Editors' Suggestion

Exotic superconducting states in altermagnets

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The interplay between magnetism and superconductivity is one of the central topics of condensed matter physics, which has recently been put into new light by the discovery of altermagnets. Here, we study this interplay from a fundamental symmetry perspective using irreducible co-representations of the altermagnetic spin-point groups. We construct and tabulate all symmetry-allowed pairing functions for altermagnets, which uncovers numerous exotic pairing states. We focus on three of them, namely, (i) a nonunitary altermagnetic superconductor with different spatial anisotropies for the spin-up and spin-down condensates, (ii) a half-and-half metal-superconductor where only electrons with one of the two spin components form Cooper pairs, and (iii) a spin chiral superconductor with spin-polarized chiral edge states. Interestingly, the first of these three superconductors exhibits an unusual fractional ac Josephson current for only one of the two spin polarizations, even though the two condensates are symmetry related. We present phenomenological Ginzburg-Landau theories for these unconventional superconductors and show that they correspond to stable minima of the free energies. We examine their topological properties, study the effects of small spin-orbit coupling, consider possible material examples, and investigate their topological responses.

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I. INTRODUCTION

The interplay between magnetism and superconductivity has been and continues to be of great interest in condensed matter physics [1–8]. On the one hand, magnetic fluctuations can serve as a pairing glue for superconductivity. On the other hand, the symmetry lowering of the magnetic order imposes restrictions on the possible pairing states. For example, ferromagnetism breaks time-reversal symmetry and lifts the spin degeneracy of the Fermi surface, leading to unconventional spin-triplet pairing [2,9,10]. In contrast, antiferromagnets exhibit spin-degenerate Fermi surfaces, such that conventional spin-singlet pairing is generally the dominant instability [2,5].

The recently discovered altermagnets exhibit both ferro- and antiferrolike characteristics [11–13]. Due to their unique symmetry properties, described by spin-point groups (SPGs), they show collinear-compensated magnetic order, while at the same time having spin-split electronic bands [14–25], as recently confirmed in photoemission spectroscopy [26–28]. As a consequence, altermagnetic metals have fully spin-polarized Fermi surfaces, whereby spin-up and spin-down parts are related by a symmetry of the SPG. Because of these unique features of both the magnetic order and the Fermi surface, one may expect a distinctive type of interplay between magnetism and superconductivity in altermagnets [29–51]. This begs the following questions: (i) What types of superconductivity can altermagnets support? (ii) Does altermagnetism lead to exotic pairing states not present in conventional ferro- or antiferromagnets? (iii) What types of unique devices and responses can be engineered using superconducting altermagnets?

In this article, we answer these questions from a fundamental symmetry point of view. Using the irreducible corepresentations [52–56] (coreps) of the SPGs [57–61], we construct and tabulate all symmetry-allowed pairing states of superconducting altermagnets. Importantly, due to negligible spin-orbit coupling (SOC), elements of the SPGs act differently on the spin and lattice degrees of freedom, in contrast to regular magnetic point groups (MPGs). Hence, the coreps of SPGs have a very different structure from those of MPGs, enabling the existence of alternative types of pairing states in altermagnets. To derive the basis functions of the superconducting orders, we employ two complementary approaches, namely, (1) constructing them from the irreducible representations of the unitary halving group using Dimmock indicators [62–64] and (2) obtaining them directly from the coreps matrices via projector operators [62]. Both methods yield identical results and unveil a number of exotic pairing states, whose spin and spatial parts are interrelated in a highly nontrivial manner.

In Ref. [46], the authors presented a general methodology for constructing superconducting basis functions from SPG coreps. In particular, they discussed two- and four-dimensional channels in altermagnets. The present work provides a comprehensive tabulation of pairing basis functions for all SPGs describing collinear compensated magnets. We study topology and physical responses associated with several exotic states. In particular, we analyze:

(i) A spin-triplet superconductor whose spin-up and spin-down pairing functions are related by a (improper) rotation

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combined with a spin flip. Since this type of pairing exhibits an altermagneticlike symmetry, we call it an altermagnetic superconductor. The state represents the most striking and important result of our analysis because of its spin-polarized topological responses. Its order parameter is nonunitary and its spin-up and spin-down parts have different spatial anisotropies, giving rise to exotic spin-polarized responses. We consider, in particular, a spin-polarized fractional ac Josephson effect [65] and identify an organic material where this could be observed. Additional details of this state are provided in the Appendix.

(ii) A superconducting state, where only the spin-up electrons form Cooper pairs, while the spin-down electrons remain metallic, which we term a half-and-half metal-superconductor. The superconducting part of this state is similar to the A_1 phase of ^{3}He [66] and ferromagnetic superconductors [67], while its normal part is a half metal [68].

(iii) A number of chiral pairing states, which break antiunitary symmetries of the normal-state altermagnet, which we call spin chiral superconductors. These occur for SPGs with coreps of dimension four or six, such that the antiunitary symmetry can be broken in each spin subspace by a complex superposition of two (or three) basis states.

For the above three examples, we discuss their topological classifications and present phenomenological Ginzburg-Landau (GL) theories [69], describing second-order phase transition from the altermagnetically ordered state into the superconducting state. We show that these unconventional superconductors arise as stable minima of the GL free-energy functional.

II. SPIN-POINT GROUP BASIS FUNCTIONS

We begin by considering the SPGs that describe the symmetries of compensated collinear magnets without SOC. In the absence of SOC, the symmetry actions $(s||g)$ on spin s and lattice g are decoupled. Collinear magnetic order exhibits an SO(2) spin rotation symmetry and a combined symmetry $(2_\perp\mathcal{T}||E)$ consisting of a unitary π spin rotation about an axis perpendicular to the ordering direction followed by time reversal. The magnetic moments are compensated due to the $(2_\perp||g)$ symmetry, where g is an operation that relates magnetic sublattices. If g is the space inversion or translation, the resulting phase is antiferromagnetic. If g is an (improper) rotation, the group corresponds to the altermagnetic order.

SPG G_S combines a unitary halving group X and an antiunitary part $X(\mathcal{T}2_\perp||E)$:

$$G_S = X + X(\mathcal{T}2_\perp||E), \tag{1}$$

$$X = X_{1/2} + (2_\perp||g)X_{1/2}, \tag{2}$$

$$X_{1/2} = \mathrm{SO}(2) \times H. \tag{3}$$

The unitary group X is further split into two cosets, $X_{1/2}$-the sublattice symmetry group, and $(2_\perp||g)X_{1/2}$, whose elements flip the spin relating spin sublattices. $X_{1/2}$ consists of a normal subgroup H, which is the symmetry group of each magnetic site, and the SO(2) spin rotational group. With the group decomposition we can construct its irreducible corepresentations [56].

To construct coreps basis functions of a group with antiunitary elements $G_S = X + XA$, we will discuss two approaches (see Fig. 1). The first approach is based on the derivation of the basis functions for the unitary group X irreducible representations (reps) $\mathbf{\Gamma}^{(i)}$. The basis functions are obtained using the projector operator

$$\mathbf{P}^{(i)}_{kl} = \frac{l_i}{|X|}\sum_{R\in X}\mathbf{D}^{(i)*}(R)_{kl}R, \tag{4}$$

where l_i is the dimension of $\mathbf{\Gamma}^{(i)}$ reps with matrices $\mathbf{D}^{(i)}$, R represents the action of the group element. For the continuous SO(2) spin-rotational symmetry, the summation is replaced by the integration with the measure $\frac{1}{2\pi}d\phi$. The projector transforms the lth basis vector to the kth. To determine the basis, we act with $\mathbf{P}^{(i)}_{ll}$ onto an arbitrary function. The $\mathbf{D}^{(i)}$ matrices were not tabulated in Ref. [56], we repeat the procedure used by the authors and obtain reps for unitary halving groups of all considered SPGs (see Supplemental Material [70] and Refs. [71–74]).

The coreps of the G_S group can be obtained from the unitary group X reps. Generally, there are three possibilities according to Dimmock rules [62]:

$$\begin{aligned}
&(a)\ \langle\boldsymbol{\eta}^{(i)} + A\boldsymbol{\eta}^{(i)}N^{-1}|,\ \mathbf{\Gamma}^{(i)} \overset{N}{\sim} \overline{\mathbf{\Gamma}^{(i)}},\ NN^* = \mathbf{D}(A^2),\\
&(b)\ \langle\boldsymbol{\eta}^{(i)}, A\boldsymbol{\eta}^{(i)}N^{-1}|,\ \mathbf{\Gamma}^{(i)} \overset{N}{\sim} \overline{\mathbf{\Gamma}^{(i)}},\ NN^* = -\mathbf{D}(A^2),\\
&(c)\ \langle\boldsymbol{\eta}^{(i)}, \overline{\boldsymbol{\eta}^{(i)}}|,\ \mathbf{\Gamma}^{(i)} \not\sim \overline{\mathbf{\Gamma}^{(i)}},
\end{aligned} \tag{5}$$

where $\langle\boldsymbol{\eta}^{(i)}|$ is the row of the basis functions of the rep $\mathbf{\Gamma}^{(i)}$. In the first two cases, the irreducible representation $\overline{\mathbf{\Gamma}^{(i)}}$ with matrices $\overline{\mathbf{D}^{(i)}}(R) = \mathbf{D}^{(i)*}(A^{-1}RA)$ is equivalent to $\mathbf{\Gamma}^{(i)}$ via a unitary transformation matrix N, i.e., $\mathbf{D}(R) = N\mathbf{D}^*(A^{-1}RA)N^{-1}$. In the latter case, the reps are not equivalent. In the cases (b) and (c) of Eq. (5) the dimensionality of the irreducible representation is increased.

There is another approach to construct the corep basis functions for a group with nonunitary elements. Instead of constructing the basis functions for the unitary halving group reps, we introduce the corep projector operator

$$P^{(i)}_{kl} = \frac{l_i}{|G_S|}\sum_{R\in X}[D^{(i)*}(R)_{kl}R + D^{(i)*}(RA)_{kl}RA], \tag{6}$$

where D are the coreps matrices in Supplemental Material [70].

We apply these approaches to construct the superconducting basis states. The Cooper pair wave function for a one-band SC can be expressed as a linear combination of basis functions

$$\begin{aligned}
\Delta_{\sigma,\sigma'}(\mathbf{k}) &= \sum_i \upsilon_i\Delta^{(i)}_{\sigma,\sigma'}(\mathbf{k}),\\
\Delta^{(i)}_{\sigma,\sigma'}(\mathbf{k}) &= i[\psi_i(\mathbf{k}) + (d_i(\mathbf{k})\cdot\hat{\tau})]\tau^y_{\sigma,\sigma'},
\end{aligned} \tag{7}$$

where $\psi_i(\mathbf{k})$ is a scalar spin-singlet part, $d_i(\mathbf{k})$ is a three-dimensional (3D) vector corresponding to the triplet pairing, and $\hat{\tau}$ is the vector of Pauli matrices. With the discussed approaches, we obtained coreps basis functions for all SPGs G_S describing collinear compensated magnetic phases in Supplemental Material [70]. The unique coreps of the SPG have even dimensionality with spin-polarized triplet basis states

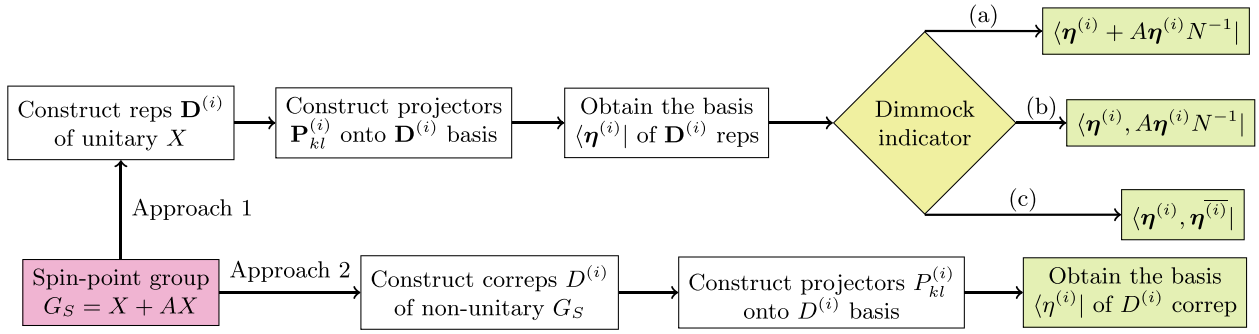


FIG. 1. Diagram describing the two approaches employed to obtain the basis functions of the superconducting order parameters in altermagnets. With approach 1, we first construct the basis functions for the unitary halving group. Approach 2 allows us to construct the basis functions directly using the corep matrices.

$d_\pm(\mathbf{k}) = (1, \pm i, 0)^T \gamma_\pm(\mathbf{k})$, related by the altermagnetic symmetry $(2_\perp||g)$. In this expression, $\gamma_\pm(\mathbf{k})$ is an odd function of $\mathbf{k}$, such that $\gamma_-(\mathbf{k}) = g\gamma_+(\mathbf{k})$. $\mathrm{D}_\pm = (1, \pm i, 0)^T$ are the eigenvectors of the SO(2) spin rotations with eigenvalues $e^{\pm i\phi}$. These states correspond to the pairing $\Delta_{\uparrow\uparrow/\downarrow\downarrow}$, see the Appendix for more details on how to construct the basis functions. The exotic superconducting phenomena uncovered in this work occur exclusively in the spin-triplet $S = \pm 1$ sector. Even though spin-singlet states can be unconventional (e.g., d wave), they do not show altermagnetic spin–space locking because the $S = 0$ order is invariant under spin transformations.

III. HALF-AND-HALF METAL-SUPERCONDUCTOR AND ALTERMAGNETIC SUPERCONDUCTOR

To obtain stable superconducting states, we minimize the GL free energy. The functional respects the same symmetries as the system's Hamiltonian. For the two-dimensional coreps describing $S = \pm 1$ states, the general free-energy expansion up to the fourth order takes the form

$$f[\upsilon_1, \upsilon_2] = \alpha(|\upsilon_1|^2 + |\upsilon_2|^2) + \beta_1(|\upsilon_1|^2 + |\upsilon_2|^2)^2 + \beta_2|\upsilon_1|^2|\upsilon_2|^2, \tag{8}$$

where υ_i determine the expression $\Delta(\mathbf{k}) = \upsilon_1\Delta_{\mathrm{D}_-}(\mathbf{k}) + \upsilon_2\Delta_{\mathrm{D}_+}(\mathbf{k})$, with the basis vectors of the coreps $\Delta_{\mathrm{D}_\pm}(\mathbf{k})$. The spin-rotational SO(2) and the charge U(1) symmetries do not allow having the term $\upsilon_1\upsilon_2^* + \upsilon_2\upsilon_1^*$. Below the phase transition, we fix $\beta_1 > 0$ and $\alpha < 0$. As an example, we consider the $\mathrm{B}_{+1}\mathrm{B}_{-1}$ corep of SPG ${}^{\bar{1}}\mathrm{m}{}^{\bar{1}}\mathrm{m}{}^{1}2$ (see Appendix). The basis functions are given by $\mathrm{D}_+(r_xk_x + r_yk_y)$and $\mathrm{D}_-(-r_xk_x + r_yk_y)$ with real coefficients $r_x, r_y \in \mathbb{R}$.

First, we consider $\beta_2 > 0$. The stable superconducting state is $|\upsilon_1| = 0, |\upsilon_2| \neq 0$ or $|\upsilon_2| = 0, |\upsilon_1| \neq 0$. In this case, the superconducting gap opens only for one of the spin subspaces, and leads to a half-and-half metal superconductor [see Fig. 2(a)]. The triplet spin-polarized pairing $\Delta_{\uparrow\uparrow/\downarrow\downarrow} \neq 0$ is nonunitary and breaks the altermagnetic symmetry $(2_\perp||g)$. A similar triplet state was discussed in the context of ^{3}He in high magnetic fields [66], corresponding to the A_1 phase. In our case, the exotic state appears in the compensated magnet without SOC and with spin-split bands.

If $\beta_2 < 0$, there is a minimum at $|\upsilon_1| = |\upsilon_2| \neq 0$. Spin-up and -down subspaces are still decoupled and the state preserves the altermagnetic symmetry $(2_\perp||g)$. The two spin-polarized pairing potentials with $S = \pm 1$ are related by the $(2_\perp||g)$ leading to the altermagnetic superconducting state [see Fig. 2(b)]. We note that the state is not a unitary state. The different anisotropies for the two spin-polarized pairings $\mathrm{D}_+(r_xk_x + r_yk_y)$and $\mathrm{D}_-(-r_xk_x + r_yk_y)$, with $S = \pm 1$, lead to the nonunitary state $d(\mathbf{k}) = \mathrm{D}_+(r_xk_x + r_yk_y) + e^{i\xi}\mathrm{D}_-(-r_xk_x + r_yk_y)$. The pair correlation is different for up and down spins, i.e., $id(\mathbf{k}) \times d(\mathbf{k})^* = 8r_xr_yk_xk_y\mathrm{D}_z$. The state is a unique state of the altermagnetic phase and cannot arise in conventional (anti)ferromagnets [75].

The minimum $|\upsilon_1| = |\upsilon_2| \neq 0$ has a large degeneracy, as the relative phase can take any value. The degeneracy is lifted in the presence of small SOC (see Appendix). In this case, we can include an additional quadratic term in the free-energy expansion $\alpha_{\mathrm{SOC}}(\upsilon_1\upsilon_2^* + \upsilon_2\upsilon_1^*)$. If $\alpha_{\mathrm{SOC}} < 0$, the minimum is at $\upsilon_1 = \upsilon_2$. When $\alpha_{\mathrm{SOC}} > 0$, it is at $\upsilon_1 = -\upsilon_2$.

The obtained superconducting states do not break the antiunitary $(\mathcal{T}2_\perp||E)$ symmetry. Together with the particle-hole symmetry, this leads to a chiral (sublattice) symmetry in each of the spin subspaces (see Appendix, SM [70], and Ref. [76]). The chiral symmetry quantizes the 1D winding number. In 2D superconductors, the symmetry protects spin-polarized point nodes at zero energy in the quasiparticle excitations bands. At the boundary of the superconductor, we expect to observe spin-polarized zero-energy modes, i.e., Majorana modes (see Fig. 4) [77]. In 3D materials, the symmetry protects spin-polarized line nodes.

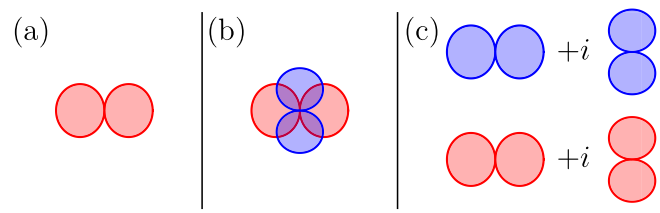


FIG. 2. Three types of exotic superconducting states in altermagnets. (a) Half-and-half metal superconductor with the gap only in one of the spin subspaces. (b) Altermagnetic superconductor with different spatial anisotropy for $S = 1$ and $S = -1$ parts of the pairing potential, related by the altermagnetic symmetry $(2_\perp||g)$. (c) Spin chiral state from the four-dimensional corep, the relative phase breaks antiunitary $(\mathcal{T}2_\perp||E)$ symmetry in each of the spin subspaces.

TABLE I. Classification of topological phases of superconducting states in altermagnets. The invariants are determined for each of the spin subspaces. If the antiunitary symmetry $(\mathcal{T}2_\perp||E)$ is broken by a complex order parameter, the two-dimensional invariant is the Chern number for each spin subspace. When $(\mathcal{T}2_\perp||E)$ is preserved, the invariant is a one-dimensional winding number.

Class	$(\mathcal{T}2_\perp\|\|E)$	1	2	3
D	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0
BDI	✓	$\mathbb{Z}$	0	0

IV. SPIN CHIRAL SUPERCONDUCTOR

Four-dimensional superconducting channels basis functions of SPGs were previously discussed in Ref. [46]. The presence of higher-dimensional coreps, i.e., four- and six-dimensional coreps, is unique to SPGs that describe collinear compensated magnets. For the four-dimensional coreps each of the spin subspaces has two basis vectors with an undetermined relative phase that can break $(\mathcal{T}2_\perp||E)$, leading to chiral superconductivity. As an example, we consider the altermagnetic SPG ${}^{\bar{1}}6/{}^{\bar{1}}m^1m^{\bar{1}}m$ describing symmetries of CrSb [27,28]. There is a four-dimensional corep $\mathrm{E}^{\mathrm{u}}_{+1}\mathrm{E}^{\mathrm{u}}_{-1}$ with four basis vectors, listed up to a common prefactor, $\mathrm{D}_-(k_x+ik_y)$, $\mathrm{D}_-(k_x-ik_y)$, $\mathrm{D}_+(k_x+ik_y)$, $\mathrm{D}_+(k_x-ik_y)$ (see Appendices). We introduce corresponding order parameters υ_1, υ_2, υ_3 and υ_4 [see Eq. (7)]. The GL free energy has five quartic terms

$$\begin{aligned} f_4 = {}& \beta_1(|\upsilon_1|^2+|\upsilon_2|^2+|\upsilon_3|^2+|\upsilon_4|^2)^2 \\ & + \beta_2(|\upsilon_1|^2|\upsilon_2|^2+|\upsilon_3|^2|\upsilon_4|^2) \\ & + \beta_3(|\upsilon_1|^2|\upsilon_3|^2+|\upsilon_2|^2|\upsilon_4|^2) \\ & + \beta_4(|\upsilon_2|^2|\upsilon_3|^2+|\upsilon_1|^2|\upsilon_4|^2) \\ & + 2\beta_5|\upsilon_1||\upsilon_2||\upsilon_3||\upsilon_4|\cos\theta, \end{aligned} \tag{9}$$

where $\theta=\theta_1-\theta_2-\theta_3+\theta_4$ is given by the phases of $\upsilon_n=|\upsilon_n|e^{i\theta_n}$. There are a number of stable minima depending on the choice of the parameters, however, there is the continuous degeneracy in the relative phase. We consider one of the solutions $|\upsilon_1|=|\upsilon_3|\neq 0$, $\upsilon_2=\upsilon_4=0$, and its opposite-chirality partner $|\upsilon_2|=|\upsilon_4|\neq 0$, $\upsilon_1=\upsilon_3=0$ (for the parameter conditions, see Appendices). The first state is $\mathrm{D}_-(k_x+ik_y)+e^{i\xi}\mathrm{D}_+(k_x+ik_y)$, [see Fig. 2(c)]. The relative phase in $k_x\pm ik_y$ breaks the antiunitary symmetry $(\mathcal{T}2_\perp||E)$, while the relative spin phase ξ remains arbitrary.

In a 2D superconductor, the gapped state is described by a Chern number [78]. In 3D material, there can be point nodes with topological charges. In Table I, we summarize the topological classification of the discussed superconducting states in Supplemental Material [70].

V. MATERIAL EXAMPLE

κ-(BEDT-TTF)$_2$Cu [N(CN)$_2$]Br is a member of quasi-2D organic compound family κ-(BEDT-TTF)$_2$X. The compound becomes altermagnetic at low temperatures and goes through the superconducting phase transition under applied pressure.

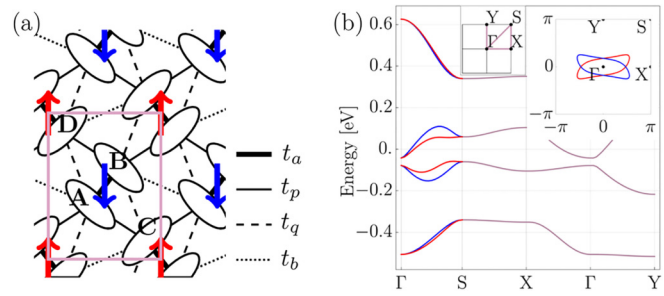


FIG. 3. (a) Quasi-two-dimensional κ-(BEDT-TTF)$_2$X compound lattice with the magnetic order. (b) The band structure of the compound tight-binding model with $(t_a, t_b, t_p, t_q, h) = (-0.207,\ -0.067,\ -0.102,\ 0.043,\ 0.2)$ eV, where h is the strength of the interaction with the magnetic moments. The BZ and Fermi surface with two spin-polarized pockets are shown in the insets.

The phase separation region appears between superconducting and compensated collinear order [79–81].

The magnetic phase has the symmetries of the 2D spin-space group ${}^1 2^{\bar{1}} g^{\bar{1}} g$ [24]. We show the 2D unit cell, band structure, and Fermi surface of the four-site tight-binding model [82] in Fig. 3 (see Appendix). The material has glide mirror symmetries that flip the spin $(2_\perp||M_{x/y}(\frac{1}{2},\frac{1}{2}))$ and the twofold rotational symmetry $(E||2_z)$. The corep table for the SPG ${}^{\bar{1}}\mathrm{m}^{\bar{1}}\mathrm{m}^1 2$ can be found in the Appendix. For the SPG, we obtained the basis functions describing singlet s-wave $\psi(\mathbf{k})=1$ and d-wave $\psi(\mathbf{k})=k_xk_y$, transforming as A_+ and A_- coreps, respectively. The basis states for the triplet $S=0$ states are $i\mathrm{D}_z k_x$ and $i\mathrm{D}_z k_y$, corresponding to B_+ and B_- coreps, $\mathrm{D}_z=(0,0,1)^T$. The unique corep $\mathrm{A}_{+1}\mathrm{A}_{-1}$ is not compatible with superconducting order, as it describes $S\neq 0$ states that are even under space inversion [here $(E||2_z)$ rotation]. The other unique corep $\mathrm{B}_{+1}\mathrm{B}_{-1}$ gives rise to spin-polarized triplet basis states $\mathrm{D}_+(r_xk_x+r_yk_y)$and $\mathrm{D}_-(-r_xk_x+r_yk_y)$ with real coefficients r_x, r_y. The two states are related by the altermagnetic symmetry $(2_\perp||M_x)$.

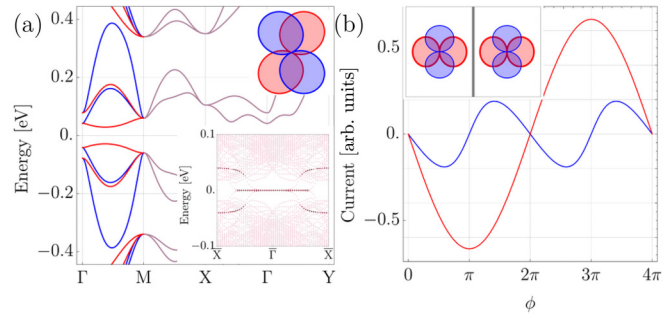


FIG. 4. (a) BdG excitation spectrum for the stable altermagnetic superconducting state (1, 1) in the material example (see Fig. 3). We show the slab model spectrum with 40 layers in the inset. The termination is along $x-y$ line. The pairing potential is shown in the inset. (b) Josephson current through the junction between two superconducting islands with $\mathrm{D}_+k_x+\mathrm{D}_-k_y$ and $e^{i\delta}(\mathrm{D}_+k_x+\mathrm{D}_-k_y)$ pairing potentials. The junction geometry is shown in the inset.

Using the free energy [Eq. (8)], we obtain stable superconducting states. The first solution (1, 0) [(0, 1)] is the half-and-half metal-superconductor $|\upsilon_1| \neq 0, |\upsilon_2| = 0$ [$|\upsilon_2| \neq 0, |\upsilon_1| = 0$], whereas the second one (1, 1) is the altermagnetic superconductor $|\upsilon_1| = |\upsilon_2| \neq 0$. We use Bogoliubov–de Gennes (BdG) formalism and consider the (1, 1) pairing potential $\mathrm{D}_+(r_x k_x + r_y k_y) + \mathrm{D}_-(-r_x k_x + r_y k_y)$ [see Appendix and Fig. 4(a)]. The excitation spectrum has spin-polarized point nodes protected by a winding number (see Table I). Each of the nodes carries a topological charge. The nontrivial topological invariant leads to localized boundary states between projections of the nodes [see Fig. 4(a)] [83,84]. Depending on the termination direction, there is a region between the nodes where the localized state is spin polarized and nondegenerate, i.e., a Majorana boundary mode. The topology of the superconducting state affects physical responses [85–87], particularly the Josephson current [65,88–92].

VI. FRACTIONAL AC SPIN JOSEPHSON EFFECT

The altermagnetic superconducting state has an exotic spatial anisotropy. For spin $S = 1$ and $S = -1$ pairing potentials, the spatial dependence is related by the altermagnetic symmetry. This different anisotropy for spin-up and spin-down states leads to spin-polarized responses related to the topology of the state. Here we consider the junction between two altermagnetic superconductors with pairing potential $\Delta_0(\mathrm{D}_+ k_x + \mathrm{D}_- k_y)$ and the phase difference δ [see Fig. 4(b)]. We compute the ac Josephson current for small applied voltage $eV \ll |\Delta_0|$ and $T = 0$ (see Appendix). The phase has the time dependence $\delta(t) = 2eVt/\hbar$ and it changes adiabatically. $S = 1$ pairs contribute similarly as in the 1D p-wave superconductor, leading to 4π periodicity [65]. The 4π periodicity is due to the topological boundary state in the spin-up subspace, whereas the spin-down subspace does not support the boundary modes for the chosen termination, consequently, the $S = -1$ pairs exhibit a 2π periodicity. In the limit of the transparent barrier, we obtain

$$I_\uparrow = -\frac{\pi \Delta_0}{4}\frac{2e}{\hbar}\sin\frac{\delta}{2}, \quad I_\downarrow = -\frac{\Delta_0}{2}\frac{2e}{\hbar}\sin\frac{\delta}{2}\frac{\cos\frac{\delta}{2}}{\left|\cos\frac{\delta}{2}\right|}. \tag{10}$$

In the tunneling regime, we have

$$I_\uparrow = -\frac{4\Delta_0}{3Z_0}\frac{2e}{\hbar}\sin\frac{\delta}{2}, \quad I_\downarrow = -\frac{\Delta_0}{2Z_0^2}\frac{2e}{\hbar}\sin\delta, \tag{11}$$

with $Z_0 = \frac{2mU_0}{\hbar^2 k_F} \gg 1$. In this limit, the main contribution is given by spin-polarized $S = 1$ Cooper pairs $I_\uparrow$.

VII. DISCUSSION

In this work, we constructed all superconducting basis functions for all spin-point groups describing collinear compensated magnets, including altermagnets. Based on this complete list, we identified a number of exotic pairing states that transform under higher-dimensional coreps of the spin-point groups. The states can break or preserve the symmetries of the altermagnet. In the latter case, the altermagnetic superconducting gap combines anisotropic pairing potentials for $S = \pm 1$. The spin-polarized pairings are related by a spatial (improper) rotation, e.g., a $\pi/2$ rotation. This is in contrast to antiferromagnets, where the potentials overlap fully. The structure of the pairing wave function is different for spin up and spin down, leading to the nonunitary state with a finite vector $id(\mathbf{k}) \times d(\mathbf{k})^*$. If the order preserves the antiunitary $(\mathcal{T}2_\perp||E)$ symmetry, the symmetry protects spin-polarized gapless point excitations in 2D systems. The nodes carry a topological charge, resulting in non-degenerate zero-energy states at the boundary, i.e., Majorana boundary modes. The topological degeneracies and anisotropic spin-space locking lead to exotic spin-polarized topological responses, e.g., fractional ac Josephson currents for only one of the two spin polarizations.

The superconducting order can break altermagnetic symmetries. The spin-polarized half-and-half metal-superconductor breaks $(2_\perp||g)$ symmetry and has the gap only in one spin-subspace. Spin chiral state breaks antiunitary symmetry of the collinear order $(\mathcal{T}2_\perp||E)$, such that the spin subspaces can carry finite Chern numbers leading to chiral boundary modes.

The discussed pairings are three examples of the large number of unique states. The complete list of symmetry-allowed pairings lays the foundation for the study of exotic superconductivity in altermagnets. It opens up research avenues, including the following: (i) Investigation of superconductivity in known altermagnetic candidate materials based on symmetry principles. It is a common practice to use point group representations to analyze the possible pairing symmetries [69]. The approach, however, includes an assumption that SOC is strong, so the spin is coupled to the lattice. For the negligible SOC [93,94], the framework is too restrictive and misses symmetry-allowed states, particularly the ones discussed in this work. (ii) Exploration of a large variety of alternative pairing states, especially those having order parameters with more than two components. The states described by higher-dimensional coreps lead to intricate physical phenomena corresponding to multicomponent order parameters [95]. The interplay and competition between several superconducting orders can result in exotic domain physics [7,96]. Additionally, the SO(2) spin rotational symmetry comes with large continuous phase degeneracy. The spatial fluctuations of the phases can lead to exotic topological defects [97–100], similar to phase vortices in 2D superconductors [101,102]. (iii) Design of superconducting devices that exploit the spin polarization of the Cooper pair condensates together with the exotic spin-point group symmetries.

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DATA AVAILABILITY

The data that support the findings of this article are openly available at [103]. The irreducible co-representations of all spin-point groups describing the symmetries of compen-

TABLE II. $\mathbf{B}_{+1}\mathbf{B}_{-1}$ rep matrices for the unitary halving group of the ${}^{\bar{1}}\mathrm{m}^{\bar{1}}\mathrm{m}^{1}2$ SPG. R_ϕ represents the SO(2) rotation.

	$R_\phi\|\|\mathrm{E}$	$2_\perp\|\|M_x$	$E\|\|2_z$	Type
$\mathbf{B}_{+1}\mathbf{B}_{-1}$	$\begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	(a)

sated collinear magnetic phases are provided in the Zenodo repository as LaTeX (.tex) files. For each irreducible corepresentation, the corresponding superconducting pairing basis functions are also provided in LaTeX format. The underlying raw data are included in the dataset as CSV (.csv) files.

APPENDIX

1. Basis construction

Here we present a step-by-step example of how to construct the basis vectors for the coreps of the ${}^{\bar{1}}\mathrm{m}^{\bar{1}}\mathrm{m}^{1}2$ SPG. The halving unitary group is $X = X_{1/2} + (2_\perp||M_x)X_{1/2}$, where $X_{1/2} = \mathrm{SO}(2) \times \{(E||E), (E||2_z)\}$ [see Eq. (2)]. The corresponding $\mathbf{B}_{+1}\mathbf{B}_{-1}$ irreducible representation (rep) matrices are listed in Table II.

To construct the basis vectors, we apply the projection operators defined in Eq. (4):

$$\mathbf{P}_{11} = \frac{2}{4\cdot 2\pi}\int_0^{2\pi} d\phi e^{-i\phi}[(R_\phi||E) - (R_\phi||C_{2z})],$$
$$\mathbf{P}_{21} = \frac{2}{4\cdot 2\pi}\int_0^{2\pi} d\phi e^{-i\phi}[(2_\perp R_\phi||M_x) - (2_\perp R_\phi||M_y)]. \tag{A1}$$

The action of symmetries on the triplet $d(\mathbf{k})$ vector [see Eq. (7)] is given by $(s||g)d(\mathbf{k}) = \Lambda(s)d(\Lambda^-(g)\mathbf{k})$, where $\Lambda(s)$ denotes the 3×3 rotation matrix associated with the s operation and $\Lambda^-(g)\mathbf{k}$ is the inverse action of the real-space operation g on the momentum $\mathbf{k}$ vector. Applying the projector $\mathbf{P}_{11}$ to a general vector $d(\mathbf{k})$, we obtain to lowest order $d_1^{\mathbf{B}_{+1}\mathbf{B}_{-1}}(\mathbf{k}) = (c_x k_x + c_y k_y)(1, i, 0)^T$, with $c_x, c_y \in \mathbb{C}$. The second basis vector follows from $d_2^{\mathbf{B}_{+1}\mathbf{B}_{-1}}(\mathbf{k}) = \mathbf{P}_{21}d_1^{\mathbf{B}_{+1}\mathbf{B}_{-1}}(\mathbf{k}) = (-c_x k_x + c_y k_y)(1, -i, 0)^T$.

Using the basis of the unitary halving group rep $\langle d_1^{\mathbf{B}_{+1}\mathbf{B}_{-1}}(\mathbf{k}), d_2^{\mathbf{B}_{+1}\mathbf{B}_{-1}}(\mathbf{k})|$, we now construct the corresponding corep. The Dimmock indicator [62] for the rep $\mathbf{B}_{+1}\mathbf{B}_{-1}$ is $\frac{1}{|X|}\sum_{R\in X}\chi([R(\mathcal{T}2_\perp||E)]^2) = +1$. This places the rep in class (a), implying that the rep $\mathbf{D}^{\mathbf{B}_{+1}\mathbf{B}_{-1}}(R)$ and its conjugate $\mathbf{D}^{\mathbf{B}_{+1}\mathbf{B}_{-1}{}^*}(A^{-1}RA)$ are equivalent [see Eqs. (5)]. Moreover, the transformation matrix $N = \left(\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right)$ relates these two sets of rep matrices. With this, the corep basis is obtained as $\langle d_1^{\mathbf{B}_{+1}\mathbf{B}_{-1}}(\mathbf{k}), d_2^{\mathbf{B}_{+1}\mathbf{B}_{-1}}(\mathbf{k})| + \langle(\mathcal{T}2_\perp||E)d_1^{\mathbf{B}_{+1}\mathbf{B}_{-1}}(\mathbf{k}), (\mathcal{T}2_\perp||E)d_2^{\mathbf{B}_{+1}\mathbf{B}_{-1}}(\mathbf{k})|N^{-1}$. The resulting corep $\mathrm{B}_{+1}\mathrm{B}_{-1}$ has the same dimensionality as the original rep, and its basis is given by

TABLE III. Coreps matrices of SPG ${}^{\bar{1}}\mathrm{m}^{\bar{1}}\mathrm{m}^{1}2$.

	$R_\phi\|\|\mathrm{E}$	$2_\perp\|\|M_x$	$2_\perp\|\|M_y$	$\mathcal{T}2_\perp\|\|\mathrm{E}$
A_+	1	1	1	1
A_-	1	-1	-1	1
B_-	1	-1	1	1
B_+	1	1	-1	1
$\mathrm{A}_{+1}\mathrm{A}_{-1}$	$\begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
$\mathrm{B}_{+1}\mathrm{B}_{-1}$	$\begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$d_+(\mathbf{k}) = (r_x k_x + r_y k_y)(1, i, 0)^T, \tag{A2}$$

$$d_-(\mathbf{k}) = (-r_x k_x + r_y k_y)(1, -i, 0)^T,\ r_x, r_y \in \mathbb{R}. \tag{A3}$$

2. Ginzburg-Landau free energy

A superconducting order parameter transforming according to the $\mathrm{B}_{+1}\mathrm{B}_{-1}$ corep can be written as $d(\mathbf{k}) = \upsilon_1 d_+(\mathbf{k}) + \upsilon_2 d_-(\mathbf{k})$. In the basis $\{d_+(\mathbf{k}), d_-(\mathbf{k})\}$, the corep matrices describe how the complex amplitudes υ_1, υ_2 transform (see Table III). For example, we have $(R_\phi||E)\{\upsilon_1, \upsilon_2\} \to \{e^{i\phi}\upsilon_1, e^{-i\phi}\upsilon_2\}$, $(2_\perp||M_x)\{\upsilon_1, \upsilon_2\} \to \{\upsilon_2, \upsilon_1\}$, $(\mathcal{T}2_\perp||E)\{\upsilon_1, \upsilon_2\} \to \{\upsilon_1^*, \upsilon_2^*\}$, and a global U(1) phase acts as $\Phi\{\upsilon_1, \upsilon_2\} \to \{e^{i\omega}\upsilon_1, e^{i\omega}\upsilon_2\}$. These symmetry operations constrain the free-energy expansion. The lowest-order invariants are the quadratic term $\alpha(|\upsilon_1|^2 + |\upsilon_2|^2)$ and the quartic terms $\beta_1(|\upsilon_1|^2 + |\upsilon_2|^2)^2$, $\beta_2|\upsilon_1|^2|\upsilon_2|^2$. For positive and large $\beta_1 > 0$, the free energy is bounded from below. If $\beta_2 < 0$, the minimum occurs at $|\upsilon_1| = |\upsilon_2| \neq 0$. This state we call the altermagnetic superconductor. There is continuous degeneracy associated with the relative phase between the two order parameter components. If instead $\beta_2 > 0$, the minimum corresponds to a single component state $|\upsilon_1| \neq 0, |\upsilon_2| = 0$ or $|\upsilon_1| = 0, |\upsilon_2| \neq 0$, which we term a half-and-half metal superconductor.

We consider SOC as a perturbation that lifts the continuous degeneracy of the altermagnetic superconductor. In the presence of SOC, we fix the magnetic order along the y axis, reducing the symmetry group to a subgroup where all operations act on both spin and lattice degrees of freedom. The resulting MPG is $mm'2'$. It contains the unitary mirror M_x and the antiunitary symmetry $\mathcal{T}M_y$. They flip magnetic moments and exchange sublattices. The antiunitary $\mathcal{T}2_z$ leaves the moments unchanged. The group generators act on the order-parameter components as $(2_x||M_x)\{\upsilon_1, \upsilon_2\} \to \{\upsilon_2, \upsilon_1\}$ and $(\mathcal{T}2_z||2_z)\{\upsilon_1, \upsilon_2\} \to \{-\upsilon_1^*, -\upsilon_2^*\}$. To lowest order in the free-energy expansion, SOC allows an additional invariant $\alpha_{\mathrm{SOC}}(\upsilon_1\upsilon_2^* + \upsilon_2\upsilon_1^*)$ which lifts the continuous degeneracy of the functional minimum for $\beta_2 < 0$. For $\alpha_{\mathrm{SOC}} < 0$, the minimum corresponds to the zero-phase state $(1, 1)$, whereas for $\alpha_{\mathrm{SOC}} > 0$, the global minimum corresponds to the π-phase state $(1, -1)$.

3. Topological classification

The $(\mathcal{T}2_\perp||E)$ symmetry enforces the reality of the gap function [see Eq. (A2)]. For the (1,1) altermagnetic super-

conductor, we have $\Delta_{(1,1)}(\mathbf{k}) = i([d_+(\mathbf{k}) + d_-(\mathbf{k})] \cdot \tau)\tau_y = -2(r_x k_x \tau_z + r_y k_y \tau_0)$. Because the system has SO(2) spin-rotational symmetry, the spin sectors decouple, resulting in real, spin-resolved gap functions $\Delta_{(1,1)\pm}(\mathbf{k}) = -2(\pm r_x k_x + r_y k_y)$. Each spin sector is then described by a BdG Hamiltonian in the symmetry class BDI [76], so the corresponding one-dimensional systems are characterized by an integer $\mathbb{Z}$ winding number. In addition, the system possesses the $(2_\perp||g)$ symmetries, which relate spin subspaces. The two subspaces have related topological invariants.

The two spin-resolved gaps $\Delta_{(1,1)\pm}(\mathbf{k})$ have nodal directions related by $(2_\perp||M_x)$ symmetry, i.e., $\pm r_x k_x + r_y k_y = 0$. This gap structure results in zero-energy excitations in the BdG spectrum [see Fig. 4(a)] protected by the $\mathbb{Z}$ winding number.

4. κ-(BEDT-TTF)$_2$X tight-binding model

The model has the sublattice degrees of freedom, corresponding to the four sites A, B, C, and D [38] (see Fig. 3),

$$H = \tau_0 \otimes \begin{pmatrix} 0 & t_b\epsilon_x^* + t_a & t_p(\epsilon_x^* + 1) & t_q(\epsilon_y^* + 1) \\ t_b\epsilon_x + t_a & 0 & t_q(\epsilon_y + 1) & t_p(\epsilon_x + 1) \\ t_p(\epsilon_x + 1) & t_q(\epsilon_y^* + 1) & 0 & \epsilon_y^*(t_a\epsilon_x + t_b) \\ t_q(\epsilon_y + 1) & t_p(\epsilon_x^* + 1) & \epsilon_y(t_a\epsilon_x^* + t_b) & 0 \end{pmatrix} + h\tau_z \otimes \mathrm{diag}[1, 1, -1, -1], \tag{A4}$$

where the first term describes the normal state model, and the second term introduces the staggered collinear magnetic order, $\epsilon_x = e^{ik_x}$, $\epsilon_y = e^{ik_y}$. The model parameters are taken: $(t_a, t_b, t_p, t_q, h) = (-0.207, -0.067, -0.102, 0.043, 0.2)$ [38,82]. To introduce superconductivity, we use BdG formalism and consider the $(1, 1)$ pairing potential $\Delta = ([\mathrm{D}_+(r_x k_x + r_y k_y) + \mathrm{D}_-(-r_x k_x + r_y k_y)] \cdot \tau) i\tau_y \otimes \mathrm{diag}[1, 1, 1, 1]$, with $(r_x, r_y) = (0.05, 0.1)$.

5. ac Josephson effect

Up to this point, we have considered the (1,1) state of the $\mathrm{B}_{+1}\mathrm{B}_{-1}$ corep given in Eq. (A2). To simplify the forthcoming calculations and make the results more transparent, we now consider the (1,1) state obtained for a system with fourfold rotational symmetry, e.g., the $\mathrm{B}^2_{+1}\mathrm{B}^1_{-1}$ corep of the ${}^{\bar{1}}4^1m^{\bar{1}}m$ spin-point group. We focus on altermagnetic superconductors described by the gap function $\Delta = \Delta_0(\mathrm{D}_+k_x + \mathrm{D}_-k_y)/k_F$, where $k_F = \sqrt{2m\mu}/\hbar$, separated by a barrier and characterized by a superconducting phase difference δ [see Fig. 4(b)]. For simplicity, we neglect anisotropies in the electronic band structure of the normal metal and assume a parabolic dispersion $\epsilon = \hbar^2(k_x^2 + k_y^2)/2m - \mu$ for both spin projections. The superconductors form an interface normal to the x direction and are periodic along the y direction. Following Ref. [65], we model the contact between them by the potential $U(x) = U_0\delta(x)$. Because the spin sectors decouple, we treat the Bogoliubov–de Gennes equations for the two spin subspaces separately:

$$H_\uparrow = \begin{pmatrix} \epsilon(\hat{k}_x, k_y) + U(x) & -2\hat{k}_x\Delta_0/k_F \\ -2\hat{k}_x^\dagger\Delta_0/k_F & -\epsilon(\hat{k}_x, k_y) - U(x) \end{pmatrix}, \tag{A5}$$

$$H_\downarrow = \begin{pmatrix} \epsilon(\hat{k}_x, k_y) + U(x) & 2k_y\Delta_0/k_F \\ 2k_y\Delta_0/k_F & -\epsilon(\hat{k}_x, k_y) - U(x) \end{pmatrix}, \tag{A6}$$

where $\hat{k}_x = -i\partial_x$. Solving the BdG equations for bound states localized at the interface, we find the states' energies

$$E_\uparrow(k_y, \delta) = \pm\frac{2\sqrt{k_F^2 - k_y^2}\Delta_0}{k_F}\sqrt{D_{k_y}}\cos\frac{\delta}{2}, \tag{A7}$$

$$E_\downarrow(k_y, \delta) = \pm\frac{2k_y\Delta_0}{k_F}\sqrt{1 - D_{k_y}\sin^2\frac{\delta}{2}}, \tag{A8}$$

where $D_{k_y} = 4/(4 + Z_{k_y}^2)$, with $Z_{k_y} = Z_0 k_F/\sqrt{k_F^2 - k_y^2}$, and $Z_0 = 2mU_0/(\hbar^2 k_F)$ characterizes the junction transparency. We now compute the dynamical ac Josephson effect. Assuming a small applied voltage $eV \ll \Delta_0$, the phase difference across the junction evolves adiabatically as $\delta(t) = 2eVt/\hbar$. At zero temperature $T = 0$, the Josephson currents for each spin component are given by $I = \int_0^{k_F}\frac{dk_y}{k_F}\frac{2e}{\hbar}\frac{\partial E(k_y,\delta)}{\partial\delta}$, where contributions from all k_y modes are summed. The integration results are given by

$$\begin{aligned} I_\uparrow =& -\frac{\Delta_0}{8}\frac{2e}{\hbar}\left[(4 - Z_0^2)\arctan\left(\frac{2}{Z_0}\right) + 2Z_0\right]\sin\frac{\delta}{2}, \\ I_\downarrow =& -\frac{e\Delta_0}{4\hbar}\tan\frac{\delta}{2}\left[\sqrt{(Z_0^2 + 4)(Z_0^2 + 2\cos\delta + 2)} - Z_0^2\right] \\ &+ \frac{e\Delta_0}{8\hbar}\tan\frac{\delta}{2}Z_0^2\frac{\cos\delta + 3}{\cos\delta/2} \\ &\times \tanh^{-1}\left[\sec\frac{\delta}{2}\sqrt{\frac{Z_0^2 + 2\cos\delta + 2}{Z_0^2 + 4}}\right] \\ &- \frac{e\Delta_0}{8\hbar}\tan\frac{\delta}{2}Z_0^2\frac{\cos\delta + 3}{\cos\delta/2}\tanh^{-1}\left(\sec\frac{\delta}{2}\right). \end{aligned} \tag{A9}$$

6. Spin chiral superconductor

We consider the corep $\mathrm{E}^{\mathrm{u}}_{+1}\mathrm{E}^{\mathrm{u}}_{-1}$ of SPG ${}^{\bar{1}}6/{}^{\bar{1}}m^1m^{\bar{1}}m$. This corep has four basis vectors $-\frac{i}{\sqrt{2}}\mathrm{D}_-(k_x + ik_y)$, $-\frac{i}{\sqrt{2}}\mathrm{D}_-(k_x -$

TABLE IV. $\mathrm{E}^{\mathrm{u}}_{+1}\mathrm{E}^{\mathrm{u}}_{-1}$ corep matrices of SPG ${}^{\bar{1}}6/{}^{\bar{1}}m^1m^{\bar{1}}m$.

$R_\phi||\mathrm{E}$

$$\begin{pmatrix} e^{i\phi} & 0 & 0 & 0 \\ 0 & e^{i\phi} & 0 & 0 \\ 0 & 0 & e^{-i\phi} & 0 \\ 0 & 0 & 0 & e^{-i\phi} \end{pmatrix}$$

$2_\perp||M_z$

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$2_\perp||M_y$

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$2_\perp||C_{6z}$

$$\begin{pmatrix} 0 & 0 & e^{i\pi/3} & 0 \\ 0 & 0 & 0 & e^{-i\pi/3} \\ e^{i\pi/3} & 0 & 0 & 0 \\ 0 & e^{-i\pi/3} & 0 & 0 \end{pmatrix}$$

$\mathcal{T}2_\perp||\mathrm{E}$

$$\begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$ik_y)$, $-\frac{i}{\sqrt{2}}\mathrm{D}_+(k_x+ik_y)$, $-\frac{i}{\sqrt{2}}\mathrm{D}_+(k_x-ik_y)$ with transformation matrices listed in Table IV. Following the procedure outlined above, we construct the GL free energy [see Eq. (9)]. In the regime defined by $\alpha<0$, $\beta_3<0$, $4\beta_1+\beta_3>0$, $\beta_2>\beta_3$, $\beta_4>\beta_3$, $\beta_2+\beta_4-\beta_3>|\beta_5|$, the free energy is minimized for $|\upsilon_1|=|\upsilon_3|=r$, $\upsilon_2=\upsilon_4=0$ or $|\upsilon_2|=|\upsilon_4|=r$, $\upsilon_1=\upsilon_3=0$, with $r^2=-\frac{\alpha}{4\beta_1+\beta_3}$. The two domains have opposite chiralities and are exchanged by $(\mathcal{T}2_\perp||E)$.

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