

Current-driven plasmons in two-layer semiconductor heterostructures: Long-wavelength features and two-stream instability

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(Received 2 April 2025; accepted 18 September 2025; published 14 October 2025)

In this work, the plasmonic oscillations in two-dimensional charge-coupled semiconductor layers, including those biased by an external current, have been thoroughly investigated by the hydrodynamic model. The relative amplitudes of the charge-density oscillations and their phase shift in the two coupled layers have been calculated. For the long wavelengths important in experimental studies, it has been found that the electron relaxation radically modifies the mode types and dispersion, providing linear portions of the wave-vector dependencies followed by a square root branching. We have shown that the hydrodynamic approach does not predict any instability for drifting electrons in two coupled identical layers, while plasmonic instability is possible for the layers with distinct characteristics. For these cases, there exists a narrow (ω, k) region with strong (resonant) interlayer interaction, where the real parts of $\omega(k)$ of the plasmon modes are identical and linear in k (a second linear portion). The imaginary parts are different and one of them can change sign, leading to the plasmonic instability effect. This instability develops as the two-stream convective instability, whose threshold and frequency critically depend on the velocity of drifting electrons. Under negligibly weak electron relaxation, there is no drift velocity threshold for the revealed instability, while electron relaxation originates the finite velocity threshold. The numerical estimations of the instability critical parameters are provided for the GaAs-based heterostructures. We suggest that this investigation of charge-coupled current-biased semiconductor layers brings the new knowledge to solid-state plasmonic physics and can be used in the practical search of terahertz electric instabilities and sources of terahertz emission.

DOI: [10.1103/PhysRevB.112.165306](https://doi.org/10.1103/PhysRevB.112.165306)

I. INTRODUCTION

Development of terahertz (THz) wave technologies aims to close the gap between infrared optics and microwave electronics. This is a difficult task because, to approach the THz band from the microwave domain, the frequency range of electronic devices must be very essentially extended, while to approach it from infrared photonics is challenging because of unusually low photon energies and long wavelengths (in free space) relevant for this frequency band. Thus, conceptions of both electronic and photonic devices should be renovated. Plasmons, fundamental wave-like excitations of charges of electrons coupled with electromagnetic fields, open a way to bridge this “THz gap.” A number of works devoted to the plasmons in conductive media, including semiconductor materials and heterostructures, constitute a new research area—plasmonics [1–4]. For THz plasmonics, especially important are surface and two-dimensional (2D) plasmons, which have been observed and studied in silicon, diamond, III-V and GaN-based heterostructures, graphene, and other 2D materials. These electronic excitations demonstrate a number of new physical phenomena, particularly, a confinement of

electromagnetic energy in subwavelength volumes compared with the free-space waves. The important advantage of the use of the 2D plasmons is the possibility to control the electron concentrations and, thus, plasmon frequencies. All of these features dictate a great interest in 2D plasmonic devices, such as THz detectors, mixers, emitters, etc. [5,6].

As for plasmonic-based emitters, the present achievements in this area are mainly associated with selective THz emission [7,8] originated from the radiative decay of 2D plasmons of nonequilibrium electrons under the grating (see also Refs. [9,10]). The development of electrically driven plasmonic coherent THz sources still remains the great challenge. One of the possibilities of amplification and generation of THz radiation lies in the use of the effects of electrically induced plasmon instabilities.

Conventionally, different mechanisms of such instabilities can be classified as follows: (i) Dyakonov-Shur instabilities in field-effect transistors [11,12] and diodes [13] with specific properties of the source and drain of the devices; (ii) Ryzhii-Satou-Shur and other transit-time instabilities, which arise for the ballistic [14–16] or quasiballistic [17] electron transport; (iii) Cherenkov-type plasmon instability [18–20]; (iv) two-stream instabilities [21–25], including instability arising for the reststrahlen frequency band in drifting plasmon-optical phonon system [26–31]. The listed instabilities are expected in short or nonuniform, spatially modulated channels. Moreover, it is accepted that in the uniform plasma with a single mobile charged component (the electrons in the case under

consideration) and with Maxwellian or shifted-Maxwellian distributions there are no instabilities [32–34].

Recent progress in the technology of 2D semiconductor materials has led in the production a variety of artificial nanosystems consisting of single-, double-, and multiquantum wells; single, double, or stacked atomically thin layers assembled into the whole heterostructure [35]. The intrinsic properties of such heterostructures become dependent on the neighboring conductive layers coupled by the electrostatic potential, interlayer transitions, and dielectric environment. This offers opportunities for convenient control of their properties in creating perspective functionalities relevant for potential applications in electronics, optics, and optoelectronics. Particularly, much research work has been devoted to plasmonic nanostructures with two spatially separated layers of various materials and configurations [23,24,36,37], which possess tunable kinetic properties and can meet the conditions of plasmonic instability. General analysis of properties of charge-coupled 2D plasmonic layers is the goal of this paper.

Previously, for two interacting plasmonic layers, it has been shown the occurrence of two plasmon modes $\omega = \omega(k)$, which dispersion relations in the long-wavelength limit are described by a square root $\omega_{\text{op}}(k) \sim \sqrt{k}$ and a linear $\omega_{\text{ap}}(k) \sim k$ dependence on the wave vector $\mathbf{k}$ ($k = |\mathbf{k}|$). In the plasmon hybridization picture, the carrier densities in the two layers oscillate in phase for the optical (symmetric) mode $\omega_{\text{op}}(k)$ and *out of phase* for the acoustic (antisymmetric) mode $\omega_{\text{ap}}(k)$. The mode spectra possess the reciprocal property, they are twofold degenerate, $\omega(\mathbf{k}) = \omega(-\mathbf{k})$. The frequencies of these modes depend on the carrier densities in the two layers n_1 and n_2 and the strength of interlayer coupling $\varepsilon(d)$, where d is the interlayer distance. When the two plasmonic layers are biased by the DC electric current, the number of distinct hybridized plasmon modes is doubled, $\omega(\mathbf{k}) \neq \omega(-\mathbf{k})$, due to the nonreciprocal character of the drifting-electron motion. Each of the modes demonstrates a Doppler shift, given by the $\mathbf{k} \cdot \mathbf{v}$ term, as well as an additional, nontrivial dependence on the electron velocity $\mathbf{v}$ caused by the interlayer coupling effect. The latter, particularly, results in the development of a two-stream convective instability upon increasing the drift velocity.

For the theoretical description, we use the set of the time-dependent hydrodynamic equations [11], which includes the equation of continuity of the carrier flow density $\mathbf{i} = n\mathbf{v}$, the equation of motion for the electrons moving with drift velocity $\mathbf{v}$, and the Poisson equation for the electrostatic potential, which couples the electron motion in both layers. Letting small variations for all the variables in the form of Fourier components ($\sim e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}$), with real wave vector $\mathbf{k}$ and complex frequency $\omega(\mathbf{k}) = \omega' + i\omega''$, and using the standard linearization procedure in the basic equations [11], we obtain the dispersion equation. The solution of this equation represents the modes of collective self-oscillations of the electron density and induced electric field. Their properties, particularly dependencies on the dc current, are analyzed below in detail.

The paper is organized as follows: In Sec. II, we describe the hydrodynamical model with the set of equations used to calculate the existing plasmon modes in a multilayer heterostructure. Section III provides analytical solutions of the dispersion equation and a thorough analysis of the obtained

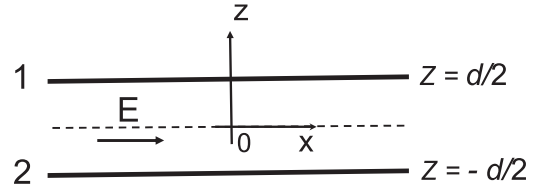


FIG. 1. Schematic of a two-layer nanostructure: layers 1 and 2 represent 2D electrons in the (x, y) plane separated by an insulating region of the width d . The electric field $\mathbf{E}$ is applied to the layers in the x direction.

plasmon modes in current-biased two-layer nanostructures. The peculiarities of long-wavelength dispersion and two-stream instability are analyzed in detail. In Sec. IV, numerical estimations are given of characteristic parameters of two-layer GaAs- and GaN-based heterostructures optimized for experimental studies of plasmons, including the plasmonic instability induced by the electric current. Section V briefly discusses and summarizes the results obtained.

II. THEORETICAL MODEL

For a multilayer heterostructure consisting of N nanolayers, the temporal evolution of spatially inhomogeneous variations of the electron densities $\delta n^{(\alpha)}(\mathbf{r}, t)$, $\alpha = 1, \dots, N$, is governed by the set of N coupled continuity equations:

$$\frac{\partial}{\partial t} \delta n^{(\alpha)}(\mathbf{r}, t) + \nabla_{\mathbf{r}} \cdot \delta \mathbf{i}^{(\alpha)}(\mathbf{r}, t) = G_{\alpha}[\delta n^{(\alpha \pm 1)}(\mathbf{r}, t)] - R_{\alpha}[\delta n^{(\alpha)}(\mathbf{r}, t)], \quad (1)$$

and the Poisson equation

$$\Delta \delta \varphi(\mathbf{r}, z, t) = \frac{4\pi e}{\epsilon_0} \sum_{\alpha=1}^N \delta n^{(\alpha)}(\mathbf{r}, t) \delta \left[z + (2\alpha - 3) \frac{d}{2} \right]. \quad (2)$$

Here, $\delta \varphi$ and $\delta \mathbf{E} = -\nabla \delta \varphi$ are variations of the electrostatic potential and electric field, respectively; the fluctuating electric field, $\delta \mathbf{E}^{(\alpha)}(\mathbf{r}, t)$, can be different in different nanolayers. $\mathbf{E}$ is the applied electric field, ϵ_0 is the dielectric permittivity of environment, $-e$ is the electron charge ($e > 0$), and $\mathbf{r} = (x, y)$ is the in-plane position vector. In Eq. (2), the Dirac δ function, $\delta(z)$, takes into account the 2D character of the electrons in nanolayers, assembled perpendicular to the z axis at equal distance d between adjacent layers (Fig. 1). The carrier motion in the neighboring layers is coupled through the electrostatic potential and possible carrier exchange. The latter is described by effective transition rates, represented by the two terms (G_{α}, R_{α}) on the right-hand side of Eq. (1).

In the absence of an external electric field ($\mathbf{E} = 0$), local electron-density excitations always decay over time. If the electric field $\mathbf{E}$ is applied, then the electron excitations drift and decay; the corresponding spatial-time Fourier components have the form of propagating and decaying waves. The specific character of these waves is determined by the dominant transport regime. We consider two regimes of electron transport differing by the intensity of dissipation processes and representing essentially distinct expressions for the electron flow density, $\delta \mathbf{i}^{(\alpha)}(\mathbf{r}, t)$, in Eq. (1). (I) Quasiballistic

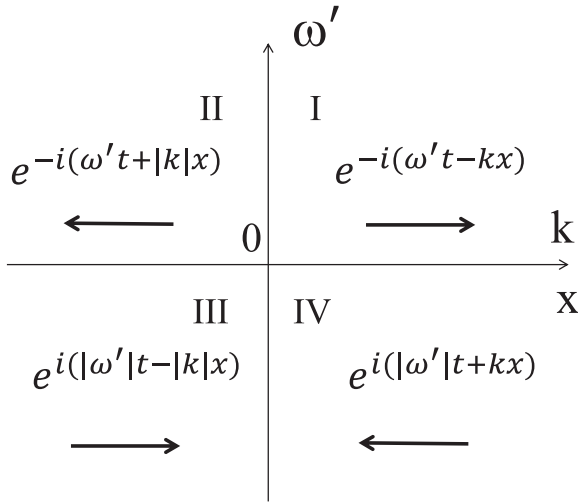


FIG. 2. Schematic diagram of plasmon wave propagation relative to an electric field applied in the x direction. The arrows indicate the direction of wave ($\approx e^{i(kx - \omega't)}$) propagation, with frequency ω' and wave vector k , in different quadrants (I–IV) of the plane (k, ω') .

transport, under which the electron density self-oscillations are weakly decaying plasmons. In the electric field $\mathbf{E}$, these plasmons can be described by the hydrodynamic approach [11]. (II) Drift-diffusion electron transport, under which the oscillating waves are dissipative self-oscillations of the electron charge density and electric field. For quasiballistic transport, the electron flow density is expressed as $\mathbf{i}^{(\alpha)} = n^{(\alpha)}\mathbf{v}^{(\alpha)}$, so that

$$\delta \mathbf{i}^{(\alpha)}(\mathbf{r}, t) = \mathbf{v}^{(\alpha)} \delta n^{(\alpha)}(\mathbf{r}, t) + n^{(\alpha)} \delta \mathbf{v}^{(\alpha)}(\mathbf{r}, t) \quad (3)$$

for the drift-diffusion transport, and we can write

$$\delta \mathbf{i}^{(\alpha)}(\mathbf{r}, t) = -\mu_\alpha [\mathbf{E} \delta n^{(\alpha)}(\mathbf{r}, t) + n^{(\alpha)} \delta \mathbf{E}^{(\alpha)}(\mathbf{r}, t)] - D_\alpha \nabla \delta n^{(\alpha)}(\mathbf{r}, t), \quad (4)$$

where μ_α and D_α are the electron mobility and diffusion coefficient, respectively. The system of Eqs. (1) and (2) is suitable for both approaches (I) and (II). In the results obtained, the transition from quasiballistic to collision-dominated regime can be followed with the aid of the well-known criteria for the validity of these approaches.

Furthermore, we use the Fourier transform $(\mathbf{r}, t) \rightarrow (\mathbf{k}, \omega)$ for the in-plane variables and solve the equations for the Fourier amplitudes $\delta \varphi_{\mathbf{k}, \omega}(z)$, $\delta \mathbf{i}_{\mathbf{k}, \omega}^{(\alpha)}$, and $\delta n_{\mathbf{k}, \omega}^{(\alpha)}$. In making use of the solution of the Poisson equation (2), $\delta \varphi_{\mathbf{k}, \omega}(z)$, together with $\delta \mathbf{E}_{\mathbf{k}, \omega}^{(\alpha)} = -i\mathbf{k} \delta \varphi_{\mathbf{k}, \omega}(z)$ and $\delta \mathbf{i}_{\mathbf{k}, \omega}^{(\alpha)}$ in the continuity equations (1), we obtain a system of N linear algebraic equations for $\delta n_{\mathbf{k}, \omega}^{(\alpha)}$. The solution of these equations allows us to derive the dispersion equation and analyze the existing modes, $\omega = \omega(\mathbf{k})$, of self-oscillations of electron density and electric field for the considered heterostructure under the applied electric field $\mathbf{E}$.

In Fig. 2, we present a schematic diagram showing distributions of the wave vector k and frequency ω' on the plane (k, ω') in regard to possible wave propagation relative to the direction of the external electric field $\mathbf{E}$ applied along the x axis. Since the (k, ω') in the first and third quadrants (also

second and fourth) correspond to the same wave propagation, shown by the right (left) arrows, it suffices to consider only two (k, ω') quadrants with the opposite wave propagations. Thus, there are two equivalent considerations: (i) $k \geq 0$, $\omega' \leq 0$ and (ii) $k \leq 0$, $\omega' \geq 0$. In what follows, we consider the first case with $k \geq 0$ and $\omega' \geq 0$. It is worth noting the existence of two possible kinds of dispersion curves for which $\omega'(k) \geq 0$ and $\omega'(k) \leq 0$. In the latter case, those parts of the dispersion curve for which $\omega'(k) > 0$ and $\omega'(k) < 0$ correspond to waves propagating in opposite directions.

III. PLASMONS IN CURRENT-BIASED TWO-LAYER NANOSTRUCTURES

The electrons in a two-layer nanostructure ($\alpha = 1, 2$) [38], which is the simplest case of a multilayer nanostructure, under an electric field $\mathbf{E}$ are characterized by the area homogeneous density $n^{(1)}$ and $n^{(2)}$, the carrier effective mass m_1 and m_2 , and the drift velocity $\mathbf{v}^{(1)}$ and $\mathbf{v}^{(2)}$. The electron motion is coupled through the electrostatic potential, $\varphi(\mathbf{r}, z, t)$, and can be affected by intralayer scattering and interlayer transitions described by effective times τ_1 and τ_2 , and τ_{tr} , respectively; $\mathbf{r}$ and z are the in-plane and vertical coordinates. The dynamics of small deviations $\delta n^{(1)}(\mathbf{r}, t)$, $\delta n^{(2)}(\mathbf{r}, t)$, $\delta \mathbf{v}^{(1)}(\mathbf{r}, t)$, $\delta \mathbf{v}^{(2)}(\mathbf{r}, t)$, and $\delta \varphi(\mathbf{r}, z, t)$ [$\sim e^{i\mathbf{k} \cdot \mathbf{r} - i\omega(\mathbf{k})t}$] from the homogeneous stationary state is governed by the set of hydrodynamic equations [11], written for the Fourier amplitudes from Eqs. (1)–(3) as

$$(-i\omega + i\mathbf{k} \cdot \mathbf{v}^{(\alpha)} + \tau_\alpha^{-1}) \delta \mathbf{v}_{\mathbf{k}, \omega}^{(\alpha)} = -\frac{e}{m_\alpha} \delta \mathbf{E}_{\mathbf{k}, \omega}^{(\alpha)}, \quad (5)$$

$$-i\omega \delta n_{\mathbf{k}, \omega}^{(\alpha)} + i\mathbf{k} \cdot \mathbf{i}_{\mathbf{k}, \omega}^{(\alpha)} = \frac{\delta n_{\mathbf{k}, \omega}^{(\beta)}}{\tau_{\beta\alpha}} - \frac{\delta n_{\mathbf{k}, \omega}^{(\alpha)}}{\tau_{\alpha\beta}}, \quad (6)$$

$$\left(\frac{\partial^2}{\partial z^2} - k^2 \right) \delta \varphi_{\mathbf{k}, \omega}(z) = \frac{4\pi e}{\epsilon_0} \left[\delta n_{\mathbf{k}, \omega}^{(1)} \delta \left(z - \frac{d}{2} \right) + \delta n_{\mathbf{k}, \omega}^{(2)} \delta \left(z + \frac{d}{2} \right) \right], \quad (7)$$

where $\delta \mathbf{i}_{\mathbf{k}, \omega}^{(\alpha)} = \mathbf{v}^{(\alpha)} \delta n_{\mathbf{k}, \omega}^{(\alpha)} + n^{(\alpha)} \delta \mathbf{v}_{\mathbf{k}, \omega}^{(\alpha)}$ is the electron flux density, and $\alpha, \beta = 1, 2$. Substituting variations of the electric field, $\delta \mathbf{E}_{\mathbf{k}, \omega}^{(\alpha)}$, obtained from the Poisson equation [39] into Eqs. (5) and (6), we get the equations for $\delta n_{\mathbf{k}, \omega}^{(1)}$ and $\delta n_{\mathbf{k}, \omega}^{(2)}$:

$$\begin{aligned} & \left(-i\omega + i\mathbf{k} \cdot \mathbf{v}^{(1)} + \tau_{12}^{-1} + \frac{\omega_{p1}^2}{A_1(\mathbf{k}, \omega)} \right) \delta n_{\mathbf{k}, \omega}^{(1)} \\ & + \left(\frac{\omega_{p1}^2}{A_1(\mathbf{k}, \omega)} e^{-kd} - \tau_{21}^{-1} \right) \delta n_{\mathbf{k}, \omega}^{(2)} = 0, \\ & \left(\frac{\omega_{p2}^2}{A_2(\mathbf{k}, \omega)} e^{-kd} - \tau_{12}^{-1} \right) \delta n_{\mathbf{k}, \omega}^{(1)} \\ & + \left(-i\omega + i\mathbf{k} \cdot \mathbf{v}^{(2)} + \tau_{21}^{-1} + \frac{\omega_{p2}^2}{A_2(\mathbf{k}, \omega)} \right) \delta n_{\mathbf{k}, \omega}^{(2)} = 0, \end{aligned} \quad (8a)$$

where

$$A_\alpha(\mathbf{k}, \omega) = -i\omega + i\mathbf{k} \cdot \mathbf{v}^{(\alpha)} + \tau_\alpha^{-1}, \quad (8b)$$

$\omega_{p\alpha} = (4\pi e^2 n^{(\alpha)} k / 2m_\alpha \epsilon_0)^{1/2} \sim k^{1/2}$ is the 2D plasmon frequency [40]. Using the solvability condition for the homogeneous system of linear equations (8a), we can derive the dispersion equation for collective charge-density oscillations, $\omega = \omega(\mathbf{k})$, in the two-layer nanostructure.

A. Two identical layers

Let us consider two identical, current-biased layers with $n^{(1)} = n^{(2)} = n$, $m_1 = m_2 = m$, $\tau_1 = \tau_2 = \tau$, and $\tau_{\alpha\beta} = \tau_{tr}$; also, we assume that the layers are equally biased so that the drift velocities in the layers are the same $\mathbf{v}^{(1)} = \mathbf{v}^{(2)} = \mathbf{v}$; from Eq. (8b), $A_1(\mathbf{k}, \omega) = A_2(\mathbf{k}, \omega) \equiv A(\mathbf{k}, \omega)$. For such heterostructure with two equally biased layers, the system of equations (8a) becomes

$$\begin{aligned} & \left(-i\omega + i\mathbf{k} \cdot \mathbf{v} + \tau_{tr}^{-1} + \frac{\omega_p^2}{A(\mathbf{k}, \omega)} \right) \delta n_{\mathbf{k}, \omega}^{(1)} \\ & + \left(\frac{\omega_p^2}{A(\mathbf{k}, \omega)} e^{-kd} - \tau_{tr}^{-1} \right) \delta n_{\mathbf{k}, \omega}^{(2)} = 0, \\ & \left(\frac{\omega_p^2}{A(\mathbf{k}, \omega)} e^{-kd} - \tau_{tr}^{-1} \right) \delta n_{\mathbf{k}, \omega}^{(1)} \\ & + \left(-i\omega + i\mathbf{k} \cdot \mathbf{v} + \tau_{tr}^{-1} + \frac{\omega_p^2}{A(\mathbf{k}, \omega)} \right) \delta n_{\mathbf{k}, \omega}^{(2)} = 0. \end{aligned} \quad (9)$$

These equations determine the plasmon dispersion and relative amplitudes in the layers.

Plasmon modes. The dispersion equation follows from Eq. (9) as

$$\begin{aligned} & \left[-i\omega + i\mathbf{k} \cdot \mathbf{v} + \frac{\omega_p^2}{A(\mathbf{k}, \omega)} (1 + e^{-kd}) \right] \\ & \times \left[-i\omega + i\mathbf{k} \cdot \mathbf{v} + 2\tau_{tr}^{-1} + \frac{\omega_p^2}{A(\mathbf{k}, \omega)} (1 - e^{-kd}) \right] = 0, \end{aligned} \quad (10)$$

where $\omega_p = (4\pi e^2 n k / 2m\epsilon_0)^{1/2} \sim k^{1/2}$.

For real wave vector $\mathbf{k}$, the frequency ω is the complex number $\omega = \omega' + i\omega''$. We have found from Eqs. (9) and (10) that the spectrum of self-oscillations in the system of drifting electrons consists of different branches (modes), which are commonly referred to the optical (OP) and acoustic (AP) plasmons [23,24,41–44]. These modes are characterized by in-phase (symmetric) oscillations of the electron density in the two layers, $\delta n_{\mathbf{k}, \omega}^{(1)} = \delta n_{\mathbf{k}, \omega}^{(2)}$, and out-of-phase (antisymmetric) oscillations, $\delta n_{\mathbf{k}, \omega}^{(1)} = -\delta n_{\mathbf{k}, \omega}^{(2)}$, respectively. Note that the latter represents quasineutral collective oscillations with $\delta n_{\mathbf{k}, \omega}^{(1)} + \delta n_{\mathbf{k}, \omega}^{(2)} = 0$. The OP mode is

$$\begin{aligned} \omega'_{op} &= \mathbf{k} \cdot \mathbf{v} \pm \frac{1}{2\tau} \sqrt{4\omega_p^2 \tau^2 (1 + e^{-kd}) - 1}, \\ \omega''_{op} &= -\frac{1}{2\tau}. \end{aligned} \quad (11)$$

The AP mode is

$$\begin{aligned} \omega'_{ap} &= \mathbf{k} \cdot \mathbf{v} \pm \frac{1}{2\tau} \sqrt{4\omega_p^2 \tau^2 (1 - e^{-kd}) - (1 - 2\tau/\tau_{tr})^2}, \\ \omega''_{ap} &= -\left(\frac{1}{2\tau} + \frac{1}{\tau_{tr}} \right). \end{aligned} \quad (12)$$

In (11) and (12), the term with the drift velocity $\mathbf{k} \cdot \mathbf{v}$ provides the Doppler shift for both the OP and AP modes.

In the absence of applied electric field ($\mathbf{E}, \mathbf{v} = 0$), we obtain from (11) and (12)

$$\omega'_{op} = \pm \frac{1}{2\tau} \sqrt{4\omega_p^2 \tau^2 (1 + e^{-kd}) - 1}, \quad \omega''_{op} = -\frac{1}{2\tau}, \quad (13)$$

$$\omega'_{ap} = \pm \frac{1}{2\tau} \sqrt{4\omega_p^2 \tau^2 (1 - e^{-kd}) - 1}, \quad \omega''_{ap} = -\frac{1}{2\tau}. \quad (14)$$

For comparison convenience of Eqs. (13) and (14), we have taken $\tau_{tr} = \infty$. In the quasiballistic regime ($\omega_p \tau \gg 1$) and long-wavelength limit ($kd \ll 1$), it follows for the OP mode that $\omega'_{op} \simeq \pm \sqrt{2}\omega_p \sim \sqrt{k}$, whereas for the AP mode, $\omega'_{ap} \simeq \pm \omega_p \sqrt{kd} \sim k$.

Long-wavelength limit. With increasing scattering, the decay rate of plasmons is increased, and the propagation length is decreased. If the electric field is applied, the occurrence of Doppler shift and dissipation can change the behavior of plasmon frequencies in the long-wavelength limit $kd \ll 1$. This limit is especially interesting because of both unusual behavior of the low-frequency plasmons in the coupled layers and facilitated experimental studies by Raman spectroscopy and scanning microspectroscopy methods. The functions of k under the square root in Eqs. (11) and (12) take negative values in the intervals $[0, k_m^{op}]$ and $[0, k_m^{ap}]$, where the upper boundaries of the intervals are determined by the roots of equations $4\omega_p^2 \tau^2 (1 + e^{-kd}) - 1 = 0$ and $4\omega_p^2 \tau^2 (1 - e^{-kd}) - 1 = 0$, respectively. An approximate analytic solution of these equations gives $k_m^{op} \simeq 1/8d(\omega_d \tau)^2$ and $k_m^{ap} \simeq 1/2d\omega_d \tau$, where

$$\omega_d = (2\pi e^2 n / m d \epsilon_0)^{1/2}. \quad (15)$$

Within these intervals, the terms with the square root become the imaginary values and provide an additional contribution to the plasmon dissipation rate. The plasmon frequency is given by the linear dependence on the wave vector $\omega' = \mathbf{k} \cdot \mathbf{v}$ for both the OPs and APs, with the same phase and group velocity equal to the electron drift velocity: $v_{ph} = v_{gr} = v$. Within the discussed intervals imaginary parts of the plasmon frequencies are

$$\begin{aligned} \omega''_{op} &= -\frac{1}{2\tau} \left[1 \mp \sqrt{1 - \frac{k}{k_m^{op}}} \right], \\ \omega''_{ap} &= -\frac{1}{2\tau} \left[1 \mp \sqrt{1 - \left(\frac{k}{k_m^{ap}} \right)^2} \right]. \end{aligned} \quad (16)$$

To analyze asymptotic behavior for $k > k_m$, we use a small deviation $\delta k > 0$ letting $k = k_m + \delta k$ to obtain for OPs

$$\delta \omega'_{op} \equiv \omega'_{op} - v k_m^{op} = v \delta k \pm \omega_d (2d \delta k)^{1/2}. \quad (17)$$

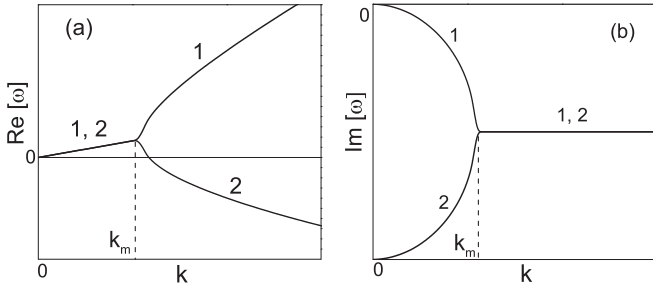


FIG. 3. A sketch of ω vs k for the long-wavelength limit (illustration is valid for both OP and AP branches plotted in arbitrary units). (a) Linear portion ($k < k_m$) and branching ($k > k_m$) of $\omega'(k)$. (b) Respective dependencies of $\omega''(k)$. Straight line (1,2) is $\omega'' = -1/2\tau$.

It follows from Eq. (17) that $\delta\omega'_{\text{op}} \simeq \pm\omega_d(2d\delta k)^{1/2} \sim (\delta k)^{1/2}$ for $\delta k \ll 2d\omega_d^2/v^2$, which is typical for OPs; and then approaches to the linear dependence $\delta\omega'_{\text{op}}(\delta k) \simeq v\delta k$ for greater δk values determined by an opposite inequality $\delta k \gg 2d\omega_d^2/v^2$. Similarly, we find for APs

$$\delta\omega'_{\text{ap}} \equiv \omega'_{\text{ap}} - vk_m^{\text{ap}} = v\delta k \pm \omega_d(2d^2k_m^{\text{ap}}\delta k)^{1/2}, \quad (18)$$

which results in $\omega'_{\text{ap}} \simeq \pm\omega_d(2d^2k_m^{\text{ap}}\delta k)^{1/2} \sim (\delta k)^{1/2}$ for $\delta k \ll 2d^2k_m^{\text{ap}}\omega_d^2/v^2$; and $\delta\omega'_{\text{ap}} \simeq v\delta k$ for δk values determined by $\delta k \gg 2d^2k_m^{\text{ap}}\omega_d^2/v^2$. As distinct from the case of $v = 0$ where $\omega'_{\text{ap}}(k) \sim k$ for $k \rightarrow 0$, we have obtained that $\omega'_{\text{ap}}(\delta k) \sim (\delta k)^{1/2}$ when $\delta k \rightarrow 0$ ($k \rightarrow k_m^{\text{ap}}$). Qualitatively, complex dependencies of $\omega'(k)$ and $\omega''(k)$ for both OP and AP plasmon branches are sketched in Fig. 3 at small $k > 0$, where (1,2) corresponds with $(\pm)$ in Eqs. (11) and (12).

Crossover quasiballistic-diffusion transport regimes. Considerable decrease of τ leads to crossover from quasiballistic ($\omega_p\tau \gg 1$, $kl_p \gg 1$) to diffusion ($\omega_p\tau \ll 1$, $kl_p \ll 1$) transport and thereby the suppression of plasmon oscillations (l_p is the electron mean-free pass). However, it can be shown that the two modes of electron-density self-oscillations (symmetric and antisymmetric) remain in effect under the drift-diffusion transport regime. Such conversion tendency from plasmons to dissipative modes can be observed by noting that, with decreasing τ the notion of electron mobility, $\mu = e\tau/m$, can be used under the inequalities $\omega\tau \ll 1$, $kl_p \ll 1$ characterizing the drift-diffusion transport regime. The corresponding self-oscillating modes are obtained by solving the system of equations (1), (2), and (4), using the same methodology with the replacement of Eq. (3) by Eq. (4).

Modification of the oscillation characteristics can be illustrated by a simple example of a single-layer nanostructure. For this case, the dispersion relations are obtained from Eqs. (11) and (12) using the limit $(d, t_{\text{tr}}) \rightarrow \infty$, which corresponds to the uncoupled layers

$$\omega_{\pm}(\mathbf{k}) = \mathbf{k} \cdot \mathbf{v} \pm \frac{1}{2\tau} \sqrt{4\omega_p^2\tau^2 - 1} - \frac{i}{2\tau}; \quad (19)$$

here, the subscripts “ $\pm$ ” in $\omega_{\pm}(\mathbf{k})$ correspond with the signs “ $\pm$ ” before the square root. For quasiballistic transport, $\omega_p\tau \gg 1$, Eq. (19) describes two weakly decaying Doppler-shifted plasmon modes with equal decay rates:

$$\omega'_{\pm}(\mathbf{k}) = \mathbf{k} \cdot \mathbf{v} \pm \omega_p(k), \quad \omega''_{\pm} = -1/2\tau. \quad (20)$$

With decreasing τ , the expression under square root in (19) may flip its sign at $4\omega_p^2\tau^2 \leq 1$, which results in plasmon modes converting into dissipative modes with oscillations driven by convective processes. These modes are characterized by equal frequencies, $\omega'_+(\mathbf{k}) = \omega'_-(\mathbf{k})$, and different decay rates:

$$\begin{aligned} \omega'_{\pm}(\mathbf{k}) &= \mathbf{k} \cdot \mathbf{v}, \quad \omega''_+(k) \simeq -\omega_p^2\tau(1 + \omega_p^2\tau^2), \\ \omega''_-(k) &\simeq -1/\tau. \end{aligned} \quad (21)$$

As $\omega''_- \sim \tau^{-1}$, this mode is strongly suppressed. Noting that $\omega_p^2\tau = \omega_{\sigma} = 2\pi/\tau_{\sigma}$, it follows that the mode with decay rate $\omega''_+(k)$ turns into a dissipative mode of the form $\omega(\mathbf{k}) = \mathbf{k} \cdot \mathbf{v} - i(\omega_{\sigma} + \tilde{D}k^2)$, where $\tilde{D}$ is the effective diffusion coefficient, and τ_{σ} is the relaxation time of a 2D electric charge, similar to the Maxwell relaxation time,

$$\tilde{D} = \left(\frac{2\pi e^2 n}{m\epsilon_0} \right)^2 \tau^3, \quad \tau_{\sigma} = \frac{m\epsilon_0}{e^2 n \tau k}. \quad (22)$$

B. Two-stream instability in two-layer nanostructures

We continue to consider the two-layer nanostructure described in Sec. III A, with an additional assumption that the electric current flows only in one of two layers; as such, we let $\mathbf{v}^{(1)} = 0$ and $\mathbf{v}^{(2)} = \mathbf{v}$. In this case, the system of equations (9) is reduced to the form

$$\begin{aligned} \left[1 - \frac{\omega}{\omega_p} \left(\frac{\omega}{\omega_p} + \frac{i}{\omega_p\tau} \right) \right] \delta n_{\mathbf{k},\omega}^{(1)} + e^{-kd} \delta n_{\mathbf{k},\omega}^{(2)} &= 0, \\ e^{-kd} \delta n_{\mathbf{k},\omega}^{(1)} + \left[1 - \frac{\omega - \mathbf{k} \cdot \mathbf{v}}{\omega_p} \left(\frac{\omega - \mathbf{k} \cdot \mathbf{v}}{\omega_p} + \frac{i}{\omega_p\tau} \right) \right] \delta n_{\mathbf{k},\omega}^{(2)} &= 0. \end{aligned} \quad (23)$$

The equality to zero of the determinant of linear equations (23) represents the dispersion equation, and its solutions define the frequencies of oscillations with given (real) values of the wave vector. These frequencies determine the asymptotic temporal behavior of the small perturbations with given values of $\mathbf{k}$.

Dispersion equation and its solutions. For further analysis, we introduce the dimensionless variables

$$\Omega = \frac{\omega}{\omega_d}, \quad |q| = kd, \quad V = \frac{v}{d\omega_d}, \quad \theta = \omega_d\tau, \quad (24)$$

where ω_d is given by Eq. (15), $v = |\mathbf{v}|$, $\tau_{\text{tr}} \rightarrow \infty$. The dispersion equation, written in dimensionless form, is

$$\begin{aligned} \left(\Omega^2 + \frac{i}{\theta} \Omega - |q| \right) \left[(\Omega - Vq)^2 + \frac{i}{\theta} (\Omega - Vq) - |q| \right] \\ = q^2 e^{-2|q|} \equiv \varepsilon(q), \end{aligned} \quad (25)$$

For example, in considering $\omega' > 0$ [$\Omega' > 0$ in Eq. (25)], the terms with V for $q > 0$ correspond to the wave propagation along the drift velocity, and $q < 0$ in the opposite direction. As noted in Sec. II, it suffices to consider the waves with $q \geq 0$ and $\Omega' \geq 0$. Noting that V is the ratio between the electron drift velocity and the wave phase velocity $v_{ph} = \omega_p/k$ ($k = 1/d$), we can suppose by analogy with the Cherenkov radiation [45] that under the electron drift with sufficiently high velocity, higher than the phase velocity v_{ph} ($V \gtrsim 1$), the

considered system of plasmons can enable a negative damping regime, when the kinetic energy of the moving electrons is transferred to the plasmon waves. Such electrically induced instability in the system of two groups of interacting carriers, moving with a relative velocity $\mathbf{v}$, is a two-stream instability,

which can also develop when the drift velocity of one group of participating carriers is zero [21,22,46].

The dispersion equation (25) is a fourth degree polynomial equation with complex coefficients whose solutions $\Omega(q) = \Omega'(q) + i\Omega''(q)$ represent four plasmon modes:

$$\Omega_1(q) = \frac{1}{2}qV + \left\{ \frac{1}{4}q^2V^2 + [((q - q_\tau)V^2 + e^{-2q})^{\frac{1}{2}} + 1]q - q_\tau \right\}^{\frac{1}{2}} - iq_\tau^{\frac{1}{2}}, \quad (26a)$$

$$\Omega_2(q) = \frac{1}{2}qV - \left\{ \frac{1}{4}q^2V^2 + [((q - q_\tau)V^2 + e^{-2q})^{\frac{1}{2}} + 1]q - q_\tau \right\}^{\frac{1}{2}} - iq_\tau^{\frac{1}{2}}, \quad (26b)$$

$$\Omega_3(q) = \frac{1}{2}qV + \left\{ \frac{1}{4}q^2V^2 - [((q - q_\tau)V^2 + e^{-2q})^{\frac{1}{2}} - 1]q - q_\tau \right\}^{\frac{1}{2}} - iq_\tau^{\frac{1}{2}}, \quad (26c)$$

$$\Omega_4(q) = \frac{1}{2}qV - \left\{ \frac{1}{4}q^2V^2 - [((q - q_\tau)V^2 + e^{-2q})^{\frac{1}{2}} - 1]q - q_\tau \right\}^{\frac{1}{2}} - iq_\tau^{\frac{1}{2}}, \quad (26d)$$

where $q_\tau = 1/4\theta^2$. The obtained modes are completely hybridized, so that they are not separated into OP and AP types of excitations. To reveal the regimes of possible instability, we analyze the behavior of modes (26a)–(26d) in a wide range of relevant values of V and θ . As it turns out from the obtained results, the increase in V results in that two of the modes approach each other and converge at some point for $V \simeq 1$. The interaction of plasmon excitations in the layers becomes resonant when the modes converge at finite values of q . This is illustrated by Figs. 4 and 5(a), 5(b), where the calculated real and imaginary parts of $\Omega(q)$ are shown in comparison for $V = 0$ (Fig. 4) and $V \gtrsim 1$ [Figs. 5(a) and 5(b)]. In Fig. 4, the curves 1, 2 correspond to OPs, whereas the curves 3, 4 correspond to APs. In the limit of small q ($q \ll 1$), the curves show different behavior as a function of q , that is $|\Omega'_{1,2}(q)| \sim \sqrt{q}$ for OPs, whereas $|\Omega'_{3,4}(q)| \sim q$ for APs; the decay rate is dispersionless and is given by $\Omega'' = -1/2\theta$ for all modes.

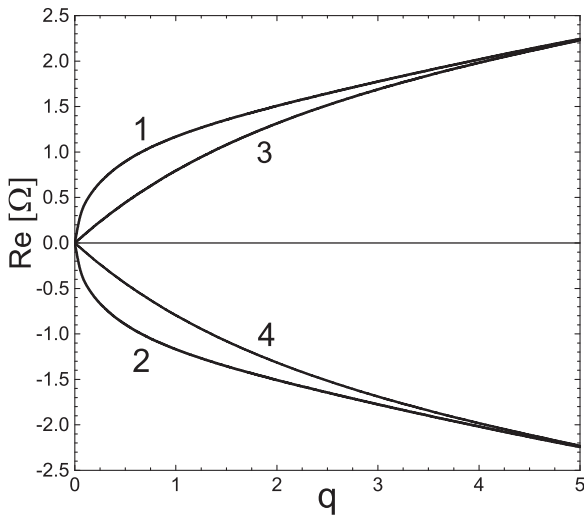


FIG. 4. Dispersion of plasmon modes, $\text{Re}[\Omega(q)]$, calculated for $V = 0$, $\theta = 50$: 1, 2 optical plasmons; 3, 4 acoustic plasmons. The mode attenuation is dispersionless: $\text{Im}[\Omega(q)] = -1/2\theta = -0.01$ (not shown).

As is seen from the comparison of Figs. 4 and 5(a), the curves 1, 2 qualitatively are weakly changed upon increasing V from $V = 0$ to $V = 1$. In contrast, the relative position of curves 3, 4 is substantially changed, so that these curves converge at $q = q_c$. In the vicinity of mode convergence, the imaginary part of frequency, $\Omega''(q)$, demonstrates a sharp peak, which maximum value can become a positive number, thereby indicating the development of instability.

Thus, unlike a single layer [34], plasmons in coupled two or multiple layers can become electrically unstable under the high electron drift.

Phase shift and relative amplitudes of plasmonic oscillations. Before proceeding to consideration of the instability, it is instructive to study charge distributions in the interacting layers. The electron-density oscillations of small amplitude in the two layers can be compared from Eq. (23) for each of the plasmon modes obtained (26a)–(26d). In the linear theory, which has been used to derive Eq. (23), the difference between such oscillations is described in terms of a phase shift Φ and a relative amplitude $\bar{f}$ so that we can write $\delta n^{(2)} = \bar{f}e^{i\Phi}\delta n^{(1)}$. The latter can be expressed, using the first of equations (23), as

$$\bar{f}e^{i\Phi} = \left(\Omega^2 + i\frac{\Omega}{\theta} - q \right) \frac{e^q}{q} \equiv f(q, \Omega). \quad (27)$$

Here, $\Phi = \arctan(f_i/f_r)$ is the phase shift, $\bar{f} = (f_r^2 + f_i^2)^{1/2}$ is the amplitude, $f_r = (\Omega_r^2 - \Omega_i^2 - q - \Omega_i/\theta)e^q/q$ and $f_i = (2\Omega_i + \theta^{-1})\Omega_r e^q/q$ are the real and imaginary parts of the function $f(q, \Omega)$, respectively. We note that it is necessary to distinguish two pairs of plasmon modes, $\Omega_1(q)$, $\Omega_2(q)$ and $\Omega_3(q)$, $\Omega_4(q)$, where the latter is characterized by the clear resonant behavior near $q = q_c$ [Figs. 5(a) and 5(b)]. Therefore, we analyze them separately.

The calculation results of the amplitudes and phase shifts for oscillating electron densities in the two layers are presented in Fig. 6 for the plasmon modes $\Omega_3(q)$ and $\Omega_4(q)$. The electron densities in the layers are shown as a function of kx for one “period” of oscillations: $0 \leq kx \leq 2\pi$, a snapshot sampling at an arbitrary time moment t . Letting $\delta n^{(1)}(kx) = a \cos(kx)$, we obtain equal amplitudes for both the modes (furthermore, we set an arbitrary amplitude $a = 1$).

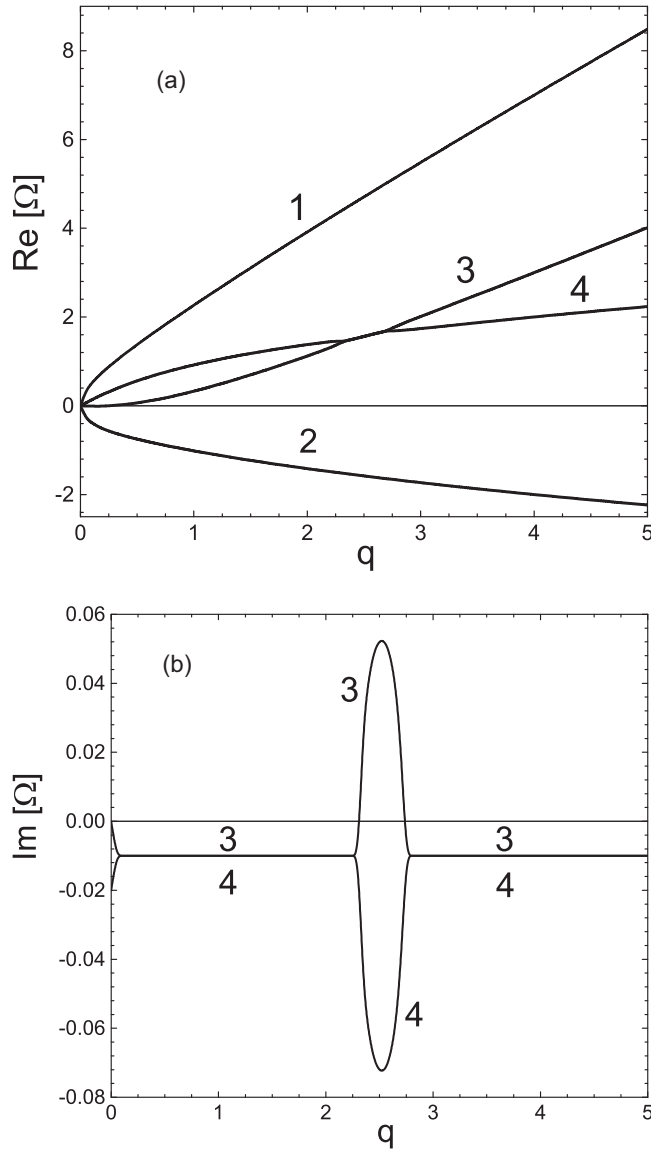


FIG. 5. Dispersion of plasmon modes (26a)–(26d) calculated for $V = 1.25$, $\theta = 50$ (numbering of curves corresponds to Fig. 4). (a) Dispersion of $\text{Re}[\Omega_1(q)] = \Omega'_1(q)$, $\text{Re}[\Omega_2(q)] = \Omega'_2(q)$, $\text{Re}[\Omega_3(q)] = \Omega'_3(q)$, and $\text{Re}[\Omega_4(q)] = \Omega'_4(q)$. (b) Dispersion of attenuation, $\text{Im}[\Omega_3(q)] = \Omega''_3(q)$ and $\text{Im}[\Omega_4(q)] = \Omega''_4(q)$, of plasmon modes shown in Fig. 5(a) by the curves 3, 4; the attenuation of modes $\Omega_1(q)$ and $\Omega_2(q)$ is dispersionless: $\Omega''_1(q) = \Omega''_2(q) = -1/2\theta = -0.01$ (not shown).

Thus, $\delta n^{(2)}(kx) = \cos(kx \mp \Phi)$ for modes $\Omega_3(q)$ and $\Omega_4(q)$, respectively. In the calculations, we used the data from Fig. 5(a): $q_c = 2.5$, $\Omega'_3(q_c) = \Omega'_4(q_c) = 1.56$. From Fig. 5(b), $\Omega''_4(q_c) = -0.073$ for the decrement, and $\Omega''_3(q_c) = 0.053$ for the maximum increment. Then, we find for layer 2 that the phase shift $\Phi = 1.25$ and the amplitude $a = 1$.

The similar consideration of modes $\Omega_1(q)$ and $\Omega_2(q)$ of Figs. 5(a) and 5(b) at $q = q_c$ [$\Omega'_1(q_c) = 4.71$, $\Omega'_2(q_c) = -1.58$, and $\Omega''_1(q_c) = \Omega''_2(q_c) = -0.01$] demonstrates that the phase shift in $\delta n^{(1)}(kx)$ and $\delta n^{(2)}(kx)$ is absent for these modes ($\Phi = 0$). However, relative amplitudes are drastically different in comparison with the case of converging modes $\Omega_3(q_c)$ and $\Omega_4(q_c)$. For $\delta n^{(1)}(kx) = \cos(kx)$, we

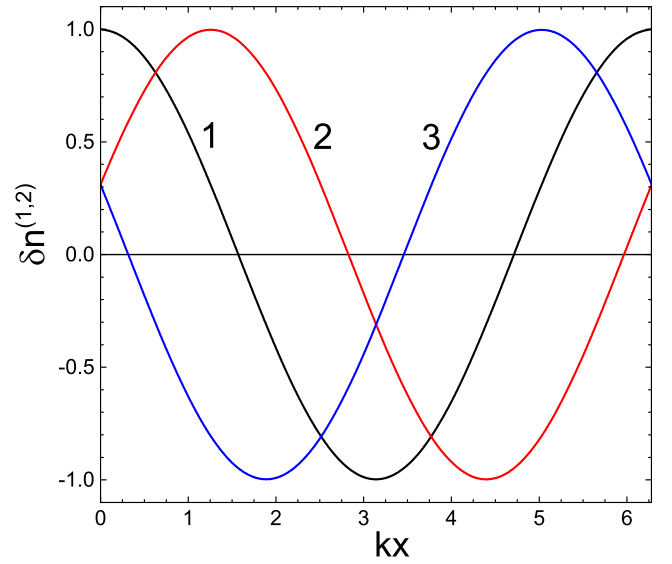


FIG. 6. Illustration of phase shift in electron-density oscillations $\delta n^{(1)}$ (curve 1) and $\delta n^{(2)}$ (curves 2, 3) for plasmon modes shown in Figs. 5(a) and 5(b) by the curves 3, 4. Layer 1: curve 1 (black). Layer 2: curve 2 (red), $\Phi = -1.25$ for plasmon mode $\Omega_3(q_c)$; curve 3 (blue), $\Phi = 1.25$ for plasmon mode $\Omega_4(q_c)$. Curves 1–3 have the same amplitude.

found that $\delta n^{(2)}(kx) = 95.8\delta n^{(1)}(kx)$ for mode $\Omega_1(q_c)$ and $\delta n^{(2)}(kx) = 0.01\delta n^{(1)}(kx)$ for mode $\Omega_2(q_c)$. Since the electron drift velocity is zero in one layer (layer 1 in Fig. 1), this results in effective detuning, which affects the coupling of the two layers so that they respond differently to the excitation. In the absence of resonance, these modes have considerably different frequencies at $q = q_c$; the oscillations behavior in the layers is as in almost noninteracting (or weakly interacting) layers.

Finally, we conclude that strong plasmonic coupling of the two layers and the instability effect are of a resonant character, require plasmon modes proximity (coincidence of the frequencies and wave vectors).

Two-stream instability. In the above accepted nomenclature, the branches $\Omega_3(q)$ and $\Omega_4(q)$ demonstrate the crossing at zero coupling [$\varepsilon \rightarrow 0$, Eq. (28)] and radical changes at $\varepsilon \neq 0$. Particularly, the expression under the square root of Eqs. (26c) and (26d),

$$R(q, V, q_\tau) = \frac{1}{4}q^2V^2 + (q - q_\tau) - q\sqrt{(q - q_\tau)^2V^2 + \exp(-2q)},$$

can be negative in some range of the wave vectors, which means that in this range $\text{Re}[\Omega_3(q)] = \text{Re}[\Omega_4(q)] = \frac{1}{2}Vq$; but $\text{Im}[\Omega_3(q)]$ and $\text{Im}[\Omega_4(q)]$ are different. As a result, the branch $\Omega_3(q)$ is weakly damped or amplified, while the branch $\Omega_4(q)$ is strongly damped:

$$\begin{aligned} \text{Im}[\Omega_3] &\equiv \Gamma_3 = \Theta[-R(q, V, q_\tau)\sqrt{|R(q, V, q_\tau)|} - q_\tau^{1/2}, \\ \text{Im}[\Omega_4] &\equiv \Gamma_4 = -\Theta[-R(q, V, q_\tau)\sqrt{|R(q, V, q_\tau)|} - q_\tau^{1/2}, \end{aligned}$$

with $\Theta(x)$ being the Heaviside step function. Figures 7(a)–7(d) illustrate the behavior of temporal

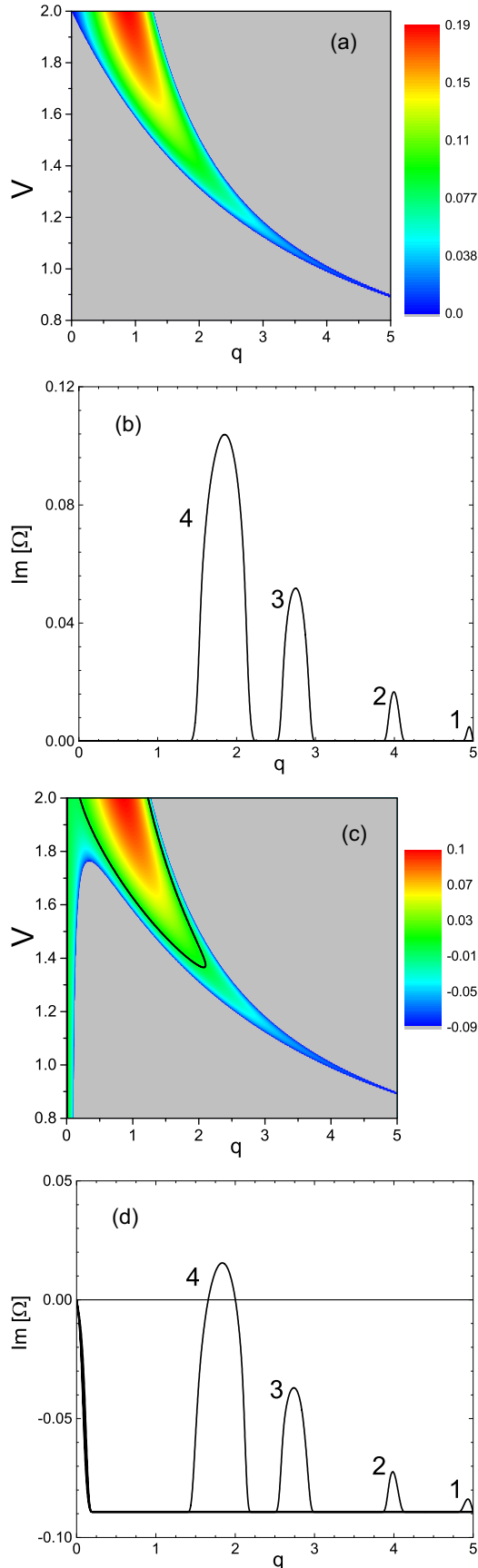


FIG. 7. (a), (c) Density plots of $\Gamma_3(q, V)$ where solid lines correspond to $\Gamma_3(q, V) = 0$, the inner part of the area (between the curves) is $\Gamma_3(q, V) > 0$, and the outer part is $\Gamma_3(q, V) < 0$. (b), (d) $\Gamma_3(q, V)$

increment or decrement, Γ_3 , of the potentially unstable branch $\Omega_3(q)$. In Figs. 7(a) and 7(c), the density plots of Γ_3 are shown in the (q, V) plane for two cases: in the absence of the electron relaxation ($q_\tau \rightarrow 0$) and for finite values of the relaxation parameter q_τ . In Figs. 7(b) and 7(d), the q dependencies of $\Gamma_3(q)$ are presented for different electron drift velocities (different V values). In the dissipativeless cases of Figs. 7(a) and 7(b), the instability exists for all presented values of the velocity, although Γ_3 sharply decreases as the velocity decreases. In the case of Figs. 7(c) and 7(d), the instability arises at large velocities, $V \geq 1.5$ (the behavior of Γ at small q is characteristic for the diffusion-transport regime discussed above).

The resonance nature of the instability facilitates a theoretical analysis for weakly coupled layers, $\varepsilon(q) \ll 1$. In the zeroth approximation, neglecting the small term on the right-hand side of Eq. (25), we get two uncoupled modes of interest:

$$\begin{aligned}\Omega_3^{(0)} &= qV - (q - q_\tau)^{1/2} - iq_\tau^{1/2}, \\ \Omega_4^{(0)} &= (q - q_\tau)^{1/2} - iq_\tau^{1/2}.\end{aligned}\quad (28)$$

The mode $\Omega_3^{(0)}$ refers to the drifting electrons, while $\Omega_4^{(0)}$ is for nondrifting electrons in the unbiased layer. At first, for simplicity, we assume the absence of dissipation ($\tau = \infty$), which can be taken into account later on. In this case, the frequencies $\Omega_3^{(0)}$ and $\Omega_4^{(0)}$ are real. The intersection of the modes $\Omega_3^{(0)}(q)$ and $\Omega_4^{(0)}(q)$ takes place at the point (q_c, Ω_c) with $q_c = 4/V^2$, $\Omega_c = 2/V$. Letting $q = q_c + \delta q$ and $\Omega = \Omega_c + \delta\Omega$, we obtain in the linear approximation: $\Omega_4(q) = \Omega_c + v_4\delta q$ and $\Omega_3(q) = \Omega_c + v_3\delta q$, where $v_4 = V/4$ and $v_3 = 3V/4$.

In the next approximation, we take into account the coupling between the two modes, which reflects in a finite value of $\varepsilon(q)$ in Eq. (25). The dispersion equation in the vicinity of the intersection point (q_c, Ω_c) takes the form

$$(\delta\Omega)^2 - (v_3 + v_4)\delta q\delta\Omega + [v_3v_4(\delta q)^2 + \bar{\varepsilon}] = 0, \quad (29)$$

where $\bar{\varepsilon} = V^{-2}e^{-8/V^2}$. The solution of Eq. (29) relative to $\delta\Omega$ is

$$(\delta\Omega)_\pm = \frac{1}{2} \left\{ V\delta q \pm \left[\left(\frac{V}{2} \right)^2 (\delta q)^2 - 4\bar{\varepsilon} \right]^{1/2} \right\}. \quad (30)$$

It follows that, for $(\delta q)^2 < (4/V)^2\bar{\varepsilon}$, the solutions $(\delta\Omega)_\pm = \frac{1}{2}V\delta q \pm i(\delta\Omega)_\pm''$ are the complex numbers, and the imaginary part of one of them is positive,

$$(\delta\Omega)_+'' \equiv \Gamma_3^{(0)} = \sqrt{\bar{\varepsilon} - \frac{V^2}{4^2}(q - q_c)^2} > 0, \quad (31)$$

i.e., the instability of plasmon waves takes place.

Differentiating (30) with respect to δq , we get the group velocity $V_{gr} = (v_3 + v_4)/2 = V/2$ of the waves ($v_{gr} = v/2$); the corresponding maximum increment of the instability also can be found: $\Gamma_{3,max}^{(0)} = \sqrt{\bar{\varepsilon}} = (Ve^{4/V^2})^{-1}$. Hence, the characteristic time of growth of initial perturbations by a factor of “ e ”

vs q at a fixed V ; curves 1, 2, 3, and 4 are for $V = 0.9, 1, 1.2$, and 1.44 , respectively. (a), (b) $q_\tau = 0$ ($\theta \rightarrow \infty$); (c), (d) $q_\tau = 8 \times 10^{-3}$ ($\theta = 5.6$).

is $\tau_i = 1/\omega_i'' = 1/\omega_d \sqrt{\bar{\epsilon}}$; then the characteristic length scale of their drift propagation, $L_i = v\tau_i$, with $\tau_i = \omega_d^{-1} V e^{4/V^2}$, is estimated to be $L_i = V^2 e^{4/V^2} d$.

Equations (30) and (31) are valid for zero electron dissipation. In considering the influence of a finite value of τ , we note that, for the quasiballistic transport regime, the parameter q_τ in Eq. (28) takes rather small values ($q_\tau \ll 1$). Then, the instability analysis accounting for the dissipation can be carried out analytically, similarly to the results obtained in Eqs. (30) and (31), considering q_τ as the small parameter. We found that, in the region of strong interaction of the branches $\Omega_3^{(0)}(q)$, $\Omega_4^{(0)}(q)$, the intersection point ($\bar{q}_c$, $\bar{\Omega}_c$) and the slopes of the curves are modified: $\bar{q}_c = q_c - q_\tau$, $\bar{\Omega}_c = \Omega_c - \frac{1}{2} V q_\tau$, $\bar{\Omega}_3^{(0)}(q) = \bar{\Omega}_c + \bar{v}_3 \delta q$, $\Omega_4^{(0)}(q) = \bar{\Omega}_c + \bar{v}_4 \delta q$, with $\bar{v}_4 = v_4(1 + v_4^2 q_\tau)$ and $\bar{v}_3 = v_3(1 + \frac{4}{27} v_3^2 q_\tau)$. For the condition of $V\sqrt{\bar{\epsilon}}q_\tau \ll 1$, the correction to the amplification or absorption of the $\Omega_3(q)$ branch is $\Gamma_3 = \Gamma_3^{(0)} - q_\tau^{1/2}$. Also, the characteristic time of growth of initial perturbations τ_i , and the characteristic length scale of drift propagation L_i , are renormalized as $\bar{\tau}_i = \omega_d^{-1} V e^{[(4/V^2) - q_\tau]}$ and $\bar{L}_i = d V^2 e^{[(4/V^2) - q_\tau]}$, respectively. The results show the tendency for suppression of the instability effect due to the electron scattering [see also Figs. 7(b) and 7(d)].

The convective character of the instability. The analyzed plasmon instability is of the *convective* kind [47]. Indeed, considering δq in Eq. (29) as the unknown, with a real $\delta\Omega$, we get the equation

$$v_3 v_4 (\delta q)^2 - (v_3 + v_4) \delta\Omega \delta q + [(\delta\Omega)^2 + \bar{\epsilon}] = 0. \quad (32)$$

This equation has two roots

$$(\delta q)_\pm = \frac{1}{2v_3 v_4} \{ (v_3 + v_4) \delta\Omega \pm [(v_3 - v_4)^2 (\delta\Omega)^2 - 4v_3 v_4 \bar{\epsilon}]^{1/2} \}. \quad (33)$$

The expression under the square root in Eq. (33) becomes negative for $(\delta\Omega)^2 < 3\bar{\epsilon}$, thus providing an imaginary part to $(\delta q)_\pm = (\delta q)' \pm i(\delta q)''$ [$(\delta q)'' > 0$]. Since the imaginary part of the root $(\delta q)_-$ is negative, the wave considered is unstable; that is, small initial perturbations in the form of plane waves will indefinitely grow with the increase of the space coordinate. For small dissipation, it is possible to estimate the characteristic length scale of amplitude growth $\lambda_e \approx \frac{\sqrt{3}}{4} V^2 e^{4/V^2} d$.

Finally, in the two-layer nanostructure with only one layer biased with the electric current, the plasmonic instability can occur, which manifests itself as the two-stream instability of convective character. The instability threshold and characteristic frequency range critically depend on the velocity of drifting electrons. Under negligibly small electron relaxation, there is no velocity threshold. The electron relaxation leads to the existence of the drift velocity threshold.

Long-wavelength limit in collision-dominated regime. The long-wavelength limit in situations when only one of two layers is biased with the current can be analyzed in the same way as described in Sec. III A. The linear portion of the dispersion curves, induced by the Doppler shift given in Eqs. (26a)–(26d), is revealed to be more pronounced for modes $\Omega_3'(q)$ and $\Omega_4'(q)$; therefore, we focus only on these

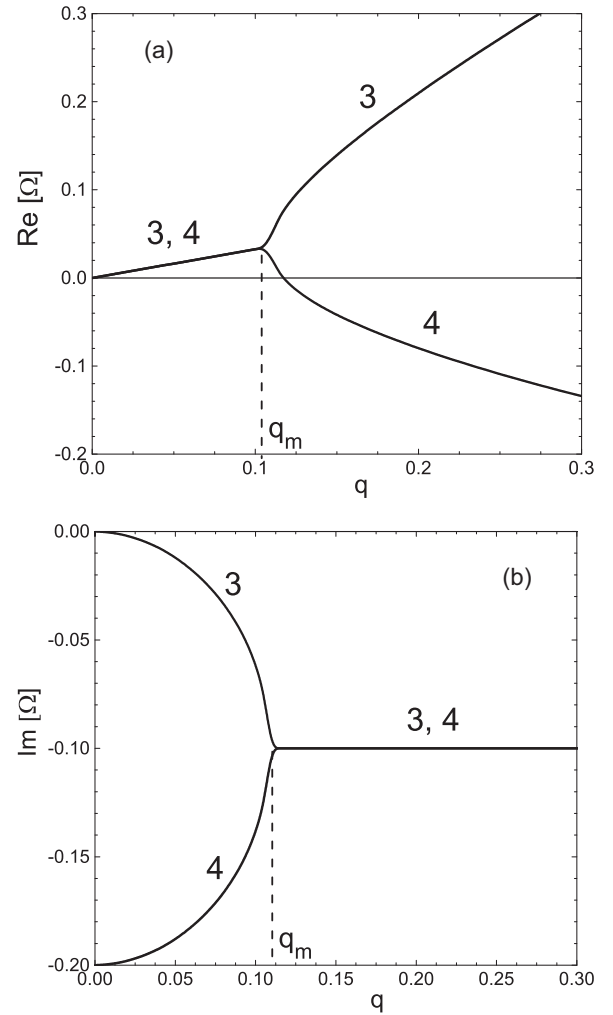


FIG. 8. Dispersion of plasmon modes $\Omega_3'(q)$ and $\Omega_4'(q)$ calculated for small q ; $V = 0.65$, $\theta = 5$. (a) Linear portion of dispersion of degenerate modes $\Omega_3'(q)$ and $\Omega_4'(q)$ for $q \leq q_m$ and branching of $\Omega_3'(q)$ and $\Omega_4'(q)$ for $q > q_m$. (b) Respective dependencies of $\Omega_3''(q)$ and $\Omega_4''(q)$. Numbering of curves corresponds to the mode indexes.

two modes. The linear dispersion $\Omega_3'(q) = \Omega_4'(q) = (1/2)qV$ is observed in the interval $0 \leq q \leq q_m$, where q_m is the root of equation $R(q, V, q_\tau) = 0$. Within this interval, the function $R(q, V, q_\tau)$ takes negative values providing an additional contribution into attenuation of the modes. The results of numerical calculations using the parameter values $V = 0.65$ and $\theta = 5$ are presented in Figs. 8(a) for $\text{Re}[\Omega_3(q)]$, $\text{Re}[\Omega_4(q)]$ and 8(b) for $\text{Im}[\Omega_3(q)]$ and $\text{Im}[\Omega_4(q)]$. It is seen that, for $q < q_m$, there are two modes of the same frequency $\Omega_3'(q) = \Omega_4'(q) = (1/2)qV$, but with different attenuation $\Omega_3''(q) = \Omega_4''(q) = \pm \sqrt{|R(q, V, q_\tau)|} - q_\tau^{1/2}$; and with the same phase or group velocity, $v_{ph} = v_{gr} = (1/2)v$. In the limit of $q \rightarrow 0$, we obtain that $\Omega_3''(q) \rightarrow 0$ and $\Omega_4''(q) \rightarrow -1/\theta = -0.2$ [Fig. 8(b)]. For $q > q_m$, they are split into two different modes, $\Omega_3'(q) \neq \Omega_4'(q)$, with the same attenuation $\Omega_3'' = \Omega_4'' = -1/2\theta$, which is independent of q . For $q = q_m$, the oscillations of electron densities in the layers are of symmetrical type $\delta n^{(1)} = \delta n^{(2)}$. For $q > q_m$, letting a small deviation $\delta q = q - q_m$, we obtain that $\Omega_3'(q) = \Omega_4'(q) \simeq (1/2)q_m V \pm$

$(1/2)R'_q(q_m)\sqrt{q-q_m}$, i.e., the wave-vector dependence characteristic for the OPs.

IV. NUMERICAL ESTIMATIONS

For numerical estimations of the characteristics of two-layer coupled plasmons and the conditions defining the two-stream plasmon instability, we consider GaAs- and GaN-based heterostructures.

GaAs-based nanostructure. We set $m = 0.067 m_0$ (m_0 is the free-electron mass), $\epsilon_0 = 12.35$, $d = 10^{-6}$ sm, and $n = 10^{11}$ cm $^{-2}$. Then, we obtain $\omega_d = 1.4 \times 10^{13}$ s $^{-1}$.

For the equilibrium conditions and low (nonheating) electric fields, the relaxation time τ can be estimated by using the low-field electron mobility. For high-quality GaAs heterostructures, one can use the mobility values $\mu = 3 \times 10^5$ sm ^{2} /V · s at a temperature $T_0 = T_1 = 77$ K, while for $T_0 = T_2 = 300$ K the typical mobility is $\mu = 800$ sm ^{2} /V · s. Respectively, we obtain the estimated relaxation times $\tau = \tau_1 \simeq 1.1 \times 10^{-11}$ s ($\theta \simeq 158$) and $\tau = \tau_2 \simeq 3 \times 10^{-14}$ s ($\theta \simeq 0.4$).

First, we consider the long-wavelength plasmons under equilibrium and at low applied fields. Using the above parameters at the temperature $T_0 = T_1$ and in the absence of the current, we find the dimensionless OP frequencies $\text{Re}[\Omega_1(q)] = \sqrt{2q}$, $\text{Re}[\Omega_2(q)] = -\sqrt{2q}$ which coincide with the dimensionless plasmon frequencies corresponding to a twice electron concentration; and the AP modes $\text{Re}[\Omega_3(q)] = q$ and $\text{Re}[\Omega_4(q)] = -q$. The signs $\pm$ correspond to the wave propagations in two opposite directions [see Fig. 2, quadrants I and IV of the $(\omega, \mathbf{k})$ plane]. At higher temperature, $T_0 = T_2$, the oscillations are overdamped in the wide intervals $0 < q < q_m^{op} = 1.2$ and $0 < q < q_m^{ap} = 1.8$ for OP and AP modes, respectively. Two different dampings are characteristic for each type of the modes because it follows from Eq. (16); remarkably, one of OP and one of AP modes have absolute values of the dimensionless damping smaller than $1/2\theta$.

In the case of low applied fields and low temperature, the OP plasmon waves are of high quality. Indeed, the parameter q_m^{op} essential for the long-wavelength limit is very small; thus, both terms in Eq. (11) are real, $\text{Re}[\Omega_1(q)] = qV + \sqrt{2q}$ and $\text{Re}[\Omega_2(q)] = qV - \sqrt{2q}$, and the plasmon damping is constant and small, $\text{Im}[\Omega_1(q)] = \text{Im}[\Omega_2(q)] = 1/2\theta$. These waves can propagate in two opposite directions. On the contrary, for the AP modes in the limit $q \ll 1$, we find $q_m^{ap} = 0.04$; thus at $0 < q < q_m^{ap}$ a linear portion of $\text{Re}[\Omega_3(q)]$ and $\text{Re}[\Omega_4(q)]$ is realized with the q -dependent dampings given by the second of Eqs. (16). Remarkably, for q less than q_m^{ap} as well as slightly above it (depending on the velocity parameter V), both AP waves are unidirectional, i.e., always propagating along the electron drift.

Consider high electric fields providing a high drift velocity of the electrons. The scattering time of hot electrons can be estimated by using the differential mobility $\mu_d(E) = dv/dE$ as $\tau = m\mu_d(E)/e$. At the lattice temperature $T_1 = 77$ K, for the electric fields of $E \approx 1$ kV/cm, the differential mobility and the drift velocity were found to be $\mu_d(E) \simeq 10^4$ cm ^{2} /V · s, $v(E) \approx 2 \times 10^7$ cm/s [48], that gives the relaxation time $\tau \simeq 4 \times 10^{-13}$ s.

To describe plasmon instability characteristics, we use the above presented estimates of v and τ [48] to obtain the dimensionless parameters values $V = 1.44$, $\theta = 5.6$, and $q_\tau = 8 \times 10^{-3}$. The branches $\Omega'_1(q)$ and $\Omega'_2(q)$ (positive and negative at $q > 0$, respectively) show the standard behavior, with positive and negative propagation directions, similarly to the branches $\Omega'_1(q)$ and $\Omega'_2(q)$ in Fig. 5.

The behavior of branches $\Omega'_3(q)$ and $\Omega'_4(q)$ is more complicated. At small $q < q_m$, there is a common linear portion of the dispersion $\Omega'_3(q) = \Omega'_4(q) = \frac{1}{2}qV$. After the branching point ($\Omega_m \approx 0.9$, $q_m \approx 0.08$), they are divided up to the region where they stick together forming a *second* (short) linear portion. It is there that the instability effect is expected. Location of the second linear portion is determined by the crossing of the $\Omega_3^0(q)$ and $\Omega_4^0(q)$ branches given by Eqs. (28): $\Omega_c = 2/V \approx 1.4$, $q_c = 4/V^2 \approx 1.9$ (the plasmon frequency ≈ 3 THz and the wavelength ≈ 33 nm). From Eq. (26c), we find that the instability of the mode $\Omega_3(q)$ occurs in the interval $q = 1.6$ – 2.1 , with maximal dimensionless amplification $\text{Im}[\Omega_3] = 0.01$. This corresponds to the temporal and spatial increments 1.4×10^{11} s $^{-1}$ and 3.5×10^3 cm $^{-1}$, respectively. Both plasmons given by the branches Ω_3 and Ω_4 propagate in the same direction. The increase in the drift velocity by 10% leads to a fourfold increase of the amplification coefficient ($\text{Im}[\Omega_3] \approx 0.04$).

GaN-based nanostructure. It is well known that, due to relatively large effective mass and strong optical phonon scattering, the high-speed electron transport in GaN-based structures occurs in very high fields (above tens of kV/cm) with large dissipation rates [49,50]. Under these conditions and high electron concentrations, the collective charge-density oscillations are strongly damped. If the concentrations are relatively small, so that these oscillations are of sub-THz or a few THz range, then the temporal high-speed electron kinetics is complex due to the *optical-phonon-transient-time resonance* in bulk [51,52] and quantum well GaN structures [53,54], when the hydrodynamic approach cannot be applied [55]. Thus, in the frame of the accepted approach, we restrict ourselves to the case of relatively low electric fields (less than 10 kV/cm).

In GaN quantum wells, the highest low-field mobility was measured at subhelium temperatures, $\mu = (1\text{--}2) \times 10^5$ cm ^{2} /(V s) [56–58] for sufficiently low electron concentrations, $n = (4\text{--}10) \times 10^{11}$ cm $^{-2}$. At $T_0 = 50\text{--}100$ K, the acoustic-phonon-limited mobility was found to be $\mu = (2.5\text{--}5.0) \times 10^4$ cm ^{2} /(V s) [59,60]. We set $m = 0.2 m_0$, $\epsilon_0 = 10$, $d = 10^{-6}$ cm and assume $n = 3 \times 10^{11}$ cm $^{-2}$. Then, we estimate $\omega_d = 1.5 \times 10^{13}$ s $^{-1}$ and the relaxation time $\tau > 10^{-12}$ s ($\theta > 15$). Under such conditions and low electric fields, the two-layer coupled plasmons are high-quality oscillations described by Eqs. (11) and (12).

For the case of a moderately strong applied field E , we can estimate the relaxation time from differential mobility $\mu_d(E)$. For example, at $E = 4$ kV/cm, the Monte Carlo simulations accounting for all actual scattering processes of the electrons give the drift velocity $v \approx 10^7$ cm/s, ($V \approx 0.65$) and $\mu_d(E) = 600$ cm ^{2} /(V s). The latter corresponds to $\tau \approx 6.8 \times 10^{-14}$ s ($\theta \approx 1$). The modes $\Omega'_1(q)$ and $\Omega'_2(q)$ (positive and negative at $q > 0$) show the standard behavior with positive and negative propagation directions, similar

to $\Omega'_1(q)$ and $\Omega'_2(q)$ presented in Fig. 5. The modes $\Omega'_3(q)$ and $\Omega'_4(q)$ at small $q < q_m = 0.58$ [$\Omega'_3(q_m) = 0.2$] have a common linear portion $\Omega'_3(q) = \Omega'_4(q) \approx 0.34q$. After the branching point, (q_m, Ω_m) , these modes are separated. Both modes are unidirectional excluding the range $0.68 < q < 2.1$, where the mode $\Omega'_4(q)$ changes the positive propagation direction to a negative one.

V. DISCUSSION AND CONCLUSION

We have studied plasmonic properties of heterostructures composed of parallel 2D semiconductor nanolayers separated by the interlayer distance d and subjected to an electric current. The electron transport in individual layers has been described in the hydrodynamic approach, taking into account the self-consistent electrostatic potential and the electron momentum dissipation characterized by a relaxation time τ . The electrostatic potential strongly couples the electrons in adjacent layers and results in radical modification of the plasmon modes; particularly, the number of modes in a two-layer nanostructure is doubled compared with that which is usually obtained in a single-layer configuration. In the absence of the electron drift, there are two types of the coupled modes, the optical $\omega_{\text{op}}(\mathbf{k})$ and acoustical $\omega_{\text{ap}}(\mathbf{k})$ modes, with the square root and linear wave-vector dependencies of the plasmon frequencies, respectively. We analyzed relative amplitudes of the charge-density oscillations in the layers and their phase shift. The analysis shows that, in the long-wavelength limit, the electron relaxation modifies both types of modes, providing linear portions of the wave-vector dependencies followed by a square root branching. The theoretical behavior of the modes in the long-wavelength limit is important, because it facilitates detail studies of the coupled oscillations by the Raman and scanning microscopy techniques.

The frequencies and damping of the two-layer coupled modes have been analyzed under the electron drift; the analysis has shown that the hydrodynamic approach does not predict any instability for the configuration with two identical layers. On the contrary, the plasmonic instability is possible for the layers with distinct characteristics. We have considered the case when only one of two charge-coupled layers is biased with the electron current, for which the plasmon dispersion equation has been solved exactly in the analytical form.

We have shown the existence of a narrow (k, ω) region with a strong interlayer interaction of a resonant character. In this region, the real parts of the respective modes are identical and linear in k (a second linear portion). The imaginary parts are different and one of them can change sign; the latter corresponds to the plasmonic instability effect.

In the two-layer nanostructure considered, the plasmonic instability occurs as the two-stream instability of convective character. The instability threshold and frequency critically depend on the velocity of drifting electrons. In fact, under negligibly small electron relaxation, there is no velocity threshold for the instability, while the electron relaxation imposes a finite velocity threshold. Meanwhile, for the relaxation time satisfying the condition $\omega_d \tau \gg 1$, the threshold velocity can be estimated as $v \gtrsim \omega_d d \sim n/\epsilon_0$. Thus, the use of semiconductor layers with low electron density ($n < 10^{12} \text{ cm}^{-2}$) and a high dielectric constant environment may be preferable for achieving the velocity threshold.

To illustrate the results obtained, we provided numerical estimates for two-layer GaAs- and GaN-based heterostructures. For both cases, we discussed the important parameters and behavior of the long-wavelength plasmons. For the case of GaAs-based heterostructures, we find that the considered plasmon instability can be realized under high-field electron transport, despite a significant increase of the relaxation rate of hot electrons. At that, for this instability, the temporal and spatial increments lie in the $\approx 10 \text{ ps}^{-1}$ and $\approx 1 \text{ } \mu\text{m}^{-1}$ ranges, respectively.

We suggest that the presented investigation brings new knowledge about solid-state plasmonic physics and can be used in the search of terahertz electric instabilities in semiconductor heterostructures composed of charge-coupled layers as the sources of terahertz emission.

ACKNOWLEDGMENTS

This work was conducted in frame of the Long-Term Program of support of the Ukrainian research teams at the Polish Academy of Sciences with financing by the U.S. National Academy of Sciences (Project LTP-NAS-A7-11-0008). The study was also supported in part by the National Academy of Sciences of Ukraine (Project No. 4.11/23-P). The authors are thankful to W. Knap for special attention and fruitful discussions of possible relevant experiments. One of the authors (V.A.K.) is partly supported by the Project ERC “TER-APLASM,” Grant No. 101053716.

Views and opinions expressed in the work are those of the author(s) only and do not necessarily reflect those of the European Union or the European Research Council Executive Agency. Neither the European Union nor the granting authority can be held responsible for them.

DATA AVAILABILITY

No data were created or analyzed in this study.

- [1] S. A. Maier, *Plasmonics: Fundamentals and Application* (Springer-Verlag US, New York, 2007), p. 224.
- [2] T. V. Shahbazyan and M. I. Stockman, *Plasmonics: Theory and Applications. Overview*, edited by J. Leszczynski, Challenges and Advances in Computational Chemistry and Physics (Springer, Dordrecht, Netherlands, 2013), pp. 577.

- [3] *Emerging Trends in Terahertz Solid-State Physics and Devices*, edited by A. Biswas, A. Banerjee, A. Acharyya, H. Inokawa, and J. N. Roy (Springer, Singapore, 2020), p. 203.
- [4] V. V. Mitin, V. A. Kochelap, M. Dutta, and M. A. Strosio, *Introduction to Optical and Optoelectronic Properties of Nanostructures* (Cambridge University Press, Cambridge, 2019).

- [5] A. Leitenstorfer, A. S. Moskalenko, T. Kampfrath, J. Kono, E. Castro-Camus, K. Peng, N. Qureshi, D. Turchinovich, K. Tanaka, A. G. Markelz, M. Havenith, C. Hough, H. J. Joyce, W. J. Padilla, B. Zhou, K.-Y. Kim, X.-C. Zhang, P. U. Jepsen, S. Dhillon, M. Vitiello, E. Linfield, A. G. Davies, M. C. Hoffmann, R. Lewis, M. Tonouchi, P. Klarskov, T. S. Seifert, Y. A. Gerasimenko, D. Mihailovic, R. Huber, J. L. Boland, O. Mitrofanov, P. Dean, B. N. Ellison, P. G. Huggard, S. P. Rea, C. Walker, D. T. Leisawitz, J. R. Gao, C. Li, Q. Chen, G. Valušis, V. P. Wallace, E. Pickwell-MacPherson, X. Shang, J. Hesler, N. Ridler, C. C. Renaud, I. Kallfass, T. Nagatsuma, J. A. Zeitler, D. Arnone, M. B. Johnston, and J. Cunningham, The 2023 terahertz science and technology roadmap, *J. Phys. D: Appl. Phys.* **56**, 223001 (2023).
- [6] M. S. Shur, Terahertz plasmonic technology, *IEEE Sens. J.* **21**, 12752 (2021).
- [7] H. Qin, Y. Yu, X. Li, J. Sun, and Y. Huang, Excitation of terahertz plasmon in two-dimensional electron gas, *Int. J. Terahertz Sci. Technol.* **9**, 71 (2016).
- [8] V. Jakstas, I. Grigelionis, V. Janonis, G. Valusis, I. Kasalynas, G. Seniutinas, S. Juodkakis, P. Prystawko, and M. Leszczynski, Electrically driven terahertz radiation of 2DEG plasmons in AlGaIn/GaN structures at 110 K temperature, *Appl. Phys. Lett.* **110**, 202101 (2017).
- [9] R. A. Hopfel, E. Vass, and E. Gornik, Thermal excitation of two-dimensional plasma oscillations, *Phys. Rev. Lett.* **49**, 1667 (1982).
- [10] M. V. Krasheninnikov and A. V. Chaplik, Plasma-acoustic waves on the surface of a piezoelectric crystal, *Sov. Phys.-JETP* **61**, 75 (1985) [*Zh. Eksp. Teor. Fiz.* **88**, 129 (1985)].
- [11] M. Dyakonov and M. Shur, Shallow water analogy for a ballistic field effect transistor: New mechanism of plasma wave generation by dc current, *Phys. Rev. Lett.* **71**, 2465 (1993).
- [12] S. Rudin and G. Rupper, Plasma instability and wave propagation in gate-controlled GaN conduction channels, *Jpn. J. Appl. Phys.* (1962–1981) **52**, 08JN25 (2013).
- [13] M. V. Cheremisin and G. G. Samsonidze, Current instability and single-mode THz generation in ungated two-dimensional electron gas, *Solid-State Electron.* **52**, 338 (2008).
- [14] V. I. Ryzhii, N. A. Banov, and V. A. Fedirko, *Sov. Phys.-Semicond.* **18**, 769 (1984) [*Fiz. Tekh. Poluprovodn.* **8**, 769 (1984)].
- [15] Z. S. Gribnikov, N. Z. Vagidov, V. V. Mitin, and G. I. Haddad, Ballistic and quasiballistic tunnel transit time oscillators for the terahertz range: Linear admittance, *J. Appl. Phys.* **93**, 5435 (2003).
- [16] V. Ryzhii, A. Satou, M. Ryzhii, T. Otsuji, and M. S. Shur, Mechanism of self-excitation of terahertz plasma oscillations in periodically double-gated electron channels, *J. Phys.: Condens. Matter* **20**, 384207 (2008). This paper proposed a possible mechanism of the THz emission observed in the experiments with grating-gated FET [61,62].
- [17] V. V. Korotyeyev, V. A. Kochelap, A. A. Klimov, G. Sabatini, H. Marinchio, C. Palermo, and L. Varani, Theory of ballistic electron transport in n^+-i-n^+ diodes. Negative dynamic resistance in THz-frequency range, *J. Nanoelectron. Optoelectron.* **6**, 169 (2011).
- [18] D. Svintsov, Emission of plasmons by drifting Dirac electrons: A hallmark of hydrodynamic transport, *Phys. Rev. B* **100**, 195428 (2019).
- [19] S. A. Mikhailov, Plasma instability and amplification of electromagnetic waves in low-dimensional electron systems, *Phys. Rev. B* **58**, 1517 (1998); Tunable solid-state far-infrared sources: New ideas and prospects, *Recent Res. Devel. Appl. Phys.* **2**, 65 (1999).
- [20] S. Boubanga-Tombet, W. Knap, D. Yadav, A. Satou, D. B. But, V. V. Popov, I. V. Gorbenko, V. Kachorovskii, and T. Otsuji, Room-temperature amplification of terahertz radiation by grating-gate graphene structures, *Phys. Rev. X* **10**, 031004 (2020).
- [21] B. Y.-K. Hu and J. W. Wilkins, Two-stream instabilities in solid-state plasmas caused by conventional and unconventional mechanisms, *Phys. Rev. B* **43**, 14009 (1991).
- [22] Z. S. Gribnikov, N. Z. Vagidov, and V. V. Mitin, Two-stream instability and oscillatory regimes induced in ballistic diodes and field effect transistors, *J. Appl. Phys.* **88**, 67366 (2000).
- [23] S. Das Sarma and A. Madhukar, Collective modes of spatially separated, two-component, two-dimensional plasma in solids, *Phys. Rev. B* **23**, 805 (1981).
- [24] E. H. Hwang and S. Das Sarma, Plasmon modes of spatially separated double-layer graphene, *Phys. Rev. B* **80**, 205405 (2009).
- [25] T. A. Morgado and M. G. Silveirinha, Negative Landau damping in bilayer graphene, *Phys. Rev. Lett.* **119**, 133901 (2017); D. Svintsov and V. Ryzhii, Comment on “Negative Landau damping in bilayer graphene,” *ibid.* **123**, 219401 (2019); T. A. Morgado and M. G. Silveirinha, Reply to the comment on “Negative Landau damping in bilayer graphene,” *ibid.* **123**, 219402 (2019).
- [26] S. M. Komirenko, K. W. Kim, V. A. Kochelap, I. Fedorov, and M. A. Strosio, Coherent optical phonon generation by the electric current in quantum wells, *Appl. Phys. Lett.* **77**, 4178 (2000); Generation of coherent confined LO phonons under the drift of two-dimensional electrons, *Phys. Rev. B* **63**, 165308 (2001).
- [27] W. Liang, K. T. Tsen, O. F. Sankey, S. M. Komirenko, K. W. Kim, V. A. Kochelap, M.C. Wu, C.-L. Ho, W.-J. Ho, and H. Morkoc, Observation of optical phonon instability induced by drifting electrons in semiconductor nanostructures, *Appl. Phys. Lett.* **82**, 1968 (2003).
- [28] K. T. Tsen, J. Kiang, D. K. Ferry, V. A. Kochelap, S. M. Komirenko, K. W. Kim, and H. Morkoc, Subpicosecond Raman studies of electric-field-induced optical phonon instability in an $\text{In}_{0.53}\text{Ga}_{0.47}\text{As}$ -based semiconductor nanostructure, *J. Phys.: Condens. Matter* **18**, 7961 (2006).
- [29] O. Sydoruk, E. Shamonina, V. Kalinin, and L. Solymar, Terahertz instability of surface optical-phonon polaritons that interact with surface plasmon polaritons in the presence of electron drift, *Phys. Plasmas* **17**, 102103 (2010).
- [30] M. Woerner, C. Somma, K. Reimann, T. Elsaesser, P. Q. Liu, Y. Yang, J. L. Reno, and I. Brener, Terahertz driven amplification of coherent optical phonons in GaAs coupled to a metasurface, *Phys. Rev. Lett.* **122**, 107402 (2019).
- [31] V. A. Kochelap, V. V. Korotyeyev, Yu. M. Lyashchuk, and K. W. Kim, Nanoscale ballistic diodes made of polar materials for amplification and generation of radiation in the 10 THz-range, *J. Appl. Phys.* **126**, 085708 (2019).
- [32] O. Penrose, Electrostatic instabilities of a uniform non-Maxwellian plasma, *Phys. Fluids* **3**, 258 (1960).

- [33] N. A. Krall and A. W. Trivelpiece, *Principles of Plasma Physics* (McGraw-Hill, New York, 1973).
- [34] In a single layer with drifting electrons, a nonhydrodynamic type of electric instability can occur due to the impact of the electric field on high-frequency electron dynamics: V. V. Korotyeyev and V. A. Kochelap, Plasma wave oscillations in a nonequilibrium two-dimensional electron gas: Electric field induced instability in terahertz frequency range, *Phys. Rev. B* **101**, 235420 (2020).
- [35] K. S. Novoselov, A. Mishchenko, A. Carvalho, and A. H. Castro Neto, 2D materials and van der Waals heterostructures, *Science* **353**, aac9439 (2016).
- [36] K. S. Thygesen, Calculating excitons, plasmons, and quasiparticles in 2D materials and van der Waals heterostructures, *2D Mater.* **4**, 022004 (2017).
- [37] R. Roldán and L. Brey, Dielectric screening and plasmons in AA-staked bilayer graphene, *Phys. Rev. B* **88**, 115420 (2013).
- [38] In chosen coordinate system, the plane $z = 0$ is in the middle of the distance d between layer 1 ($z = d/2$) and layer 2 ($z = -d/2$).
- [39] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/7vjy-36zl> for the details of solution of the Poisson equation for the two-layer nanostructure.
- [40] F. Stern, Polarizability of a two-dimensional electron gas, *Phys. Rev. Lett.* **18**, 546 (1967).
- [41] A. V. Chaplik, Absorption and emission of electromagnetic waves by two-dimensional plasmons, *Surf. Sci. Rep.* **5**, 289 (1985).
- [42] T.-H. Park and P. Nordlander, On the nature of the bonding and antibonding metallic film and nanoshell plasmons, *Chem. Phys. Lett.* **472**, 228 (2009).
- [43] N. J. Halas, S. Lal, W.-S. Chang, S. Link, and P. Nordlander, Plasmons in strongly coupled metallic nanostructures, *Chem. Rev. (Washington, DC, U. S.)* **111**, 3913 (2011).
- [44] A. Politano and G. Chiarello, Plasmon modes in graphene: status and prospect, *Nanoscale* **6**, 10927 (2014).
- [45] L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Medium*, 2nd ed. (Pergamon Press Inc., New York, 1984), Vol. 8, Chap. 6, p. 406.
- [46] V. Pinhão, P. Cosme, and H. Terças, Anomalous conductivity due to two-stream instability in bilayer graphene, *Phys. Rev. B* **108**, L081406 (2023).
- [47] L. D. Landau and E. M. Lifshitz, *Course of Theoretical Physics. Physical Kinetics* (Pergamon Press Inc., Maxwell House, New York, 1981). Vol. 10, Chap. 6, p. 276.
- [48] V. V. Koroteev, V. A. Kochelap, S. Danylyuk, and L. Varani, Spatial dispersion of the high-frequency conductivity of two-dimensional electron gas subjected to a high electric field: Collisionless case, *Appl. Phys. Lett.* **113**, 041102 (2018).
- [49] J. M. Barker, D. K. Ferry, S. M. Goodnick, D. D. Koleske, A. Allerman, and R. J. Shul, High field transport in GaN/AlGaIn heterostructures, *J. Vac. Sci. Technol., B: Microelectron. Nanometer Struct.–Process., Meas., Phenom.* **22**, 2045 (2004).
- [50] S. K. O’Leary, B. E. Foutz, M. S. Shur, and L. F. Eastman, Steady-state and transient electron transport within the III-V nitride semiconductors, GaN, AlN, and InN: A Review, *J. Mater. Sci.: Mater. Electron.* **17**, 87 (2006).
- [51] E. Gornik and A. A. Andronov, Far-infrared semiconductor lasers, *Opt. Quantum Electron.* **23**, v (1991).
- [52] L. E. Vorob’ev, S. N. Danilov, V. N. Tulupenko, and D. F. Firsov, Generation of millimeter radiation due to electric-field-induced electron-transit-time resonance in indium phosphide *JETP Lett.* **73**, 219 (2001) [*Pis’m’a Zh. Eksp. Teor. Fiz.* **73**, 253 (2001)].
- [53] V. V. Korotyeyev, V. A. Kochelap, K. W. Kim, and D. L. Woolard, Streaming distribution of two-dimensional electrons in III-N heterostructures for electrically pumped terahertz generation, *Appl. Phys. Lett.* **82**, 2643 (2003).
- [54] P. Shiktorov, E. Starikov, V. Gružinskis, L. Varani, C. Palermo, J.-F. Millithaler, and L. Reggiani, Frequency limits of terahertz radiation generated by optical-phonon transit-time resonance in quantum wells and heterolayers, *Phys. Rev. B* **76**, 045333 (2007).
- [55] Note, detailed kinetic analysis predicts charge-wave excitations of nonhydrodynamical type: V. V. Korotyeyev, V. A. Kochelap, and L. Varani, Wave excitations of drifting two-dimensional electron gas under strong inelastic scattering, *J. Appl. Phys.* **112**, 083721 (2012).
- [56] M. J. Manfra, K. W. Baldwin, A. M. Sergent, K. W. West, R. J. Molnar, and J. Caissie, Electron mobility in very low density GaN/AlGaIn/GaN heterostructures, *Appl. Phys. Lett.* **85**, 1722 (2004).
- [57] C. Skierbiszewski, Z. Wasilewski, M. Siekacz, A. Feduniewicz, B. Pastuszka, I. Grzegory, M. Leszczynski, and S. Porowski, High magnetic field studies of AlGaIn/GaN heterostructures grown on bulk GaN, *Phys. Status Solidi, A* **201**, 320 (2004).
- [58] C. Skierbiszewski, K. Dybko, W. Knap, M. Siekacz, Z. R. Krupczynski, G. Nowak, M. Boćkowski, J. Lusakowski, Z. R. Wasilewski, D. Maude, T. Suski, and S. Porowski, High mobility two-dimensional electron gas in heterostructures grown on bulk GaN by plasma assisted molecular beam epitaxy, *Appl. Phys. Lett.* **86**, 102106 (2005).
- [59] E. A. Henriksen, S. Syed, Y. Ahmadian, M. J. Manfra, K. W. Baldwin, A. M. Sergent, R. J. Molnar, and H. L. Stormer, Acoustic phonon scattering in a low density, high mobility AlGaIn/GaN field-effect transistor, *Appl. Phys. Lett.* **86**, 252108 (2005).
- [60] A. Asgari, S. Babanejad, and L. Faraone, Electron mobility, Hall scattering factor, and sheet conductivity in Al-GaN/AlN/GaN heterostructures, *J. Appl. Phys.* **110**, 113713 (2011).
- [61] Y. M. Meziani, T. Otsuji, M. Hanabe, T. Ishibashi, T. Uno, and E. Sano, Room temperature generation of terahertz radiation from a grating-bicoupled plasmon-resonant emitter: Size effect, *Appl. Phys. Lett.* **90**, 061105 (2007).
- [62] T. Otsuji, Y. M. Meziani, M. Hanabe, T. Ishibashi, T. Uno, and E. Sano, Grating-bicoupled plasmon-resonant terahertz emitter fabricated with GaAs-based heterostructure material systems, *Appl. Phys. Lett.* **89**, 263502 (2006).