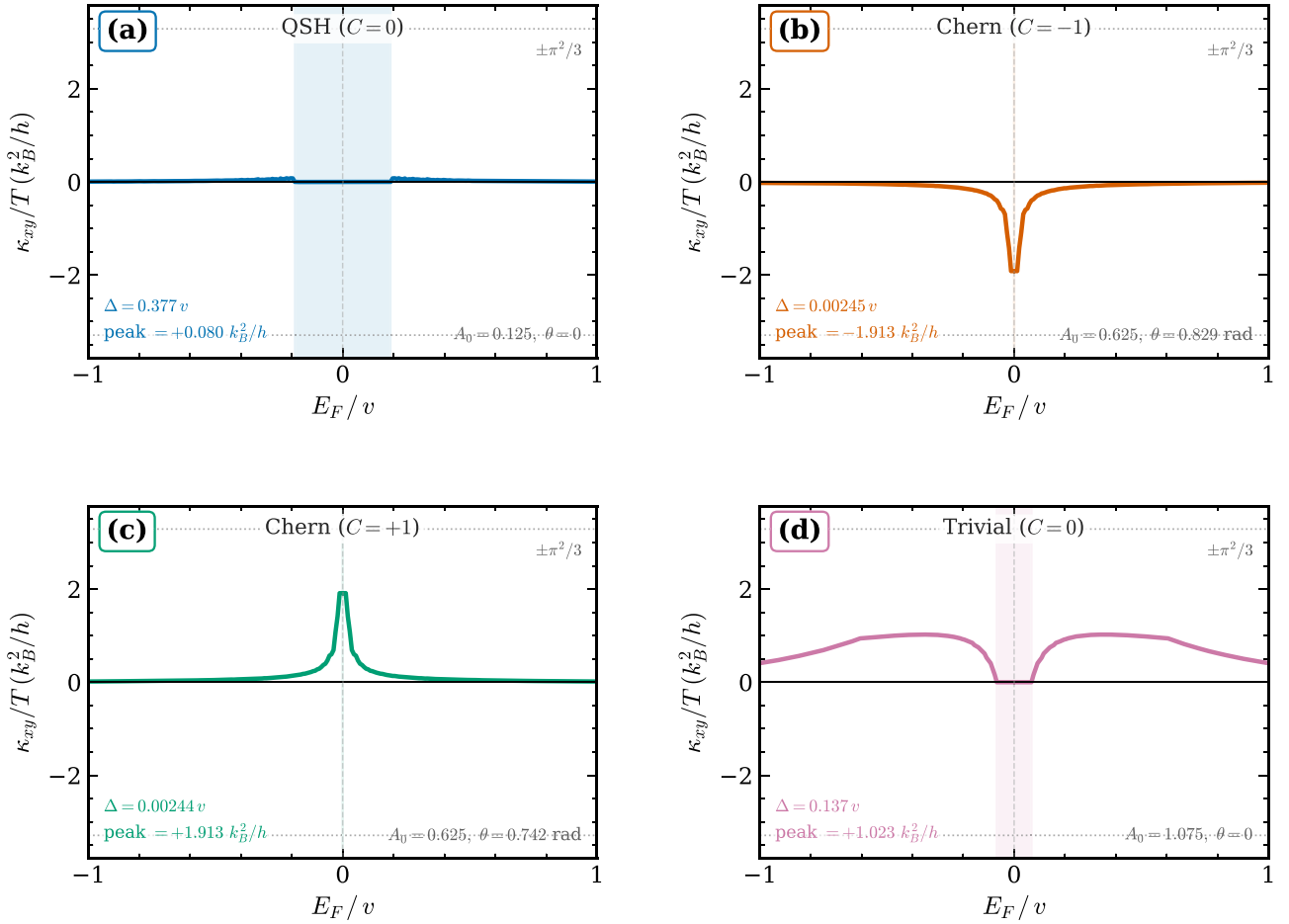


FIG. 12. Anomalous thermal Hall conductivity  $\kappa_{xy}^{(0)}/T$  at  $T = 0$  across all four topological phases. Top panel: Overlay of all four phases; dotted lines at  $\pm\pi^2/3 k_B^2/h$  mark the WF-predicted quantized values. Individual panels (a)–(d) use the same parameters as Fig. 10. The  $C = +1$  Chern phase (c) shows near-quantization  $\kappa_{xy}^{(0)}/T \approx +1.913 k_B^2/h \approx \pi^2/3 k_B^2/h$ , confirming the anomalous Wiedemann-Franz law  $\kappa_{xy}^{(0)} = -L_0 T \sigma_{xy}$ . The sign of  $\kappa_{xy}^{(0)}/T$  tracks the sign of  $C$ , providing an unambiguous thermal Hall fingerprint of each topological phase. Panel (d) is the trivial phase, which shows a broad, non-quantized positive response peaking at  $+1.023 k_B^2/h$  outside the gap and no plateau at the quantized value.

actively explored in several material platforms. We discuss the most promising candidates and the conditions under which our theoretical predictions may be experimentally accessible.

**MnTe-based systems.** MnTe is one of the most studied altermagnetic candidates, with spin-resolved band splitting confirmed by angle-resolved photoemission spectroscopy [13]. Thin-film growth and heterostructure engineering with appropriate substrate-induced strain could drive band inversion in this system. The resulting altermagnetic QSH phase would then be an ideal platform for the Floquet engineering studied here.

**RuO<sub>2</sub>-based thin films.** RuO<sub>2</sub> realizes  $d$ -wave spin splitting on a square lattice [6], directly matching the symmetry of our model Hamiltonian. Epitaxial strain and substrate engineering provide knobs to tune the effective  $m/b$  ratio toward the topological regime. Mid-infrared laser irradiation of such films would then allow the light-controlled topological phase transitions predicted here to be tested.

**Artificial platforms.** Moiré superlattices and cold-atom optical lattices with square-lattice geometry allow direct engineering of all hopping parameters  $v$ ,  $b$ , and  $t_a$  in the

model Hamiltonian. These platforms can be straightforwardly tuned to satisfy the QSH condition Eq. (3), making them natural testbeds for the Floquet-engineered topology studied here.

We stress that while the qualitative phenomenology described in this work, including the sequential topological phase transitions, the  $d$ -wave polarization dependence, and the anomalous Wiedemann-Franz law, is symmetry-driven and expected to be generic across this class of altermagnetic systems, quantitative predictions for specific materials will require first-principles input and are left for future work.

The off-resonant (high-frequency) approximation is valid because midinfrared to terahertz radiation can be used to enter the Floquet driving regime considered in this work, where the photon energy exceeds the electronic bandwidth. Driving frequencies between 10 and 100 THz are adequate for typical hopping amplitudes on the order of 0.1 to 1 eV. The required field strengths correspond to dimensionless amplitudes  $A_0 \sim 0.5$ –1, which are achievable with currently available pulsed laser systems. Crucially, the anticipated topological transi-

TABLE I. Comparison of transport properties and topological phase transitions in the  $d$ -wave AM and conventional AFM under LPL irradiation ( $\theta = 0$ , QSH initial phase).

| Observable                                      | AM ( $t_a = \sqrt{3}$ )                                    | AFM ( $t_a = 1$ )                                          |
|-------------------------------------------------|------------------------------------------------------------|------------------------------------------------------------|
| Symmetry under LPL                              | $C_{4z}\mathcal{T}$ broken                                 | $\mathcal{PT}$ preserved                                   |
| Spin degeneracy                                 | Lifted (except $\theta = \pm\pi/4$ )                       | Preserved everywhere                                       |
| Phase sequence with $A_0$                       | QSH $\rightarrow$ Chern ( $C = -1$ ) $\rightarrow$ Trivial | QSH $\rightarrow$ Trivial                                  |
| $A_0^{c1}$ (first transition)                   | $\approx 0.521$                                            | $\approx 0.640$                                            |
| $A_0^{c2}$ (second transition)                  | $\approx 0.918$                                            | —                                                          |
| $\sigma_{xy}$                                   | Nonzero, light-controlled                                  | Zero                                                       |
| $\alpha_{xy}$                                   | Nonzero, thermally activated                               | Zero                                                       |
| $\kappa_{xy}^{(0)}$                             | Nonzero; quantized in Chern phase                          | Zero                                                       |
| WF law $\kappa_{xy}^{(0)} = -L_0 T \sigma_{xy}$ | Satisfied at $T \rightarrow 0$                             | Trivially ( $0 = 0$ )                                      |
| Mott formula                                    | Satisfied at low $T$                                       | Trivially                                                  |
| Sign control by $\theta$                        | All three coefficients                                     | None                                                       |
| Nodes at $\theta = \pm\pi/4$                    | All three coefficients                                     | All zero regardless                                        |
| QSH spin transport                              | $\sigma_{xy}^s, \alpha_{xy}^s, \kappa_{xy}^{(0),s} \neq 0$ | $\sigma_{xy}^s, \alpha_{xy}^s, \kappa_{xy}^{(0),s} \neq 0$ |

tions rely on the field amplitude to frequency ratio, which permits experimental tuning flexibility without the need for extreme circumstances. Floquet-engineered phases can endure across picosecond timeframes, which is adequate for transport measurements, according to recent pump-probe experiments.

We emphasize that the validity of the high-frequency expansion,  $\hbar\omega \gg W$ , and the suppression of heating are related but distinct requirements. Even when  $\hbar\omega$  is only moderately larger than  $W$ , the heating rate in the off-resonant regime is exponentially suppressed,  $\Gamma_{\text{heat}} \sim e^{-c\omega/J}$  for some constant  $c$  of order unity and  $J$  the relevant hopping scale, as established for periodically driven systems [34,35]. This exponential suppression underlies the existence of the long-lived Floquet prethermal regime in which our effective Hamiltonian description applies, with lifetimes extending to picosecond or nanosecond timescales. For field amplitudes approaching  $A_0 \sim 1$ , multiphoton resonances between Floquet sidebands become more likely, and virtual photon absorption could in principle introduce additional heating channels not captured by the leading-order effective Hamiltonian. We therefore restrict our quantitative analysis to  $A_0 \lesssim 1.2$  and note that the qualitative phenomenology, sequential gap closings and the associated transport signatures, is expected to survive within the prethermal window even if higher-order corrections shift the precise critical amplitudes  $A_0^{c1}$  and  $A_0^{c2}$  quantitatively.

Standard measurement geometries allow for experimental access to the predicted transport signatures. Transverse voltage measurements under optical irradiation in a Hall bar design can be used to identify the anomalous Hall conductivity. By creating a longitudinal temperature gradient achievable by integrated microheaters or local laser heating and measuring the transverse voltage that results, one can quantify the anomalous Nernst effect. Using well-established thermometry methods, the thermal Hall conductivity can be derived from transverse temperature gradients. Notably, all three reactions exhibit a distinctive dependency on the input light's polarization angle, reversing sign under orthogonal rotation and disappearing at symmetry-restoring directions. In contrast to

traditional antiferromagnets, this angular dependence offers a strong and clear indication of altermagnetic order.

Lastly, the sign reversal of the Hall responses by polarization rotation and the tunability of the Nernst peak temperature provide useful paths for device applications. For ultrafast spin caloritronics, it is very appealing to be able to control heat and charge transport solely optically, without the use of magnetic fields. The qualitative behaviors described here are symmetry-driven and so anticipated to be widely applicable across a broad class of altermagnetic systems, while quantitative predictions will require material-specific modeling.

## VII. CONCLUSION

We have presented a comprehensive theoretical study of intrinsic thermal transport in a  $d$ -wave altermagnetics under linearly polarized light (LPL). We find that the Berry curvature can be expressed in closed form in both spin sectors, and that all anomalous transport responses in the altermagnetic (AM) phase have a microscopic origin due to the differential renormalization  $j_1 \neq j_2$ .

A main result is the emergence of a sequential topological phase transition, QSH  $\rightarrow$  spin-polarized Chern insulator ( $C = -1$ ) at  $A_0^{c1} \approx 0.521$ , followed by Chern  $\rightarrow$  trivial at  $A_0^{c2} \approx 0.918$ . This hierarchy is unique to altermagnets, in contrast to conventional antiferromagnets (AFMs), which exhibit a direct QSH  $\rightarrow$  trivial transition.

Within the Chern insulating regime, both charge and thermal responses become topologically quantized, with  $\sigma_{xy} = -e^2/h$  and  $\kappa_{xy}/T = \pi^2 k_B^2/(3h)$ , and satisfy the anomalous Wiedemann-Franz relation  $\kappa_{xy} = -L_0 T \sigma_{xy}$ . In contrast, the anomalous Nernst conductivity  $\alpha_{xy}$  is purely thermally driven, vanishing at  $T = 0$  and peaking at a characteristic temperature  $T^*$  tunable by  $A_0$  and  $\theta$ , in agreement with the Mott relation  $\alpha_{xy} \approx -(\pi^2 k_B^2 T/3e) \partial_\mu \sigma_{xy}$ .

All anomalous transport coefficients exhibit a universal  $d$ -wave dependence on the polarization angle, vanishing at  $\theta = \pm\pi/4$ , reaching extrema at  $\theta = 0, \pm\pi/2$ , and reversing sign under  $\theta \rightarrow \pi/2 - \theta$ . This provides a purely optical route

to control and reverse the thermal Hall response, with no counterpart in conventional AFMs.

Crucially, thermal Hall and Nernst measurements are established as definitive experimental probes of altermagnetic order, even in regimes where charge transport signals may be misleading, due to the total absence of anomalous transport in AFMs under LPL, in sharp contrast to AMs.

With direct implications for spin caloritronics and the optical identification of altermagnetic order in quantum materials, these findings establish Floquet engineering of altermagnetic topological insulators as a viable platform for all-optical control of thermal and spin transport. The model parameters used throughout satisfy the QSH condition  $|m| < |b|(1 + t_a^2)$  and define a symmetry-allowed minimal-model regime close to the topological phase boundary. While an intrinsic QSH phase in a monolayer altermagnetic compound has not yet been confirmed experimentally, MnTe-based heterostructures, strained RuO<sub>2</sub> thin films, and artificial moiré or cold-atom lattices represent promising candidate platforms where the conditions of our model may be realized and the predicted effects may be tested.

The present work also extends the rapidly growing field of externally controlled Berry-curvature transport to the setting of Floquet-engineered altermagnetic systems. Recent studies have shown that Berry-curvature-related Hall and spin-dependent responses in low-dimensional materials and compensated magnetic systems can be tuned by strain, electric fields, and heterostructure geometry [30,31]. Our results demonstrate that linearly polarized light provides an analogous and complementary all-optical control knob in the altermagnetic setting, with the additional advantage that the polarization angle directly encodes the  $d$ -wave symmetry of the altermagnetic order parameter through the nodal structure of all anomalous transport coefficients.

### DATA AVAILABILITY

There are no publicly available research data or software supporting this paper. Requests for further information or data should be sent to the authors.

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