

Boltzmann theory of the inverse Edelstein effect in a two-dimensional Rashba gas

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We investigate the inverse Edelstein effect in a nonhomogeneous system consisting of a ferromagnetic layer coupled to a Rashba two-dimensional electron gas. Within a semiclassical Boltzmann framework, we derive analytical expressions for the charge and spin currents and analyze their dependence on key parameters such as the chemical potential and the Rashba coupling strength. We show how interfacial exchange and spin-orbit interactions jointly control the efficiency of spin-to-charge conversion, leading to distinct regimes characterized by qualitatively different transport responses. A central outcome of our work is the availability of closed-form analytical results, which provide direct physical insight and enable a transparent and quantitative benchmarking with experiments on complex oxide interfaces, such as $\text{LaAlO}_3/\text{SrTiO}_3$.

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I. INTRODUCTION

In recent years, spin-charge interconversion phenomena have become a central topic in spintronics, as they enable the electrical generation, manipulation, and detection of spin currents [1–7]. In this context, interfaces have emerged as particularly effective platforms, owing to the strong symmetry breaking and enhanced spin-orbit coupling. Among these systems, oxide-based interfaces have proven especially promising. For example, $\text{LaAlO}_3/\text{SrTiO}_3$ and $\text{LaAlO}_3/\text{KTaO}_3$ have garnered significant attention due to their unique electronic properties [8–12]. At these interfaces, an electric field perpendicular to the plane breaks the inversion symmetry, leading to the formation of a two-dimensional electron gas (2DEG) with a Rashba spin-orbit coupling (RSOC) [13], which plays a crucial role in spin-charge interconversion phenomena [14–24]. Thus, the ability to manipulate the spin and charge in these systems opens up avenues for spin-orbitronic applications [12,25–32].

From a theoretical perspective, spin-charge conversion at interfaces has been extensively investigated in 2DEGs with RSOC, which provide a minimal and well-controlled platform to capture the essential microscopic mechanisms. In such systems, the lack of inversion symmetry leads to spin-split Fermi surfaces characterized by chiral spin texture [see Fig. 2(a)], whereby each Fermi contour carries a distinct spin

polarization. This peculiar momentum–spin locking naturally enables spin-charge interconversion phenomena, most notably the direct and inverse Edelstein effects (DEE and IEE, respectively).

The DEE, also referred to as the inverse galvanic effect, describes the generation of a nonequilibrium in-plane spin polarization in response to an applied electric field and has been the subject of extensive theoretical investigation [10,20,24,30,33–38]. By Onsager reciprocity, its reciprocal counterpart, i.e., the IEE, corresponds to the conversion of an injected spin accumulation into a transverse charge current [39–44]. Owing to its direct relevance for electrical spin detection and its sensitivity to the underlying spin texture, the IEE represents a particularly powerful probe of spin-orbit-coupled two-dimensional systems and constitutes the primary focus of the present work. On the experimental side, spin-charge interconversion driven by the Edelstein effect has been intensively explored at oxide interfaces. In particular, $\text{LaAlO}_3/\text{SrTiO}_3$ -based heterostructures have emerged as a versatile platform for investigating both DEE and IEE, owing to their gate-tunable carrier density, interfacial inversion symmetry breaking, and sizable Rashba interaction. Pioneering experiments have demonstrated current-induced spin polarization and efficient spin-to-charge conversion at these oxide interfaces, establishing the relevance of the Edelstein mechanism beyond conventional semiconductor systems. A series of works in the context of oxide heterostructures [19,29,45], such as $\text{LaAlO}_3/\text{SrTiO}_3$ [11], $\text{LaAlO}_3/\text{KTaO}_3$ [17], and $\text{FeNi}/\text{Cu}/\text{Ta}/\text{MgO}$ [46], provided key experimental evidence of the IEE by means of spin pumping from an adjacent ferromagnet, revealing significant conversion efficiencies and their strong dependence on interfacial electronic reconstruction and spin-orbit coupling strength. These studies also highlighted the crucial role of orbital hybridization and multiband effects in determining the spin-charge conversion response [10,36], setting oxide interfaces apart from simpler Rashba 2DEGs

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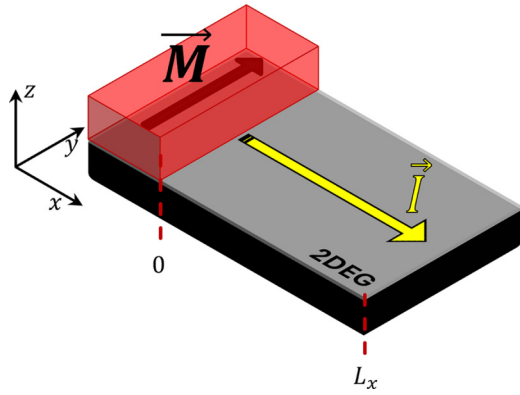


FIG. 1. Schematic of the setup used to study the IEE. A ferromagnetic metal is placed in proximity to a 2DEG with strong spin-orbit coupling. The magnetization, generated by the ferromagnet, injects spin-angular momentum into the interfacial region of the 2DEG, generating a nonequilibrium spin accumulation polarized along y . Through the IEE, the interfacial spin accumulation is converted into a charge current flowing along the x direction in the 2DEG channel.

and uncovering their potential for oxide-based spin-orbitronic devices.

Even though the IEE is experimentally more accessible via a measurable charge current, the development of a simple theoretical description remains challenging. This difficulty stems from the fact that, in the IEE, the magnetization is introduced as a localized perturbation at the boundary of the system, thereby breaking translational invariance along one spatial direction. This situation differs fundamentally from the DEE, where the driving electric field can be treated as a homogeneous external perturbation. Several theoretical approaches have been proposed to address this issue [21,39–44,47]. Microscopic descriptions based on generalized Buttiker formalism [48,49] have been employed for confined geometries such as Hall bars, using numerical calculations [29]. In parallel, more elementary semiclassical approaches based on the Boltzmann equation have also been developed, mainly in the context of topological insulators [50]. In fact, in the diffusive regime, where the scale of the electronic mean free path is much smaller than the size of the system, the semiclassical description provides an accurate and physically transparent account of the transport properties.

In this work, we study the IEE in a 2DEG confined along the $\hat{y}$ direction, in proximity to a ferromagnet, as depicted in Fig. 1. We formulate a theoretical description based on the semiclassical Boltzmann approach with proper boundary conditions, and this framework allows for the analytical evaluation of key physical observables, such as the induced charge current and the spin torque. This central outcome of our work provides direct physical insight and enables a transparent and quantitative benchmarking with experiments. This is particularly advantageous for complex oxide interfaces, such as LAO/STO, where material-specific parameters and interfacial inhomogeneities play a crucial role and where analytical modeling offers a robust reference framework for interpreting and guiding spin-orbitronic measurements. The paper is organized as follows. In Secs. II and III, we introduce, respectively, the

Rashba model and the semiclassical Boltzmann framework employed in our analysis. In Sec. IV, we find the analytical expression for the electric current by adopting an appropriate Boltzmann distribution function, and we discuss its behavior as a function of the chemical potential and the Rashba coupling strength. In Sec. V, we study the spin current and its related continuity equation in the context of Boltzmann approach. Finally, in Sec. VI, we discuss our results and give the perspectives of our work.

II. THE TWO-DIMENSIONAL RASHBA GAS

We consider the setup schematically shown in Fig. 1, which is designed to investigate the IEE at the interface between a ferromagnet and a 2DEG with RSOC. In the physical situation of interest, the ferromagnet is driven into ferromagnetic resonance, giving rise to a time-dependent magnetization that injects spin-angular momentum into the adjacent 2DEG. This spin-pumping mechanism leads to a nonequilibrium spin accumulation in the interfacial region, whose steady-state value results from the dynamical balance between spin injection and spin diffusion. In fact, in experimental realizations, the spin injection takes place in the presence of an external rotating magnetic field. Thus, the magnetization consists of two contributions: a static component (DC) and an oscillating one (AC). Thus, in principle, the Boltzmann distribution expanded to linear order formally includes the corresponding DC and AC corrections, which are generally coupled. Because the system is harmonically driven, both contributions persist in the long-time steady state. However, for the observables considered here, the time-averaged response is fully captured—within the retained perturbative order—by the DC component since the AC contribution averages to zero over time. Therefore, to capture the time-independent consequences of the nonequilibrium spin accumulation at the interface, we adopt a simplified description in which the effect of the magnetization dynamics is represented by an effective static magnetization oriented along the $\hat{y}$ direction. This static exchange field should be understood as the temporal average of the precessing magnetization and provides an effective description of the stationary spin accumulation established in the 2DEG under dynamical equilibrium conditions. Importantly, this approximation does not imply that a static ferromagnet in proximity to a Rashba 2DEG would generate a stationary charge current. Rather, the time-independent treatment is intended to isolate the steady-state response associated with the presence of a spin accumulation maintained by spin-pumping and spin-relaxation processes. Within this framework, the system can be modeled as a Rashba electron gas in contact with a ferromagnet in the region $x \leq 0$. In the steady state, the nonequilibrium magnetization is established in the conducting region for $x > 0$, which in principle should be determined self-consistently. However, we will treat the problem in linear response, and this correction can therefore be neglected within the present level of approximation. Our primary interest is to investigate how the resulting spin accumulation affects the region $x > 0$ where the Hamiltonian of the Rashba 2DEG reads

$$\hat{H} = \frac{k^2}{2m} + \alpha \hat{z} \cdot (\boldsymbol{\sigma} \times \mathbf{k}), \quad (1)$$

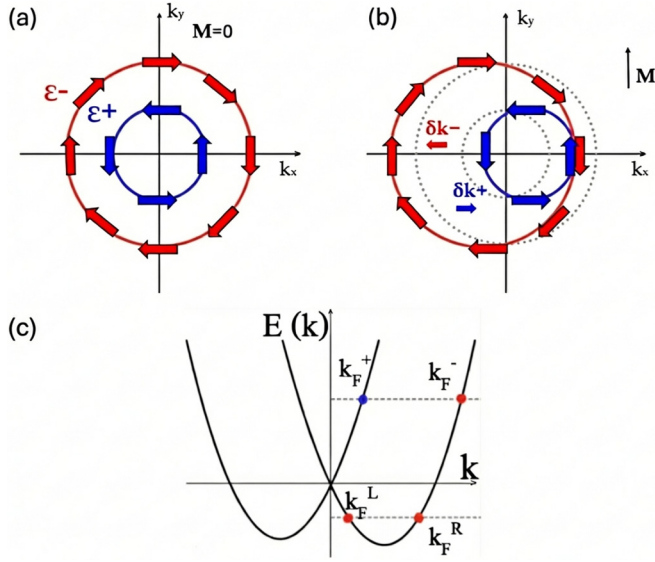


FIG. 2. Schematic representation of Fermi surfaces in the high-density regime (HDR) (a) at the equilibrium and (b) with an applied magnetic field $\mathbf{M} = M_y \hat{y}$. If an external magnetization is applied, the Fermi surfaces are shifted by δk in opposite directions. (c) Energy dispersion of the Rashba 2DEG and Fermi momenta at fixed energy for the HDR and low-density regime (LDR). Blue and red dots indicate the branches of the energy dispersion, respectively + and -.

where the first term represents the kinetic energy, $\mathbf{k}$ is the momentum, k its modulus, m is the effective carrier mass, σ denotes the Pauli matrices, and we set $\hbar = 1$ throughout. The second term corresponds to the RSOC contribution, where α is the coupling strength. As we said, the spin-pumping mechanism occurring at the interface leads to a spin accumulation, which acts as the source of the IEE. Since we are interested in studying how spin injection from the ferromagnet affects the nonmagnetized 2DEG, the effect of the ferromagnet will enter the model through the boundary conditions of the Boltzmann equation, but this aspect will be clarified in the next section. Regarding the resulting band structure of Eq. (1), it is illustrated in Fig. 2. The spectrum consists of two chiral bands, whose dispersion is given by

$$\varepsilon_{\mathbf{k}}^v = \frac{k^2}{2m} + vk\alpha, \quad (2)$$

where $v = \pm$ is the chiral index. The occupation of the bands depends on the chemical potential μ . For $\mu \geq 0$, above the band crossing, both chiral bands provide carriers that contribute to transport, whereas for $\mu < 0$, only one band is occupied [51]. Therefore, it is convenient to distinguish between a high-density regime (HDR), defined by $\mu \geq 0$, and a low-density regime (LDR), defined by $\mu < 0$. This distinction is particularly relevant since transport properties are determined by the Fermi momenta k_F , whose expressions differ in the two regimes. In the HDR, both chiral bands contribute to transport, and the corresponding Fermi momenta read

$$k_F^v = -vk_0 + \sqrt{k_0^2 + 2m\mu} \quad (\text{HDR}), \quad (3)$$

where $k_0 = \alpha m$. The two values of k_F^v correspond to the blue and red dots shown in Fig. 2(c). In the LDR, only a single band is occupied, yet two distinct Fermi momenta are associated with it. In this case, one can distinguish the left (k_F^L) and right (k_F^R) Fermi momenta corresponding to the branch $\varepsilon_{\mathbf{k}}^-$, whose expressions read

$$k_F^\eta = k_0 - \eta \sqrt{k_0^2 + 2m\mu} \quad (\text{LDR}), \quad (4)$$

which are both indicated by the red dots in Fig. 2 and where $\eta = \pm = R/L$ distinguishes between the left and right carriers for the lower band. The difference between these two regimes comes from the band chirality and the direction of the group velocity of the charge carriers. While the standard chiral operator $S = \hat{n} \cdot (\sigma \times \mathbf{k})/k$ successfully discriminates between the upper and lower Rashba bands, it does not provide an equivalent classification at the level of carrier transport. Above the Rashba band-crossing point, the two Fermi surfaces possess opposite chiralities; however, below the crossing, they are the same. For this reason, it is essential to introduce a transport-chiral index, defined as $\eta = -v(\hat{v} \cdot \hat{\mathbf{k}})$, with $\hat{v} = \nabla_{\mathbf{k}} \varepsilon^v / |\nabla_{\mathbf{k}} \varepsilon^v|$ being the normalized group velocity, and $\hat{\mathbf{k}} = \mathbf{k}/k$. This is the same index that appears in Eq. (4). In the following, we use benchmark parameters in agreement with experimental measurements on oxide interfaces [52], i.e., an effective mass $m = 0.7m_e$, a RSOC coupling strength $\alpha = 0.006\text{--}0.01 \text{ eV \AA}$, and a lattice parameter $a = 3.905 \text{ \AA}$. Considering the electron mean free path at oxide interfaces, the transport time is fixed at $\tau = 10^{-11} \text{ s}$ [50]. To properly employ a semiclassical approach, it is necessary to work in the diffusive regime. Looking at Fig. 1, we therefore choose a length that is much larger than the electron mean free path and accordingly set $L_x = 500 \text{ \mu m}$. The parameter values adopted in our analysis are motivated by the materials, such as $\text{LaAlO}_3/\text{KTaO}_3$, while the theoretical framework remains applicable to any Rashba 2D gas. In the following section, we introduce the semiclassical Boltzmann formalism to investigate the IEE.

III. SEMICLASSICAL BOLTZMANN APPROACH

In this section, we discuss the Boltzmann transport equation and how we account for the presence of the ferromagnet. As mentioned before, linear response theory within the semiclassical Boltzmann framework provides an appropriate description of the system under consideration. The dilute limit ensures that scattering processes can be treated as weakly correlated, so that the leading contribution to the response is captured by the semiclassical approach. Moreover, since the electronic mean free path is much smaller than the system size L_x , the system operates in the diffusive regime. This allows for a macroscopic description of transport, enabling the calculation of observables such as the charge current. As shown in Fig. 1, a ferromagnet covering the region $x \leq 0$ is placed in proximity to a Rashba 2DEG. We will model the effect of the ferromagnet on the system by considering different populations of forward-moving carrier—induced by the ferromagnet—that enter the nonmagnetic region and by studying how the distribution function in the Boltzmann equation is modified by them. In this framework, the effect of the magne-

tization dynamics can be described in terms of an effective static exchange field oriented along the $\hat{y}$ direction, namely, $\hat{H}_{\text{FM}} = h_y \sigma \cdot \hat{y}$, representing the time-averaged magnetization under ferromagnetic resonance conditions. Since we will work in the linear response regime, we will treat the Zeeman coupling as a perturbation of the system. This effective field will induce a steady nonequilibrium spin accumulation in the 2DEG, which is converted into a charge current via the IEE.

To study the spin and charge transport, we adopt the semiclassical Boltzmann formalism. The inhomogeneous and stationary Boltzmann equation reads

$$\mathbf{v}_{\mathbf{k}} \cdot \frac{\partial f(\mathbf{r}, \mathbf{k})}{\partial \mathbf{r}} = -\frac{f(\mathbf{r}, \mathbf{k}) - \langle f \rangle}{\tau}, \quad (5)$$

where the classical nonequilibrium distribution function $f(\mathbf{r}, \mathbf{k})$ of electrons in the system depends on the phase-space point $(\mathbf{r}, \mathbf{k})$, with $\mathbf{k}$ and $\mathbf{r}$ being the momentum and coordinate of an electron. The parameter τ is the transport time, and $\langle f \rangle$ represents the angular average of the distribution function. We thus assume that the scattering processes involved are elastic and do not introduce any preferred direction in momentum space; as a result, they are isotropic and drive the distribution function toward its angular average. Since f is translationally invariant along $\hat{y}$, the left-hand term of the Boltzmann equation can be rewritten as $v_{x\mathbf{k}} \frac{\partial f(x, \mathbf{k})}{\partial x}$, with $v_{x\mathbf{k}} = \frac{\partial \varepsilon_{\mathbf{k}}}{\partial k_x}$ being the x component of the velocity $\mathbf{v}_{\mathbf{k}}$. We consider a regime of weak nonequilibrium and assume that the distribution function can be written in terms of a local deviation from equilibrium. Specifically, we adopt the ansatz

$$f(x, \mathbf{k}) \approx f_0 + \left(-\frac{\partial f_0}{\partial \varepsilon_{\mathbf{k}}} \right) g(x, v_{x\mathbf{k}}), \quad (6)$$

where f_0 is the equilibrium Fermi-Dirac distribution function, and $g(x, v_{x\mathbf{k}})$ encodes the spatially dependent correction associated with a local shift of the chemical potential. Within this framework, the stationary Boltzmann equation determines the function g , describing the first correction due to the external magnetization. Here, we make explicit the dependence of g on the group velocity $v_{x\mathbf{k}}$ since it selects the carriers of the Rashba 2DEG that effectively contribute to scattering processes. We note that, at zero temperature, $-\frac{\partial f_0}{\partial \varepsilon_{\mathbf{k}}} = \delta(\varepsilon_{\mathbf{k}} - \mu)$ restricts the contribution to carriers at the Fermi energy, with a fixed polar angle $\theta = \arctan(\frac{k_y}{k_x})$. Therefore, in the following, we will evaluate the group velocity at the Fermi momenta given in Eqs. (3) and (4). The equation for the distribution correction g reads

$$v_x^{v/\eta} \frac{\partial g(x, v_x^{v/\eta})}{\partial x} = -\frac{g(x, v_x^{v/\eta}) - \langle g^{v/\eta} \rangle}{\tau}, \quad (7)$$

where $v_x^{v/\eta} = |v_F^{v/\eta}| \cos \phi$, with ϕ denoting the angle between the group velocity and the $\hat{x}$ direction, and

$$v_F^{v/\eta} = \left. \frac{\partial \varepsilon_{\mathbf{k}}}{\partial k} \right|_{k=k_F^{v/\eta}} = \left(\frac{k_F^{v/\eta}}{m} + v\alpha \right). \quad (8)$$

Finally, the angular average of the distribution function is then defined as

$$\langle g^{v/\eta} \rangle = \frac{1}{2\pi} \int_0^{2\pi} d\phi g(x, v_x^{v/\eta}). \quad (9)$$

The indices v and η are to be chosen according to the HDR and LDR, respectively; see Eqs. (3) and (4).

The problem is fully specified once appropriate boundary conditions are imposed, which determine the form of the function $g(x, v_{x\mathbf{k}})$. The modeling of the ferromagnet plays a central role in this context. In the limit $x \rightarrow 0^-$, the proximity of the ferromagnet induces a spin injection polarized along the y direction into the 2DEG. The presence of such a perturbation at $x \rightarrow 0^-$ induces an energy splitting $\delta\varepsilon^v = -v h_y \cos(\theta)$ on the eigenvalues of the Hamiltonian in Eq. (1), where θ is the angle formed by the magnetization with the $\hat{y}$ direction. This energy splitting generates two spin-polarized carrier populations that enter the nonmagnetic region. These populations are strongly anisotropic and, due to elastic scattering, progressively isotropize within the 2DEG region. This can be conveniently considered a boundary condition on $g(x, v_{x\mathbf{k}} > 0)$ for the incoming carriers, which is opposite for the two bands (see Appendix A for a detailed derivation),

$$g(x=0, v_{x\mathbf{k}}^v > 0) = -v h_y \cos(\theta) \Theta(v_{x\mathbf{k}}^v). \quad (10)$$

On the other hand, at the boundary $x = L_x$, electrons incoming from the right side of the system are in equilibrium; therefore, the corresponding boundary condition for the function g reads

$$g(x=L_x, v_{x\mathbf{k}}^v < 0) = 0. \quad (11)$$

In this sense, referring to Figs. 2(a) and 2(b), where the magnetization induced by the ferromagnet produces a transverse shift Δk , the latter mechanism—namely, the magnetic field-induced displacement of the Fermi surface—is illustrated. This shift results in different nonequilibrium distribution functions for the two Rashba bands, which are subsequently injected into the system. On the other hand, Fig. 2(c) shows the energy spectrum of the pure Rashba 2DEG in the region $x > 0$, where the magnetic field is absent. In fact, as we said before, in this region, we account for the anisotropic distributions associated with incoming carriers at the boundary conditions, thus effectively considering a purely 2DEG with no intrinsic shift of the Fermi surfaces.

Now integrating the first-order linear differential Eq. (7) and considering the boundary conditions for g , it is possible to obtain a formal solution for the distribution function

$$\begin{aligned} g(x, v_{x\mathbf{k}}^v) &= \Theta(v_{x\mathbf{k}}^v) \left\{ (\mp h_y \cos \theta) e^{-\frac{x}{v_{x\mathbf{k}}^v \tau}} + \int_0^x \langle g^v \rangle e^{-\frac{x-\xi}{v_{x\mathbf{k}}^v \tau}} \frac{1}{v_{x\mathbf{k}}^v \tau} d\xi \right\} \\ &+ \Theta(-v_{x\mathbf{k}}^v) \left\{ \int_{L_x}^x \langle g^v \rangle e^{-\frac{x-\xi}{v_{x\mathbf{k}}^v \tau}} \frac{1}{v_{x\mathbf{k}}^v \tau} d\xi \right\}. \end{aligned} \quad (12)$$

In Refs. [38,50], an approximate self-consistent solution of Eq. (12) is provided by a linear approximation of $\langle g^v \rangle$. Although the solution for the function g is given by an integral and self-consistent expression, it can only be solved numerically, considering the boundary conditions mentioned above. In our work, instead, we choose to use a different expression for the function g . In fact, it is straightforward to verify that, for a Rashba 2DEG, a monopole expansion of g provides a solution to Eq. (12), as a consequence of the isotropic nature of the model. Owing to the strongly anisotropic boundary conditions, however, this solution is valid only in the region

where the distribution function is isotropic $\lambda < x < L - \lambda$, with $\lambda = |v_F|\tau$ being the mean free path. We note that λ is the only intrinsic length scale of the problem and therefore sets the spatial range over which this solution applies. This consideration also explains why we neglect the penetration of the magnetization at the interface with the 2DEG and why this enters only through the boundary conditions. Within this range of validity, the approach with the linearized $\langle g^v \rangle$ and the monopole expansion are equivalent; however, the latter allows for a fully analytical treatment, providing additional insight into the structure of the distribution function. Since in our geometry $v_x \propto \cos(\phi)$, the main anisotropic correction is captured by retaining only the isotropic component and the first angular harmonic, whereas higher harmonics, generated by the boundary conditions, are naturally suppressed over distances of order of λ . Thus, an analytic solution can be obtained by means of the monopole expansion

$$g^{v/\eta} = g_0^{v/\eta} + g_1^{v/\eta} \cos(\phi), \quad (13)$$

where we introduced the notation $g^{v/\eta} \equiv g(x, v_x^{v/\eta})$ and v/η , which are used respectively in the HDR and LDR.

Substituting the expression of $g^{v/\eta}$ in Eq. (7) and considering the boundary conditions, we can find the coefficients $g_0^{v/\eta}$ and $g_1^{v/\eta}$ (see Appendix B for the details)

$$\begin{aligned} g_0^{v/\eta} &= -\frac{c^{v/\eta}}{v_F^{v/\eta} \tau} x + c_0^{v/\eta}, \\ g_1^{v/\eta} &= c^{v/\eta}, \end{aligned} \quad (14)$$

where

$$\begin{aligned} c^{v/\eta} &= \frac{2B^{v/\eta} v_F^{v/\eta} \tau}{4v_F^{v/\eta} \tau + \pi L_x}, \\ c_0^{v/\eta} &= B^{v/\eta} \left(\frac{2}{\pi} - \frac{4v_F^{v/\eta} \tau}{4v_F^{v/\eta} \tau \pi + \pi^2 L_x} \right), \end{aligned} \quad (15)$$

with $B^v = -v\hbar$, for the HDR and $B^\eta = -\eta\hbar$ for the LDR. The use of the monopole expansion of the function g makes it possible to calculate analytically the charge current that flows in the system due to the IEE.

IV. ELECTRIC CURRENT DUE TO THE IEE

Within the semiclassical Boltzmann approach, the electric current generated by the IEE can be written as

$$I_c = I_0 \sum_{v/\eta} \int v_x^{v/\eta} g^{v/\eta} \left(-\frac{\partial f_0}{\partial \varepsilon_{\mathbf{k}}^v} \right) k dk d\theta, \quad (16)$$

where we set $I_0 = \frac{eL_y}{4\pi^2 m a^3}$, while we use dimensionless variables in the integral. As discussed in Sec. III, the contributions of g and $\langle \hat{v}_x \rangle_{\mathbf{k}}^v$ are already evaluated at the Fermi momenta selected by $-\frac{\partial f_0}{\partial \varepsilon_{\mathbf{k}}}$.

Using Eq. (13) for $g(x, v_x^{v/\eta})$, we can find an analytical expression for the electric current, which differs for HDR and

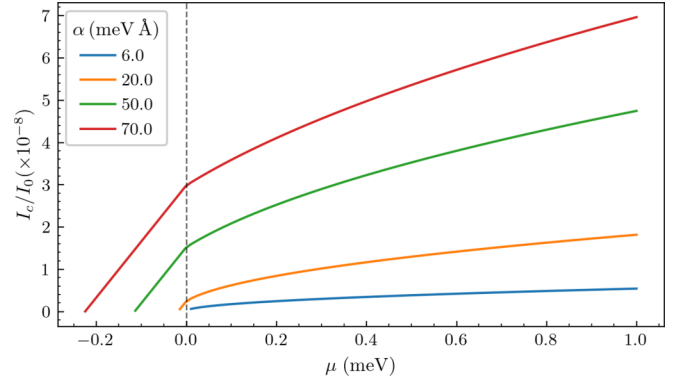


FIG. 3. Electric current as a function of the chemical potential μ for different values of the Rashba parameter α , where $I_0 = 5.63$ A. The dashed line at $\mu = 0$ represents the crossover between the LDR and HDR.

LDR and reads

$$\begin{aligned} I_c(\mu \geq 0) &= I_0 \pi \sum_v g_1^v k_F^v \\ &= \frac{4h_y m \pi \alpha \sqrt{m(2\mu + m\alpha^2)} \tau}{L_x m \pi + 4\sqrt{m(2\mu + m\alpha^2)} \tau}, \end{aligned} \quad (17)$$

and

$$\begin{aligned} I_c(\mu < 0) &= I_0 \pi \sum_\eta g_1^\eta k_F^\eta \\ &= \frac{4h_y m \pi (2\mu + m\alpha^2) \tau}{L_x m \pi + 4\sqrt{m(2\mu + m\alpha^2)} \tau}. \end{aligned} \quad (18)$$

Let us note that the current is independent of the spatial coordinate x , and therefore, it is conserved along the x direction. This follows from the fact that the analytic expression depends only on the g_1 coefficient given in Eq. (13) and is consistent with the isotropization requirement for the equilibrium distribution function in Eq. (5). In Fig. 3, we show the electric current within the entire range of chemical potential and for different benchmark parameters of the Rashba coupling α . For our parameter choice, $I_c \sim \mu$ in LDR since $L_x m \gg 4\sqrt{m(2\mu + m\alpha^2)} \tau$. This differs from the HDR behavior, which goes as $I_c \sim \sqrt{\mu}$. For $\mu < -m\alpha^2/2$, the expressions lose their validity. In Fig. 4, we show the electric current as a function of α for fixed values of the chemical potential. When $\mu \gg m\alpha^2/2$, the current increases linearly with α . Within the HDR, for $\mu < m\alpha^2/2$, the predicted behavior becomes quadratic in α , as indicated by Eq. (17). The LDR, on the other hand, is characterized by the range $-m\alpha^2/2 < \mu < 0$, where the current also exhibits an approximately quadratic dependence on α , see Eq. (18). As expected, when $\alpha = 0$, i.e., in the absence of RSOC, spin-to-charge conversion cannot occur and the current vanishes. Also, this limit is captured by Eq. (17). Finally, we note that, since we are working within the linear response regime, both expressions for the charge current depend linearly on $\hbar$, which is fixed to $\hbar = 0.001$ eV, a realistic and representative value for a given ferromagnetic material.

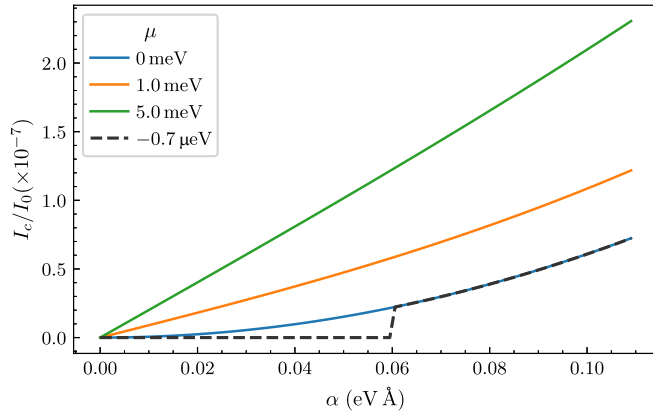


FIG. 4. Electric current as a function of the Rashba parameter α for different values of the chemical potential. Here, $I_0 = 5.63$ A.

V. SPIN-CURRENT AND SPIN-CONTINUITY EQUATION

Having access to the distribution function gives us the opportunity to also study the spin response of the system, which is indeed expected due to the shift of the Fermi surfaces. In systems with spin-orbit coupling, the spin density is not a conserved quantity; therefore, a consistent definition of spin current is problematic [53] and requires some caution. As widely done in the literature, we introduce the spin-current operator, defined as the symmetrized product of the velocity and spin operators $j_i^j = \{v_i, \sigma_j\}/2$, and then calculate its average [54]. This operator describes the rate at which the spin component j is transported along the spatial direction i by the motion of the electronic wave packet. The symmetrization ensures hermiticity and accounts for the fact that, in the presence of spin-orbit interactions, v_i and σ_j do not generally commute. Therefore, the definition of the spin current reads

$$I_S = I_{S_0} \sum_v \int \frac{1}{2} \langle \{v_x, \sigma_y\} \rangle^{v/\eta} g^{v/\eta} \left(-\frac{\partial f_0}{\partial \varepsilon_{\mathbf{k}}^v} \right) k dk d\theta, \quad (19)$$

where $I_{S_0} = \frac{I_0}{2e}$, and we choose all quantities in the integral to be dimensionless. Interestingly, the mean value of the anti-commutator is analytically given by

$$\frac{1}{2} \langle \{v_x, \sigma_y\} \rangle^{v/\eta} = -v \frac{k_F^{v/\eta}}{m} \cos^2(\theta) - \alpha, \quad (20)$$

for which we can identify two contributions: one depending on the carrier momentum and the chirality of the band, and a constant term coming from the presence of RSOC. We can explicitly calculate the spin-current expression and obtain in the HDR

$$\begin{aligned} I_S(\mu > 0) &= I_{S_0} \sum_v \left\{ \left(-v \frac{\pi}{m} k_F^v - 2\pi\alpha \right) g_0^v \frac{k_F^v}{|v_F^v|} \right\} \\ &= \frac{8h_y \mu (m\pi(L_x - x) + 2\sqrt{m(2\mu + m\alpha^2)} \tau)}{L_x \pi \sqrt{m(2\mu + m\alpha^2)} + 8\mu\tau + 4m\alpha^2\tau}, \end{aligned} \quad (21)$$

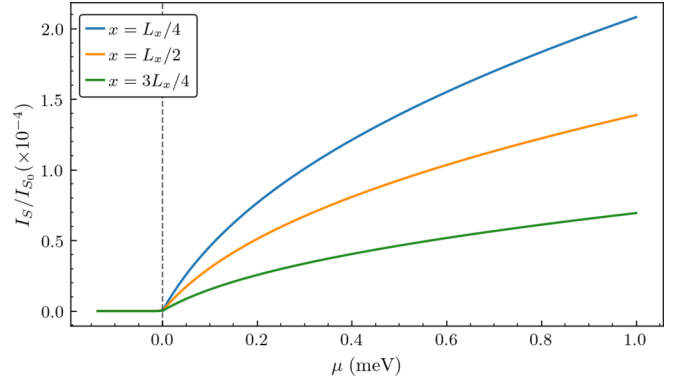


FIG. 5. Spin current as a function of the chemical potential for different values of x , for a fixed value of the Rashba parameter $\alpha = 54$ meV $\text{\AA}$, considering the electronic correlation between the spin and the group velocity.

and in LDR

$$\begin{aligned} I_S(\mu < 0) &= I_{S_0} \sum_{\eta} \left\{ \eta^2 \left(\frac{\pi}{m} k_F^{\eta} - 2\pi\alpha \right) g_0^{\eta} \frac{k_F^{\eta}}{|v_F^{\eta}|} \right\} \\ &= 0. \end{aligned} \quad (22)$$

In Fig. 5, we show the spin current for benchmark values of x . In contrast with the electric currents defined in Eqs. (3) and (4), the spin current is not conserved due to the spin precession along the x direction. This is clearly reflected in the spin-current expression in Eq. (21) that depends on x via the coefficient g_0 [see Eq. (14)]. Moreover, as for the expressions of the charge current, the spin current also exhibits a linear dependence on h_y , which is again fixed to $h_y = 0.001$ eV, consistent with the linear response regime. In Fig. 5, the spin current is zero below the band crossing and grows only for chemical potentials above it. Equation (22) vanishes due to the presence of the factor α in Eq. (20), which does not depend on the transport-chirality index of the carrier. Moreover, it is interesting to note that a vanishing spin current is a characteristic feature of the LDR, i.e., when the Fermi surfaces do not experience a relative shift with respect to each other but are displaced in the same direction in momentum space.

Since the IEE gives rise to a spin-polarized current along the $\hat{y}$ direction, which decays along the $\hat{x}$ direction of the system, it is natural to ask whether torqueline phenomena can emerge in such a system. Therefore, to derive the expression for this quantity, it is convenient to start from the continuity equation for the spin $\partial S_i / \partial t = -\nabla \cdot \mathbf{J}_i + T_i$ [55], where S_i is the spin density along $i = x, y, z$, $\mathbf{J}_i$ is the spin-current density carrying the i th spin component, and T_i is the torque density. To derive such an equation in the diffusive regime, we may directly use the expression for the magnetization

$$S_i = \sum_{\mathbf{k}, v} \langle \sigma_i \rangle_v^v f^v(x, \mathbf{k}). \quad (23)$$

By taking the time derivative of Eq. (23) and considering the Boltzmann equation

$$\frac{\partial f}{\partial t} = -v_x \frac{\partial f}{\partial x} - \frac{\delta f}{\tau}, \quad (24)$$

we obtain

$$\frac{\partial S_i}{\partial t} = \sum_{v/\eta} \int d^2k \langle \sigma_i \rangle^{v/\eta} \left(-\frac{\partial f_0}{\partial \varepsilon_{\mathbf{k}}^v} \right) \times \left[-v_x^{v/\eta} \frac{\partial g^{v/\eta}}{\partial x} - \frac{1}{\tau} g^{v/\eta} \right], \quad (25)$$

where we have already evaluated the quantities at the Fermi momenta selected by $\delta(\varepsilon_{\mathbf{k}}^v - \mu)$. We can identify the second term in Eq. (25) as the torque. Let us note that, since we are considering a static magnetization along the y axis, the only component of the torque that is different from zero is along $\hat{y}$, and its expression is

$$T_y = -T_0 \sum_v \int d^2k \langle \sigma_y \rangle^{v/\eta} v_x^{v/\eta} \frac{\partial g^{v/\eta}}{\partial x} \left(-\frac{\partial f_0}{\partial \varepsilon_{\mathbf{k}}^v} \right) = -4T_0 \pi h_y m \frac{\sqrt{m(2\mu + \alpha^2 m)}}{4\tau \sqrt{m(2\mu + \alpha^2 m)} + \pi m L_x}, \quad (26)$$

in HDR, and

$$T_y = -\frac{4T_0 \pi h_y m^2 \alpha}{4\tau \sqrt{m(2\mu + \alpha^2 m)} + \pi m L_x}, \quad (27)$$

in LDR, where $T_0 = 1/ma^4$. The fact that T_y is the only torque component indicates that, rather than a torque, we observe a modulation of the spin current along the x direction in the system. This effect is commonly known as a spin sink.

By reconsidering the spin-continuity equation, Eq. (25), we find that the first term can be identified with the divergence of the expectation value of the spin-current density operator defined in Eq. (19) only if v_x commutes with σ_y and both commute with the Hamiltonian $\hat{H}$. Upon integration along the transverse direction, this term reduces to

$$I_S = I_{S_0} \sum_{v/\eta} \int d^2k \langle \sigma_y \rangle^{v/\eta} v_x^{v/\eta} g^{v/\eta} \left(-\frac{\partial f_0}{\partial \varepsilon_{\mathbf{k}}^v} \right), \quad (28)$$

where I_{S_0} accounts for the integration performed along the y direction. In Appendix C, we show that this condition holds when the RSOC becomes progressively less relevant, i.e., for higher values of the chemical potential μ .

VI. CONCLUSIONS

In this work, we investigate spin-charge conversion in a 2DEG with RSOC, focusing on the IEE. The system is described by the Rashba Hamiltonian, which captures the essential consequences of inversion-symmetry breaking at oxide interfaces, such as $\text{LaAlO}_3/\text{SrTiO}_3$ and $\text{LaAlO}_3/\text{KTaO}_3$.

Nonequilibrium transport is treated within the semiclassical Boltzmann formalism, with boundary conditions accounting for spin injection from an adjacent ferromagnet and relaxation processes within the 2DEG. Within the diffusive regime and the relaxation time approximation, the inhomogeneous Boltzmann problem admits a fully analytical treatment: By retaining the first angular harmonic in the angular expansion of the nonequilibrium distribution function, we derive closed-form expressions for the IEE, the charge current (and the corresponding spin-current response) in both the HDR and LDR. These analytical formulas provide benchmark

estimates of spin-charge conversion as a function of chemical potential, Rashba coupling, scattering time, and device length. While we have referred to specific oxide interfaces, such as $\text{LaAlO}_3/\text{SrTiO}_3$ and $\text{LaAlO}_3/\text{KTaO}_3$, the model we employ is fully general and does not rely on the details of a particular two-dimensional material. The only essential requirements are the presence of RSOC and a system size L_x much larger than the electronic mean free path. Under these conditions, the semiclassical framework adopted here provides a consistent description of the transport properties. The induced electric current exhibits a strong dependence on the chemical potential, with a marked qualitative change when the chemical potential crosses the band-touching point, as shown in Fig. 3. As the chemical potential increases, the current initially grows linearly and displays an abrupt change upon crossing the band-touching point. This behavior reflects a discontinuity in the density of states when transitioning between the LDR and HDR. From a quantitative perspective, a charge current on the order of 10 nA is consistent with values expected in real materials exhibiting spin-charge conversion. Our results may therefore provide useful guidance for future experiments, for instance, in identifying the parameter range required to observe measurable charge currents in the nA- μ A regime.

The spin current, which is not conserved in the presence of spin-orbit coupling, was evaluated starting from its operator definition and reformulated within the Boltzmann framework. We find that the spin current vanishes below the band-crossing point and becomes finite only for positive chemical potentials. This result can be traced back to the nature of the IEE: At low carrier densities and in the presence of a finite magnetization, the Fermi contours undergo a rigid shift without generating a relative displacement between the Rashba-split bands. Thus, although it is physically expected that both charge and spin currents increase with increasing Rashba parameter, chemical potential, and proximity to the interface, the way these parameters affect the observables is far from trivial. In fact, neither quantity increases monotonically with the chemical potential, which clearly reveals the existence of two distinct regimes reflecting the different physical behavior below and above the Rashba band crossing. Our analytical treatment naturally captures these nontrivial features. We also highlight the limitations of the Boltzmann approach in this context. A formulation of the spin-current continuity equation within semiclassical Boltzmann theory, including spin-torque contributions, is reliable only in the regime of weak RSOC or when the chemical potential lies sufficiently far from the band-crossing point. A more general description for the low-energy regime would likely require a theory based on the density matrix capable of properly capturing spin-coherence effects [56,57]. Overall, our results show how spin-orbit coupling, band filling, and interface-induced boundary conditions jointly determine the efficiency of spin-charge conversion in Rashba systems. These findings provide a transparent framework for interpreting inverse Edelstein measurements through oxide interfaces. Extensions of this work could include a self-consistent treatment of the Boltzmann equation and the incorporation of magnetization dynamics, which would allow access to nonlinear and time-dependent spin-orbitronic effects.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: DETERMINATION OF THE DISTRIBUTION FUNCTION VARIATION

As we pointed out in the main text, the system is not uniform along the x direction, so it is convenient to distinguish two regions: $x \leq 0$ and $x > 0$. In the first region, the ferromagnet is placed on top of the 2DEG. Here, a magnetization emerges and leads to a spin-pumping mechanism at the interface at $x = 0$. On the other hand, in the second region, there is only the 2DEG. In this picture, we start by writing the Hamiltonian of the Rashba 2DEG at the interface with the ferromagnet

$$\hat{H} = \frac{k^2}{2m} + \alpha \hat{z} \cdot (\sigma \times \mathbf{k}) + h_y \sigma_y. \quad (\text{A1})$$

This is the Hamiltonian that describes the system in the region $x \leq 0$. Under this condition, the general distribution function $f_{\mathbf{k}}$ in the Boltzmann equation can be expanded for small values of h_y

$$f_{\mathbf{k}} \approx f_0 + \delta f_{\mathbf{k}} = f_0 + \left(\frac{\partial f_{\mathbf{k}}}{\partial \varepsilon_{\mathbf{k}}^{\pm}} \frac{\partial \varepsilon_{\mathbf{k}}^{\pm}}{\partial h_y} \right) \Big|_{h_y=0} h_y, \quad (\text{A2})$$

where

$$\varepsilon_{\mathbf{k}}^v = \frac{k_x^2 + k_y^2}{2m} + v \sqrt{h_y^2 - 2h_y \alpha k_x + (k_x + k_y)^2 \alpha^2} \quad (\text{A3})$$

is the energy spectrum. Performing the derivative of the energy with respect to the parameter h_y and substituting $k_x = k \cos \theta$ and $k_y = k \sin \theta$, we find that $\delta f_{\mathbf{k}}$ takes the form

$$\delta f_{\mathbf{k}} = \mp \left(- \frac{\partial f_0}{\partial \varepsilon_{\mathbf{k}}^{\pm}} \right) h_y \cos \theta, \quad (\text{A4})$$

where $\mathbf{k} = k (\cos \theta, \sin \theta)$. This result is consistent with our assumption for the boundary condition at $x = 0$ of Eq. (10). At this stage, the effect of the ferromagnet can be described in terms of two distinct populations of forward-moving carriers injected into the region $x > 0$. These strongly anisotropic distributions, due to elastic scattering, progressively relax and become isotropic within the 2DEG; accordingly, they are accounted through boundary conditions. In principle, in the

steady-state, a nonequilibrium magnetization emerges in the channel, leading to a modification of the Hamiltonian for $x > 0$ that should be determined self-consistently. However, within the linear response regime considered here, such corrections are of higher order and can be safely neglected at the present level of approximation. It is therefore sufficient to employ the translationally invariant Rashba Hamiltonian without additional corrections when considering regions sufficiently far from the interface. Indeed, our analytical treatment applies in the regime $x \gg \lambda$, where λ is the mean free path, and the injected carriers have already experienced multiple elastic scattering events, resulting in an isotropic distribution. In this regime, interfacial proximity effects and spatial variations of the exchange field are negligible, so that the Hamiltonian in Eq. (1) provides an accurate description of the system.

APPENDIX B: ANALYTICAL CALCULATION OF THE MONOPOLE SOLUTION OF THE FUNCTION $g^{v/\eta}$

In this appendix, we explicitly derive the coefficients for the monopole expansion of the function $g^{v/\eta} = g_0^{v/\eta} + g_1^{v/\eta} \cos(\phi)$. We begin by computing the function $\langle g^{v/\eta} \rangle$

$$\langle g^{v/\eta} \rangle = \frac{1}{2\pi} \int_0^{2\pi} d\phi (g_0^{v/\eta} + g_1^{v/\eta} \cos \phi) = g_0^{v/\eta}. \quad (\text{B1})$$

Substituting the expressions for $g(x, v_x^{v/\eta})$ and $\langle g^{v/\eta} \rangle$ into Eq. (7), we obtain

$$v_x^{v/\eta} \frac{\partial}{\partial x} (g_0^{v/\eta}(x) + g_1^{v/\eta}(x) \cos \phi) = - \frac{g_1^{v/\eta}(x) \cos \phi}{\tau}. \quad (\text{B2})$$

Multiplying by $\cos \phi$ and integrating the left- and right-hand sides of Eq. (B2) with respect to ϕ , we obtain

$$|v_F| \frac{\partial g_0^{v/\eta}}{\partial x} = - \frac{g_1^{v/\eta}}{\tau} \Rightarrow g_0^{v/\eta} = - \frac{g_1^{v/\eta}}{|v_F| \tau} x + c_0^{v/\eta}. \quad (\text{B3})$$

A second equation can be obtained by integrating the equation directly with respect to ϕ , so we can write

$$\frac{\partial g_1^{v/\eta}}{\partial x} = 0 \Rightarrow g_1^{v/\eta} = c^{v/\eta}. \quad (\text{B4})$$

Thus, we can see that $g_1^{v/\eta}$ does not depend on x . To impose our boundary conditions, we must distinguish the calculations performed in the HDR and LDR, due to the different group velocity directions of the carriers. Indeed, the group velocity and momentum have the same orientation on both Fermi surfaces in the HDR, whereas in the LDR, the group velocity of the inner Fermi surface is opposite in sign to its momentum. Therefore, particular care must be taken when evaluating integrals involving a change of variables between ϕ , the angle in velocity space, and θ , the angle in momentum

space. In the HDR, $\theta = \phi$ for both carriers, so we can write

$$\begin{aligned}\langle g(x=0, v_x^v > 0) \rangle &= \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} d\phi (-v h_y \cos(\phi)) \\ &= -\frac{v h_y}{\pi} = \frac{B^v}{\pi}, \\ \langle g(x=L_x, v_x^v < 0) \rangle &= 0.\end{aligned}\quad (\text{B5})$$

Instead, in the LDR, $\phi = \theta + \pi$ for $\eta = 1$, while $\phi = \pi$ for $\eta = -1$. Therefore, we obtain

$$\begin{aligned}\langle g(x=0, v_x^\eta > 0) \rangle &= \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} d\phi (-\eta h_y \cos \phi) \\ &= -\frac{\eta h_y}{\pi} = \frac{B^\eta}{\pi}, \\ \langle g(x=L_x, v_x^\eta < 0) \rangle &= 0.\end{aligned}\quad (\text{B6})$$

We can then set the expressions derived from the boundary conditions equal to the original definition of $g(x, v_x^{v/\eta})$, thereby obtaining, for example, in the HDR

$$\begin{aligned}\frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} d\phi (g_0^v + g_1^v \cos \phi) \\ &= \frac{-v h_y}{\pi} \Rightarrow \pi c_0^v + 2c^v = -2v h_y, \\ \frac{1}{2\pi} \int_{\pi/2}^{3\pi/2} d\phi (g_0^v + g_1^v \cos \phi) \\ &= 0 \Rightarrow \left(-\frac{L_x \pi}{|v_F| \tau} - 2 \right) c^v + c_0^v \pi = 0,\end{aligned}\quad (\text{B7})$$

and the same procedure can be applied in the LDR. Solving the system of two equations yields the coefficients $c^{v/\eta}$ and $c_0^{v/\eta}$ in the two regimes

$$\begin{aligned}c^v &= \frac{2B^v |v_F| \tau}{4|v_F| \tau + \pi L_x}, \\ c_0^v &= B^v \left(\frac{2}{\pi} - \frac{4|v_F| \tau}{4|v_F| \pi \tau + \pi^2 L_x} \right),\end{aligned}\quad (\text{B8})$$

in the HDR, and

$$\begin{aligned}c^\eta &= \frac{2B^\eta |v_F| \tau}{4|v_F| \tau + \pi L_x}, \\ c_0^\eta &= B^\eta \left(\frac{2}{\pi} - \frac{4|v_F| \tau}{4|v_F| \pi \tau + \pi^2 L_x} \right),\end{aligned}\quad (\text{B9})$$

in the LDR.

APPENDIX C: THE EVALUATION OF THE SPIN CURRENT FROM THE CONTINUITY EQUATION

In this appendix, starting from the definition of the spin current in Eq. (19), we show that, when reduced to the single-band limit or to weak spin-orbit coupling, it recovers Eq. (28); namely, the spin-current expression coming from the continuity equation coincides with the expectation value of the

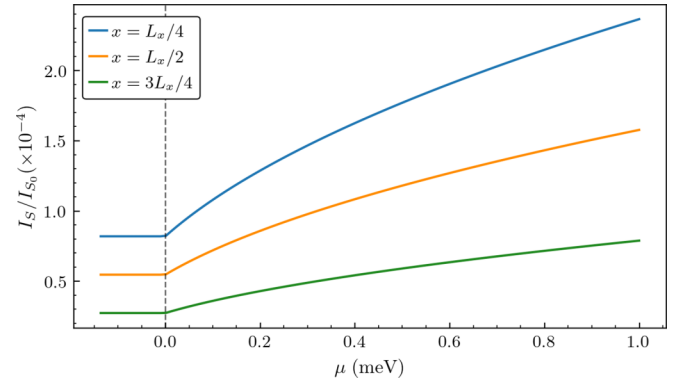


FIG. 6. Normalized spin current as a function of the chemical potential for different values of x and a fixed value of $\alpha = 54 \text{ meV } \text{\AA}$.

spin-current operator. It is straightforward to demonstrate the identity

$$\frac{1}{2} \langle \{v_x, \sigma_y\} \rangle^{v/\eta} = v_x^{v/\eta} \langle \sigma_y \rangle^{v/\eta} - \alpha \sin^2(\theta). \quad (\text{C1})$$

Therefore, if we consider a system in which the RSOC is sufficiently weak, we may also consider the group velocity and the spin to be decoupled and mutually independent, and Eqs. (19) and (28) coincide. However, since the Fermi momenta at which these expressions are evaluated depend on the chemical potential, within the energy window considered near the band crossing, the RSOC is far from negligible. Indeed, as we can see in Fig. 6, which shows the spin current as a function of the chemical potential, a clearly nonphysical result emerges: At the band bottom—where the group velocity vanishes, as discussed above—the spin current does not go to zero. This behavior suggests that writing the continuity equation directly in terms of the magnetization constitutes an approximation that breaks down in Rashba systems at low energies, calling for a more refined treatment based on kinetic theory or on a density-matrix formulation.

However, as shown in Fig. 7, for large values of the chemical potential (i.e., when $k_F \gg k_0$), the correct expression of Eq. (19) accounting for the contribution of the SOC and the pathological one, the expression in Eq. (28), derived from the

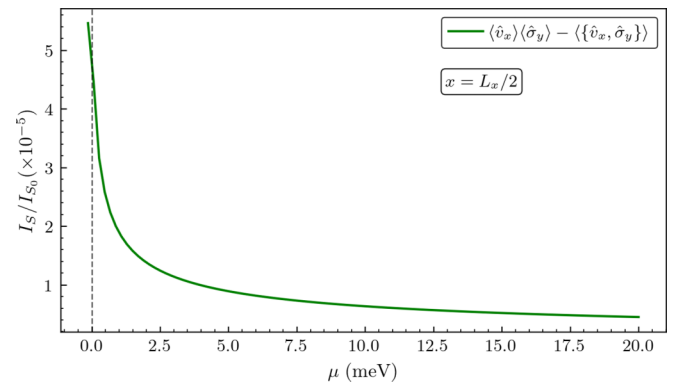


FIG. 7. Difference of the spin current computed using Eqs. (21), (22), and (28) as a function of the chemical potential for $\alpha = 54 \text{ meV } \text{\AA}$ and $x = L_x/2$.

continuity equation, gradually converge. In fact, as expected, as the chemical potential increases, the RSOC becomes pro-

gressively less relevant and the two expressions effectively coincide.

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- [1] S. A. Wolf, D. D. Awschalom, R. A. Buhrman, J. M. Daughton, S. von Molnár, M. L. Roukes, A. Y. Chtchelkanova, and D. M. Treger, Spintronics: A spin-based electronics vision for the future, *Science* **294**, 1488 (2001).
- [2] M. N. Baibich, J. M. Broto, A. Fert, F. N. Van Dau, F. Petroff, P. Etienne, G. Creuzet, A. Friederich, and J. Chazelas, Giant magnetoresistance of (001)Fe/(001)Cr magnetic superlattices, *Phys. Rev. Lett.* **61**, 2472 (1988).
- [3] G. Binasch, P. Grünberg, F. Saurenbach, and W. Zinn, Enhanced magnetoresistance in layered magnetic structures with antiferromagnetic interlayer exchange, *Phys. Rev. B* **39**, 4828 (1989).
- [4] I. Žutić, J. Fabian, and S. Das Sarma, Spintronics: Fundamentals and applications, *Rev. Mod. Phys.* **76**, 323 (2004).
- [5] S. Maekawa, *Concepts in Spin Electronics* (Oxford University Press, Oxford, 2006).
- [6] A. Soumyanarayanan, N. Reyren, A. Fert, and C. Panagopoulos, Emergent phenomena induced by spin-orbit coupling at surfaces and interfaces, *Nature (London)* **539**, 509 (2016).
- [7] J. Sinova, D. Culcer, Q. Niu, N. A. Sinitsyn, T. Jungwirth, and A. H. MacDonald, Universal intrinsic spin Hall effect, *Phys. Rev. Lett.* **92**, 126603 (2004).
- [8] S. Gariglio, A. Caviglia, J.-M. Triscone, and M. Gabay, A spin-orbit playground: Surfaces and interfaces of transition metal oxides, *Rep. Prog. Phys.* **82**, 012501 (2019).
- [9] M. Trama, V. Cataudella, C. A. Perroni, F. Romeo, and R. Citro, Gate tunable anomalous Hall effect: Berry curvature probe at oxides interfaces, *Phys. Rev. B* **106**, 075430 (2022).
- [10] M. Trama, V. Cataudella, C. A. Perroni, F. Romeo, and R. Citro, Tunable spin and orbital Edelstein effect at (111) LaAlO₃/SrTiO₃ interface, *Nanomaterials* **12**, 2494 (2022).
- [11] F. Trier, P. Noël, J.-V. Kim, J.-P. Attané, L. Vila, and M. Bibes, Oxide spin-orbitronics: Spin-charge interconversion and topological spin textures, *Nat. Rev. Mater.* **7**, 258 (2022).
- [12] A. Maiellaro, M. Trama, J. Settino, C. Guarcello, F. Romeo, and R. Citro, Engineered Josephson diode effect in kinked Rashba nanochannels, *SciPost Phys.* **17**, 101 (2024).
- [13] E. I. Rashba, Spin-orbit coupling in condensed matter physics, *Sov. Phys. Solid State* **2**, 1109 (1960).
- [14] V. P. Amin and M. D. Stiles, Spin transport at interfaces with spin-orbit coupling: Phenomenology, *Phys. Rev. B* **94**, 104420 (2016).
- [15] D. C. Vaz, P. Noël, A. Johansson, B. Göbel, F. Y. Bruno, G. Singh, S. McKeown-Walker, F. Trier, L. M. Vicente-Arche, A. Sander, *et al.*, Mapping spin-charge conversion to the band structure in a topological oxide two-dimensional electron gas, *Nat. Mater.* **18**, 1187 (2019).
- [16] J.-i. Inoue, G. E. W. Bauer, and L. W. Molenkamp, Diffuse transport and spin accumulation in a Rashba two-dimensional electron gas, *Phys. Rev. B* **67**, 033104 (2003).
- [17] L. Vicente-Arche, *et al.*, Spin-charge interconversion in KTaO₃ 2D electron gases, *Adv. Mater.* **33**, 2102102 (2021).
- [18] S. Varotto, A. Johansson, B. Göbel, L. M. Vicente-Arche, S. Mallik, J. Bréhin, R. Salazar, F. Bertran, P. L. Fèvre, N. Bergeal, *et al.*, Direct visualization of Rashba-split bands and spin/orbital-charge interconversion at KTaO₃ interfaces, *Nat. Commun.* **13**, 6165 (2022).
- [19] M. Bibes, J. E. Villegas, and A. Barthélémy, Ultrathin oxide films and interfaces for electronics and spintronics, *Adv. Phys.* **60**, 5 (2011).
- [20] P. Gambardella and I. M. Miron, Current-induced spin-orbit torques, *Philos. Trans. R. Soc. A* **369**, 3175 (2011).
- [21] G. Seibold, S. Caprara, M. Grilli, and R. Raimondi, Theory of the spin galvanic effect at oxide interfaces, *Phys. Rev. Lett.* **119**, 256801 (2017).
- [22] L. J. D'Onofrio, M. T. Mercaldo, W. Brzezicki, A. Kłosiński, F. Mazzola, C. Ortix, and M. Cuoco, Filtering spin and orbital moment in centrosymmetric systems, *Phys. Rev. B* **112**, 085428 (2025).
- [23] M. Trama, V. Cataudella, C. A. Perroni, F. Romeo, and R. Citro, Effect of confinement and Coulomb interactions on the electronic structure of the (111) LaAlO₃/SrTiO₃ interface, *Nanomaterials* **13**, 819 (2023).
- [24] M. Trama, I. Gaiardoni, C. Guarcello, J. I. Facio, A. Maiellaro, F. Romeo, R. Citro, and J. van den Brink, Nonlinear anomalous Edelstein response at altermagnetic interfaces, *Phys. Rev. B* **112**, 184404 (2025).
- [25] J. Zhai, M. Trama, H. Liu, Z. Zhu, Y. Zhu, C. A. Perroni, R. Citro, P. He, and J. Shen, Large nonlinear transverse conductivity and Berry curvature in KTaO₃ based two-dimensional electron gas, *Nano Lett.* **23**, 11892 (2023).
- [26] C. Guarcello, A. Maiellaro, J. Settino, I. Gaiardoni, M. Trama, F. Romeo, and R. Citro, Probing topological superconductivity of oxide nanojunctions using fractional Shapiro steps, *Chaos, Solitons Fractals* **189**, 115596 (2024).
- [27] A. Maiellaro, J. Settino, C. Guarcello, F. Romeo, and R. Citro, Hallmarks of orbital-flavored Majorana states in Josephson junctions based on oxide nanochannels, *Phys. Rev. B* **107**, L201405 (2023).
- [28] Y. A. Bychkov and É. I. Rashba, Properties of a 2D electron gas with lifted spectral degeneracy, *JETP Lett.* **39**, 78 (1984).
- [29] A. Maiellaro, F. Romeo, M. Trama, I. Gaiardoni, J. Settino, C. Guarcello, N. Bergeal, M. Bibes, and R. Citro, Theory of charge-to-spin conversion under quantum confinement, *Phys. Rev. Res.* **7**, 043100 (2025).
- [30] S. Ando, Y. Tanaka, M. Cuoco, L. Chirulli, and M. T. Mercaldo, Spin and orbital Edelstein effect in spin-orbit coupled non-centrosymmetric superconductors, *Phys. Rev. B* **110**, 184503 (2024).
- [31] B. Zhao, B. Karpiak, D. Khokhriakov, A. Johansson, A. M. Hoque, X. Xu, Y. Jiang, I. Mertig, and S. P. Dash, Unconventional charge-spin conversion in Weyl-semimetal WTe₂, *Adv. Mater.* **32**, 2000818 (2020).
- [32] A. Maleki Sheikhabadi, I. Miatka, E. Y. Sherman, and R. Raimondi, Theory of the inverse spin galvanic effect in quantum wells, *Phys. Rev. B* **97**, 235412 (2018).
- [33] V. M. Edelstein, Spin polarization of conduction electrons induced by electric current in two-dimensional asymmetric electron systems, *Solid State Commun.* **73**, 233 (1990).

- [34] S. Leiva-Montecinos, J. Henk, I. Mertig, and A. Johansson, Spin and orbital Edelstein effect in a bilayer system with Rashba interaction, *Phys. Rev. Res.* **5**, 043294 (2023).
- [35] A. Aronov and Y. B. Lyanda-Geller, Nuclear electric resonance and orientation of carrier spins by an electric field, *Sov. J. Exp. Theor. Phys. Lett.* **50**, 431 (1989).
- [36] A. Johansson, B. Göbel, J. Henk, M. Bibes, and I. Mertig, Spin and orbital Edelstein effects in a two-dimensional electron gas: Theory and application to SrTiO₃ interfaces, *Phys. Rev. Res.* **3**, 013275 (2021).
- [37] A. Johansson, J. Henk, and I. Mertig, Theoretical aspects of the Edelstein effect for anisotropic two-dimensional electron gas and topological insulators, *Phys. Rev. B* **93**, 195440 (2016).
- [38] I. Gaiardoni, M. Trama, A. Maiellaro, C. Guarcello, F. Romeo, and R. Citro, Edelstein effect in isotropic and anisotropic Rashba models, *Condens. Matter* **10**, 15 (2025).
- [39] K. Shen, G. Vignale, and R. Raimondi, Microscopic theory of the inverse Edelstein effect, *Phys. Rev. Lett.* **112**, 096601 (2014).
- [40] Y. K. Kato, R. C. Myers, A. C. Gossard, and D. D. Awschalom, Current-induced spin polarization in strained semiconductors, *Phys. Rev. Lett.* **93**, 176601 (2004).
- [41] A. Y. Silov, P. Blajnov, J. Wolter, R. Hey, K. Ploog, and N. Averkiev, Current-induced spin polarization at a single hetero-junction, *Appl. Phys. Lett.* **85**, 5929 (2004).
- [42] C. Gorini, R. Raimondi, and P. Schwab, Onsager relations in a two-dimensional electron gas with spin-orbit coupling, *Phys. Rev. Lett.* **109**, 246604 (2012).
- [43] J. R. Sánchez, L. Vila, G. Desfonds, S. Gambarelli, J. Attané, J. De Teresa, C. Magén, and A. Fert, Spin-to-charge conversion using Rashba coupling at the interface between non-magnetic materials, *Nat. Commun.* **4**, 2944 (2013).
- [44] G. Vignale and I. V. Tokatly, Theory of the nonlinear Rashba-Edelstein effect: The clean electron gas limit, *Phys. Rev. B* **93**, 035310 (2016).
- [45] S. Massabeau, O. Paull, A. Pezo, F. Miljevic, M. Mićica, A. Grisard, P. Morfin, R. Lebrun, H. Jaffrès, S. Dhillon, *et al.*, Inverse Rashba Edelstein THz emission modulation induced by ferroelectricity in CoFeB/PtSe₂/MoSe₂/LiNbO₃ systems, *APL Mater.* **13**, 041102 (2025).
- [46] F. Liu, C. Zhou, R. Tang, G. Chai, and C. Jiang, Controllable charge-spin conversion by Rashba-Edelstein effect at Cu/Ta interface, *J. Magn. Magn. Mater.* **540**, 168462 (2021).
- [47] C. Monaco, A. Ferreira, and R. Raimondi, Spin Hall and inverse spin galvanic effects in graphene with strong interfacial spin-orbit coupling: A quasi-classical Green's function approach, *Phys. Rev. Res.* **3**, 033137 (2021).
- [48] M. Büttiker, Scattering theory of current and intensity noise correlations in conductors and wave guides, *Phys. Rev. B* **46**, 12485 (1992).
- [49] M. Büttiker, Four-terminal phase-coherent conductance, *Phys. Rev. Lett.* **57**, 1761 (1986).
- [50] H. Geng, W. Luo, W.-Y. Deng, L. Sheng, R. Shen, and D.-Y. Xing, Theory of inverse Edelstein effect of the surface states of a topological insulator, *Sci. Rep.* **7**, 3755 (2017).
- [51] V. Brosco, L. Benfatto, E. Cappelluti, and C. Grimaldi, Unconventional dc transport in Rashba electron gases, *Phys. Rev. Lett.* **116**, 166602 (2016).
- [52] Y. Y. Ilani, A. Tylan-Tyler, P. Irvin, and J. Levy, Physics of SrTiO₃-based heterostructures and nanostructures: A review, *Rep. Prog. Phys.* **81**, 036503 (2018).
- [53] J. Shi, P. Zhang, D. Xiao, and Q. Niu, Proper definition of spin current in spin-orbit coupled systems, *Phys. Rev. Lett.* **96**, 076604 (2006).
- [54] N. Sugimoto, S. Onoda, S. Murakami, and N. Nagaosa, Spin Hall effect of a conserved current: Conditions for a nonzero spin Hall current, *Phys. Rev. B* **73**, 113305 (2006).
- [55] F. Romeo and R. Citro, Spin-torque generation by dc or ac voltages in quasi-one-dimensional magnetic layered structures, *Phys. Rev. B* **81**, 045307 (2010).
- [56] P. Schwab, R. Raimondi, and C. Gorini, Spin-charge locking and tunneling into a helical metal, *Europhys. Lett.* **93**, 67004 (2011).
- [57] L. Sheng, D. Y. Xing, Z. D. Wang, and J. Dong, Quasiclassical approach to magnetotransport in magnetic inhomogeneous systems, *Phys. Rev. B* **55**, 5908 (1997).