

Stabilization of the skyrmion in a hybrid magnetic-superconducting nanostucture

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Stabilizing magnetic skyrmions typically requires a Dzyaloshinskii–Moriya interaction or competing interactions that produce magnetic frustration. An alternative route relies on external fields. In conventional magnetic materials, spatially inhomogeneous magnetic fields—such as demagnetizing or Oersted fields—can stabilize skyrmions. Here, we propose using a superconducting nanoring to locally control and tune the magnetic field in an adjacent ferromagnetic layer. The field generated by a persistent supercurrent stabilizes a Néel skyrmion. We analyze the stabilization conditions in ferromagnetic layers with perpendicular magnetic anisotropy as functions of nanoring size and induced supercurrent. We show that skyrmion stability requires the supercurrent to exceed a critical threshold. Consistent results are presented from analytical modeling and micromagnetic simulations for thin Co and Ga:YIG films.

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I. INTRODUCTION

In recent decades, inhomogeneous magnetic textures such as domain walls, skyrmions, vortices, etc., have attracted much attention because of their potential applications in spintronics and magnonics [1,2]. Domain walls and skyrmions can be used to store and transmit the information in race-track memory systems [3,4]. Moreover, the information can be processed on the neuromorphic platforms based on magnetic vortices or skyrmions [5,6].

Conventionally, magnetic skyrmions [3,7] used to be stabilized by the Dzyaloshinskii–Moriya interaction (DMI) [8–10] or by the magnetic frustration due to competing exchange and Ruderman–Kittel–Kasuya–Yosida (RKKY) interactions [11–13], which imposes rather strong constraints on skyrmion-hosting materials. To overcome this problem it has been proposed to stabilize the skyrmions by the demagnetizing field [14–17]. Alternatively, it was shown that the Bloch skyrmions can be stabilized in artificial crystals based upon a combination of an out-of-plane magnetized film and nanopatterned arrays of nanodisks [18] or in sub-micron dots with moderate out-of-plane magnetic anisotropy

[19,20]. Additionally, it was demonstrated the stabilization of the Néel-like skyrmions by means of a strong stray dipolar field generated by a patterned hard magnetic layer placed near the soft magnetic film was demonstrated [21]. However, all these methods, which play with material parameters, state of interfaces, or the sizes of the nanostructures, do not allow creating and destroying the skyrmions on demand, making their application limited. To gain such functionality, an external bias is used. However, the local control of nanometer-sized skyrmions, e.g., by spin polarized current from a scanning tunneling microscope (STM) tip [22], is challenging. The simpler approach is to operate with dipolarly stabilized skyrmions of larger sizes and to apply a local magnetic field produced by the electric current. This idea was exploited to steer domains walls [23], control magnetization configuration of ferromagnetic (FM) rings [24], and induce hopfions in a FM torus [25].

The skyrmions can be used to design neuromorphic systems [26]. Recently, Qiu *et al.* [27] proposed an on-chip skyrmion artificial synaptic device, where the radii of skyrmions in flat circular nanodots exhibiting interfacial DMI were influenced by the pulses of Oersted field generated by the current flowing in Ω -shaped wires, while the output signal of each synapse was read out from the magnetic tunnel junction with a free layer being a manipulated skyrmion [28]. The authors reasonably point out that one of the problems in this spintronic system is the local heating generated by the currents. Here, we propose an alternative approach to stabilize Néel skyrmions in a continuous layer made of conventional FM materials (Co, YIG), using persistent currents induced in superconducting (SC) rings [29]. Such on-demand control

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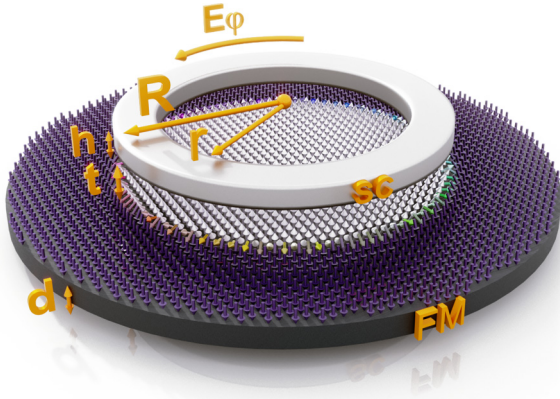


FIG. 1. The magnetic stray field of the superconducting (SC) ring stabilizes the magnetization texture in the ferromagnetic (FM) layer with out-of-plane magnetic anisotropy. Eddy currents in the SC ring of external radius R , inner radius r , and height h , circulate in one direction across the entire cross section due to the electric field component E_ϕ . The SC ring is separated from the FM layer by a gap t . The thickness d of the FM layer is small compared to the spatial variations in magnetization. The illustrated magnetization texture represents a skyrmion stabilized in gallium-doped YIG—see Fig. 3.

brings closer the realization of a magnonic neuromorphic system [30], where the magnetic skyrmions serve as weights and interconnections of the neural network, while spin waves handle signal routing and nonlinear activation.

The paper has the following structure. The theoretical models developed to calculate the stray magnetic field produced by the SC ring and to determine the radius of a skyrmion stabilized by this field are presented in Sec. II. The procedure for stabilizing a skyrmion in the micromagnetic simulations, carried out with our in-house version of mumax3 [31], amumax [32], is summarized at the end of Sec. II B (“Micromagnetic simulations”). The subsequent Sec. III presents and discusses the outcomes of the theoretical and numerical studies investigating the influence of the SC ring geometry, the magnitude of the SC current, and the material parameters of the FM layer concerning skyrmion stabilization. The studies are summarized in Sec. IV with our Conclusions. Additional details are provided in the Supplemental Material [33], where we show how the dimensions and current of the SC ring affect the skyrmion size, and discuss how the critical current required for the skyrmion stabilization depends on material parameters.

II. MODEL

We consider a single superconducting ring with a thin ferromagnetic layer underneath (see Fig. 1). However, our results are valid for an array of SC rings if the distance between them is large enough to exclude inter-ring interactions. The SC ring and the FM layer are electrically isolated from each other. We assume that the SC ring is in the Meissner state and the FM layer is characterized by an effective out-of-plane magnetic anisotropy. To simplify the calculations, we consider a magnetic layer of finite in-plane size, i.e., a circular disk with radius R_d , which is larger than the sizes of the SC ring:

$R_d > R > r$, where R and r denote the outer and inner radii of the ring, respectively. The SC ring has a rectangular cross section: $(R - r) \times h$, where h is its height, and is separated from the FM layer by the gap of the width t . The thickness of the FM layer d is small. Therefore, both the magnetic field generated by the SC ring and the FM layer magnetization can be considered as constants over the thickness of the FM layer. Taking into account the cylindrical symmetry of the system, we use the cylindrical coordinates with the radius vector $\mathbf{r} = (\rho, \phi, z)$, where z is the out-of-plane axis.

The SC current can be induced in a superconducting ring using various schemes [34], resulting in different spatial distributions of the current density. We consider the case in which a unidirectional flow of SC current is formed throughout the entire cross section of the ring in the absence of an external magnetic field. In this state, the only source of the static magnetic field is the persistent current, which produces the magnetic field of opposite directions inside and outside the SC ring, thereby enhancing the stabilization of the skyrmion. To reach such a state, we consider the application of an electric field pulse with nonzero curl in the plane of the ring, $\mathbf{E}(t) = E(t)\mathbf{e}_\phi$. We assume a rectangular pulse shape; i.e., the electric field is constant and nonzero during a finite time interval t_0 . This effect is clearly demonstrated by the first London equation [35,36],

$$\mathbf{E} = -\frac{\lambda^2}{\mu_0} \partial_t \mathbf{j}, \quad (1)$$

where the electric field should be turned on only for a short time to change the value of SC current density $\mathbf{j}$. The rate of this change depends on the London penetration depth λ . The symbol μ_0 denotes the permeability of vacuum. The same state can be reached when the SC ring is taken out of the region of the static magnetic field after being cooled down in the presence of this field.

The currents in the SC ring create a nonuniform distribution of the stray magnetic field with radial symmetry in the FM layer $\mathbf{B}_{sc}(\rho) = B_\rho(\rho)\mathbf{e}_\rho + B_z(\rho)\mathbf{e}_z$; see Fig. 2. We consider the FM layer (thin disk) with an effective perpendicular magnetic anisotropy (PMA) constant $K = K_u - \mu_0 M_s^2 > 0$, where the magnetocrystalline (K_u) anisotropy overcomes the shape anisotropy of the layer.

The magnetic stray field generated by the SC ring, $\mathbf{B}_{sc}$, stabilizes the Néel-type skyrmion in the FM layer. In principle, one should also take into account the effect of the magnetic field generated by the skyrmion’s magnetization texture on the SC currents, and consequently, on the field produced by the SC ring. However, as we confirmed by micromagnetic simulation, this feedback is weak—the field produced by the Néel-type skyrmion is at least an order of magnitude smaller than the stray field of the SC ring. The small magnitude of the skyrmion’s stray field follows from the fact that a flat, infinitely extended, and uniformly magnetized FM film produces no external field. The stray field arises here from magnetostatic volume charges in regions where the magnetization is nonuniform (see Fig. 1). Moreover, owing to the Halbach effect [37], the stray field above the FM layer (where the SC ring is located) is smaller than below it. Consequently, the stray field of the skyrmion texture perceived by the SC

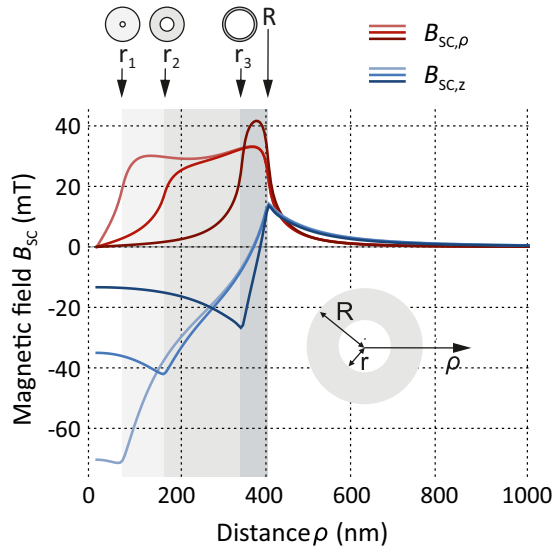


FIG. 2. Profiles of the magnetic field components $B_{sc,z}$ (blue lines) and $B_{sc,\rho}$ (red lines) generated by SC rings with the same external radius $R = 405$ nm and different internal radii r : $r_1 = 61$ nm, $r_2 = 160$ nm, and $r_3 = 344$ nm. The field profiles are plotted along the radial direction, starting from the center of the SC ring at $\rho = 0$. The gray vertical strips indicate the region occupied by the SC ring: $r < \rho < R$.

ring remains about 1% of the field generated by the ring itself (the Supplemental Material [33]). Therefore, we consider only the influence of the SC ring on the FM layer, neglecting the mutual interaction between the two subsystems. Our study was carried out in two consecutive steps. First, we semianalytically calculate the magnetic stray field generated by the SC ring. Then we investigate, both analytically and numerically, the stabilization of the Néel-type skyrmion in a thin FM disk placed in the magnetic field produced by the SC ring.

A. Static magnetic field produced by superconducting ring

To find the magnetic field distribution created by the SC current induced by the electric field, we have modified a semianalytical approach developed in Ref. [36]. Application of the electric field $\mathbf{E}$ along the circumferential direction in the SC ring in the Meissner state induces a current of density $\mathbf{j} = j\mathbf{e}_\varphi$. According to the Ampere equation the current density is connected with magnetic field $\mathbf{B}_{sc}$ and vector magnetic potential $\mathbf{A}_{sc}$:

$$\mu_0 \mathbf{j} = \nabla \times \mathbf{B}_{sc} = \nabla \times \nabla \times \mathbf{A}_{sc} = -\Delta \mathbf{A}_{sc}, \quad (2)$$

where the Coulomb gauge was used for the vector potential. Equation (2) has a form of Poisson equation for vector potential $\mathbf{A}_{sc} = A_{sc}\mathbf{e}_\varphi$:

$$\Delta A_{sc} = -\mu_0 j + \frac{1}{\rho^2} A_{sc}. \quad (3)$$

The total vector potential $\mathbf{A} = \mathbf{A}_{sc} + \mathbf{A}_E$ also includes the contribution $\mathbf{A}_E$ originating from the rectangular pulse of applied electric field $\mathbf{E}(t) = E(t)\mathbf{e}_\varphi$ [36]:

$$A_E = -\int_0^{t_0} E dt, \quad (4)$$

where t_0 and E are the duration and intensity of the pulse, respectively. The total vector potential is related to the SC current by the formula [35]

$$\mathbf{A} = \mathbf{A}_{sc} + \mathbf{A}_E = -\mu_0 \lambda^2 \mathbf{j}, \quad (5)$$

derived from the London equation (1), and the relation $\mathbf{E} = -\partial_t \mathbf{A}$. Combining Eqs. (5), (4), and (3) allows us to formulate the following integral equation for the density of the SC current:

$$\lambda^2 j(\mathbf{r}) = \int_V d^3 \mathbf{r}' Q(\mathbf{r}, \mathbf{r}') \left(j(\mathbf{r}') + \frac{\lambda^2}{\rho'^2} (j(\mathbf{r}') - j_0) \right) + \lambda^2 j_0, \quad (6)$$

where V is the volume of the SC ring, $j_0 = Et_0/(\mu_0 \lambda^2)$, and the function $Q = -\frac{1}{4\pi|\mathbf{r}-\mathbf{r}'|}$ is the integral kernel for the Laplace operator. In the cylindrical coordinate system, it can be expressed in k space using Fourier expansion inside the SC ring [38]:

$$\frac{1}{|\mathbf{r}-\mathbf{r}'|} = \sum_{n=-\infty}^{+\infty} \int_0^{+\infty} dk J_n(k\rho) J_n(k\rho') e^{-k|z-z'|} e^{in(\varphi-\varphi')}, \quad (7)$$

with J_n being the Bessel function of the first kind of the n th order. Using (7) and taking into account that j does not depend on the φ coordinate and weakly depends on the z coordinate, Eq. (6) can be simplified to the form

$$\lambda^2 j(\rho) = \int_r^R d\rho' \tilde{Q}(\rho, \rho') j(\rho') + j_0 \lambda^2 \left(1 - \int_r^R d\rho' \tilde{C}(\rho, \rho') \right), \quad (8)$$

where $\tilde{C}(\rho, \rho') = -\int_0^\infty dk \frac{1}{k\rho'} J_0(k\rho) J_0(k\rho') (1 - \frac{1-e^{-kh}}{kh})$ and $\tilde{Q}(\rho, \rho') = \tilde{C}(\rho, \rho')(\rho'^2 + \lambda^2)$. Here, we average the integral (7) over the z coordinate, because we assumed that the current density j does not change significantly across the SC ring thickness and consider it as a function of only the ρ coordinate. Equation (8) can be integrated numerically by sampling the function $j(\rho)$ on a grid of N equidistant points ρ_i in the interval (r, R) . In this way the function $j(\rho)$ becomes a vector $\mathbf{j}$ with N components $j_i = j(\rho_i)$, and the integral kernels are transformed into the $N \times N$ matrices $\mathcal{Q}$ and $\mathcal{C}$ of the elements $\mathcal{Q}_{i,j} = \tilde{Q}(\rho_i, \rho'_j)$ and $\mathcal{C}_{i,j} = \tilde{C}(\rho_i, \rho'_j)$, respectively. Then Eq. (8) can be reduced to the matrix equation for the components of the vector $\mathbf{j}_i$ which has the following solution (for details see Refs. [36,39]):

$$j_i = j_0 \lambda^2 \sum_j \mathcal{P}_{i,j}. \quad (9)$$

The matrix $\mathcal{P}$ is defined as

$$\mathcal{P} = (\lambda^2 \mathcal{I} - \Delta \rho \mathcal{Q})^{-1} (\mathcal{I} - \Delta \rho \mathcal{C}), \quad (10)$$

where $\Delta \rho = (R - r)/N$ and $\mathcal{I}$ denotes the identity matrix. Finally, the distribution of the magnetic stray field produced

by the eddy currents of the SC ring can be determined from the Biot-Savart law:

$$\begin{aligned} B_{\text{sc},\rho}(\rho, z) &= -\frac{\mu_0}{4\pi} \int_{V_r} d^3\mathbf{r}' S(\rho, \rho', z, z') \\ &\quad \times j(\rho') \cos \varphi'(z - z'), \\ B_{\text{sc},z}(\rho, z) &= \frac{\mu_0}{4\pi} \int_{V_r} d^3\mathbf{r}' S(\rho, \rho', z, z') \\ &\quad \times j(\rho')(\rho \cos \varphi' - \rho'), \end{aligned} \quad (11)$$

where the integration goes over the volume V_r of the SC ring with $d^3\mathbf{r}' = \rho' d\rho' d\varphi' dz'$ and $S(\rho, \rho', z, z') = [\rho^2 + \rho'^2 - 2\rho\rho' \cos \varphi' + (z - z')^2]^{-\frac{3}{2}}$.

The exemplary profiles of the magnetic stray field components B_{sc} produced by the SC ring of the external radius $R = 405$ nm and different internal radii r , $r_1 = 61$ nm, $r_2 = 160$ nm, and $r_3 = 344$ nm, are presented in Fig. 2. Calculations were done for the rectangular pulse of applied electric field $E t_0 = 11.25 \times 10^{-9}$ V/m s. We assumed the London penetration depth $\lambda = 50$ nm, which is realistic for Nb [40]. Blue and orange solid lines correspond to the $B_{\text{sc},\rho}$ and $B_{\text{sc},z}$ components, respectively, plotted along the radial direction. It is clear from Fig. 2 that the radial component of the SC field, $B_{\text{sc},\rho}$, promotes in-plane static magnetization configuration in FM disk under the SC ring, while the normal component $B_{\text{sc},z}$ makes favorable out-of-plane magnetization near the ring edges, which has opposite directions inside and outside of the ring. In other words, the magnetic stray field of the SC ring has the same symmetry as Néel-type skyrmion magnetization, which allows us to expect that application of the magnetic field generated by the SC ring to the FM disk can stabilize the skyrmion texture. To check this hypotheses and establish the conditions for the skyrmion stabilization, we have formulated a theoretical model, which is described in Sec. II B.

B. Skyrmion stabilization by the field of superconducting ring

Let us investigate the energy of the ferromagnetic layer (or disk of the radius $R_d \gg R$) to find the magnetization configurations that can be stabilized in the FM layer by the magnetic field generated by the SC ring. We assume that the layer is thin, so that the magnetization can be considered uniform along the out-of-plane z coordinate. The magnetic energy functional for the disk of the radius R_d and thickness d can be written as

$$E[\mathbf{M}] = d \int_{S_d} d^2\mathbf{r} \varepsilon(\mathbf{M}(\mathbf{r})), \quad (12)$$

where the integration goes over the area S_d of the disk's cross section in the ρ - φ plane with $d^2\mathbf{r} = \rho d\rho d\varphi$. The energy density ε consists of the exchange ε_{ex} , magnetostatic ε_{ms} , perpendicular magnetic anisotropy ε_{an} , and Zeeman ε_z contributions.

To derive the explicit expression for the magnetic energy density, it is convenient to represent the magnetization in spherical coordinates with the azimuthal and polar angles $\theta(\rho, \varphi)$ and $\phi(\rho, \varphi)$: $\mathbf{M} = M_s(\sin \theta \cos \phi \hat{\mathbf{x}} + \sin \theta \sin \phi \hat{\mathbf{y}} + \cos \theta \hat{\mathbf{z}})$, where M_s is the saturation magnetization. Then the

exchange energy density takes the form [41]

$$\varepsilon_{\text{ex}} = A_{\text{ex}}((\nabla \theta)^2 + (\nabla \phi)^2 \sin^2 \theta), \quad (13)$$

where A_{ex} is exchange stiffness. The magnetostatic energy density within the limit of the ultrathin disk is close to the energy density of the infinite film: $\varepsilon_{\text{ms}} = \frac{1}{2} \mu_0 M_s^2 \cos^2 \theta$ —we neglected here the contribution of volume magnetic charges. The energy density of uniaxial perpendicular magnetic anisotropy is $\varepsilon_{\text{an}} = -K_u \cos^2 \theta$, where K_u is the anisotropy constant. Since the magnetostatic interactions are responsible for the sample shape anisotropy, we can define the effective anisotropy energy density:

$$-K_u^{\text{eff}} \cos^2 \theta = \varepsilon_a + \varepsilon_{\text{ms}} = -(K_u - \frac{1}{2} \mu_0 M_s^2) \cos^2 \theta. \quad (14)$$

The Zeeman energy is defined by the applied magnetic field, which here is the magnetic stray field produced by the SC ring. Remembering that the SC stray field creates conditions favorable for stabilization of the Néel-like skyrmion texture (Fig. 2), we can conclude that the Zeeman interaction is responsible for the skyrmion formation in the absence of DMI. Since the SC magnetic stray field has radial symmetry, the Zeeman energy density can be written in the following form:

$$\varepsilon_z = -M_s(B_\rho \sin \theta + B_z \cos \theta). \quad (15)$$

To find the stable magnetization texture, we have to minimize the functional $E[\mathbf{M}]$ with respect to the θ and ϕ magnetization angles solving the Euler-Lagrange equations for these angles. Due to the symmetry of the stray field (11), we look for axially symmetric magnetization configurations, where $\mathbf{M}(\rho, \varphi)$ is a specific function. It means that the polar and azimuthal angles for the magnetization vector are $\theta(\rho, \varphi) = \theta(\rho)$ and $\phi(\rho, \varphi) = \varphi + \varphi_0$, where $\varphi_0 = 0$ or π for the Néel skyrmion. The magnetization texture $\mathbf{M}(\mathbf{r})$ which extremizes the functional (12) is then expressed by the scalar function $\theta(\rho)$ which depends on one coordinate only. It simplifies the variational problem for Eq. (12) to the solution of a one-dimensional Euler-Lagrange equation for the $\theta(\rho)$ function (see Ref. [42]) with boundary conditions $\theta(\rho)|_{\rho=0} = \pi$, $\frac{d}{d\rho} \theta(\rho)|_{\rho=R_d} = 0$ [43]. Despite this simplification, analytical solution of the Euler-Lagrange equation is not possible. For this reason, we looked for a solution in the form of a trial function. We have used the so-called DeBonte ansatz [44]:

$$\tan \left(\frac{1}{2} \theta(\rho) \right) = \frac{R_s}{\rho} \exp \left(-\frac{1}{\Delta} (\rho - R_s) \right), \quad (16)$$

which is a quite accurate description of the texture for the Néel-type skyrmions with two parameters: the width of skyrmion domain wall Δ and its radius R_s (defined as a center of domain wall, where $\theta(\rho = R_s) = \pi/2$). In our studies, we fixed the width of the domain wall because it depends mostly on the competition between the material parameters' out-of-plane anisotropy and exchange stiffness [45]: $\Delta = \sqrt{A_{\text{ex}}/K_u^{\text{eff}}}$. For the Néel skyrmion ansatz (16), the energy (12) is a function of one parameter—the radius of skyrmion domain wall R_s

(see Refs. [21,42]):

$$E(R_s) = 2\pi d \int_0^{R_d} \rho d\rho \left[\frac{2A_{\text{ex}}}{M_s^2} \left(\frac{1}{\rho^2} + \frac{1}{\rho\Delta} + \frac{1}{\Delta^2} \right) M_\rho^2 - (B_{\text{sc},\rho} M_\rho + B_{\text{sc},z} M_z) \right] - \pi d K_u^{\text{eff}} R_d^2, \quad (17)$$

where $M_\rho(\rho) = M_s \sin[\theta(\rho)]$ and $M_z(\rho) = M_s \cos[\theta(\rho)]$ are the components of the magnetization vector in the cylindrical coordinate system: $\mathbf{M} = M_\rho \hat{\rho} + M_z \hat{z}$. The components of the magnetic stray field (11) $B_{\text{sc},\rho}(\rho)$ and $B_{\text{sc},z}(\rho)$ are taken in the middle plane of the thin FM layer.

Using the standard procedure (i.e., looking for the solutions of the equation $dE/dR_s = 0$ satisfying the condition $d^2E/dR_s^2 > 0$), we can minimize the energy (17) with respect to the radius of skyrmion domain wall R_s to find the radius $R_s = R_{s,0}$ of the stable skyrmion. We consider that the skyrmion can be generated in the system if the energy (17) has a minimum for some finite value of R_s inside the FM disk: $0 < R_s < R_d$.

Micromagnetic simulations. The theoretical studies of skyrmion stabilization are cross checked by micromagnetic simulations. We use our own version of mumax3 [31,46] called amumax [32], which solves the Landau-Lifshitz-Gilbert equation:

$$\frac{d\mathbf{m}}{dt} = \frac{|\gamma|\mu_0}{1 + \alpha^2} [\mathbf{m} \times \mathbf{H}_{\text{eff}} + \alpha \mathbf{m} \times (\mathbf{m} \times \mathbf{H}_{\text{eff}})], \quad (18)$$

where $\mathbf{m} = \mathbf{M}/M_s$ is the normalized magnetization, $\mathbf{H}_{\text{eff}}$ is the effective magnetic field acting on the magnetization, γ is the gyromagnetic ratio, μ_0 denotes the vacuum permeability, and α is the Gilbert damping. The following components contribute to the effective magnetic field $\mathbf{H}_{\text{eff}}$: demagnetizing field in magnetostatic approximation $\mathbf{H}_{\text{ms}}$, exchange field $\mathbf{H}_{\text{ex}}$, uniaxial magnetic anisotropy field $\mathbf{H}_{\text{an}}$, and external magnetic field $\mathbf{H}_{\text{sc}} = 1/\mu_0 \mathbf{B}_{\text{sc}}$ produced by the SC ring.

$$\mathbf{H}_{\text{eff}} = \mathbf{H}_{\text{ms}} + \mathbf{H}_{\text{ex}} + \mathbf{H}_{\text{an}} + \mathbf{H}_{\text{sc}}. \quad (19)$$

The exchange and anisotropy fields are defined as

$$\mathbf{H}_{\text{ex}} = \frac{2A_{\text{ex}}}{\mu_0 M_s} \Delta \mathbf{m}, \quad \mathbf{H}_{\text{an}} = \frac{2K_u}{\mu_0 M_s} m_z \hat{z}, \quad (20)$$

and the magnetostatic field is computed numerically from the general formula

$$\mathbf{H}_{\text{ms}}(\mathbf{r}) = -M_s \int \mathbf{N}(\mathbf{r} - \mathbf{r}') \cdot \mathbf{m}(\mathbf{r}') d^3 \mathbf{r}', \quad (21)$$

where the demagnetizing tensor is

$$\mathbf{N}(\mathbf{r} - \mathbf{r}') = \frac{1}{4\pi} \nabla \nabla' \frac{1}{|\mathbf{r} - \mathbf{r}'|}. \quad (22)$$

The external field $\mathbf{H}_{\text{sc}} = 1/\mu_0 \mathbf{B}_{\text{sc}}$ is calculated as it is presented in Sec. II A and introduced to the micromagnetic solver. We neglect the thermal effects.

To calculate the skyrmion magnetic energy, we use the frozen spin technique with the *frozen spin function* built into mumax3 and amumax. In the sequence of calculations, we select the different trial radii of skyrmion domain wall R_s and frozen magnetization ($\theta = \pi/2$, $\phi = \varphi$) in a very narrow ring

(one cuboid of the mesh) of the radius R_s . By varying the radius of the artificial ring, we compute the dependence energy of the skyrmion as a function of its radius $E(R_s)$. This allows us to find the radius for which the energy is minimized. Then we remove the constraint of frozen magnetization and allow the whole texture to relax freely. As a result, we are able to find a stable Néel skyrmion magnetization configuration in the magnetic field produced by the SC ring.

The material parameters and the dimensions of the considered structure, used in the semianalytic calculations and micromagnetic simulations, are given in Sec. III.

III. RESULTS

In our calculations, we have used a SC ring with thickness $h = 100$ nm and varying outer radius $R = 50 - 500$ nm and inner radius $r = 0.1R - 0.9R$. The SC ring is made of Nb with the London penetration depth $\lambda = 50$ nm [40]. The higher ring is better for creating a stronger field, but we are limited in our model by the assumption of uniform current density in the z direction. The assumed height of the ring, $h = 2\lambda$, is a reasonable compromise, because for $h < \lambda$ we enter the regime of small superconductors [47], where λ becomes size dependent [48], and the magnetic response of the SC ring is reduced. The ring is separated from the FM disk by a nonconductive spacer of thickness $t = 5$ nm. Decreasing t enhances the stabilization of the skyrmion by strengthening the SC ring's field inside the FM disk, but it would also promote tunneling to a conducting ferromagnet—an effect excluded from our model. For the FM disk, we explored two commonly used materials with significantly different parameters: Ga:YIG (saturation magnetization $M_s = 160$ kA/m, exchange stiffness $A_{\text{ex}} = 1.37$ pJ/m, PMA $K_u = 756$ J/m³, disk thickness $d = 6$ nm [49,50]) and Co ($M_s = 1.4$ MA/m, $A_{\text{ex}} = 30$ pJ/m, $K_u = 1.39$ MJ/m³, $d = 0.75$ nm [51]). To eliminate the influence of the disk edge on the skyrmion, we set the radius of the disk to $R_d = 3R$. Calculations, whose results are presented in Fig. 3, were done for the fixed profile of the electric field pulse $E t_0 = 11.25 \times 10^{-9}$ V s/m.

For the skyrmion described by the DeBonte ansatz, we calculated the energy of its texture assuming a fixed domain wall width: $\Delta = \sqrt{A_{\text{ex}}/K_u^{\text{eff}}}$, and examined how it depends on the assumed radius of the skyrmion domain wall: $E(R_s)$ —see Eq. (17) and Figs. 3(c)–3(f). We then searched for the minimum of the $E(R_s)$ dependence and identified its position $R_{s,0}$ and depth ΔE . The parameter $R_{s,0}$ is considered as a radius of a stable skyrmion and ΔE a measure of its stability against thermal or microwave excitation, according to the Arrhenius law. It is worth noting that at the boundaries of the domain, $0 < R_s < R_d$, we also observe the energy wells. However, when $R_s = 0$ or $R_s = R_d$, the FM disk of radius R_d is in single-domain out-of-plane magnetization configuration, and the skyrmion texture vanishes. The energy minimum $E(R_{s,0})$, corresponding to the stable skyrmion, is separated by two maxima: a lower one (for R_s close to 0) and a higher one (for R_s close to R_d). Therefore, the depth of the energy well, ΔE , is determined with respect to the lower maximum—see Fig. 3(c). The results of the semianalytic calculations [solid green lines in Figs. 3(c)–3(f)] were compared to the outcomes of micromagnetic simulations [dashed blue lines in

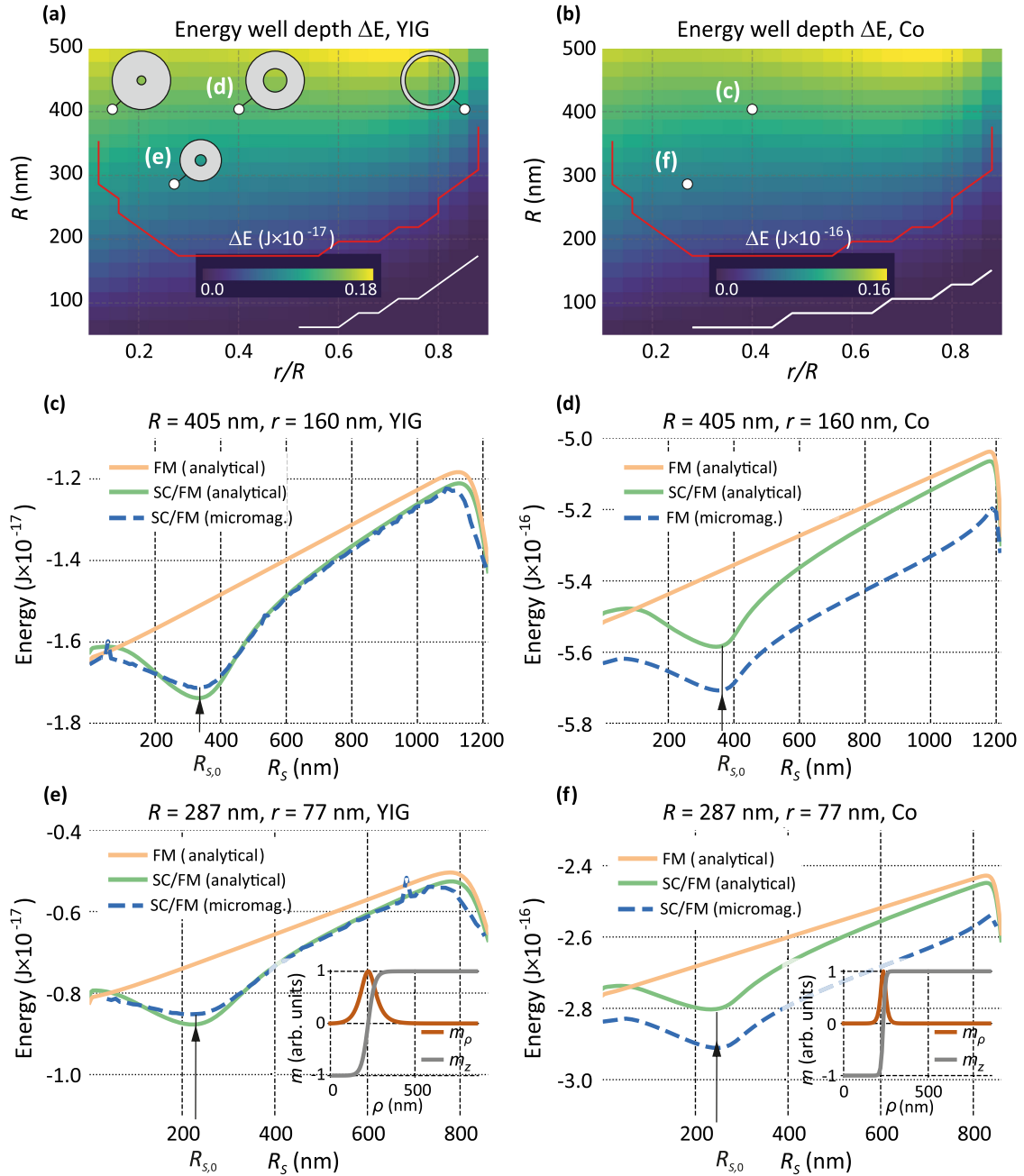


FIG. 3. Dependencies of the energy well depth for the skyrmion texture, ΔE , on the geometry of the SC ring: the outer ring radius R and the ratio between the inner and outer radii r/R , calculated for Ga:YIG (a) and Co (b). The uniformly colored dark blue area below the white line denotes the region where the energy well is absent ($\Delta E = 0$), and the skyrmion cannot be stabilized. In the region between the red and white lines, the energy well has a finite depth, but the current exceeds the critical value for Nd, for given the electric field pulse profile $E t_0 = 11.25 \times 10^{-9}$ V s/m. In the region above the red line, the skyrmion can be stabilized by the superconducting current. For both Ga:YIG (c,e) and Co (d,f), we show the dependence of the skyrmion energy on the assumed radius of skyrmion domain wall R_s , for two selected SC ring sizes [indicated by white points in (a,b)]. The energy well is deeper for larger outer radius R and for narrower ring width (i.e., $r/R \approx 0.7$)—compare (c,d) to (e,f). In the absence of the SC ring's stray field, the energy well does not exist [see orange lines in (c)–(f) for analytical calculations] and the skyrmion cannot be stabilized. Green and dashed blue lines in (c)–(f) represent the results in the presence of the SC ring, for semianalytical and micromagnetic calculations, respectively. Insets in (e),(f) show the profile of the stable skyrmion texture, with R_s corresponding to the energy minimum: $R_s = R_{s,0}$.

Figs. 3(c)–3(f)]. The values of $R_{s,0}$ and ΔE are in good agreement, confirming that the presented semianalytic method can be reliably used to investigate the size and stability of magnetic skyrmions in the considered SC/FM hybrid nanostructures. However, the analytical approach tends to

overestimate the magnetic energy of the skyrmion textures (i.e., it does not minimize the energy for a given R_s), due to the assumptions and constraints introduced (due to the fixed width of the domain wall). For several selected skyrmion textures stabilized in the micromagnetic simulations [Figs. 3(c)

and 3(d)], we calculated the skyrmion number using the discretized form of the formula $\frac{1}{4\pi M_s} \int \mathbf{M} \cdot (\partial_x \mathbf{M} \times \partial_y \mathbf{M}) dx dy$ [52]. The obtained skyrmion numbers were approximately 1, with a numerical precision of around 1%. This is in good agreement with the value for a Néel skyrmion.

First, we aim to investigate how the size of the SC ring influences skyrmion stability. The dependence of the depth of the energy well ΔE on the outer ring radius R and the ratio between the inner and outer radii r/R were calculated using the semianalytic model for Ga:YIG [Fig. 3(a)] and Co [Fig. 3(b)]. Assuming that the SC ring is excited the same way, regardless of its size, we fixed the height and duration of the electric field pulse that induces the SC current: $E t_0 = 11.25 \times 10^{-9}$ V s/m. In the dark blue area, below the white line, $\Delta E = 0$ and the skyrmion cannot be stabilized. Area of nonzero ΔE (i.e., the region above the white line) can be extended by increasing the electric field pulse value, which leads to the enhancement of the SC stray magnetic field, wherein, one should remember, induced current cannot exceed the critical value, above which the Meissner state is destroyed in the SC ring. For the taken profile of the electric field pulse $E t_0$, the region where the maximum current density is smaller than the critical value for Nb is delimited by the red line. This area (i.e., the region above the red line) will increase with the decrease of $E t_0$. It is evident that, in the case of a small and narrow SC ring [bottom-right corner of the plots in Figs. 3(a) and 3(b)], stabilizing the skyrmion becomes more challenging, with the primary limiting factor being the critical current density of the superconducting material.

We found also that the radius of the skyrmion domain wall R_{s0} is mostly determined by the outer size of SC ring R , and its relative width $[(R - r)/R = 1 - r/R]$ plays a minor role (see Supplemental Material [33]). This finding is easy to understand if one looks at Fig. 2: the maximum of the radial component of the SC stray field B_{ρ} , which corresponds to the middle of the domain wall (in-plane magnetization configuration), is always close to R .

It is clear from Figs. 3(a) and 3(b) that, for a fixed ratio r/R , a larger outer ring radius R results in a more stable skyrmion. In such a case, the skyrmion is simply larger (higher values of R_{s0}). Therefore, any changes in the magnetic configuration of the stabilized skyrmion domain wall will require more energy and the depth energy minimum ΔE must be larger. Moreover, the skyrmion stability increases as the ring becomes thinner (for fixed R), up to a certain value of r/R (about 0.7 for YIG and 0.75 for Co). However, for very thin SC rings, we see the essential decrease of the skyrmion stability. This effect can be attributed to the manner in which the SC ring generates the stray magnetic field—see Fig. 2. For narrow rings (i.e., larger r/R ratios), the radial component of the stray field, $B_{sc,\rho}$, is enhanced in the vicinity of the domain wall at $R_{s0} \approx R$, which slightly improves skyrmion stabilization. Conversely, for the SC wider rings (i.e., smaller r/R ratios), a nonzero $B_{sc,\rho}$ persists within the skyrmion core, which tends to destabilize the texture. The decrease of ΔE for $r/R \approx 1$ is dictated by the reduction of the SC current in the ring when its width $R(1 - r/R)$ is comparable to the London penetration depth λ . We note that the width of the SC ring is the smallest in the bottom-right corner of the plots in Figs. 3(a)

and 3(b), below the white line. In this region the stray magnetic field is too weak to induce any minimum in the $E(R_s)$ dependence.

Let us now discuss in more detail the dependence of the energy of skyrmion texture on the assumed size of the skyrmion, $E(R_s)$, as presented in Figs. 3(c)–3(f). The energy profiles $E(R_s)$ were plotted for selected superconducting ring geometries: $R = 405$ nm, $r = 160$ nm (with $r/R = 0.4$)—see Figs. 3(c) and 3(d); and $R = 287$ nm, $r = 77$ nm (with $r/R = 0.27$)—see Figs. 3(e) and 3(f). Two materials were considered: Ga:YIG—Figs. 3(c) and 3(e), and Co—Figs. 3(d) and 3(f). The orange lines show, for reference, the energy of a skyrmion texture in a FM disk in the absence of the external magnetic field generated by the SC ring. In this case, no energy minimum appears for $0 < R_s < R_d$, indicating that the skyrmion cannot be stabilized. The positive slope of $E(R_s)$ implies that, once imposed, the skyrmion texture gradually annihilates by shrinking its radius to zero. This linear dependence of $E(R_s)$ arises from the fact that, without an external field, the energy is determined solely by the domain wall, and is therefore proportional to its circumference: $2\pi R_s$. However, placing the FM disk in the stray magnetic field generated by the SC ring results in the appearance of an energy minimum (dashed blue and solid green lines), which enables the skyrmion stabilization at the radius corresponding to this minimum. For both YIG and Co, increasing the SC ring size R and narrowing its relative width, $(R - r)/R = 1 - r/R$, deepens the energy well—i.e., enhances the skyrmion stabilization—as discussed above. This demonstrates a universal mechanism for controlling skyrmion stability via the geometry of the SC ring and the SC current, which can be modified by electric field pulses.

The insets in Figs. 3(e) and 3(f) present normalized magnetization profiles of the skyrmion texture at the energy minimum: $R_s = R_{s,0}$. Red and gray lines show m_{ρ} and m_z components, respectively, plotted using Eq. (16), wherein $\rho = R_s$ corresponds to the zero value of m_z , i.e., the center of the radial domain wall. As one can see, the width of the domain wall, the depth of the energy well and, therefore, the absolute stability of the skyrmion, expressed by the value of ΔE , are significantly different for systems based on YIG and Co ferromagnetic materials, even if they have the same geometrical parameters. Thus we are smoothly approaching the question of how the material parameters of a ferromagnet affect skyrmion stabilization and which materials would be optimal for applications.

There are three main material parameters whose interplay determines the suitability of a material for stabilizing skyrmions using the stray magnetic field of the SC nanostructure: saturation magnetization (M_s), exchange stiffness (A_{ex}), and uniaxial magnetic anisotropy (K_u) (see Supplemental Material [33] for details). Note again that the considered materials do not exhibit any DMI. Our analysis has shown that larger values of M_s and smaller values of A_{ex} increase the energy difference ΔE , and are therefore favorable for the skyrmion stabilization. This behavior can be understood by considering that the skyrmion is stabilized via Zeeman interaction, where the stray magnetic field of the SC ring affects the magnetization. As a result, increasing M_s

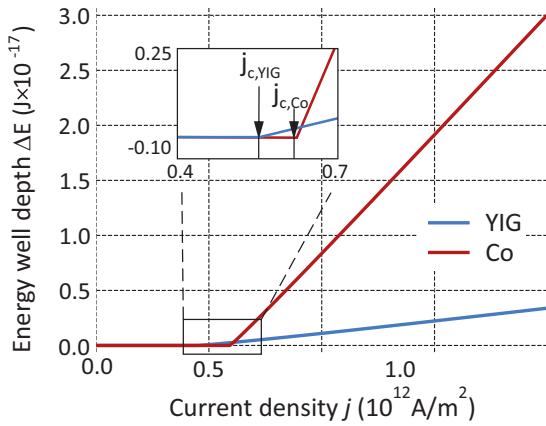


FIG. 4. Dependence of the energy well depth for the skyrmion texture, ΔE , on the maximum current density in the SC ring of fixed size (outer radius: $R = 405$ nm; inner radius: $r = 160$ nm), calculated for Ga:YIG (blue line) and Co (red line). The inset shows a zoomed-in region near the threshold current value, where the stray field generated by the SC ring becomes sufficient to stabilize the skyrmion.

enhances the strength of this interaction and promotes the skyrmion stabilization. The requirement for small A_{ex} stems from the need to reduce the stiffness of the magnetic texture, which has to be reversed in the center of the skyrmion relative to its outer regions. The influence of the out-of-plane magnetic anisotropy is such that lower anisotropy values facilitate the skyrmion stabilization, as they allow the magnetization to rotate more easily. However, the uniaxial anisotropy constant must still exceed the shape anisotropy energy in order to maintain an out-of-plane magnetization configuration far from the skyrmion domain wall.

The choice of the investigated FM materials pursued the goal of showing these features. Ga:YIG is characterized by relatively small values of both M_s and A_{ex} , wherein this material has low damping of spin waves, which makes it attractive for using in real magnonic devices. Cobalt is characterized by both relatively high values of M_s and A_{ex} . It is one of the most popular and investigated FM materials with well established production technology. For the considered parameters Co provides significantly larger stability of the skyrmion (larger ΔE) and a broader range of suitable geometrical parameters of an SC ring [Figs. 3(a) and 3(b)] due to the much larger value of M_s in comparison with Ga:YIG, despite the smaller A_{ex} and lower K_u of Ga:YIG. Thus, in this case, the Zeeman interaction plays a crucial role in the determination of the skyrmion stability.

Finally, let us discuss the most notable advantage of skyrmion stabilization by the stray magnetic field produced by the superconducting ring: the possibility to control skyrmion stability via the current flowing through the ring, which can be tuned by pulses of the applied electric field. In particular, an increase in current enhances the SC stray magnetic field and, consequently, the Zeeman energy, which in turn deepens the energy potential well. The dependence of the energy barrier ΔE on the maximum current density in the ring with $R = 405$ nm and $r = 160$ nm is presented in Fig. 4 for Ga:YIG (blue line) and Co (red line). The inset shows

a zoomed-in region near the current densities at which ΔE becomes nonzero, indicating the critical value j_c , above which the skyrmion can be stabilized in the system. The value of j_c , as well as the rate of increase of ΔE with the current density j increasing, is determined by the interplay between the FM parameters M_s and A_{ex} , as discussed above. Interestingly, the $\Delta E(j)$ curves for Ga:YIG and Co intersect. This behavior arises from the fact that, at low currents, the SC stray magnetic field and the resulting Zeeman interaction are weak, so the stability is primarily governed by the exchange interaction. However, at higher current densities, the contribution of the Zeeman energy becomes more significant, strongly affecting skyrmion stability. This explains why, at low current values, the skyrmion is more stable in Ga:YIG due to its low exchange stiffness A_{ex} , while at higher current densities, Co becomes more favorable for the skyrmion stabilization because of its large saturation magnetization M_s .

IV. CONCLUSIONS

The stabilization of Néel-type skyrmions in a hybrid system consisting of ferromagnetic disk placed under a superconducting ring has been studied both analytically and numerically. It has been demonstrated that the application of an electric field pulse to the superconducting ring in the Meissner state generates a persistent current, which, in turn, induces a nonuniform magnetic field in the ferromagnetic disk. This field exhibits a spatial symmetry compatible with that of a Néel-type magnetic skyrmion, thus facilitating its stabilization.

To establish criteria for selecting magnetic materials optimal for skyrmion formation in hybrid superconductor/ferromagnet structures, we investigated how key material parameters—saturation magnetization, out-of-plane magnetic anisotropy, and exchange stiffness—affect the skyrmion stabilization. We found that higher saturation magnetization enhances the Zeeman interaction between the superconductor ring and ferromagnet, promoting the skyrmion formation, while lower exchange stiffness facilitates the necessary magnetization reversal at the skyrmion core. Reduced out-of-plane magnetic anisotropy favors easier magnetization rotation, but must remain strong enough to overcome shape anisotropy and maintain out-of-plane magnetization alignment. We studied two typical ferromagnetic materials, Ga:YIG (low saturation magnetization and exchange stiffness) and Co (high values of both), and found that the skyrmion stability is significantly higher in Co, highlighting the key role of the Zeeman interaction and saturation magnetization.

Finally, the most significant advantage of skyrmion stabilization by the superconducting ring's stray magnetic field is the ability to control the skyrmion stability through the current flowing in the ring, which can be adjusted by electric field pulses. In particular, an increase in the current density enhances the SC stray magnetic field, thereby enhancing the Zeeman energy and stabilizing the skyrmion, wherein there is a critical current density above which the skyrmion can be stabilized, offering a pathway for energy-efficient information writing and erasing through skyrmion nucleation and annihilation with a specified polarity, thus advancing the development of magnonic neuromorphic systems [30], where the spin

waves can scatter on the adjustable pattern of skyrmions in the FM layer.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [53].

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