

## Stochastic dynamics of skyrmions on a racetrack: Impact of equilibrium and nonequilibrium noise

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Current-driven motion of domain walls and skyrmions is central to the operation of nonvolatile magnetic memory devices. Racetrack memory requires current densities high enough to generate velocities above 50 m/s, but such conditions also enhance spin-current noise. We develop a theoretical framework based on the stochastic Thiele equation to analyze the effects of equilibrium (thermal) and nonequilibrium (spin-current) fluctuations on skyrmion dynamics. From this approach, we derive diffusion coefficients and mean-squared displacements that quantify stochastic motion under both noise sources. Micromagnetic simulations and analytical results demonstrate that spin-current noise dominates skyrmion dynamics in typical racetrack structures up to room temperature. We further address the first-passage-time problem, obtaining the mean first-passage time and its standard deviation along and across the racetrack. These results quantify how random displacements affect skyrmion propagation and detection, providing insights into error sources in high-speed racetrack memory devices.

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Developments in the controlled motion of magnetic textures, such as domain walls on racetracks driven by electric currents, have opened the way to alternative nonvolatile memory devices with significantly higher performance than conventional solid-state memories [1]. Advances in racetrack memory technology now enable a wide range of applications, from ultrafast to ultradense data storage [2,3].

An alternative to domain-wall magnetic textures is skyrmions [4], which can provide denser information storage due to their nanoscale size (down to a few nanometers). These particlelike spin configurations can exist either in magnetic materials with noncentrosymmetric crystal lattices or at the interfaces of magnetic films. Correspondingly, two types of skyrmion states arise from different forms of the Dzyaloshinskii-Moriya interaction (DMI). In noncentrosymmetric lattices [5–7], the bulk DMI stabilizes vortexlike (Bloch-type) skyrmions [8]. In contrast, interfacial DMI leads to the stabilization of “hedgehog” (Néel-type) skyrmions [9,10]. Moreover, Néel-type skyrmions can also be stabilized in centrosymmetric ferromagnets by imposing a vertical strain gradient via epitaxial growth [11].

Historically, the first racetrack memories were realized in permalloy nanowires [1], where domain walls were driven by nanosecond current pulses via the spin-transfer torque

(STT) mechanism [12–15]. The next generation of racetracks [2,16,17] was achieved by injecting a spin current from a heavy-metal layer into an adjacent ferromagnetic layer. The resulting spin-current-induced motion of the magnetic texture arises from spin-orbit scattering of conduction electrons via the spin Hall effect in the heavy-metal layer [18–20] and is described by the spin-orbit torque (SOT) mechanism. In this work, we focus on racetracks with SOT current injection, which enables higher velocities of magnetic textures. However, our theoretical framework can be readily extended to the STT case.

Real magnetic systems are inherently subject to random fluctuations, which may include both equilibrium and nonequilibrium sources of noise. Equilibrium thermal agitation leads to fluctuations of the magnetization field [21–29]. In contrast, the presence of an external current gives rise to nonequilibrium noise [30–40]. In high-speed spintronic devices (with typical velocities of at least about 50 m/s or higher [2,41–43]), large current densities can lead to enhanced spin-current fluctuations. In conventional electrical conductors, such as heavy-metal layers interfaced with nanoscale ferromagnets, spin-current noise increases magnetization noise through fluctuating spin torque [34,40]. Anomalous behavior of magnetization fluctuations in current-controlled spin valves has already been demonstrated at cryogenic temperatures [44], and this effect is expected to remain significant even at room temperature.

One of the major technological challenges in racetrack memory is achieving the highest possible packing density of nanoscale magnetic textures [45–47], while still enabling reliable detection of domain walls or skyrmions when they are densely packed. In this work, we focus on describing the effects of both thermal noise and spin-current noise on

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skyrmion dynamics. Our results reveal a pronounced impact of spin-current fluctuations on the stable operation of racetrack memory devices. For typical racetrack geometries and operational conditions, trajectory offsets caused by spin-current noise dominate over thermal noise. This stochastic spreading of skyrmion trajectories is quantified in terms of the mean-squared displacement (MSD), which we obtain analytically using the Thiele approach and numerically through micromagnetic simulations. We also address the first-passage-time (FPT) problem for skyrmion motion on the racetrack, estimating the mean and standard deviation of the random times at which a skyrmion reaches either the racetrack side boundary or the detection region. Interaction of a skyrmion with a boundary can lead to its annihilation [48] and consequent loss of stored information, thereby compromising racetrack stability. Conversely, time uncertainty in reaching the detection region affects signal decoding quality. A deeper understanding of the roles of equilibrium and nonequilibrium fluctuations in skyrmion motion is therefore essential for the design of next-generation, high-speed, and high-density spintronic devices.

The dynamics of magnetization  $\mathbf{M}(\mathbf{r}, t) = [M_1(\mathbf{r}, t), M_2(\mathbf{r}, t), M_3(\mathbf{r}, t)]$  is described by the Landau-Lifshitz-Gilbert (LLG) equation [49,50], which includes an additional term accounting for the spin transfer moment:

$$\partial_t \mathbf{M} = -\gamma \mathbf{M} \times \mathbf{H}_{\text{eff}} + \frac{\alpha}{M_s} \mathbf{M} \times \partial_t \mathbf{M} + \mathbf{T}_{\text{SOT}}. \quad (1)$$

Here  $\mathbf{H}_{\text{eff}}$  is the effective field (its explicit form is given in the Supplemental Material [51], Sec. I),  $\gamma$  is the gyromagnetic ratio,  $\alpha$  is the damping parameter, and  $M_s$  is the saturation magnetization. The spin-orbit torque  $\mathbf{T}_{\text{SOT}}$  can be expressed as a combination of Slonczewski's torque [12] and the dampinglike torque

$$\mathbf{T}_{\text{SOT}} = -\frac{\gamma}{M_s^2 \mathcal{V}} \mathbf{M} \times (\mathbf{M} \times \mathbf{I}_s) - \frac{\gamma \beta}{M_s \mathcal{V}} \mathbf{M} \times \mathbf{I}_s, \quad (2)$$

where  $\mathbf{I}_s = \mathbf{P} \hbar / (2e)$  [ $\mathbf{P} = P \cdot \mathbf{p} = P(\cos \psi, \sin \psi, 0)$ ] the spin-polarization vector,  $\hbar$  the reduced Planck constant,  $e$  the electron charge,  $I$  the electric current,  $\beta$  the amplitude of the dampinglike torque, and  $\mathcal{V}$  the sample volume.

Next, we generalize the LLG equation to account for both thermal noise and spin-current noise. Following Ref. [21], the thermal noise is represented by a fluctuating magnetic field  $\delta \mathbf{h}(\mathbf{r}, t)$  with zero mean as  $\mathbf{H}_{\text{eff}} \rightarrow \mathbf{H}_{\text{eff}} + \delta \mathbf{h}$ . In addition to thermal fluctuations, we introduce spin-current noise (spin-shot noise)  $\delta \mathbf{I}(\mathbf{r}, t)$  as  $\mathbf{I}_s \rightarrow \mathbf{I}_s + \delta \mathbf{I}$ , following Ref. [36]. This noise originates from the discrete nature of electron transport in the heavy-metal layer and the resulting stochastic transfer of angular momentum to the local magnetic moments in the ferromagnet [34,36]. It is therefore analogous to charge shot noise [52]. The stochastic LLG equation reads as

$$\begin{aligned} \partial_t \mathbf{M} = & -\gamma \mathbf{M} \times (\mathbf{H}_{\text{eff}} + \delta \mathbf{h}) + \frac{\alpha}{M_s} \mathbf{M} \times \partial_t \mathbf{M} \\ & - \frac{\gamma}{M_s^2 \mathcal{V}} \mathbf{M} \times [\mathbf{M} \times (\mathbf{I}_s + \delta \mathbf{I})] \\ & - \frac{\gamma \beta}{M_s \mathcal{V}} \mathbf{M} \times (\mathbf{I}_s + \delta \mathbf{I}). \end{aligned} \quad (3)$$

The autocorrelation functions of the fluctuation terms take the form [21,34,40]

$$\begin{aligned} \langle \delta h_i(t, \mathbf{r}) \delta h_j(t', \mathbf{r}') \rangle &= \frac{2\alpha k_B T}{\gamma M_s} \delta_{ij} \delta(\mathbf{r} - \mathbf{r}') \delta(t - t'), \\ \langle \delta I_i(t, \mathbf{r}) \delta I_j(t', \mathbf{r}') \rangle &= \frac{\alpha M_s \mathcal{V}^2}{\gamma} \delta_{ij} \delta(\mathbf{r} - \mathbf{r}') \delta(t - t') \\ &\times \frac{\delta \varepsilon}{E_F} \left( eU \coth \frac{eU}{2k_B T} - 2k_B T \right). \end{aligned} \quad (4)$$

Here  $i, j = \{x, y, z\}$ ,  $\delta_{ij}$  is the Kronecker delta function,  $\delta(\cdot)$  is the Dirac delta function,  $U$  and  $T$  denote the applied voltage and temperature,  $E_F$  is the Fermi energy in the heavy-metal layer, and  $\delta \varepsilon$  is the exchange splitting in the ferromagnetic layer, which can be estimated as  $\delta \varepsilon \sim E_F/5$  [34].

The strength of the current noise is thus determined by both material parameters ( $E_F, \delta \varepsilon$ ) and external conditions ( $U, T$ ), reflecting the crossover from thermal noise at low bias to spin-shot-noise-dominated fluctuations at large applied voltage. Consequently, different skyrmion systems are characterized by quantitatively different noise amplitudes depending on their electronic structure and transport regime. In Eq. (4), we separate these contributions following Ref. [53]. At the same time, thermal and spin-current-induced fluctuations can be described equivalently in terms of either  $\delta \mathbf{h}$  or  $\delta \mathbf{I}$  using the relation  $\langle \delta h_i(t, \mathbf{r}) \delta h_j(t', \mathbf{r}') \rangle = \frac{1}{M_s^2 \mathcal{V}^2} \langle \delta I_i(t, \mathbf{r}) \delta I_j(t', \mathbf{r}') \rangle$  [38]. We consider equilibrium and nonequilibrium noise sources of different origins and assume that they are uncorrelated,  $\langle \delta h_i(t, \mathbf{r}) \delta I_j(t', \mathbf{r}') \rangle = 0$ . In the zero-temperature limit, current fluctuations persist, while in the absence of an applied voltage only thermal fluctuations remain.

We employ the Thiele approach to evaluate the contribution of equilibrium and nonequilibrium noise to skyrmion displacements. The Thiele formalism [54] uses the traveling-wave Ansatz  $\mathbf{M}(\mathbf{r}, t) = \mathbf{M}[\mathbf{r} - \mathbf{R}(t)]$ , where  $\mathbf{R}(t)$  is the position of the center of the skyrmion. In our study, we neglect the higher derivatives of  $\mathbf{R}(t)$  [55], which have only a minor effect on the main results, and reduce Eq. (3) to the form of the stochastic Thiele equation (see details in the Supplemental Material [51], Sec. I)

$$\begin{aligned} \widehat{G} \cdot \dot{\mathbf{R}} + \alpha \widehat{D} \cdot \dot{\mathbf{R}} + \mathbf{F} &= 0, \\ \mathbf{F} &= \mathbf{F}_{\text{eff}} + \mathbf{F}_{\text{SOT}} + \mathbf{F}_{\delta h} + \mathbf{F}_{\delta I}, \end{aligned} \quad (5)$$

where  $\widehat{G}$  is the gyrotropic tensor and  $\widehat{D}$  is the dissipation tensor, which are given by  $G_{ij} = -1/(\gamma M_s^2) \int \mathbf{M} \cdot (\nabla_i \mathbf{M} \times \nabla_j \mathbf{M}) d\mathbf{r}$  and  $D_{ij} = 1/(\gamma M_s) \int \nabla_i \mathbf{M} \cdot \nabla_j \mathbf{M} d\mathbf{r}$ , respectively. The effective force is  $F_{\text{eff},i} = \int \mathbf{H}_{\text{eff}} \cdot \nabla_i \mathbf{M} d\mathbf{r}$ , and  $F_{\text{SOT},i} = 1/(\gamma M_s^2) \int (\mathbf{M} \times \nabla_i \mathbf{M}) \cdot \mathbf{I}_s d\mathbf{r} + \beta/(\gamma M_s) \int \mathbf{I}_s \cdot \nabla_i \mathbf{M} d\mathbf{r}$ . The two uncorrelated stochastic forces take the form

$$\begin{aligned} F_{\delta I,i} &= \frac{1}{\gamma M_s^2} \int (\mathbf{M} \times \nabla_i \mathbf{M} + M_s \beta \nabla_i \mathbf{M}) \cdot \delta \mathbf{I} d\mathbf{r}, \\ F_{\delta h,i} &= \int \delta \mathbf{h} \cdot \nabla_i \mathbf{M} d\mathbf{r}, \end{aligned} \quad (6)$$

and their autocorrelation functions are given by

$$\begin{aligned}\langle F_{\delta h,i}(t)F_{\delta h,j}(t') \rangle &= \alpha k_B T D_{ij} \delta(t - t'), \\ \langle F_{\delta I,i}(t)F_{\delta I,j}(t') \rangle &= \alpha(1 + \beta^2) D_{ij} \delta(t - t') \\ &\quad \times \frac{\delta \varepsilon}{E_F} \left( eU \coth \frac{eU}{2k_B T} - 2k_B T \right).\end{aligned}\quad (7)$$

The resulting Eqs. (5)–(7) describe two-dimensional skyrmion diffusion in thin ferromagnetic films in the ( $x$ - $y$ ) plane. Owing to the rotational symmetry of a skyrmion at rest we neglect  $D_{xy}$  and  $D_{yx}$ , and set  $D_{xx} = D_{yy} = D$  and  $G_{xy} = G$  [56], which is also confirmed by our micromagnetic simulations. The solution of Eq. (5) for the velocity  $\mathbf{R}(t) = \mathbf{v}(t) = \{v_x(t), v_y(t)\}$  of the skyrmion is given by

$$v_x = \frac{-GF_y - \alpha DF_x}{G^2 + \alpha^2 D^2}, \quad v_y = \frac{GF_x - \alpha DF_y}{G^2 + \alpha^2 D^2}. \quad (8)$$

After evaluating the velocity correlation functions and integrating over time  $\mathbf{R}(t) = \int_0^t \mathbf{v}(t') dt'$  (see details in the Supplemental Material [51], Sec. II) one obtains the MSD  $\langle \delta \mathbf{R}^2(t) \rangle = \langle (\mathbf{R}(t) - \langle \mathbf{R}(t) \rangle)^2 \rangle$ ,

$$\langle \delta R_x^2(t) \rangle = \langle \delta R_y^2(t) \rangle = 2\mathcal{D}t, \quad (9)$$

where the total diffusion coefficient  $\mathcal{D} = \mathcal{D}_T + \mathcal{D}_I$  accounts for the respective contributions of thermal and spin-current noise fluctuations,

$$\begin{aligned}\mathcal{D}_T &= \frac{\alpha D}{2(G^2 + \alpha^2 D^2)} 2k_B T, \\ \mathcal{D}_I &= \frac{\alpha D(1 + \beta^2)}{2(G^2 + \alpha^2 D^2)} \frac{\delta \varepsilon}{E_F} \left( eU \coth \frac{eU}{2k_B T} - 2k_B T \right).\end{aligned}\quad (10)$$

Note the nonmonotonic dependence of the diffusion coefficient  $\mathcal{D}$  on the damping parameter  $\alpha$ , which results in diffusion suppression as  $\alpha \rightarrow 0$ . This behavior is characteristic of skyrmions, in contrast to domain walls, for which  $G = 0$ , and  $\mathcal{D} \sim 1/\alpha$  [56,57].

Micromagnetic simulations were performed using the micromagnetic module of comsol multiphysics [58,59] for a two-dimensional system. A comparison between this module and MuMax3, [60] is presented in the Supplemental Material [51], Sec. III. We extended the comsol module with a user-defined code that incorporates thermal and spin-current fluctuations in accordance with Eqs. (4). The following parameters of the effective field  $\mathbf{H}_{\text{eff}}$  (see Supplemental Material, Sec. III) were used in the simulations: exchange coefficient  $A_{\text{ex}} = 10^{-11}$  J/m, anisotropy  $K_u = 10^6$  J/m<sup>3</sup>, saturation magnetization  $M_s = 0.6 \times 10^6$  A/m, interfacial DMI constant  $D_{\text{DMI}} = -3.5 \times 10^{-3}$  J/m<sup>2</sup>, and damping parameters  $\alpha = \beta = 0.3$ . The racetrack sample has an ( $x$ - $y$ ) cross section of  $500 \times 150$  nm<sup>2</sup> and thickness  $\Delta_f = 1$  nm. The applied current density is  $J = 4 \times 10^{11}$  A/m<sup>2</sup>, the voltage is  $U = 0.5$  V, and a linear current-voltage relationship  $J \propto U$  is assumed. The temperature is set to  $T = 300$  K. These parameters are typical of experimental racetrack systems [16,45,46]. The topological properties of skyrmions are fully characterized by three quantities ( $Q$ ,  $Q_v$ ,  $Q_h$ ), representing the topological charge, vorticity number, and helicity number, respectively [61,62]. We restrict our analysis to the DMI, that stabilizes Néel-type skyrmions with  $(1, 1, \pi)$ .

Using micromagnetic modeling, the gyrocoupling constant  $G$  and dissipation tensor component  $D$  take the following values:  $G = 4\pi M_s \Delta_f / \gamma$  and  $D = 18.1 M_s \Delta_f / \gamma$  (see details in the Supplemental Material, Sec. III). In this work, we focus on the current-driven motion of skyrmions along the  $x$  axis, which can be achieved by suppressing the transverse motion along the  $y$  axis induced by the skyrmion Hall effect [63]. Following Ref. [64], the condition  $\langle v_y \rangle = 0$  can be satisfied by choosing the angle  $\psi$  of the polarization vector  $\mathbf{p}$  as

$$\psi = \arctan \frac{G}{\alpha D} + \frac{\pi}{2} - Q_h. \quad (11)$$

For the Néel skyrmion, this yields  $\psi = -23.36^\circ$ , which is consistent with our micromagnetic simulations (note that the angle  $\delta$  in Ref. [64] corresponds to  $\psi$  as  $\delta = \psi - \pi/2$ ). For the chosen set of parameters the resulting drift velocity along the  $x$  axis is estimated as  $\langle v_x \rangle = 64.4$  m/s (see Supplemental Material, Sec. III).

Exemplary skyrmion trajectories obtained from micromagnetic simulations using comsol multiphysics are shown in Fig. 1(a), where both thermal and spin-current noises are taken into account. The current pulse duration is set to 3 ns, resulting in a mean displacement along the  $x$  axis of 193.2 nm. The final skyrmion positions after 3 ns are marked by black asterisks. It can be seen that thermal and spin-current fluctuations cause significant random deviations around the mean position in both the  $x$  and the  $y$  directions. The MSDs along both axes, obtained from simulated trajectories, are shown by solid lines in Fig. 1(b) together with the corresponding MSDs calculated from the Thiele approach, Eqs. (9) and (10), shown by the dashed lines. The Figure demonstrates excellent agreement between the micromagnetic simulations and the analytical predictions based on the Thiele formalism. For the selected system parameters, nonequilibrium fluctuations dominate over equilibrium (thermal) fluctuations.

We now proceed to study the first-passage problem for skyrmions on the racetrack in the situation where the skyrmion Hall effect and therefore the transverse drift across the racetrack, are absent. Using Eqs. (5)–(10), we rewrite the Thiele equation (5) in the form of a standard stochastic equation of motion

$$\dot{\mathbf{R}} = \langle \mathbf{v} \rangle + \sqrt{2\mathcal{D}} \boldsymbol{\xi}(t). \quad (12)$$

Here,  $\langle \mathbf{v} \rangle = \{\langle v_x \rangle, 0\}$  is the mean drift velocity, whose components are determined from Eq. (8), and  $\boldsymbol{\xi}(t)$  is a white Gaussian noise with the zero mean and autocorrelation function  $\langle \xi_i(t) \xi_j(t') \rangle = \delta_{ij} \delta(t - t')$ . The form (12) is particularly convenient for analyzing first-passage phenomena [65,66]. Here, we focus on the mean FPTs  $\langle \tau_{x,y} \rangle$  along and across the racetrack, as well as their standard deviations (SDs)  $\sigma_{x,y} = \sqrt{\langle \tau_{x,y}^2 \rangle - \langle \tau_{x,y} \rangle^2}$ . The details of derivation for semi-infinite and finite domains are provided in the Supplemental Material, Sec. IV. Figure 2(a) illustrates the nontrivial behavior of the normalized SD  $\sigma_x / \langle \tau_x \rangle = \sqrt{2\mathcal{D}(J) / (\langle v_x(J) \rangle d)}$  for the FPT along the racetrack as the function of the current density  $J$ , where  $d$  denotes the distance between the initial position at  $t = 0$  and the detector. For skyrmion diffusion across the racetrack, the mean FPT to reach the racetrack side boundaries  $\pm a$  and its SD are given by  $\langle \tau_y \rangle = a^2 / (2\mathcal{D})$  and  $\sigma_y = a^2 / (\sqrt{6}\mathcal{D})$ ,

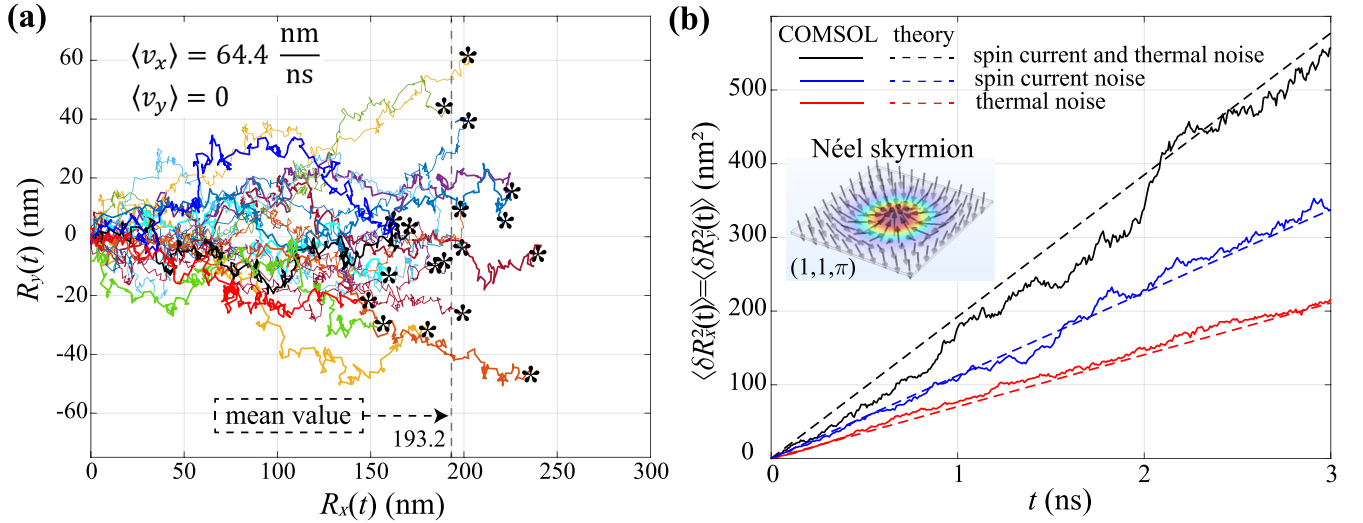


FIG. 1. (a) Example trajectories of Néel skyrmion motion under the combined influence of thermal and spin-current noise. The racetrack width of 150 nm is chosen to be sufficiently large to prevent skyrmion interaction with the racetrack boundaries during a 3 ns current pulse. (b) MSD of the skyrmion position obtained from both micromagnetic simulations (solid lines, 100 trajectories) and the analytical calculations (dashed lines). The results are shown for the cases with only thermal noise, only spin-current noise, and both noise sources (from bottom to top, respectively).

respectively, differing only by a constant prefactor. Figure 2(b) presents the mean FPT  $\langle \tau_y(J) \rangle$  across the racetrack with  $a = 75$  nm as a function of the current density  $J$  in the presence of thermal and/or spin-current fluctuations.

The analytical results presented in Fig. 2(a) indicate that the spread of the mean FPT along the racetrack is governed by thermal fluctuations at low drift velocities, whereas at higher, yet still experimentally typical, velocities for present-day racetracks, nonequilibrium spin-current fluctuations become dominant. Remarkably, at zero temperature the normalized standard deviation of the FPT is independent of the current density. As the current density increases, the normalized total

spread of the mean FPT approaches a constant value determined by spin-current fluctuations at zero temperature. These trends can be understood in more detail by considering the interplay between thermal and spin-current-induced fluctuations. The curve corresponding to thermal fluctuations only ( $\mathcal{D}_I \rightarrow 0$ ) exhibits a decreasing trend: when drift velocity tends to zero, the motion of the skyrmion becomes purely diffusive, which leads to the increase of normalized SD of the FPT. As the drift velocity increases, the diffusion time to the detector decreases, producing a strictly decreasing SD. Two limiting cases of spin-current noise are also shown. In the first case (nearly constant, blue dot-dashed line),  $\mathcal{D}_T \rightarrow 0$ ,  $T \rightarrow 0$ ,

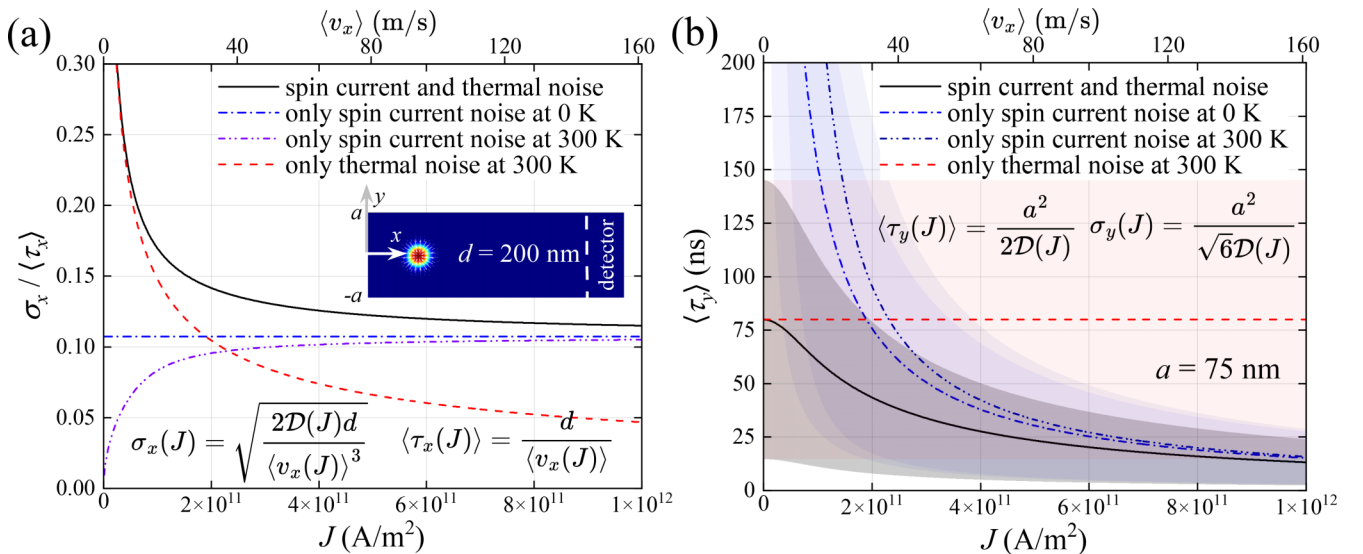


FIG. 2. (a) Normalized SD of the FPT to reach the detector position (x direction), and (b) Mean FPT to reach the racetrack boundary  $\pm a$  (y direction) as functions of the current density  $J$  for various combinations of noises. Shaded regions in panel (b) indicate the standard deviations of the mean FPT along the y axis. Here  $d = 200$  nm,  $a = 75$  nm.

and  $\mathcal{D}_I \sim eU$ . In the second case (increasing, purple dot-dot-dashed line),  $\mathcal{D}_T \rightarrow 0$  and  $\mathcal{D}_I \sim eU \coth(eU/2k_B T) - 2k_B T$ . At zero current, the normalized SDs differ, but with increasing current, the finite-temperature SD approaches the  $T = 0$  curve, explaining the observed trends of all curves in Fig. 2(a).

Figure 2(b) shows the mean FPT across the racetrack as a function of the current density. Owing to the suppression of the skyrmion Hall effect, the motion along the  $y$  axis is purely diffusive in nature. Similar to the motion along the racetrack, at low current densities (corresponding to low drift velocities) the mean FPT in the transverse direction is governed by thermal fluctuations, whereas at higher currents, spin-current fluctuations dominate the process of reaching the boundaries. For the parameters used here, the mean FPT along the  $y$  axis exceeds the typical duration of the current-pulse sequence [46]; however, hitting the boundary can become significant for narrow racetracks due to the quadratic dependence on the racetrack width.

We developed an analytical framework to describe the combined effects of equilibrium (thermal) and nonequilibrium (spin-current) fluctuations on skyrmion dynamics in racetrack systems. Using the Thiele approach, we reduced the stochastic Landau-Lifshitz-Gilbert equation to a Langevin-like form for the skyrmion center of mass, allowing analytical evaluation of the mean-squared displacement and first-passage time. The skyrmion exhibits normal diffusion with a coefficient gov-

erned by thermal fluctuations at low current densities, while spin-current noise dominates at higher, yet experimentally relevant, currents. Even in the absence of the skyrmion Hall effect, diffusive motion can drive the skyrmion to the racetrack boundaries, with the mean FPT across the racetrack becoming comparable to the measurement time for low currents and narrow geometries. Micromagnetic simulations including both fluctuation sources confirm these analytical results.

Random displacements of magnetic textures represent a key source of bit-position and timing errors in high-speed racetrack memory devices [47,67–69]. Apart from thermal and spin-current-induced noises, spatial disorder in the energy landscape [70,71] can induce pinning and, as a consequence, anomalous diffusion, thereby affecting skyrmion stochastic dynamics in realistic racetrack systems [72–76]. Together with stochastic texture-creation errors [47], these effects call for extending the present theory to include diverse noise mechanisms that ultimately limit the reliability of racetrack operation.

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*Data availability.* The data that support the findings of this Letter are openly available [77], embargo periods may apply.

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