

Comment on “Extension of the adiabatic theorem”

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In Damerow and Kehrein [Phys. Rev. B **113**, 165102 (2026)] proposed the conjecture that, for quantum quenches within the same phase, the overlap between the initial ground state and postquench eigenstates is maximal for the postquench ground state. We show that this statement is not valid in general. An explicit local, translationally invariant, gapped free-fermion counterexample exists even though the pre- and postquench Hamiltonians are connected by a symmetry-preserving gapped path and the thermodynamic-limit spectrum is continuous. The example therefore indicates that phase equivalence must be supplemented by an additional dynamical or geometric condition, beyond the absence of an equilibrium phase boundary.

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In Ref. [1], the following conjecture was formulated [Eq. (2) of that work]:

$$\max_n |\langle \psi_n^{(f)} | GS_i \rangle|^2 = |\langle GS_f | GS_i \rangle|^2, \quad (1)$$

for quenches between Hamiltonians in the same phase. Here $|GS_i\rangle$ and $|GS_f\rangle$ denote the pre- and postquench ground states, respectively, and $\{|\psi_n^{(f)}\rangle\}$ are the eigenstates of the postquench Hamiltonian. We provide a counterexample.

Consider a one-dimensional translationally invariant two-band free-fermion model

$$H_\mu = \sum_k \Psi_k^\dagger h_\mu(k) \Psi_k, \quad \Psi_k = \begin{pmatrix} a_k \\ b_k \end{pmatrix}, \quad (2)$$

with

$$h_f(k) = \varepsilon(k) \sigma_z, \quad (3a)$$

$$h_i(k) = \varepsilon(k)(\cos \phi \sigma_z + \sin \phi \sigma_x), \quad (3b)$$

and

$$\varepsilon(k) = m + t \cos k, \quad m > |t| > 0. \quad (4)$$

Because $\varepsilon(k)$ contains only a constant and a cosine harmonic, both Hamiltonians are local nearest neighbor models in real space.

At half filling, both H_i and H_f are gapped. Their single-particle eigenvalues are

$$E_\pm(k) = \pm \varepsilon(k), \quad (5)$$

so the gap is finite for all k . Moreover, the interpolation

$$h_s(k) = \varepsilon(k)[\cos(s\phi)\sigma_z + \sin(s\phi)\sigma_x], \quad s \in [0, 1], \quad (6)$$

preserves translation symmetry and particle-number conservation, while its spectrum remains $\pm \varepsilon(k)$ for all s . Hence H_i and H_f lie in the same phase according to the definition adopted in Ref. [1]. In the thermodynamic limit, k becomes continuous, and the spectrum is correspondingly continuous as required by the premise of Ref. [1].

Let $\beta_{k,\pm}^\dagger$ create the negative- and positive-energy eigenstates of $h_f(k)$. The occupied negative-energy state of $h_i(k)$ is then

$$d_k^\dagger = \cos \frac{\phi}{2} \beta_{k,-}^\dagger - \sin \frac{\phi}{2} \beta_{k,+}^\dagger. \quad (7)$$

For a finite system with N occupied momenta, the initial ground state is

$$|GS_i\rangle = \prod_k d_k^\dagger |0\rangle, \quad (8)$$

whereas the postquench ground state is

$$|GS_f\rangle = \prod_k \beta_{k,-}^\dagger |0\rangle. \quad (9)$$

Any postquench eigenstate at the same filling is specified by a subset S of momenta promoted to the upper band:

$$|S\rangle = \prod_{k \in S} \beta_{k,+}^\dagger \prod_{k \notin S} \beta_{k,-}^\dagger |0\rangle. \quad (10)$$

Since different momentum sectors factorize independently, one immediately finds

$$|\langle S | GS_i \rangle|^2 = \left(\cos^2 \frac{\phi}{2} \right)^{N-|S|} \left(\sin^2 \frac{\phi}{2} \right)^{|S|}. \quad (11)$$

In particular, for the postquench ground state,

$$|\langle GS_f | GS_i \rangle|^2 = \left(\cos^2 \frac{\phi}{2} \right)^N. \quad (12)$$

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Now consider a single particle-hole excitation $|q\rangle$, corresponding to $|S| = 1$. Equation (11) gives

$$\frac{|\langle q|GS_i\rangle|^2}{|\langle GS_f|GS_i\rangle|^2} = \tan^2 \frac{\phi}{2}. \quad (13)$$

Therefore, whenever $\phi > \pi/2$, one has

$$|\langle q|GS_i\rangle|^2 > |\langle GS_f|GS_i\rangle|^2, \quad (14)$$

so the conjecture in Eq. (1) is violated.

For a concrete choice, take

$$\phi = \frac{3\pi}{4}. \quad (15)$$

Then

$$\tan^2 \frac{\phi}{2} = \tan^2 \frac{3\pi}{8} = (1 + \sqrt{2})^2 > 1, \quad (16)$$

and thus even a single excited eigenstate has larger overlap with the initial ground state than the final ground state.

In fact, Eq. (11) shows more. If $\phi > \pi/2$, then

$$\sin^2 \frac{\phi}{2} > \cos^2 \frac{\phi}{2}, \quad (17)$$

so the overlap increases monotonically with $|S|$. The maximal overlap is therefore attained for the highly excited state in which all occupied momenta are promoted to the upper band:

$$|S_{\max}\rangle = \prod_k \beta_{k,+}^\dagger |0\rangle, \quad (18)$$

for which

$$|\langle S_{\max}|GS_i\rangle|^2 = \left(\sin^2 \frac{\phi}{2}\right)^N > \left(\cos^2 \frac{\phi}{2}\right)^N = |\langle GS_f|GS_i\rangle|^2. \quad (19)$$

We conclude that the conjecture proposed in Ref. [1] does not hold in general, even within the class of local, translationally invariant, gapped free-fermion Hamiltonians connected by a symmetry-preserving gapped path and having a continuous spectrum in the thermodynamic limit. The positive results obtained in Ref. [1] for the transverse field Ising model and special cases of the axial next nearest neighbor Ising model must therefore rely on an additional structure not captured by the conjecture in its present form. In particular, a thermodynamically gapped path is not sufficient on its own; a valid refinement must also exclude the relevant dynamical phase-transition obstruction, which in factorized free-fermion systems is captured by the sectorwise geometric criterion for ground-state overlap dominance.

Note added. Recently, we became aware of Ref. [2], which derives an exact necessary and sufficient Bloch-vector criterion for ground-state overlap dominance in translationally invariant free-fermion systems and relates the breakdown of this dominance to Fisher-zero crossings and dynamical quantum phase transitions. This provides independent support for the conclusion that membership in the same thermodynamic phase is not by itself the correct criterion.

Data availability. There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

[1] S. Damerow and S. Kehrein, Extension of the adiabatic theorem, *Phys. Rev. B* **113**, 165102 (2026).

[2] T. Haque, Exact criterion for ground-state overlap dominance after quantum quenches, [arXiv:2604.11420](https://arxiv.org/abs/2604.11420).