

Orientation-dependent exceptional points in non-Hermitian altermagnetic Josephson junctions

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(Received 2 March 2026; revised 11 June 2026; accepted 17 June 2026; published 13 August 2026)

Motivated by the orientation-dependent spin splitting and vanishing net magnetization of altermagnets, we study d -wave altermagnetic Josephson junctions (JJs) with a focus on the emergence and controllability of non-Hermitian degeneracies [or exceptional points (EPs)] where both eigenvalues and eigenvectors coalesce. Unlike conventional JJs, the altermagnetic order induces spin- and orientation-dependent Andreev bound states, offering additional control of the Josephson effect. We show that the position of EPs as a function of the superconducting phase difference ϕ can be tuned by the crystal orientation, strength of the altermagnetic order, Rashba spin-orbit coupling, and the degree of non-Hermiticity. Furthermore, the number of EPs is found to be directly tunable by altermagnetic properties. The total supercurrent associated with different transverse momentum k_y shows a smooth behavior but sensitive to the crystal orientation and other properties of the altermagnetic JJ. These findings demonstrate a pathway to realizing distinctive non-Hermitian phenomena arising from the interplay among non-Hermiticity, Rashba interaction, superconductivity, and altermagnetism.

DOI: [10.1103/7wps-4ryn](https://doi.org/10.1103/7wps-4ryn)

I. INTRODUCTION

Since the mid-2010s, a distinct class of magnetic materials has emerged exhibiting a behavior qualitatively different from that typical of conventional ferromagnets (FMs) and antiferromagnets. These materials, now referred to as altermagnets (AMs), are remarkable in that they host spin-split electronic bands despite possessing zero net magnetization [1–3]. Their defining feature lies in the underlying spin-group symmetry: Although ordinary time-reversal symmetry is broken, opposite-spin sublattices remain connected through appropriate crystal symmetries, resulting in momentum-dependent spin splitting despite vanishing macroscopic net magnetization. As a consequence, AMs display unconventional Fermi surface geometries and intricate spin textures [4–6]. At present, a rapidly growing number of candidate materials have been predicted to host altermagnetic order, including MnTe [5,7], RuO₂ [8], and Mn₅Si₃ [9].

Because of the vanishing macroscopic net magnetization, AMs offer a clear advantage when combined with superconductors (SCs), opening possibilities for unconventional proximity effects and novel quantum states. Furthermore, the robustness of the AM proximity effect to disorder makes such systems experimentally relevant [10,11]. This synergy has sparked growing interest, and the investigation of transport phenomena in AM-SC junctions has rapidly become a vibrant research direction [12–19]. For instance, Lu *et al.*

demonstrated that an altermagnetic Josephson junction (JJ) can realize a tunable ϕ junction, with the tunability directly linked to the intrinsic properties of the AM [20].

The JJs themselves represent one of the most fundamental superconducting platforms, consisting of a nonsuperconducting barrier sandwiched between two SCs. A phase difference between the superconducting leads modifies the Andreev bound state (ABS) spectrum, enabling the coherent transfer of Cooper pairs and producing a dissipationless supercurrent through the JJ [21]. Owing to these characteristics, JJs not only serve as powerful probes for identifying different superconducting states [22,23] but also function as indispensable building blocks for emerging quantum technologies [24,25].

Non-Hermitian (NH) systems have recently become a central focus of research, largely because they enable novel quantum phases that cannot exist within conventional Hermitian frameworks [26,27]. A defining feature of NH physics is its complex energy spectrum, which can host unconventional degeneracies known as exceptional points (EPs): singularities where both eigenvalues and eigenvectors simultaneously merge [28–30]. These EPs are now recognized as inherently topological entities, characterized not by traditional band topology but by the concept of point gaps unique to complex spectra [31]. In the context of phase-dependent superconductivity, the standard classification schemes [32,33] have to be adapted due to the differences between momentum and phase variables [34]. Moreover, the NH spectrum displays an extraordinary sensitivity to perturbations, leading to enhanced responses in the vicinity of EPs and paving the way for sensing applications that lack any Hermitian counterpart [35].

The role of NH physics in superconducting systems with and without altermagnetic field has been actively explored in recent studies. Kornich and Trauzettel showed that in

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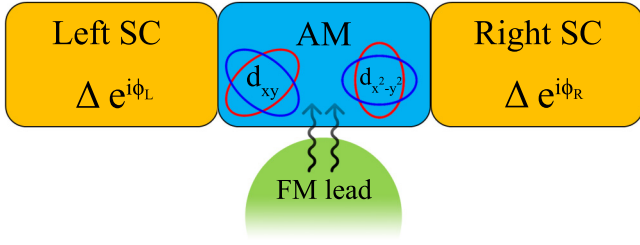


FIG. 1. Schematics of NH JJ formed by coupling a tunnel region composed of a d -wave AM (blue) to a ferromagnetic lead (green). The source of non-Hermiticity is indicated by wiggly arrows.

PT-symmetric NH SCs distinctive signals are revealed in angle-resolved photoelectron fluctuation spectroscopy as well as signatures in the current-voltage characteristic [36,37]. It was demonstrated that the low-energy spectrum can host topologically protected EPs, which may arise at either zero or finite real energies depending on the superconducting phase difference [34,38,39]. Furthermore, the appearance and orientation-dependent tunability of EPs in FM-AM open junctions have been reported [29]. Control of EPs in such hybrid platforms has been shown to be feasible through external magnetic fields [29,30] as well as circularly polarized light [40]. The Meissner response also shows signatures of NH physics [41]. An attribution of diverging real and imaginary supercurrents [38] to signatures of non-Hermiticity has been shown to be unphysical and result in a simple reduction of the supercurrent due to additional dephasing [42–44].

Although JJs naturally provide a fertile setting for NH phenomena, the impact of altermagnetic manipulation on EPs and supercurrent remains unexplored. The present work is a contribution to filling this gap by studying the formation of EPs in a d -wave altermagnetic JJ in the presence of Rashba spin-orbit coupling (RSOC) and their controllability via the JJ's properties. The presence of altermagnetism renders the ABSs both spin and orientation dependent, providing an additional degree of control over EPs compared to conventional JJs. We show that the position and number of EPs relative to the superconducting phase difference can be tuned by crystal orientation, altermagnetic vector strength and symmetry, RSOC strength, and the degree of non-Hermiticity. Finally, we analyze the resulting supercurrent following Ref. [42]. The total supercurrent of the JJ remains smooth, with modest influence from the EPs. These findings introduce altermagnetic JJs as a promising platform for realizing distinctive NH phenomena arising from the interplay of non-Hermiticity, superconductivity, RSOC, and altermagnetism.

II. NH EFFECTIVE MODEL

We study spinful NH JJs incorporating a short altermagnetic segment connected to an FM, as depicted in Fig. 1. The junction's central region forms an AM-FM block, which effectively generates spin-selective loss and gain within the open system [29].

Restricting ourselves to single-site regions, the proposed altermagnetic JJ can be effectively modeled by [38,39]

$$H_{\text{eff}} = H + \Sigma^r(\omega = 0), \quad (1)$$

where H denotes the Hermitian Hamiltonian of JJ formed by conventional SCs with a finite phase difference ϕ between their order parameters. The Hermitian Hamiltonian H can be expressed as [38,39]

$$H = \begin{bmatrix} H_L & V & 0 \\ V & H_M & V \\ 0 & V & H_R \end{bmatrix}. \quad (2)$$

Since our analysis focuses on the formation and structure of ABSs, which are primarily governed by the intrinsic properties of the middle region and the superconducting pairing, we simply model the interface coupling by a momentum-independent hopping matrix $V = \hbar\sigma_0 \otimes \tau_z$ [38,39]. In this description, $\hbar$ effectively controls the interface coupling. However, in realistic heterostructures, the tunneling amplitudes may depend on momentum due to orbital and symmetry matching at the interface, specifically in AMs. Such momentum-dependent coupling $V(k)$ would introduce anisotropic transparency and modify the detailed dispersion of the ABS spectrum, potentially shifting or deforming EP contours in momentum space. However, it might not alter the intrinsic mechanism responsible for ABS formation.

In Eq. (2), H_L , H_M , and H_R correspond to the Hamiltonians of the left SC region, middle altermagnetic region, and right SC region of the JJ, respectively, and are defined as [1,29,38]

$$H_L = \epsilon_L \sigma_0 \otimes \tau_z + \text{Re}[\Delta_L] \sigma_y \otimes \tau_y - \text{Im}[\Delta_L] \sigma_y \otimes \tau_x, \quad (3)$$

$$H_R = \epsilon_R \sigma_0 \otimes \tau_z + \text{Re}[\Delta_R] \sigma_y \otimes \tau_y - \text{Im}[\Delta_R] \sigma_y \otimes \tau_x, \quad (4)$$

$$H_M = [t(k_x^2 + k_y^2) - \mu] \sigma_0 \otimes \tau_z + t_j k_x k_y \sigma_z \otimes \tau_z + H_{\text{RSOC}}, \quad (5)$$

where Δ_L and Δ_R denote the gap functions of the left and right SCs, respectively, and

$$H_{\text{RSOC}} = \lambda k_y \sigma_x \otimes \tau_z - \lambda k_x \sigma_y \otimes \tau_z, \quad (6)$$

represents the RSOC with coupling strength λ , originating from the ferromagnetic substrate. Here $\sigma_{x,y,z}$ and $\tau_{x,y,z}$ are Pauli matrices acting in spin and particle-hole spaces, respectively, and σ_0 refers to identity matrix in spin space. In Eqs. (3) and (4), we set $\epsilon_{R,L} = 0$, similarly to Ref. [38]. In the Bogoliubov–de Gennes formalism, RSOC couples with opposite signs to particles and holes due to the particle-hole transformation. This sign reversal is encoded by multiplying the RSOC term with τ_z , that acts on the Nambu space. To focus on the effect of pure altermagnetism and RSOC, we consider $t(k_x^2 + k_y^2) = \mu$ henceforth, because these terms can be found in various systems and are not specific to the Rashba-perturbed AM ones. This assumption holds only for fixed momentum, as we desired here.

In Eqs. (5) and (6), k_x and k_y denote the x and y components of the wave vector, respectively; t is the hopping amplitude given by $\hbar^2/2m$; and t_j characterizes the altermagnetic vector strength. It should be noted that the symmetry of the AM-wave can be changed from d_{xy} to $d_{x^2-y^2}$ by replacing the second term in H_M with $t_j(k_x^2 - k_y^2)$ (see Fig. 1). Noting that some references, such as Ref. [16], have used a factor of 2 for the $d_{x^2-y^2}$ wave's term, this factor does not affect the physics behind the AMs and only changes some of the numerical

results, so we ignore it here for easier comparison between two wave symmetries.

The second term in Eq. (1) represents the retarded zero-frequency self-energy induced by coupling to the FM layer, which is responsible for introducing non-Hermiticity into the proposed system. The self-energy matrix is defined as [38]

$$\Sigma^r(\omega = 0) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \Sigma & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad (7)$$

where

$$\Sigma = -i\Gamma\sigma_0 \otimes \tau_0 - i\gamma\sigma_z \otimes \tau_0, \quad (8)$$

with τ_0 showing the identity matrix in the particle-hole space. Here Σ is taken to be spin dependent, acting identically on both particles and holes. Γ and γ correspond to the symmetric and asymmetric contributions to the self-energy, respectively, and are defined as $\Gamma = (\Gamma_\uparrow + \Gamma_\downarrow)/2$ and $\gamma = (\Gamma_\uparrow - \Gamma_\downarrow)/2$, where $\Gamma_{\uparrow(\downarrow)}$ denotes the coupling of spin $\uparrow(\downarrow)$ to the lead and explicitly defined as $\Gamma_{\uparrow,\downarrow} = \pi|t'|^2\rho^{\uparrow,\downarrow}$, where t' and ρ refer to the hopping amplitude into the lead from the AM and the surface density of states of the lead, respectively. Further details regarding these definitions and the derivation of the self-energy can be found in Refs. [28,38]. In the following, we assume that $\Gamma_\downarrow = 0$, so that the NH terms are spin polarized. This is necessary, based on a general classification argument [34].

Although the self-energy Σ^r in Eq. (1) generally depends on momentum and frequency, it can be taken as frequency independent in the wideband limit [38,42,45,46], as we assumed here. While this approximation captures the essential physics of reservoir-induced dissipation and EP formation, it neglects the energy dependence of quasiparticle lifetimes and coupling strengths, which can become significant near the superconducting gap edges. Incorporating these might quantitatively modify the ABS spectrum and potentially shift the location of EPs; however, the qualitative emergence and tunability of EPs remain robust as long as the NH coupling is not suppressed by this energy dependence.

In addition to frequency dependence, tunneling across a realistic AM-FM interface may exhibit momentum filtering, which would render the induced gain/loss terms effectively momentum-dependent. Such a k -dependent self-energy could introduce anisotropic decay rates among quasiparticle states and modify the precise shape or position of the EP contours. However, within the wideband limit adopted here, a momentum-independent self-energy captures the leading dissipative effects. Since the formation of EPs originates from the generic competition between the Hermitian terms and reservoir-induced dissipation, their existence and qualitative tunability via altermagnetic or RSOC terms are expected to remain robust against such momentum-dependent corrections.

Substituting Eqs. (3)–(6) into Eq. (2) yields a 12×12 matrix that fully characterizes the system and its interactions (see Sec. A of the Supplemental Material for the explicit form of the Hamiltonian [47]). Consequently, Eq. (2) does not allow for simple analytic eigenvalues. While in simpler configurations, such as SC-normal-SC JJs with energies truncated up to second order, the Hamiltonian may be diagonalized to obtain approximate solutions [38]; in our case the interplay between

RSOC and altermagnetic field mixes the spin degrees of freedom, making the higher-order contributions indispensable. Therefore, we use the full Hamiltonian in Eq. (1) to compute the energy spectrum and identify the ABSs via the condition $\text{Re}[E/\Delta] \leq 1$ (see Fig. S1 and the corresponding discussion in the Supplemental Material [47]). In general, this constraint leads to a complex energy spectrum composed of four real and four imaginary bands, which can be pairwise degenerate. The emergence of EPs is then recognized by the simultaneous coalescence of both the real and imaginary ABS energies, mostly accompanied by a $\text{Re}[E] = 0$ profile between them.

In NH open junctions, the Josephson supercurrent can be derived from the NH effective Hamiltonian as [42,43]

$$I(\phi) = -\frac{1}{\pi} \frac{d}{d\phi} \text{Im Tr}(H_{\text{eff}} \ln[H_{\text{eff}}]), \quad (9)$$

which for diagonalizable Hamiltonians is equivalent to

$$I(\phi) = -\frac{1}{\pi} \frac{d}{d\phi} \text{Im} \sum_n (E_n \ln[E_n]), \quad (10)$$

with $e/\hbar = 1$.

The supercurrent defined in Eq. (9) depends on the wave-vector components as well as on the phase difference, i.e., $I(\phi, k_x, k_y)$. For brevity, however, we briefly denote it as $I(\phi)$. Because AM-induced anisotropy requires integration over k_y due to the intrinsic anisotropy of AMs, the total supercurrent $[I_{\text{tot}}(\phi)]$ can be expressed as

$$I_{\text{tot}}(\phi) = \int_{-k_{\text{max}}}^{k_{\text{max}}} I(\phi) dk_y, \quad (11)$$

where $k_{\text{max}} = 1/\sqrt{1-t_j^2}$ for the d_{xy} wave AM symmetry [16].

Notably, some previous works [38,48] have proposed to extend the Hermitian expression $\tilde{I} \propto dE/d\phi$ to the NH regime by allowing the ABS energies to take complex values. Under this extension, the derivatives can be found as

$$\tilde{I}(\phi) = \frac{e}{\hbar} \sum_{n \leq 0} \left(\frac{d\text{Re}[E_n]}{d\phi} - i \frac{d\text{Im}[E_n]}{d\phi} \right). \quad (12)$$

Since the ABSs merge at EPs, this phase derivative of the corresponding eigenvalue $[\tilde{I}(\phi)]$ is expected to produce divergent features in both the real and imaginary components, which are unrelated to the measurable supercurrent in the NH junctions. A representative example of using Eq. (12) is shown in Fig. S2 of the Supplemental Material [47], where the divergences coincide precisely with the EPs in the ABS spectrum; a feature which can be inferred from the infinite slope of the ABS energy bands at EPs [cf. Figs. 2(c1)–2(d2)]. In contrast, the supercurrent defined in Eq. (9) remains finite and physically meaningful, providing a robust global observable that captures the influence of the non-Hermiticity without relying on branch-resolved spectral derivatives.

Noting that throughout this work, unless otherwise stated, wave vectors are given in units of a^{-1} , energies are expressed in units of t/a^2 , where a is the lattice constant of the assumed square lattice. The RSOC strength is also given in units of t/a . The AM hopping term t_j is mostly used as a dimensionless factor and, thus, the second term in Eq. (5) has a factor of t [16], which is not written there.

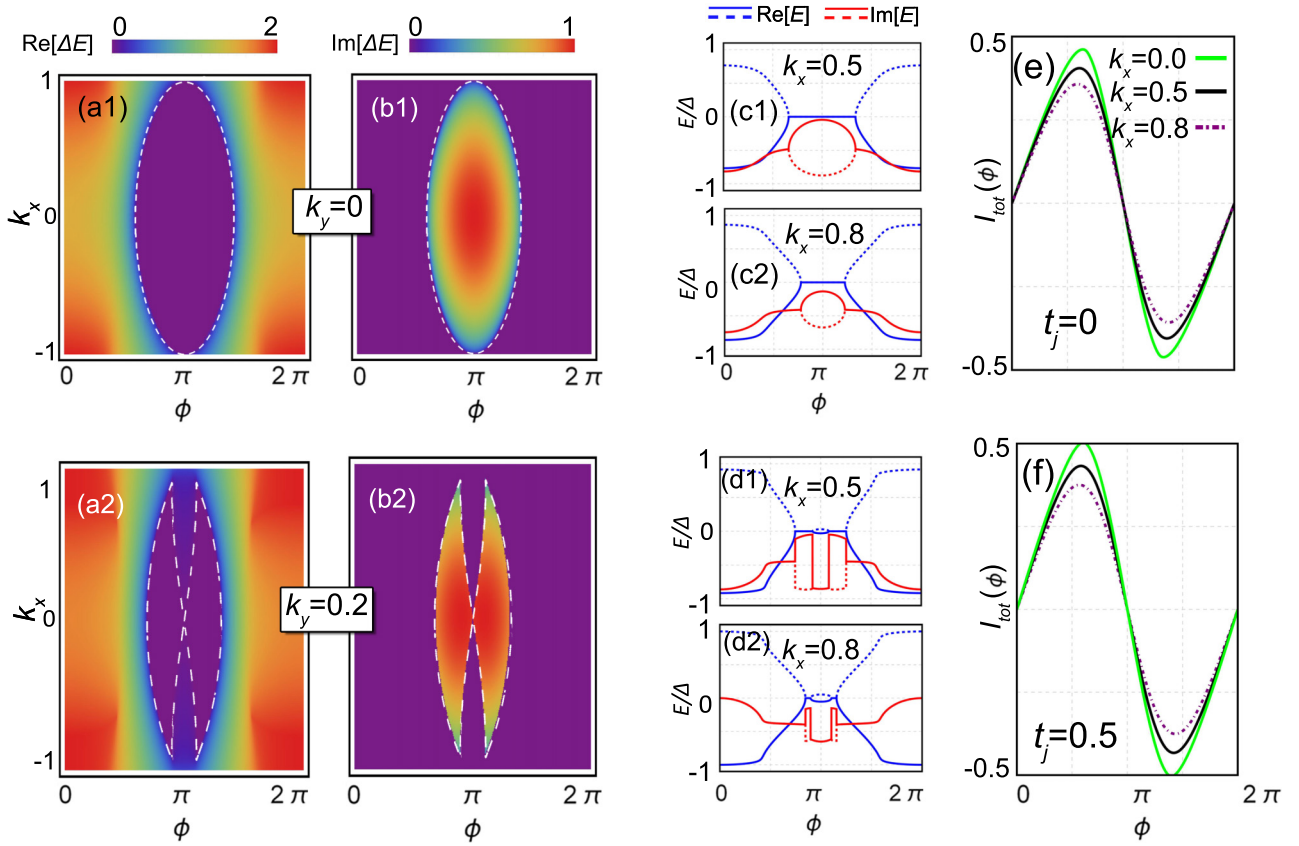


FIG. 2. Density plots of the real (a1) and (a2) and imaginary (b1) and (b2) parts of the energy difference $\Delta E = E_2 - E_1$ versus k_x and ϕ . Results are shown for $k_y = 0$ in panels (a1) and (b1) and for $k_y = 0.2$ in (a2) and (b2). Panels (c1) and (c2) present the ABSs versus ϕ at $k_x = 0.5$ and $k_x = 0.8$, respectively, for $k_y = 0$ and $t_j = 0.3$. Panels (d1) and (d2) are the same as (c1) and (c2) but for $k_y = 0.2$. Panels (e) and (f) represent the total supercurrent computed by Eq. (11) for (e) $t_j = 0$ and (f) $t_j = 0.5$. Here we use $\lambda = \Delta = h = 1$. The white dashed lines in the density plots illustrate the variation of the EP position, schematically.

III. RESULTS AND DISCUSSION

A. JJ with d_{xy} AM

First, we investigate the emergence of EPs in the proposed JJ structure. The coexistence of superconductivity and altermagnetism markedly enriches the EP conditions compared with the simpler FM-AM case [29,30]. We aim to identify the parameter regimes supporting EPs in the $(E-\phi)$ spectrum, with $\phi = \phi_L - \phi_R$ the superconducting phase difference. The combined action of altermagnetism and RSOC introduces internal tuning knobs, e.g., $k_{x,y}$, RSOC strength, altermagnetic vector amplitude, and altermagnetic wave symmetry, that control both the number and position of EPs. Almost all of these variations can be achieved without external magnetic fields.

Figures 2(a1) and 2(b1) show density plots of the real and imaginary parts of the energy difference $\Delta E = E_2 - E_1$ (E_2 and E_1 are two branches of the ABS energies) as functions of ϕ and k_x when the altermagnetic field is switched off due to setting $k_y = 0$. The border line of the zero-valued region [purple interval in Fig. 2(a1) which is surrounded by white-dashed line] identifies the (k_x, ϕ) points at which the real and imaginary parts of the ABSs simultaneously merge; a manifestation of EPs. A comparison between Figs. 2(a1) and 2(b1), indicates whenever the real part becomes nonzero, the

imaginary one vanishes, and vice versa, a contrasting trend widely observed in NH systems containing FM [29,30,39,40].

Evidently, increasing $|k_x|$ at $k_y = 0$ reduces the separation between pairs of EPs corresponding to the same k_x due to the RSOC manipulation, resulting in an elliptical zero-contour profile. However, the symmetrical behavior of the energy difference with respect to ϕ remains unchanged over all values of k_x .

To further illuminate this behavior, the linear ABS spectra as functions of ϕ are displayed in Figs. 2(c1) and 2(c2) for different values of k_x . As k_x grows, the phase values at which the real and imaginary components of the energy coalesce shift, consistent with the trends observed in Figs. 2(a1) and 2(b1), confirming the feasibility of EP engineering through control of the crystal orientation, even in the absence of altermagnetic manipulation, thanks to the RSOC.

Notably, our calculation describes a true junction geometry, and k_x is used only as a nominal momentum parameter of the central region: Each panel in Fig. 2 is obtained for a fixed k_x , providing a momentum-resolved view of the junction spectrum rather than a bulk band structure. Thus, varying k_x across density plots serves only as a probe of how the EPs evolve through momentum space rather than scanning k_x continuously.

In a d_{xy} AM, setting either k_x or k_y to zero eliminates the altermagnetic contribution, such that the spectrum is governed solely by RSOC. We now demonstrate, however, that these effects—together with additional tunable features—remain accessible for finite momenta $k_x, k_y \neq 0$. In particular, for $k_x = k_y = 0$, the system reduces effectively to a conventional SC-normal-SC junction without altermagnetic or Rashba perturbations, analogously to the setup studied in Ref. [38]. The extra terms retained in our full Hamiltonian induce minor quantitative modifications of the $(E-\phi)$ dispersion (see Sec. B of the Supplemental Material for details [47]). Importantly, the central results—namely, the emergence of EPs in this structure and their manipulation—remain robust against these corrections.

Figures 2(a2) and 2(b2) present the density plots of the energy difference as a function of ϕ at $k_y = 0.2$. Introducing a nonzero k_y activates the altermagnetic term, which in turn generates four EPs, in contrast to the two observed in the $k_y = 0$ case. A comparison between the density plots in Figs. 2(a1) and 2(b1) and Figs. 2(a2) and 2(b2) reveals that in the presence of RSOC, the AM induces an additional nonzero (zero) feature in the real (imaginary) part of the energy difference, thereby creating more opportunities for the emergence of EPs.

For a fixed nonzero k_y , increasing $|k_x|$ broadens this newly opened nonzero region in $\text{Re}[\Delta E]$, pushing the corresponding EP lines farther apart. At the same time, the EPs located on each side of the nonzero window move closer together, forming a butterfly-like pattern, as visible in Figs. 2(a2) and 2(b2). The distinct roles played by k_x and k_y in shaping the ABS spectrum originate from the different commutation relations of the Pauli matrices appearing in the Hamiltonian. The impact of altermagnetic manipulation on the number and location of EPs can be further observed and confirmed in linear plots of Figs. 2(d1) and 2(d2).

In the absence of AM, this additional nonzero window does not appear; therefore, the AM can modify both the number and the positions of the EPs. Activating the AM introduces a perpendicular component to the effective field (which, in the absence of AM, lies solely in the x - y plane due to the RSOC term), thereby opening a new gap in the ABS spectrum as a function of ϕ .

In Fig. 2(e) we compute the total supercurrent $[I_{\text{tot}}(\phi)]$ using Eq. (11). Despite the divergent phase-derivatives of certain ABS branches [Eq. (12)], the total supercurrent obtained from Eq. (11), which includes different transverse k_y channels, shows a smooth dependence on ϕ , with zero crossings at $\phi = n\pi$, generally consistent with Refs. [42,44]. The localized singularities associated with ABS phase derivatives at EPs (cf. Fig. S2 of Supplemental Material [47]) are largely averaged out when integrating over transverse modes, resulting in a robust macroscopic current that exhibits only a slight phase shift and depends moderately on the longitudinal momentum k_x .

Although varying k_x neither alters the positions of the $I(\phi) = 0$ crossings nor changes the current polarity, it (i) reduces the overall current amplitude and (ii) induces a slight shift of the peak positions. These effects reflect the role of RSOC in the absence of altermagnetic manipulation. Indeed, in this limit the modification originates solely from the RSOC term, due to the absence of t_j .

As shown in Fig. 2(f), which displays the total supercurrent obtained from Eq. (11) as a function of ϕ , activating the AM slightly modifies both the amplitude and the pattern of $I_{\text{tot}}(\phi)$. These features are sensitive to the wave-vector components. Similarly to the $t_j = 0$ case, increasing k_x in the presence of nonzero t_j slightly shifts the peak positions and modifies their amplitudes.

Although the current-phase relation for a fixed and sufficiently large transverse momentum k_y is strongly anharmonic and markedly nonsinusoidal (not shown), the integration over k_y suppresses higher harmonics, rendering the total supercurrent nearly sinusoidal. Similarly to the effect of k_x , varying t_j also leads to slight changes in both the positions and magnitudes of $I_{\text{tot}}(\phi)$. This is evident from a comparison of Figs. 2(e) and 2(f), which indicates that increasing t_j enhances the maximum of $I_{\text{tot}}(\phi)$. This enhancement can be attributed to the enlargement of k_{max} with increasing t_j , an effect that is also present at $k_x = 0$, where the AM is absent.

Despite global symmetries governing in conventional NH systems, for the d_{xy} altermagnetic order, which transforms according to the B_{2g} irreducible representation of the tetragonal point group, the behavior of the EPs is governed by how crystalline symmetries constrain the momentum dependence of the effective Hamiltonian. Because $k_x k_y$ changes sign under the vertical mirrors $M_x : (k_x, k_y) \rightarrow (-k_x, k_y)$ and $M_y : (k_x, k_y) \rightarrow (k_x, -k_y)$, the altermagnetic field vanishes along the coordinate axes where either $k_x = 0$ or $k_y = 0$. Along these high-symmetry lines the altermagnetic spin splitting is suppressed and the remaining symmetry prevents the appearance of altermagnetism induced EPs; only the nonaltermagnetic ones survive. RSOC modifies the spin texture but does not alter the symmetry condition that forces the altermagnetic field to vanish. At $(k_x, k_y) = (0, 0)$, the momentum is invariant under the full C_{4v} point-group symmetry and both the altermagnetic field and the RSOC vanish at this point, and the junction effectively reduces to the symmetric SC-normal-SC limit (cf. Fig. S1 in the Supplemental Material [47]). The EP therefore originates from the intrinsic NH coupling to the reservoirs rather than from the altermagnetic field itself, and it is protected by the complete restoration of rotational and mirror symmetries. For $k_x \neq 0$ and $k_y \neq 0$, none of the operations M_x , M_y , or C_{4v} leave the momentum invariant. In this regime the B_{2g} altermagnetic field becomes finite, while RSOC simultaneously produces a momentum-dependent spin texture. The coexistence of these two spin-dependent terms lifts the spin degeneracy in a highly anisotropic manner and allows the ABSs to coalesce at isolated momentum points. Away from the high-symmetry axes these EPs split and evolve in momentum space, forming the butterfly-like structure observed in Fig. 2.

Next we investigate the influence of AM on the system and the tunability of EPs via AM- or NH-related parameters. Figure 3 presents density plots of $\text{Re}[\Delta E]$ versus t_j and ϕ for different values of $\Gamma_{\uparrow}$. The dark-purple regions in Fig. 3(a), which is plotted for $\Gamma_{\uparrow} = 2$, indicate the intervals of zero real energy difference, at the boundaries of which EPs may emerge. When the altermagnetic field is absent ($t_j = 0$), two EPs appear at symmetric phase differences, due to non-Hermiticity. The number of these EPs remains unchanged, but

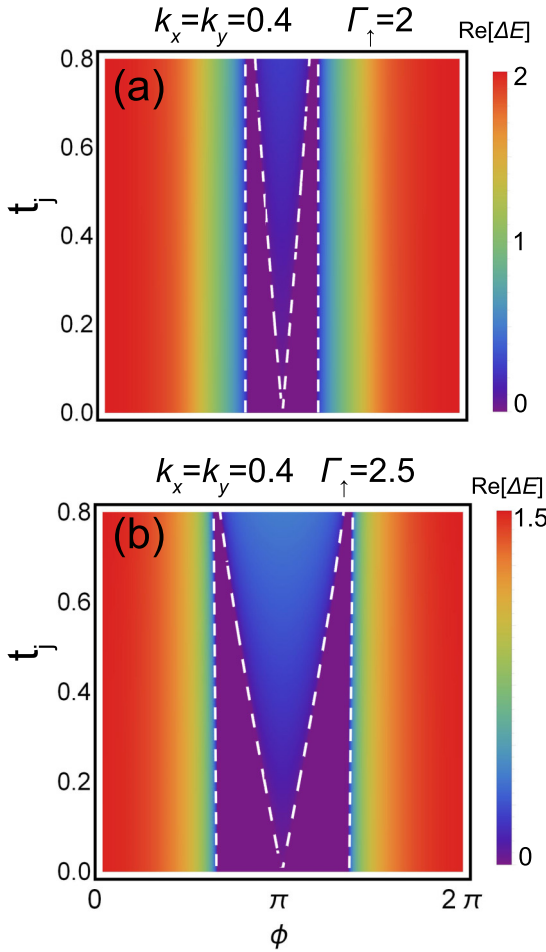


FIG. 3. Density plots of real energy difference $\text{Re}[\Delta E]$ as a function of altermagnetic vector strength t_j and ϕ for (a) $\Gamma_{\uparrow} = 2$ and (b) $\Gamma_{\uparrow} = 2.5$. In both panels $k_x = k_y = 0.4$ and other parameters are the same as Fig. 2. The white-dashed guide lines illustrate the variation of the EP position, schematically.

their position could be controlled via RSOC, even when AM is absent, cf. Fig. 4.

Increasing t_j introduces an additional nonzero interval (the triangular dark-blue region encircled by the white-dashed contour) leading to four EPs. With stronger altermagnetism, the EPs approach each other in pairs while the pairwise separation enlarges. Two of these EPs, indicated by the straight vertical white-dashed lines, are independent of t_j and remain unaffected by AM since they mainly originate from NH effects combined with RSOC tuning; they are therefore termed non-altermagnetic EPs. In contrast, the altermagnetic perturbation generates two additional EPs located between the nonaltermagnetic ones, whose mutual separation can be tuned through t_j . Clearly, choosing different values of k_x and k_y modifies the EP pattern in the $(t_j - \phi)$ plane, as illustrated and discussed in Fig. S5 of the Supplemental Material in detail [47].

In the proposed JJ, NH modulation also influences the effects of altermagnetism. To illustrate this, Fig. 3(b) shows a density plot of $\text{Re}[\Delta E]$ in the $(t_j - \phi)$ plane for $\Gamma_{\uparrow} = 2.5$. A similar trend is observed as in Fig. 3(a); however, a higher non-Hermiticity factor broadens the zero-value region at $t_j =$

0, thereby increasing the separation between nonaltermagnetic EPs. As in Fig. 3(a), increasing t_j opens a nonzero interval and raises the number of EPs to four. Indeed, the higher non-Hermiticity separates the nonaltermagnetic EPs more and therefore controls the EP location thanks to altermagnetic modifications, demonstrating that the EPs can be tuned via both NH- and AM-related parameters.

In addition to modifying the altermagnetic effects, the non-Hermiticity strength itself can also shift the positions of EPs in both cases; with and without AM. To further clarify this, Figs. 4(a) and 4(b) present density plots of $\text{Re}[\Delta E]$ versus $\Gamma_{\uparrow}$ and ϕ for $t_j = 0$ and $t_j = 0.5$, respectively. Clearly, the strength of non-Hermiticity significantly affects the EP positions even at $t_j = 0$, while the number of EPs remains unchanged, indicating that the higher number of EPs mainly originates from the altermagnetic manipulation. Specifically, increasing $\Gamma_{\uparrow}$ nonlinearly separates the EPs by widening the zero-valued regions. When $\Gamma_{\uparrow}$ is sufficiently large, the zero-profile extends over the entire $0-2\pi$ interval, whereas for negligibly small $\Gamma_{\uparrow}$, the zero-profile closes, indicating the absence of EPs in Hermitian regime (not shown here). In this case ($k_x = k_y = 0.4$), the disappearance of EPs occurs at higher $\Gamma_{\uparrow}$ ($\Gamma_{\uparrow} \approx 1.1$), due to the contribution of other energy scales raised by AM and RSOC.

These observations are further supported by the linear plots in Figs. 4(c1) and 4(c2), which display the real and imaginary ABSs as functions of ϕ for two representative values of $\Gamma_{\uparrow}$. As $\Gamma_{\uparrow}$ increases, the zero-profile of the real energy broadens, and the nondegenerate imaginary profile also expands. Additionally, the maximum value of the real ABS at $\phi = 0$ decreases with increasing non-Hermiticity, while the imaginary part generally enhances, which can be attributed to the more loss features at higher values of non-Hermiticity.

When the altermagnetic field is switched on, a similar trend is observed upon varying $\Gamma_{\uparrow}$. However, a nonzero t_j introduces a new nondegenerate (degenerate) window in the real (imaginary) ABSs, consistent with the behavior seen in Figs. 2 and 3. This results in four EPs, whose positions can be tuned via $\Gamma_{\uparrow}$, in the way that increasing $\Gamma_{\uparrow}$ can enhance the separation of the EPs. A clearer representation of this effect is provided by the linear ABS plots shown in Figs. 4(d1) and 4(d2), which show that growing $\Gamma_{\uparrow}$ widens the middle circle in real ABS diagram and also widens the zero-profile around it, consequently separating the EPs.

To investigate the effect of RSOC on the EPs, Figs. 4(e) and 4(f) show density plots of $\text{Re}[\Delta E]$ as functions of λ and ϕ for $t_j = 0$ (nonaltermagnetic) and $t_j \neq 0$ (altermagnetic) cases, respectively. When AM is absent, increasing RSOC reduces the separation between nonaltermagnetic EPs by narrowing the zero profile of the real ABS energies and the nondegenerate window of the imaginary ABS components. This effect is clearly seen by comparing Figs. 4(g1) and 4(g2), where increasing λ from 0.6 to 1.2 brings the EPs closer together. Interestingly, the EPs can be found even at $\lambda = 0$, indicating that the RSOC is not necessary for EP emergence in nonaltermagnetic JJ, coinciding with Ref. [38].

When AM is present ($t_j \neq 0$), an additional nondegenerate profile appears in the ABS spectrum, increasing the number of EPs [see Fig. 4(f)]. In this case, varying RSOC changes the EP separations via two mechanisms: first by modifying

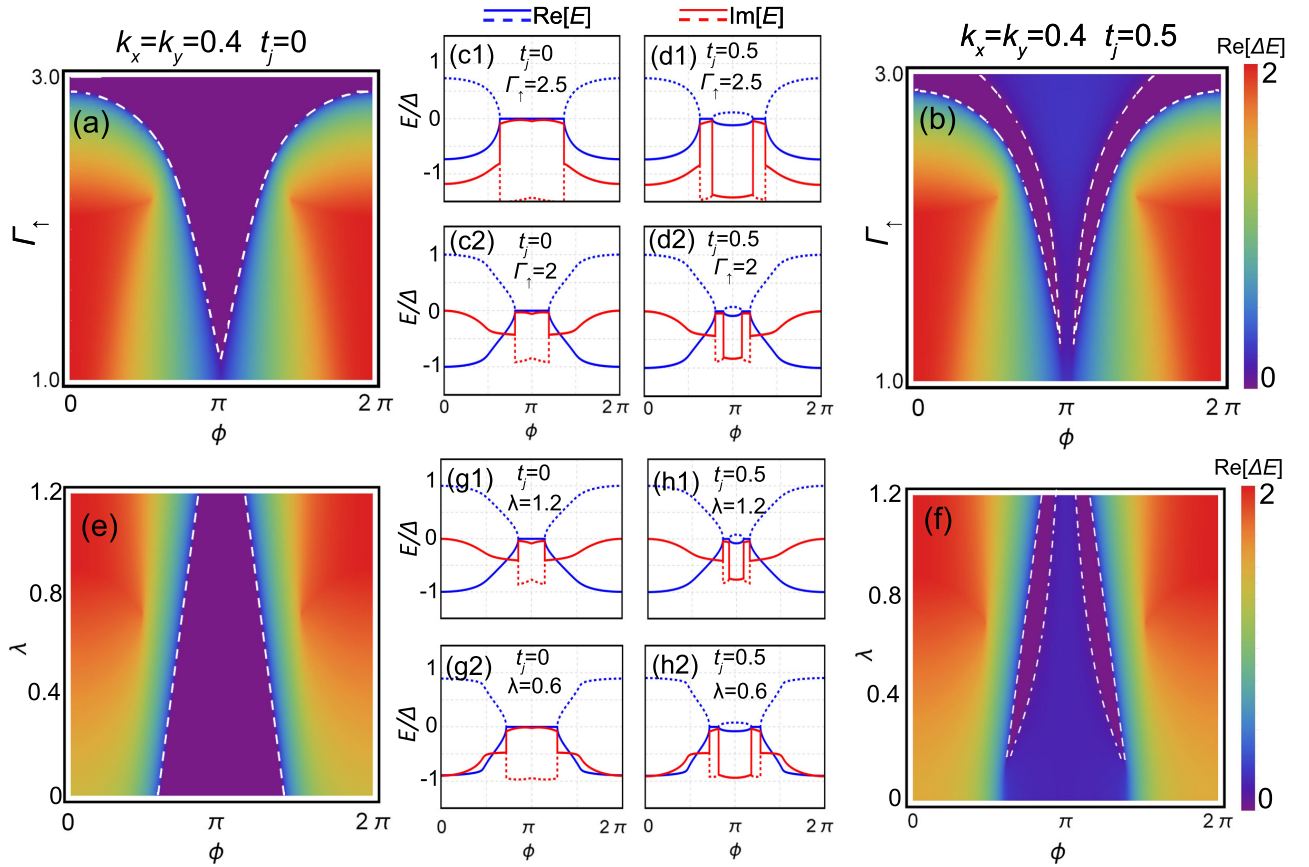


FIG. 4. Density plots of real energy difference $\text{Re}[\Delta E]$ as a function of [(a) and (b)] $\Gamma_{\uparrow}$ and ϕ and [(e) and (f)] λ and ϕ . The k_x and k_y are fixed at 0.4 and t_j is labeled in each panel. Other parameters are the same as Fig. 2. The white-dashed lines in the density plots illustrate the variation of the EP position, schematically. Linear plots of [(c1), (c2), (d1), and (d2)] show the ABSs versus ϕ at [(c1) and (d1)] $\Gamma_{\uparrow} = 2.5$ and [(c2) and (d2)] $\Gamma_{\uparrow} = 2$, confirming the behavior seen in (a) and (b). Linear plots of [(g1), (g2), (h1), and (h2)] show the ABSs versus ϕ at [(g1) and (h1)] $\lambda = 1.2$ and [(g2) and (h2)] $\lambda = 0.6$, confirming the behavior seen in (e) and (f).

the nondegenerate (degenerate) window in the real (imaginary) diagram and, second, by varying the distance between altermagnetic and nonaltermagnetic EPs. These two effects can be seen clearly in linear plots presented in Figs. 4(h1) and 4(h2) where both the nonzero circle and its surrounding zero lines are modified by λ , indicating the tunability of the EPs via this parameter in the proposed JJ. Noting that, since the nonaltermagnetic EPs [indicated by the white-dashed lines in Figs. 4(a) and 4(e)] depend on the presence and strength of non-Hermiticity and RSOC, these EPs can be effectively tuned by these two parameters, in contrast to the behavior observed for t_j in Fig. 3. Interestingly, in the altermagnetic JJ, when RSOC is weak enough, the EPs disappear, indicating that the emergence of EPs in this structure requires a finite RSOC. This is also found in nonaltermagnetic Rashba semiconductors [28]. Reducing t_j , reduces the required RSOC for the EP emergence, until the number of EPs is reduced from 4 to 2, which belong to the NH structure (*not shown here*).

B. JJ with $d_{x^2-y^2}$ AM

In this section, we show that the other AM-wave symmetry, $d_{x^2-y^2}$, can also host tunable EPs, with positions distinct from those found in the d_{xy} wave, highlighting the possibility of

controlling the EP properties via wave changing. To illustrate this, Fig. 5 presents linear plots of the ABS energies as a function of ϕ for different waves.

As seen in Figs. 5(a) and 5(b), in a $d_{x^2-y^2}$ altermagnetic JJ, switching on only k_x (even at infinitesimal values) destroys the ABSs. This behavior can be attributed to the competition between the subspaces, which drives the corresponding energies outside the range supporting ABS formation. In contrast, turning on k_y alone allows the EPs to be tuned. More precisely, the key difference between the d_{xy} and $d_{x^2-y^2}$ symmetries lies in how the RSOC, altermagnetic, and superconducting terms populate the $\sigma_i \tau_j$ operator basis, and whether these operators can provide the required conditions for EP emergence. The absence of EPs in the $k_y = 0$ case in $d_{x^2-y^2}$ symmetry mainly originates from the fact that RSOC and the superconducting pairing share the same spin structure (σ_y) and act along three mutually noncommuting Nambu directions (τ_x, τ_y, τ_z), while the altermagnetic terms quadratically ($\propto k_x^2$) act on the $\sigma_z \tau_z$ space and remain algebraically independent of this set. The commutation rules between these spaces prevent the Hermitian Hamiltonian from collapsing into a situation in which the competition between Hermitian and NH parts leads to EP emergence. In contrast, on the $k_x = 0$ axis the RSOC, altermagnetic, and superconducting terms occupy mutually

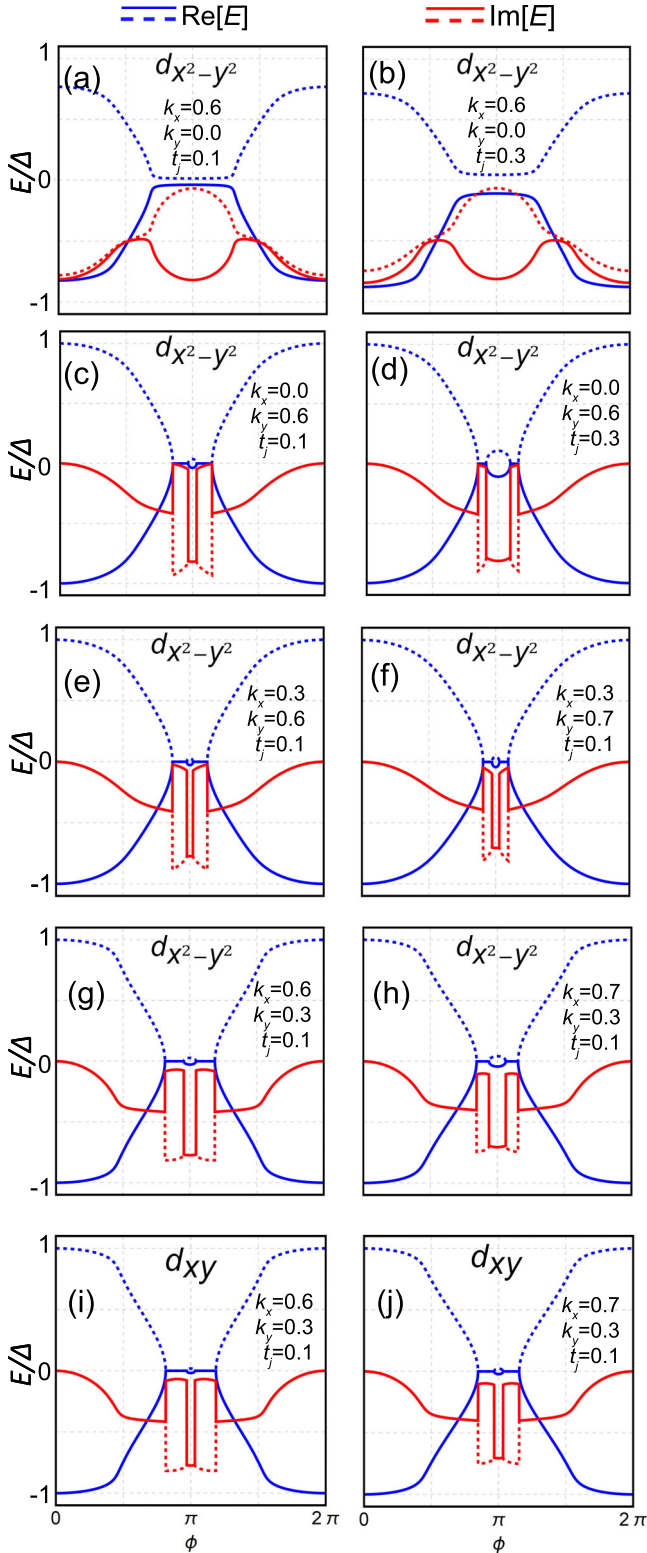


FIG. 5. The linear plots of ABSs with respect to ϕ for [(a)–(h)] $d_{x^2-y^2}$ and [(i) and (j)] d_{xy} . The other fixed parameters are the same as Fig. 2, unless they are labeled.

orthogonal spin directions. In this reduced setting the NH contribution can compete with the Hermitian terms in a way that permits eigenvalue coalescence, thereby allowing EPs to emerge. As seen in Figs. 5(c) and 5(d), increasing t_j at

$k_x = 0$ and $k_y \neq 0$ in this wave enlarges the nonzero profile of the real ABSs (the central circle) while simultaneously narrowing the zero-profile region, thereby bringing the EPs closer together.

When both k_x and k_y are nonzero, the JJ exhibits behavior similar to the d_{xy} case, with only minor differences. This is illustrated in Figs. 5(e)–5(h), which show the ABS spectra as a function of ϕ for different values of k_x and k_y . The following observations can be made:

(i) Comparing Figs. 5(c) and 5(e) shows that activating k_x does not destroy the ABSs when k_y is finite. Increasing k_x instead brings the EPs closer by closing the nondegenerate central circle of $\text{Re}[E]$.

(ii) Comparing Figs. 5(e) and 5(f) indicates that increasing k_y at $k_x \neq 0$ brings outer EPs closer, while further enlarging the nondegenerate central circle.

(iii) Comparing Figs. 5(g) and 5(h) shows that when $k_x > k_y$, increasing k_x expands the central nondegenerate circle in the real ABS spectrum while leaving the outer EPs nearly unchanged.

Comparing Figs. 5(i) and 5(j) with Figs. 5(g) and 5(h) reveals a strong similarity between the two waves. The main difference is the position of the EPs, which can be traced to the different orders of k_x and k_y in the altermagnetic terms of two different waves. Another distinction in this wave is that turning on the $d_{x^2-y^2}$ does not need both $k_x \neq 0$ and $k_y \neq 0$, and setting only k_y to a nonzero value is sufficient. Since higher-order AMs (such as g -wave AM) can host a larger number of nodal structures in their Fermi surfaces [29,49], they may exhibit distinct EP patterns and multiplicities, which could motivate future investigations of altermagnetic JJs.

Notably, all calculations in this work are performed at zero temperature ($T = 0$). At finite temperature, however, the supercurrent in Eq. (9) is modified due to thermal occupation of quasiparticle states governed by the Fermi-Dirac distribution. In this case, the supercurrent can be expressed as

$$I(\phi, \beta) = \frac{2}{\beta} \frac{d}{d\phi} \text{ReTr} \left[\ln \left[\Gamma \left(\frac{1}{2} + \frac{i\beta}{2\pi} H_{\text{eff}} \right) \right] \right], \quad (13)$$

where $\beta = 1/k_B T$ [42]. This formulation shows that increasing temperature generally suppresses the supercurrent. A similar conclusion has been obtained using an alternative approach in Ref. [44]. Nevertheless, as long as the temperature remains well below the critical temperature of the layers, the ABSs underlying the EP structure remain well defined. Therefore, the phase shift and sensitivity predicted here are expected to remain qualitatively robust at low but finite temperatures, although their magnitude or pattern may be changed due to thermal effects.

IV. CONNECT TO EXPERIMENT

Although an experimental realization of our setup has not yet been implemented, several of its key ingredients have been reported in the literature. Altermagnetism has been predicted or observed in materials such as RuO_2 , Mn_5Si_3 , and MnTe [5,7–9,50]. Josephson junctions with controlled crystallographic orientations have been realized experimentally [51], and orientation-dependent transport has been extensively studied. Signatures of ABSs might be probed via scanning

tunneling spectroscopy [52,53] or conductance measurements [54], while phase-biased Josephson transport can be implemented following approaches developed in SC-semiconductor platforms [55,56].

Junctions involving FMs and SOC semiconductors such as InAs [57] and InSb [58] have been experimentally fabricated, suggesting that analogous FM-AM interfaces may be achievable, although significant materials optimization and interface engineering will likely be required. In our effective description, the spin-dependent coupling strengths $\Gamma_{\uparrow,\downarrow}$ are tunable via both the spin-resolved density of states in the ferromagnetic lead and the tunneling probability across the interface [38,59]. However, a fully quantitative connection between microscopic device design and the effective couplings cannot be captured in our simplified model; here we treat them as phenomenological parameters to capture the essential mechanism.

We have also neglected disorder effects in our model and assumed reasonably clean interfaces. Nevertheless, disorder and interface imperfections may influence the quantitative properties of the JJ and could modify the detailed EP characteristics, which may not be applicable in this simplified model. Although external control of altermagnetic phases and properties remains challenging, recent works have suggested promising routes toward their external tuning [60–62], while the RSOC considered here can also be electrically controlled [30]. Our work does not rely on any specific tuning protocol; instead, it highlights how changes in system parameters would be reflected in the Andreev spectrum and Josephson response, which might serve as a qualitative guideline.

V. CONCLUSIONS

In conclusion, we investigated orientation-dependent exceptional points in non-Hermitian d -wave altermagnetic Josephson junctions, demonstrating that the junction serves as a versatile platform for realizing and controlling these

exceptional points through the interplay of non-Hermiticity, superconductivity, Rashba interaction, and altermagnetism. Owing to the orientation-dependent spin splitting and the absence of net magnetization intrinsic to altermagnets, the Andreev bound states acquire a pronounced spin- and orientation-dependent structure that is fundamentally distinct from that of conventional Josephson junctions. This enables a high degree of tunability of exceptional points through accessible parameters, including the crystal orientation, the strength of the altermagnetic order, Rashba spin-orbit coupling, superconducting phase difference, and the degree of non-Hermiticity. In particular, we find that both the location and the number of exceptional points can be directly engineered by the altermagnetic properties of the junction. Unlike the divergent phase-derivatives associated with individual Andreev bound state branches, the total Josephson current remains smooth. Although the exceptional points induce pronounced non-Hermitian bifurcations in the Andreev spectrum, their influence on the physical Josephson current remains modest. Accordingly, the most direct experimental signatures of the exceptional points may not be easily found in the current-phase relation itself. Our results point to altermagnetic Josephson junctions as a promising setting for engineering non-Hermitian degeneracies and exploring unconventional superconducting transport and topological phenomena in altermagnetic systems.

ACKNOWLEDGMENTS

W.B. was supported by Deutsche Forschungsgemeinschaft (DFG; German Research Foundation) through Project ID No. 417034116 (SPP2244) and Project ID No. 465140728.

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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