

Universal quench dynamics of q -fermion Yukawa-Sachdev-Ye-Kitaev models

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We study the quantum quench of the Yukawa-Sachdev-Ye-Kitaev model and one of its random hopping extensions with q fermions and one boson. Several equilibrium properties are computed for general q with different parameter scaling within the large- N dynamical mean-field scheme. The non-Fermi liquid quench dynamics are studied by integrating the Kadanoff-Baym equations for switching off the random hopping term with constant Yukawa coupling or quenching to different final Yukawa couplings. The postquench oscillations and relaxation dynamics are insensitive to the quench amplitudes deep inside the non-Fermi liquid phase. With weak hopping coupling quenches, we find universal thermalization dynamics similar to the SYK model; however, with two temperatures and two distinct relaxation rates for bosons and fermions, both signal Planckian relaxations without quasiparticles, as in strange metals.

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I. INTRODUCTION

The Yukawa Sachdev-Ye-Kitaev (YSYK) model has been introduced recently [1,2] as a model hosts both the non-Fermi liquid phase [3–7] and the pairing phase [2], features that also appear in high temperature superconductors. The similarity is pushed forward by several recent studies for the equilibrium properties of the lattice extension of this model, by considering either the two-dimensional Fermi surface [8–13] or by computing the superfluid stiffness [5] with a random coupling lattice. In both cases, qualitative agreements were found with those observed in cuprates.

Closely related to the Sachdev-Ye-Kitaev (SYK) model [14,15]—in which maximal quantum chaos, holographic duality to black holes, and non-Fermi liquid behavior play central roles in understanding dynamics [16–21]—the Yukawa SYK model and its extensions [22] inherit many of these properties. They have also been studied in the contexts of quantum chaos [23,24], holography [25], thermalization [26], and tunable non-Fermi liquid behavior [27–31].

Away from the equilibrium, the dynamics of YSYK models are of significant interest from both the fundamental and phenomenological perspectives. They may give insights into the light-driven unconventional superconductors and also may give us further understanding of the nonequilibrium processes in strange metals [32] with interesting non-Fermi liquid behaviors [33], such as the “Planckian” transport [32]. Motivated by this, the quantum quenched YSYK model has been

studied previously both for the superconducting state [34] in a closed system and the normal state [26] with an external phonon bath coupled. However, the quench dynamics of isolated YSYK models remain largely unexplored, even in the case of the original YSYK model.

In this work, we study the equilibrium and nonequilibrium properties of a random q -point hopping extension of the YSYK model with q -fermion interaction. We mainly focus on half-filling where the chemical potential $\mu = 0$. After the disorder average our model is effectively a SYK_q -YSYK $_q$ model, where q is a positive even integer. Note the effective action of our model is still local after the disorder average [5]. Numerically, we integrate the corresponding Kadanoff-Baym equations of the large- N saddle point dynamical mean-field equations, starting from a correlated thermal initial state. In this setup, the random hopping strength v , the boson-fermion Yukawa coupling g , and the number of fermion pairs in the Yukawa interaction can be varied.

While analytical large- q solutions of the SYK model [15,35] have been extensively studied and successfully applied, analytically controllable solutions for YSYK models are still lacking. Some finite- q results may give insights into the large- q solutions. From a broader point of view, the YSYK is also closely related to questions from Kondo physics [36,37], the Holstein model [38,39], and the Migdal-Eliashberg theory [40,41].

Although in real materials, driving with light will lead to much more involved light matter couplings than simple quantum quenches, to get insight into the nonequilibrium behaviors of non-Fermi liquids that without quasiparticles, e.g., for studying the linear in T relaxations, quenches provide the simplest and one of the most tractable approaches. We present numerical results for the quench dynamics for the random hopping YSYK model from different initial temperatures and several quench protocols. Although there are a lot of freedoms to choose quench protocols, we focus on two cases: quench g with v fixed and quench v with g fixed for simplicity. In

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the main text, the latter case, quench v with g fixed, will be discussed in detail because it shows clear Planckian dynamics.

This work is organized as follows. In Sec. II, we present the model and effective action, the saddle point equation, and observables. We also give some numerical details. In Sec. III, we start our analysis with the equilibrium properties of the YSYK $_q$ and random hopping YSYK $_q$ model. In Sec. IV, is our central result. We show nonequilibrium quench dynamics of our model when the Yukawa coupling g is fixed while the coupling v varies during the quench. Here, we fit the intermediate time temperature relaxation rates and find they are linear in the final temperature. In Sec. V, we discuss possible explanations of those universal relaxation dynamics where we further derive some kinetic equations. Section VI is the conclusion. In Appendix C, we present the results for quenches with the coupling v fixed. Several detailed derivations are also collected in the Appendixes A, B, D, E, and F.

II. THE MODEL

Here we derive our q fermion YSYK model with random q -point hopping, where we will present the Lagrangians, effective actions, saddle point equations, and definitions of observables. We also give some details about numerics. More details on the disorder average can be found in Appendix A. From now on we use $\hbar = k_B = 1$.

A. Model

The Lagrangian of the Yukawa-Sachdev-Ye-Kitaev model with q fermion boson interaction can be defined as $L = L_0 + L_g + L_v$. The noninteracting part is given by

$$L_0(t) = \sum_i^N \sum_{\sigma} \psi_{i\sigma}^\dagger(t, \mathbf{x})(i\partial_t + \mu)\psi_{i\sigma}(t, \mathbf{x}) + \sum_k^M \sum_{\mathbf{x}} \frac{1}{2} \phi_k(t, \mathbf{x})(-\partial_t^2 - \omega_0^2)\phi_k(t, \mathbf{x}), \quad (1)$$

where ϕ represent scalar bosons with bare frequency ω_0 , and ψ complex fermions with chemical potential μ . Here i, j are indices of N complex fermion flavors, $\sigma = \uparrow, \downarrow$ for the spins and k for M boson flavors. The boson-fermion interaction part is

$$L_g(t) = \sum_{Ijk\sigma\mathbf{x}} \frac{1}{\sqrt{M}} \left(\frac{1}{\sqrt{N}} \right)^{q-1} g_{Ijk}(t) [\psi_{I\sigma}^\dagger \psi_{J\sigma} \phi_k](t, \mathbf{x}). \quad (2)$$

Here we defined compact indices $I = i_1 \dots i_{\frac{q}{2}}$ and $\psi_J = \psi_{j_1} \dots \psi_{j_{\frac{q}{2}}}$. The random q -point hopping ($\frac{1}{2}q$ -body hopping) part is

$$L_v(t) = \sum_{(\mathbf{x}_1, \mathbf{x}_2)} \sum_{IJ\sigma} \frac{v_{IJ}(t, \mathbf{x}_1, \mathbf{x}_2)}{(\sqrt{N})^{q-1}} \psi_{I\sigma}^\dagger(t, \mathbf{x}_1) \psi_{J\sigma}(t, \mathbf{x}_2). \quad (3)$$

The g_{Ijk} and v_{IJ} are random couplings. The q is an even number. We assume the Gaussian random couplings

$$\begin{aligned} \mathbf{E}[g_{Ijk}^* g_{LPk}] &= g^2 \frac{2}{q} \delta_{IL} \delta_{JP} \delta_{kl}, \\ \mathbf{E}[v_{Ijk}(\mathbf{x}_1)^* v_{LPk}(\mathbf{x}_2)] &= v^2 \frac{2}{q} \delta_{IL} \delta_{JP} \delta_{kl} \delta_{\mathbf{x}_1, \mathbf{x}_2}, \end{aligned} \quad (4)$$

which gives only normal state solutions and excludes superconducting pairing states. Here, the $\mathbf{E}[\dots]$ means disorder average.

B. Disorder averaged effective action and saddle point equations

The local disorder averaged G - Σ - D - Π effective action $S_{\text{eff}} = S_{\text{eff},x}$ is

$$\begin{aligned} \frac{1}{N} iS_{\text{eff}} &= \text{tr} \log (G_0^{-1} - \Sigma) - \frac{\lambda}{2} \text{tr} \log (D_0^{-1} - \Pi) \\ &+ \frac{2}{q} \int_{t_1, t_2} v(t_1) v(t_2) [G(t_1, t_2) G(t_2, t_1)]^{\frac{q}{2}} \\ &+ i \frac{2}{q} \int_{t_1, t_2} g(t_1) g(t_2) [G(t_1, t_2) G(t_2, t_1)]^{\frac{q}{2}} D(t_1, t_2) \\ &+ \int_{t_1, t_2} \Sigma(t_1, t_2) G(t_1, t_2) - \frac{\lambda}{2} \int_{t_1, t_2} \Pi(t_1, t_2) D(t_1, t_2). \end{aligned} \quad (5)$$

Here $G_0^{-1} = i\partial_t + \mu$ and $D_0^{-1} = -\partial_t^2 - \omega_0^2$, and the trace tr is taken over time variables. Note that the time integration contour in the effective action can be chosen as the three-branch Keldysh contour. Here, we suppressed spin and coordinate indices and defined

$$\lambda = \frac{M}{N}. \quad (6)$$

The disorder-averaged partition function can now be written as

$$Z = \int d[G, \Sigma, D, \Pi] \exp(iNS_{\text{eff}}). \quad (7)$$

If N is large, the saddle point contributions are dominated and are exact for $N \rightarrow \infty$. They can be derived by functional variations over G, Σ, D, Π dynamical mean fields. It is convenient to define

$$R_{q-2}(t_1, t_2) \equiv [G(t_1, t_2) G(t_2, t_1)]^{\frac{q-2}{2}}. \quad (8)$$

With this, the saddle point equations are

$$\begin{aligned} G(t_1, t_2) &= [G_0^{-1} - \Sigma]^{-1}(t_1, t_2), \\ \Sigma(t_1, t_2) &= (v(t_1) v(t_2) + g(t_1) g(t_2) iD(t_1, t_2)) \\ &\quad \times R_{q-2}(t_1, t_2) G(t_1, t_2), \\ D(t_1, t_2) &= [D_0^{-1} - \Pi]^{-1}(t_1, t_2), \\ \Pi(t_1, t_2) &= -i \frac{2g(t_1) g(t_2)}{\lambda} \frac{2}{q} R_{q-2}(t_1, t_2) G(t_1, t_2) G(t_2, t_1). \end{aligned} \quad (9)$$

See also (A8) in the Appendix. If $q = 2$ and $v = 0$, the saddle point equations reduce to the usual YSYK ones [1,2]. For $q = 2$ and nonzero v , it is the model considered recently in Ref. [5]. In this work, we usually set $\lambda = 4/q^2$. The bare boson frequency is fixed by

$$\omega_0 = 1. \quad (10)$$

From now on, it will serve as the energy unit.

C. Definitions of observables

Here we briefly define and discuss the observables we use to analyze the dynamics.

a. Occupation density. Occupation densities of the fermion and boson are defined as equal-time functions

$$n^{(f)}(t) = -iG^<(t, t), \quad n^{(b)}(t) = -iD^<(t, t). \quad (11)$$

In numerics, those quantities are accessible for the full computed time range because they do not require Fourier transforms.

b. Wigner transform. Here we use G as an example to define the Wigner transform

$$G(t_1, t_2) = G(t_a + t_r/2, t_a - t_r/2). \quad (12)$$

The relative and average time are $t_r = t_1 - t_2$ and $t_a = (t_1 + t_2)/2$. One can then perform a Fourier transform on the relative time $t_r \rightarrow \omega$ and write

$$G(\omega, t_a) = \int dt_r e^{i\omega t_r} G(t_1, t_2). \quad (13)$$

The same is defined for other two-point functions, e.g., $D(t_1, t_2)$, $\Sigma(t_1, t_2)$ and $\Pi(t_1, t_2)$. From two-point functions in Wigner coordinates, one can obtain time-dependent spectral functions and effective temperatures.

c. Spectral functions. We define the time-dependent fermion spectral function as

$$A^{(G)}(\omega, t_a) = -2\text{Im } G^R(\omega, t_a), \quad (14)$$

and the boson spectral function as

$$A^{(D)}(\omega, t_a) = -2\text{Im } D^R(\omega, t_a) \quad (15)$$

and similarly for $A^{(\alpha)}$, where $\alpha \in \{\Sigma, \Pi\}$.

d. Distribution functions. The time-dependent effective distributions $f^{(G)}$ and $b^{(D)}$ can be defined as

$$1 - 2f^{(G)} = G^K(\omega, t_a)/(G^R(\omega, t_a) - G^A(\omega, t_a)), \quad (16)$$

$$1 + 2b^{(D)} = D^K(\omega, t_a)/(D^R(\omega, t_a) - D^A(\omega, t_a)). \quad (17)$$

The superscripts indicate the Green's functions they are from. Note, it is also possible to define distribution functions from self-energies.

e. Effective temperature. The effective distributions and the effective temperatures can be defined through Wigner Green's functions by assuming the distribution functions have the same form as those in equilibrium. For instance, the effective temperature extracted from the propagators can be written as [26]

$$\tanh(\beta^{(G)}\omega/2) = G^K(\omega, t_a)/(G^R(\omega, t_a) - G^A(\omega, t_a)), \quad (18)$$

and the boson effective temperature through

$$\tanh(\beta^{(D)}\omega/2) = (D^R(\omega, t_a) - D^A(\omega, t_a))/D^K(\omega, t_a). \quad (19)$$

In numerics, the $\beta^{(G,D)}$ are extracted from numerical partial by using the frequency points $\omega = \pm 0.0167\omega_0$.

f. Interaction energy. The interaction energy [26] for the YSYK model can be derived by adding a source that couples to the H_{int} then perform disorder average and G - Σ trick, and finally setting $J = 0$. By adding a source term to the

action $S[v, g, JH_{\text{int}}] = S[v, g] + i \int_x J(x) H_{\text{int}}[v, g](x)$, the interaction energy becomes

$$\langle H_{\text{int}} \rangle = \left[\frac{\delta}{\delta J(x)} \int d[\psi, \phi] d[v, g] \mathcal{P}[v, g] e^{iS[v, g, JH_{\text{int}}]} \right]_{J=0}, \quad (20)$$

where $\mathcal{P}[v, g]$ is the classical disorder distribution of v and g couplings. The result is identical to the Galitskii-Migdal equation [42,43]. This equation can be used in imaginary time or real-time computations. The correlation energy is

$$E_{\text{corr}}^{(b)}(t) = \frac{1}{2} \text{Im} [\Pi \circ D]^<(t, t). \quad (21)$$

Here, the $\circ$ is a convolution along the chosen time contour. For instance, the purely imaginary time version is supposed to be $E_{\text{corr}}^{(b)} = \frac{1}{2} \text{Im} [\Pi \circ D]^<(\tau = 0)$. We have also checked that $E_{\text{corr}}^{(b)} = -2E_{\text{corr}}^{(f)}$, the factor of 2 is from the spin.

g. Relaxation time. The relaxation time can be extracted from the effective temperatures or occupation numbers, etc. In many cases, those observables have damping behavior, and it may be convenient to assume oscillations on top of exponential relaxations. For an observable O , we define

$$O(t) = C_O \cos(\omega_O t) e^{-\Gamma_O t}, \quad (22)$$

where C_O is a constant, ω_O is the damping oscillation frequency and Γ_O is the relaxation rate. For temperature relaxation fittings, we usually set $\omega_O = 0$.

h. Renormalized phonon frequency. We define the renormalized boson frequency as

$$\omega_r^2 = \omega_0^2 + \text{Re } \Pi(\omega = 0, t_a), \quad (23)$$

which can be directly seen from the full boson propagator $D^R(\omega, t_a) = [-\omega^2 - \omega_0^2 - \Pi(\omega, t_a)]^{-1}$.

D. Numerical details

The Kadanoff-Baym equations, with the three-branch contour, are integrated by using codes based on *The Non-Equilibrium Systems Simulation* (NESSi) package [43], which provides a well-tested framework. For time stepping, the fermion equations are integrated with order 5 backward differentiation formula (BDF) as the predictor and Gregory integration for the Volterra integrodifferential equation as the predictor. The correctors are iterations at the same time point until convergence. The boson equations are stepped through the Volterra integral equations into linear equations of order 5. In most cases, we use imaginary time points $N_\tau = 2000$ and real-time step $h = 0.08$ with real-time points $N_t = 4000$. The total evolution time is typically $t_{\text{total}} = 160$. The quench time t_0 is usually set as the middle point of the full evolution period. Several cases are checked with $h = 0.02$ without free evolutions before the quenches.

The initial correlated Matsubara Green's functions for a given inverse temperature β_i are computed first by a standalone self-consistent iteration code in imaginary time and Matsubara frequencies using fast Fourier transforms (FFT), with the imaginary time arrays over-padded to 3 times the length of the frequencies. In most cases, we choose ~ 4000 Matsubara frequencies in a symmetric interval around zero. Note that the boson and fermion have different Matsubara

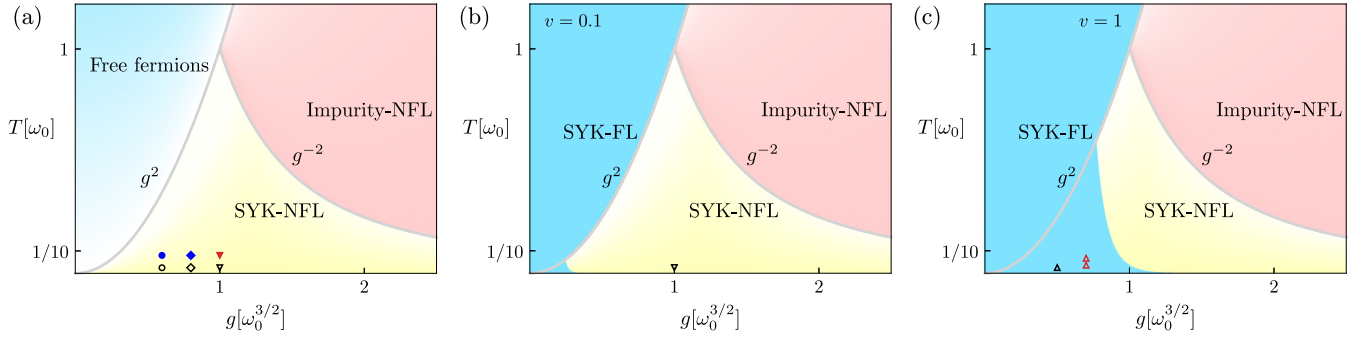


FIG. 1. The half-filling phase diagram for $q = 2$ according to Ref. [5] using the equations provided there. (a) $v = 0$. The two crossovers separate three phases: the free fermion phase, the non-Fermi liquid phase (SYK-NFL), and the impurity-like non-Fermi liquid phase (Impurity-NFL). (b) $v = 0.1$. The Fermi liquid phase (SYK-FL) is marked blue. (c) $v = 1$. The SYK-FL phase is larger than $v = 0.1$. Based on our study, we find that the v quench from (b) to (a) results in an instance of the Planckian relaxation, meaning inverse effective temperature relaxation rates for fermions and bosons $\Gamma_{\beta}^{(G),(D)} \sim T_f$, see Fig. 7. However, it is important to note that $\Gamma_{\beta}^{(G)} \neq \Gamma_{\beta}^{(D)}$. In panel (a), we also marked the initial temperature $T_i = 0.025$ (empty symbols), and the typical highest final temperature $T_f = 0.08$ (filled symbols), with the same symbols used in Fig. 7(b). The left most symbols, for $g = 0.6$ indicate the quench that deviates from Planckian relaxations as shown in Fig. 7(b). In contrast, g quenches occurring near the settings in (c) can lead to prethermal relaxation, characterized by rates significantly lower than Planckian relaxation rates, with $\Gamma_{\beta}^{(G)} \neq \Gamma_{\beta}^{(D)}$. See also Fig. 12. In (c), we marked the T_i (in black) and T_f (in red) with triangles, corresponding to Fig. 12(e) with $g_i = 0.5, g_f = 0.7$. The T_f is determined from the $t_a = 80$ temperatures. When quenching within the SYK-FL phase, prethermalization results in two $t_a = 80$ temperatures for bosons and fermions respectively, indicated by two triangles in (c), which signifies the system relaxes slowly.

frequency point positions due to statistics. The absolute tolerance is set to 10^{-7} . The resulting two-point functions are interpolated to 2000 imaginary time points and sent to NESSi. The equilibrium problem is then solved again with the fixed-point method to the absolute tolerance 10^{-7} for fermion and boson propagators, which gives a double check of the initial state solutions. The Kadanoff-Baym equation is then evolved to quench time t_0 and after the quench to the final time t_f . We have also checked that the equal time observables, such as occupations and correlation energies, stay constant before the quench.

III. YSYK- q IN EQUILIBRIUM

Focusing on normal states, we first revisit several equilibrium properties of the Yukawa-Sachdev-Ye-Kitaev model with additional random q -point hopping. We also include results for finite q . The equilibrium solutions are later used as initial conditions of the Kadanoff-Baym equations.

A. Phase diagram for $q = 2$

The phase diagram of $q = 2$ YSYK with the random hopping term has been previously studied [2,5], where we show it in Figs. 1(a)–1(c) in a temperature-coupling, i.e., the $T - g$, diagram. Note that due to our choice of the complex disorder couplings, no superconducting pairing phase can appear, see (4). If $v = 0$, as in Fig. 1(a), two crossoverlines are separating three phases, and we call them, following the Refs. [2,5], the Free fermion phase, the SYK non-Fermi liquid (SYK-NFL) phase of the YSYK model, and the impurity-like non-Fermi liquid phase (impurity NFL). The crossover lines are qualitatively $\sim g^{-2}$ and g^2 , shown in light grey solid lines in Fig. 1. Switch on the coupling v , as those in Figs. 1(b) and 1(c), we have the free fermion phase been replaced by a SYK Fermi liquid (SYK-FL) phase [5]. Those SYK-FL phases are colored

blue in Figs. 1(b) and 1(c). One can see that for larger v the area of the SYK-FL phases becomes larger. We can see that with $g = 1$ and $T < 1/10$ and $v = 0$, the system is deep in the SYK-NFL phase. This is indeed the phase we will mainly consider.

B. Finite q two point functions in imaginary time

If the system is deep inside the SYK-NFL phase, e.g., with low temperatures $T < 1/10$ and $g > 1$, the long-time parts of the imaginary time Green's functions can be approximated well by conformal solutions. Solving the saddle point equations numerically by using Matsubara formalism, we can also get fermion and boson Green's functions, G and D . The long (imaginary) time conformal solutions [1,2] are

$$G_c(\tau) \sim |\tau|^{-\frac{4\Delta}{q}} \stackrel{q=2}{\simeq} |\tau|^{-0.84}, \quad (24)$$

$$D_c(\tau) \sim |\tau|^{-(2-4\Delta)} \stackrel{q=2}{\simeq} |\tau|^{-0.32}, \quad (25)$$

as for $q = 2$, $\Delta \simeq 0.420374$. For finite q other than 2, the scaling exponent Δ can be determined through the equation

$$1 = \frac{-4 \tan(2\pi \Delta)(q - 4\Delta) \tan\left(\frac{2\pi \Delta}{q}\right)}{(4\Delta - 1)\lambda q^2}. \quad (26)$$

One can find more details about this equation in Appendix F2.

In Fig. 2, we show solutions from the saddle point equations in imaginary time and Matsubara frequencies numerically for different q with $\beta = 160\omega_0^{-1}$. The scaling exponents with $\lambda = \frac{M}{N} = \frac{4}{q^2}$ are computed and listed in Table I. From the figures, we can see that in an energy scale where ω is much smaller than T , the numerically computed Green's functions follow the expected scaling behaviors.

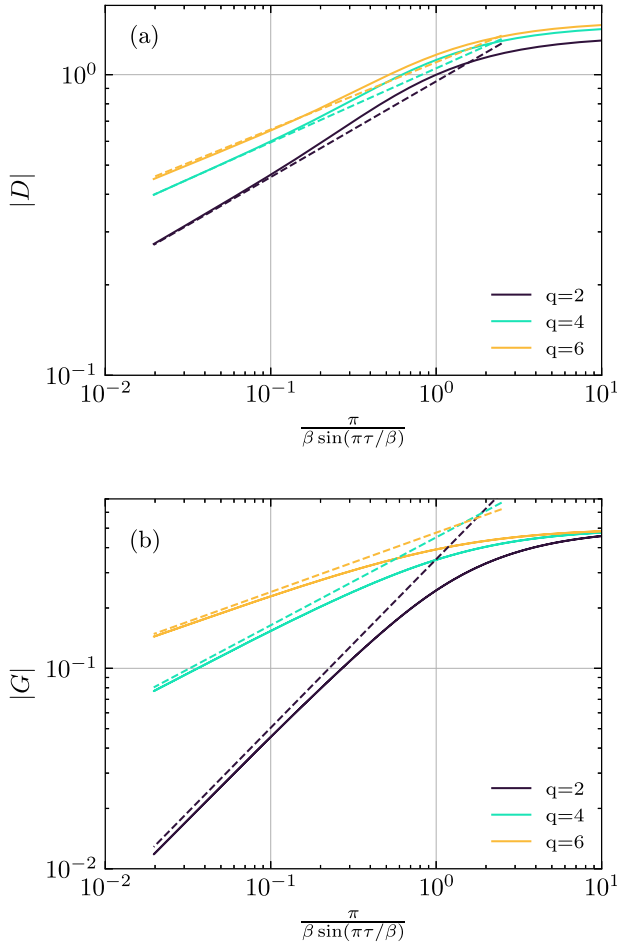


FIG. 2. Imaginary time Green's functions, (a) for fermion and (b) for boson. Solid lines are numerics and dashed lines are conformal solutions with the scaling dimensions in Table I. Here $\beta = 160\omega_0^{-1}$, $g^2 = \omega_0^3 2^{q-1}/q$, $\lambda = 4/q^2$, and $v = 0$. One can see the computed q dependent exponents fit well. Note for reaching low temperature, we used 10^4 Matsubara frequencies. One expects the agreement when $\frac{\pi}{\beta \sin(\pi \tau / \beta)}$ is small.

IV. QUENCH WITH FIXED YUKAWA COUPLING g

Here, we study the SYK interaction quench

$$v \rightarrow 0, \quad g \rightarrow g. \quad (27)$$

The quench is realized through a Heaviside theta $v_i + \theta(t_0)\delta v$ where t_0 is the quench time. For finite q other than 2, we use the scaling

$$g_{q|i,f} = g \sqrt{\frac{2^{q-1}}{q}}, \quad v_{q|i,f} = v \sqrt{\frac{2^{q-1}}{q}}, \quad \lambda_q = \lambda \frac{4}{q^2} \quad (28)$$

TABLE I. Scaling exponent Δ for different q with $\lambda = \frac{4}{q^2}$, from solving Eq. (26).

q	2	4	6	8	10
Δ	0.420374	0.438401	0.444108	0.446943	0.448643

to get a more uniform q dependence. Note that by using these scalings, the q dependent coupling is fully determined by the values of g, v, λ , which simplify the numbers in our discussions. Note that this also means the quench parameters are scaled as well.

A. Spectral function and distribution

In Fig. 3, we show spectral functions of fermions, $A^{(G)}(\omega, t_a)$ and bosons, $A^{(D)}(\omega, t_a)$, and their corresponding effective distributions for several centers of mass time t_a , where $g = 1$. The spectral functions and distributions are obtained from Fourier transforms of the real-time two-point functions. This is why the earliest time we choose is $t = -80$, where a sufficiently large relative time t_r window is available. Note the KBEs starting time is $t = -160$. From Figs. 3(a) and 3(c), the shape of the spectral function agrees with that in previous studies [5]. One can see that for the small quench of the v , only the low-frequency part of the spectral functions reduces their height; this applies similarly to the fermions and bosons. The spectral functions also converge to their final value rather rapidly. In Figs. 3(b) and 3(d), we show the effective distribution functions extracted from the corresponding two-point functions through the fluctuation-dissipation relation. For visibility we show the quantities $1 - 2f^{(G)}(\omega, t_a)$ and $(1 + 2b^{(G)}(\omega, t_a))^{-1}$, since they should be equal to $\tanh \frac{\beta\omega}{2}$ in equilibrium, thus well behaved. We can see from the figures that distributions are smooth curves without sharp peak structures and with a fast relaxation to their final values. In the insets of Figs. 3(b) and 3(d), we show the zoomed-in low-frequency part of the distributions, where one can see that they are straight curves and give well-defined effective inverse temperatures β_{eff} through their slopes around $\omega = 0$. We will use the effective temperature extracted from those slopes to study the nonequilibrium temperature relaxations later in this section.

B. Dynamics of boson occupations, renormalized frequencies, and correlation energies

In Fig. 4, we show boson occupation $n^{(b)}$, the absolute value of boson correlation energy $|E_{\text{corr}}^{(b)}|$, the squared boson renormalized frequencies ω_r^2 and a log-log plot of those three quantities. The overall picture is that their postquench behavior is almost identical, with similar frequencies and relaxation rates. This can be best seen from Fig. 4(d) where both decay rate and the oscillation periods of $n^{(b)}$, $|E_{\text{corr}}^{(b)}|$, and ω_r^2 match each other. From Fig. 4(a), we can see that for different initial v , the occupation oscillations share the same frequency. This can be understood [38] by assuming that the boson field oscillates as $\phi(t) \sim \cos(\frac{1}{2}\omega_{\phi^2}t)$ where $\frac{1}{2}\omega_{\phi^2}$ is the oscillation frequency, then occupation is then $\langle \phi(t)\phi(t) \rangle$ oscillates with the frequency ω_{ϕ^2} . The relaxation rates are robust against different quench parameters. This is also true for quench protocols where v is fixed and g is varying, see Appendix C. The other two quantities are shown in Figs. 4(b) and 4(c) give similar dynamics. Figure 4(c) shows that for larger quenches the system is driven farther from the quantum criticality since the critical point is the soft boson limit $\omega_r \rightarrow 0$. This is consistent with the quench heating the system. We will see in the

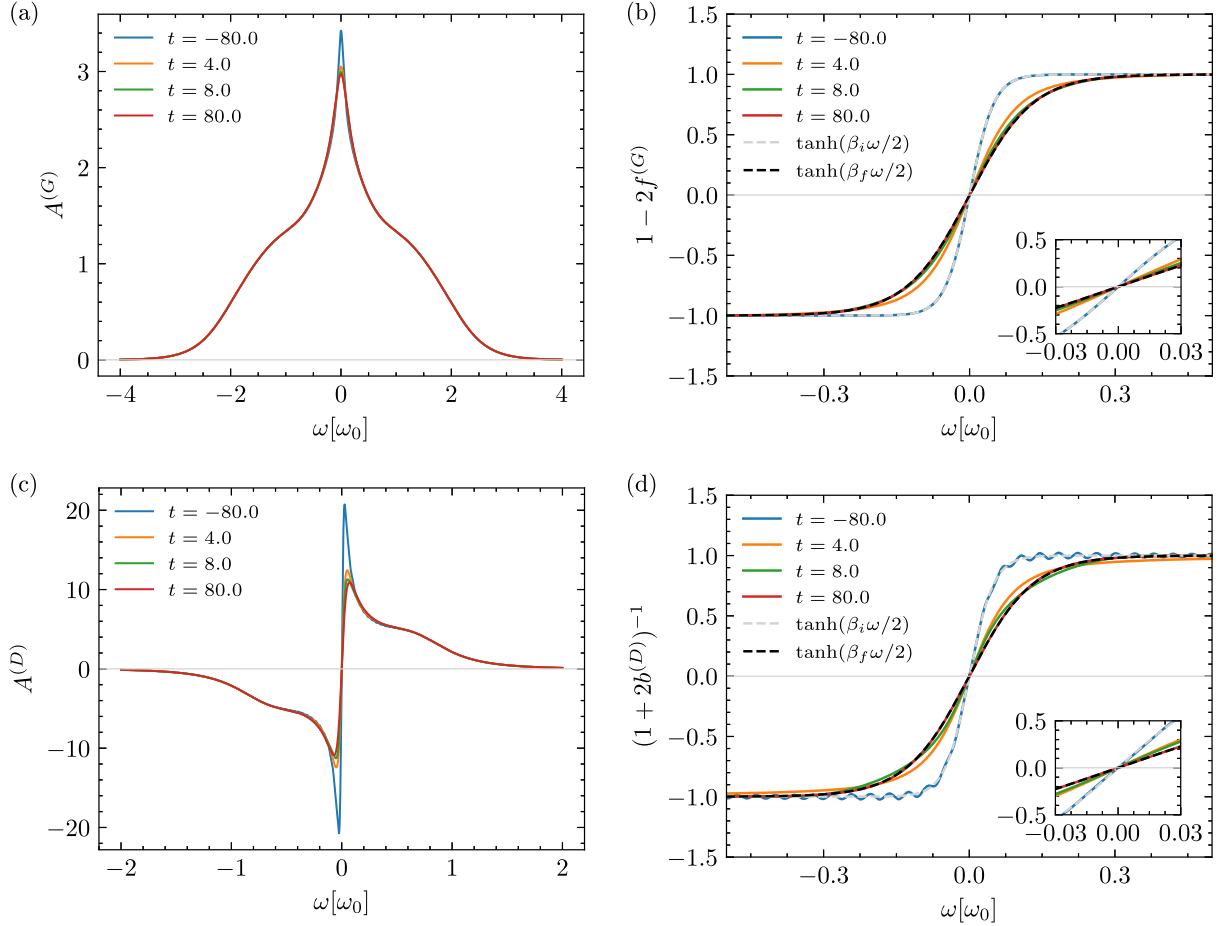


FIG. 3. Spectral functions (a) for fermions and (c) for bosons. Effective distributions (b) for fermions and (d) for bosons. Here we fixed $g = 1$, $q = 2$ with quench protocol $v_i = 0.1$, $v_f = 0$, and $\beta_i = 40$. The insets of (c) and (d) are zoomed-in pictures. The spectral function changes mainly in the low-frequency region for this small quench. The distribution functions have smooth time dependence and can give well-defined effective temperatures that can be extracted from the slopes of effective distributions around $\omega = 0$ and be used for the analysis of temperature dynamics. Here, the $\beta_i = 40$ is the initial temperature, and the $\beta_f \simeq 15.3$ is the fit temperature at the latest time. One can see that the initial and final distributions match the expected equilibrium distribution. Note that all the real-frequency quantities are obtained by Fourier transforming the real-time two-point functions.

next section that compared with the $n^{(b)}$, $|E_{\text{corr}}^{(b)}|$ and ω_r^2 , the effective temperature dynamics behave differently.

C. Time dependent effective temperature and linear in T_f relaxation rate

In Fig. 5, we first show the time center of mass time dependence of effective temperature for bosons and fermions for five different initial v_i and $v_f = 0$. Note that the strongest quench we show in this plot is $v_i = 0.5$, which is for a broader view of the effect of the quench parameter amplitudes and their resulting temperature relaxations. One can see from Fig. 5(a) that for $v_i = 0.1$ the boson and fermion effective inverse temperatures β_{eff} exponentially relax to their final values, however the intermediate values and relaxation rates are distinct to each other. This is true for other quench parameters, however, the difference is that for larger quench, there are temperature oscillations, which make the analysis harder. There are again precursors of the center of mass time t_a , which is again because the t_a slices cross both the t_1 and

t_2 axes and thus can contain postquench information when $t_a < 0$. Here we always quench at $t_1 = t_2 = 0$ and the fermion Green's function $G(t_1, t_2)$. In Fig. 5(b), we plot the effective temperatures T_{eff} for the same set of initial v and only show the postquench part. It is clear that for large quench, e.g., $v_i = 0.5$, the temperature oscillations are pronounced and the oscillation amplitudes are different for boson and fermion. Also, the effective temperature relaxations are faster when quench amplitudes are larger.

For a closer look at the temperature relaxations, we consider the small quenches $v_i < 0.12$ and fit the intermediate time inverse temperatures to the exponential functions. In Figs. 6(a)–6(c), we show the logarithmic scale plot, for three different initial temperatures $\beta_i = 40, 60, 80$, of the fermion (boson in insets) effective inverse temperature differences

$$\Delta\beta^{(G,D)}(t_a) = \beta^{(G,D)}(t_a) - \beta_f, \quad (29)$$

where β_f is the inverse temperature of the latest time in the computation, typically $\beta_f^{(G)} = \beta_f^{(D)} = \beta(t_a = 80/\omega_0)$. The

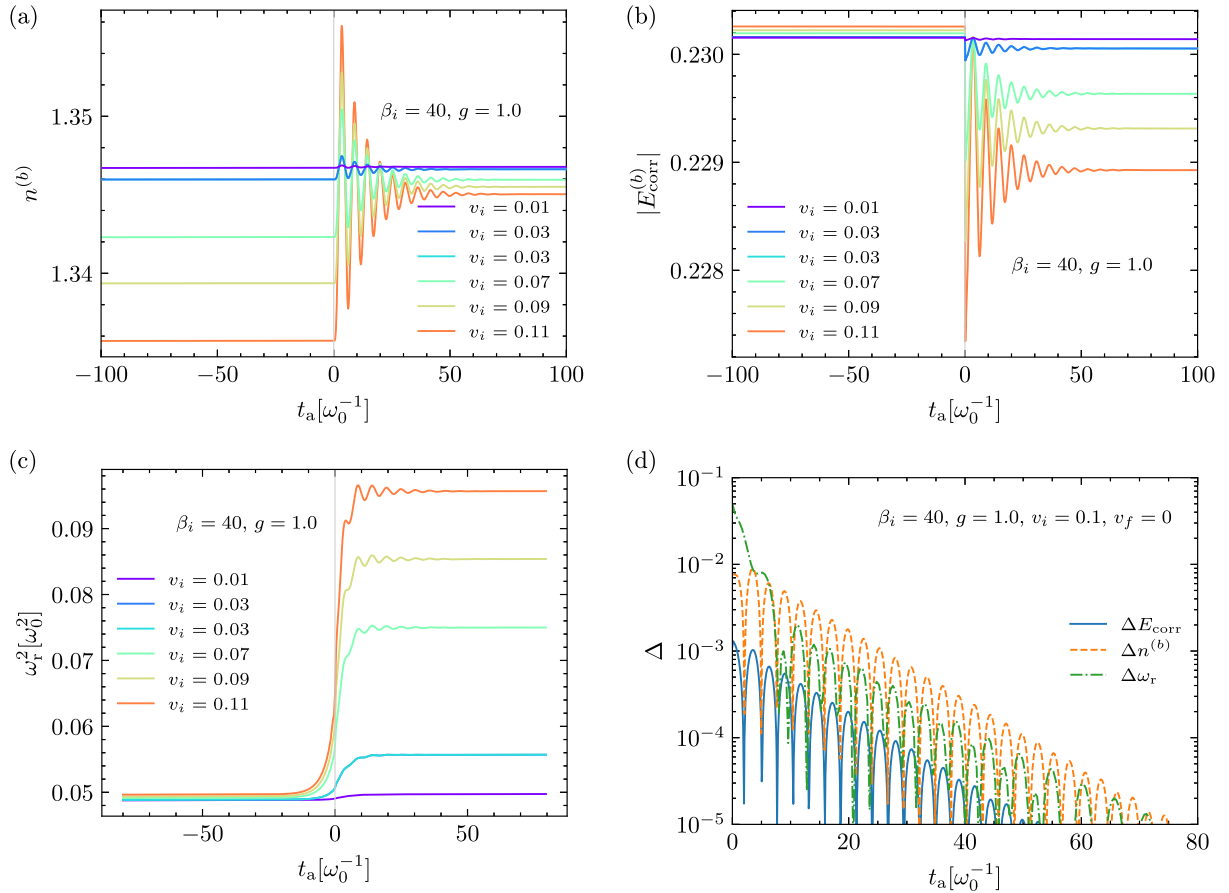


FIG. 4. Time dependencies of boson occupations $n^{(b)}(t)$, boson correlation energies $|E_{\text{corr}}^{(b)}(t)|$ and boson renormalized frequencies $\omega_r^2(t_a)$. The protocol is v_i to $v_f = 0$ with fixed $g = 1, q = 2$. The lower right figure shows the log-log plot for $v_i = 0.1$ of those three quantities, and the agreement of their time dependence can be directly observed. For $n^{(b)}(t)$ and $|E_{\text{corr}}^{(b)}(t)|$, one can see that they stay constant before the quench at $t = 0$. The precursors of the ω_r before the quenches are because of using the center of mass slices two-point functions, which can contain the postquenched part even when $t_a < 0$. We find damping oscillation dynamics are different from the temperature dynamics.

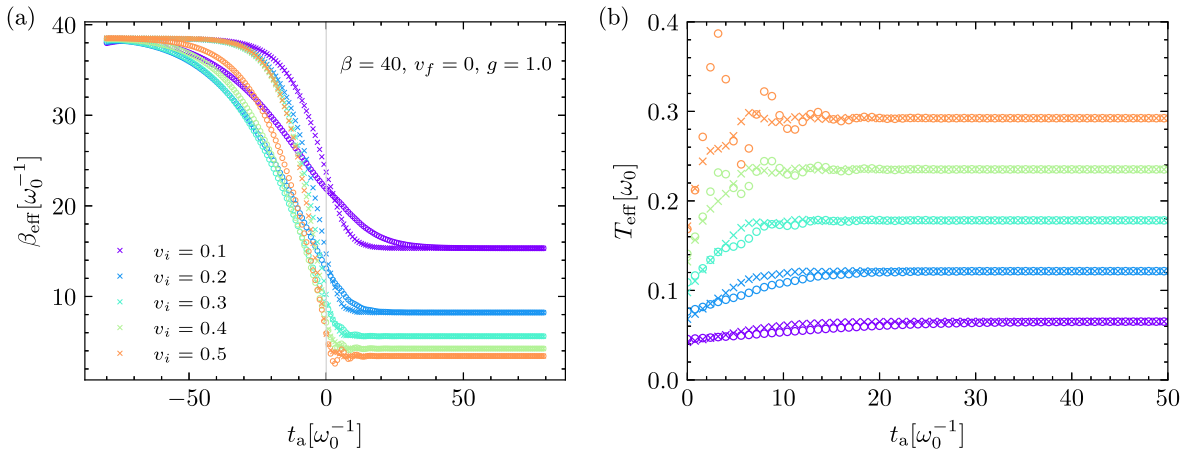


FIG. 5. Center of the mass time dependence of effective inverse temperature and temperature. We fixed $g = 1, q = 2$. The crosses are for fermions and circles for bosons. Note here we use strong quenches $v_i = 0.1, 0.2, 0.3, 0.4, 0.5$ to give a broader view of the quench parameter region. The data shown is sparse for visibility. From (a), we can see precursors of the effective inverse temperatures decaying. This is because the center of mass, i.e., Wigner coordinate, two-point function, is used for temperature extraction, which means even for $t_a < 0$, there can be postquenched effects. For $t_a > 0$, the two-point functions are purely postquenched. The effective temperatures are shown in (b). We can see that the bosons and fermions have different relaxation rates, and the quench affects the different subsystems differently.

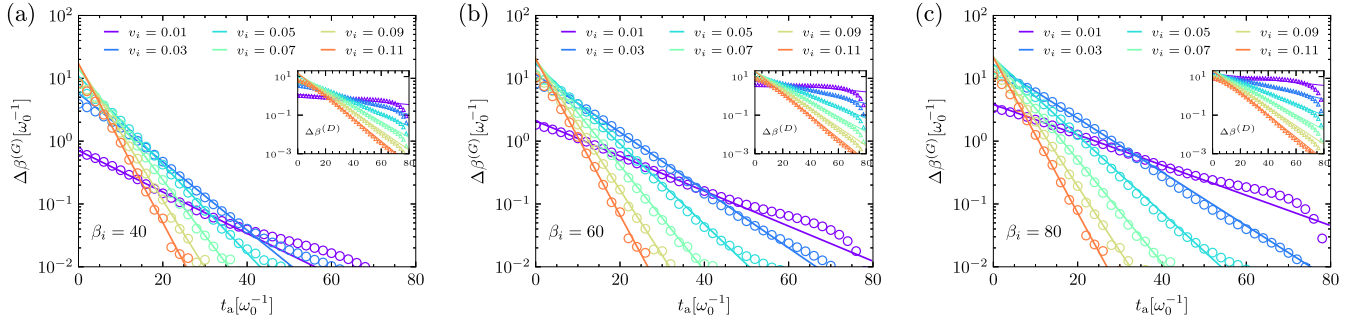


FIG. 6. Temperature relaxations for $q = 2$, $g = 1$, and different initial temperatures. Note $\Delta\beta(t) = \beta(t) - \beta_f$. Here, markers are data, and solid lines are fitted curves or the guide of the eye. From (a)–(c), we can see that the exponentials fit well in a long intermediate time window. The effective inverse temperature relaxation rates $\Gamma_{\beta}^{(G,D)}$ are extracted from the propagators G or D . The fitting results of this figure are summarized in Fig. 7(a).

$\beta^{(G)}$ and $\beta^{(D)}$ are defined previously in equations (18) and (19). In Figs. 6(a)–6(c), we show sparser data than the actual computed data for visibility. The solid lines in Figs. 6(a)–6(c) are fitted curves where we assume the exponential decay of the inverse temperature

$$\Delta\beta^{(G,D)}(t_a) \sim e^{-\Gamma_{\beta}^{(G,D)} t_a} \quad (30)$$

From Figs. 6(a)–6(c), we can see that apart from the very long time behaviors, which may be from the numerical errors, the effective inverse temperatures of fermions show two stages of dynamics: a high-frequency nonuniversal change at short times and a long-time exponential decay. For small v , we find good agreement of the fits, whereas for larger v , there are some oscillations, especially for $\Delta\beta^{(G)}$.

The extracted fermion (boson in inset) inverse temperature relaxation rates versus final temperature T_f are shown in Fig. 7(a), both fermions and bosons give linear dependence of the final temperature T_f . For the pure SYK₄ the linear dependence of the T_f is observed in Ref. [16]. Here, the crucial difference is that we consider two species of particles, namely bosons and fermions, where both show strongly coupled relaxation behaviors, however, with distinct relaxation rates.

In Fig. 7(a), to estimate the uncertainties of the relaxation rates, we do linear regressions of the $\log \Delta\beta(t)$ with lower cutoffs for $\Delta\beta$ at 10^{-1} , 10^{-2} , and 10^{-3} , and get fitting results Γ_1 , Γ_2 , Γ_3 . Then the upper bounds of the error bars are $\max(\Gamma_1, \Gamma_2, \Gamma_3, \Gamma)$ and lower bounds are $\min(\Gamma_1, \Gamma_2, \Gamma_3, \Gamma)$. Note that in this way, the error bars do not reflect a normal distribution, since the data points for a given cutoff are highly correlated. The error bars should be understood as a qualitative estimation of the fitting uncertainties and cross checks of the nonlinear fittings. Bosons' data are from the same methods as fermions, except that due to slower relaxation, the starting time is $t_a = 20$ and linear regression cutoffs are 10^{-1} , 10^{-2} , and 10^{-3} . We also checked that the fitting standard deviations are much smaller than the estimated uncertainties.

D. Temperature relaxations for $q \geq 4$ or $g \neq 1$

To further clarify the origin of the linear T_f dependence of the temperature relaxation rates, we further numerically evaluate two more setups: $q = 2$ and with different fixed g , and $q = 4, 6$ with $g = 1$.

In Fig. 7(b), we show the relaxation rates for different Yukawa couplings g from 0.6 to 1.1. The overall tendency

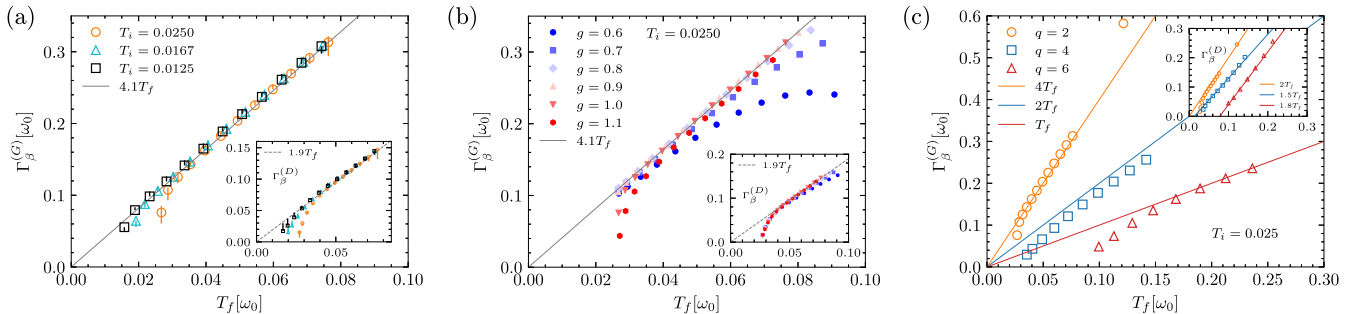


FIG. 7. Temperature relaxations by quenching from finite v to 0. (a) v quenches with different initial temperatures $T_i = 1/40$, $1/60$, $1/80$, and $g = 1$. For fermions, the data are taken after $t_a = 10$. Center values Γ are from nonlinear fits to exponential functions. The bars are from linear regressions with different lower cutoffs; see the main text for explanations. (b) Temperature relaxations for fermions and bosons (inset) with different constant g . The $T_i = 1/40$. For increasing g , the curves converge to be linear in T_f and share a single slope of about 4.1. (c) Temperature relaxations for fermions and bosons (inset) with different q where $q = 2, 4, 6$, and $g = 1$. Note g^2/ω_0^3 is dimensionless, and $\omega_0 = 1$. Here $\beta_i = 40$, i.e., $T_i = 0.025$. Note the actual couplings for different q are scaled as described in Eq. (28). The solid lines are the guide for the eye. We can see that the relaxation rates follow the tendencies of scaling dimensions, where for fermions $\sim \Delta/q$ and for bosons $\sim \Delta$. For each $q > 2$, the linear dependencies can still be resolved. The top orange data in (c) is that with initial $v = 0.2$ and has a strong effective temperature oscillation. Otherwise, the $q = 2$ data in (c) are the same as that in (a).

is clear: for higher g , the curves converge to a single slope $\sim 4.1T_f$ and show linear dependencies. The low T_f parts of the larger g curves have deviations from the linear behavior, especially for the points close to the initial temperature $T = 0.025$. For the curves with $g \leq 0.8$, deviations from the line $4.1T_f$ become pronounced. This is because the postquenched parameters are closer to the free fermion phase, thus giving much slower relaxations, i.e., smaller Γ , and are pulled downwards from the bound linear in T_f .

We also computed the relaxation rates for $q = 4, 6$. The $q > 6$ cases require a lower temperature to get deep inside the non-Fermi liquid phase, which is numerically not as feasible as the $q \leq 6$ cases. In Fig. 7(c), we show the temperature relaxation rates for different q , $q = 2, 4, 6$, using the parameter scaling (28) and the saddle point equations with $q \geq 2$. The extraction technique is the same as that used in Figs. 6 and 7(a). For the $q = 4, 6$, we have $\Gamma_{\beta}^{((G))} \sim 1/q$ and $\Gamma_{\beta}^{(D)} \sim 1$ which is following the scaling dimension of the propagators (24) and (25). This is expected since the postquench Hamiltonian is a q fermion YSYK without the v coupling. For g quenches with finite coupling v , we evaluated the temperature relaxation, see Fig. 13 in Appendix C, where the linear dependence is less pronounced, which can be a result of the competition of the random hopping term.

V. MORE ABOUT UNIVERSAL RELAXATIONS

Here we continue discussions about the universal quench dynamics or relaxations further. The hope is that for some time region, the approximated kinetic equations, i.e., approximations of the Kadanoff-Baym equations, may give some partial analytical insights. We will see that the equations remain rather complicated, and it is challenging to find analytical solutions.

A. Temperature relaxation

Inspired by the solutions obtained from the Kadanoff-Baym equation, we may approximate the intermediate time dynamics by assuming time local equilibrium and using time-dependent effective temperatures. The exponential growth of the effective temperature may be written as

$$\partial_t (T^{(f)}(t) - T_f^{(f)}) = \Gamma_T^{(f)} (T^{(f)}(t) - T_f^{(f)}), \quad (31)$$

$$\partial_t (T^{(b)}(t) - T_f^{(b)}) = \Gamma_T^{(b)} (T^{(b)}(t) - T_f^{(b)}), \quad (32)$$

where we used $T^{(f,b)} \equiv T_{\text{eff}}^{(f,b)}$. The relaxation rate is expected to be $\Gamma_T^{(f,b)} = -\Gamma_{\beta}^{(G,D)}$. From the evolution of the spectral functions and effective distributions, which look quasistatic, we expect Γ_T may have some relation with the generalized collision integral and the quantum Boltzmann equations. However, we will see that it is rather difficult to derive precise analytical relations.

B. About temperature relaxation: approximate kinetic equations with scale separation

For temperature relaxations, we may try to get differential equations of effective distributions. One way to achieve this is to apply the scale separation assumption. After Wigner

transform and expanding to the lowest order of the gradient expansion [44–47], we get generalized quantum Boltzmann equations [17,44,48] without assuming quasiparticles, see also Appendix D. The resulting equations are

$$-i\partial_{t_a} F(\omega, t_a) = I_{\text{col}}^{(f)}[F, B], \quad (33)$$

$$-2i\omega\partial_{t_a} B(\omega, t_a) = I_{\text{col}}^{(b)}[F, B], \quad (34)$$

where F and B are related to distribution functions. The collision integrals under the same approximations are

$$I_{\text{col}}^{(f)} = \Sigma^K(\omega, t_a) - F(\omega, t_a)(\Sigma^R(\omega, t_a) - \Sigma^A(\omega, t_a)), \quad (35)$$

$$I_{\text{col}}^{(b)} = \Pi^K(\omega, t_a) - B(\omega, t_a)(\Pi^R(\omega, t_a) - \Pi^A(\omega, t_a)). \quad (36)$$

We can also write Green's functions into time-dependent distributions and spectral functions under the scale separation approximation [44,45] $G^K(\omega, t_a) = F(G^R - G^A) = F(\omega, t_a)A^{(G)}(\omega, t_a)$ and $D^K(\omega, t_a) = B(D^R - D^A) = B(\omega, t_a)A^{(D)}(\omega, t_a)$, which gives the greater and lesser functions immediately in the form

$$G^{\gtrless}(\omega, t_a) = -iA^{(G)} \frac{F \pm 1}{2}, \quad D^{\gtrless}(\omega, t_a) = -iA^{(D)} \frac{B \pm 1}{2}, \quad (37)$$

where we suppressed some variables for short. One can easily identify that $F = 1 - 2f$ and $B = 1 + 2b$, which are exactly the definitions of F and B .

We can write out the collision integrals with the help of $\Sigma^R - \Sigma^A = \Sigma^> - \Sigma^<$. Defining $[A \bullet B](v) = \int_{\omega} A(v - \omega)B(\omega)$ the results are

$$\begin{aligned} \frac{-2i}{g^2} I_{\text{col}}^{(f)} &= F((A^{(D)}B) \bullet A^{(G)} + A^{(D)} \bullet (A^{(G)}F)) \\ &\quad - (A^{(D)}B) \bullet (A^{(G)}F) - A^{(D)} \bullet A^{(G)}, \end{aligned} \quad (38)$$

$$\begin{aligned} \frac{-\lambda i}{g^2} I_{\text{col}}^{(b)} &= F((A^{(G)}F) \bullet A_-^{(G)} - A^{(G)} \bullet (A_-^{(G)}F_-)) \\ &\quad + (A^{(G)}F) \bullet (A_-^{(G)}F_-) - A^{(G)} \bullet A_-^{(G)}, \end{aligned} \quad (39)$$

here $A_- = A(-\omega, t_a)$ and similarly for F_- . See also (D29) and (D30) in the Appendix. From the above collision integrals, we can find immediately that for $q = 2$ the coupling v does not contribute to the kinetic equation. This is understandable because when $q = 2$, the v terms are quadratic and collisionless. For larger q , we also computed the collision integrals, which can be found from the Appendix, Eqs. (D31) and (D32). The v terms now appear in the collision integrals and are expected to give SYK dynamics, which is competing with the YSYK part. One can see that those equations are rather involved and difficult to solve analytically. Since we already have the results from the Kadanoff-Baym equations, we will not try to solve those kinetic equations numerically here.

C. Occupation relaxation: equal-time equations

Here we discuss more about the boson occupation $n^{(b)}(t)$ oscillation with the frequency ω_{ϕ^2} and decay rate Γ_{ϕ^2} . The relevant equation will be the equal-time version of the Kadanoff-Baym equations, which can be found mostly in the literature about the generalized Kadanoff-Baym ansatz

as an approximation. Here, we will not further apply any approximations but only use the general exact equations, and we will focus on the equations for bosons.

We start with the Kadanoff-Baym equations

$$[-\partial_{t_1}^2 - \omega_0^2]D^<(t_1, t_2) = K_1(t_1, t_2), \quad (40)$$

$$[-\partial_{t_2}^2 - \omega_0^2]D^<(t_1, t_2) = K_2(t_1, t_2), \quad (41)$$

where the K_1 and K_2 are terms from usual Langreth rules, for which the full equations can be found in Eqs. (E3) and (E4). To get the equal time equations, some operations are needed to convert the second derivatives to first-order ones by factorizing the products and inverting one derivative to the right-hand side. We show how we do this in Appendix E. The equal time equation can then be written as

$$-\partial_t n^{(b)}(t) = C_{\text{col}}^{(b)}(t). \quad (42)$$

The right-hand side convoluted collision integral is defined as

$$C_{\text{col}}^{(b)}(t) = [P_+ \circ K_1](t, t) - [K_2 \circ M_+](t, t), \quad (43)$$

where P_+ is a retarded function and M_+ is an advanced function, defined by

$$P_+(t_1, t_2) = -i\theta(t_1 - t_2)e^{+i\omega_0(t_1 - t_2)}, \quad (44)$$

$$M_+(t_1, t_2) = i\theta(t_2 - t_1)e^{+i\omega_0(t_1 - t_2)}. \quad (45)$$

The simplest check is that $C_{\text{col}}(t) = 0$ if we have free collisionless bosons $K_1 = 0$ and $K_2 = 0$.

One can also see by taking a step further that with suitable choices of $K_1(t_1, t_2)$ and $K_2(t_1, t_2)$, the above equations can give $n^{(b)}$ oscillations with the frequency ω_0 . Plugging in the P_+ and M_+ to $C_{\text{col}}^{(b)}(t)$, we get

$$C_{\text{col}}^{(b)}(t) = -ie^{i\omega_0 t} \alpha_1(t) - ie^{-i\omega_0 t} \alpha_2(t) \quad (46)$$

with $\alpha_1(t) \equiv \int_{-\infty}^t dt_2 e^{-i\omega_0 t_2} K_1(t_2, t)$ and $\alpha_2(t) \equiv \int_t^{\infty} dt_2 K_2(t, t_2) e^{i\omega_0 t_2}$. Now if we can have $\alpha_1(t) \sim \alpha_2(t) = \alpha$, which may be achieved by assuming K_1 and K_2 are peaked around t , we get $-\partial_t n^{(b)}(t) = -2i\alpha \cos(\omega_0 t)$ which has the solution $n^{(b)}(t) \sim \alpha \sin(\omega_0)/\omega_0$ up to boundary terms and prefactors. Of course, the actual solutions of the damped oscillations should be found by computing the $C_{\text{col}}^{(b)}(t)$, and it will give frequency corrections and decay rates. We leave this analysis to future studies.

VI. DISCUSSION AND CONCLUSION

In this work, we investigated the equilibrium two-point functions and quantum quench dynamics of the finite- q extension of the normal state of the YSYK model and one of its extensions. In contrast to previous studies [26], no external bath is coupled to the system, and the relaxation dynamics are entirely governed by the intrinsic interactions of the YSYK model. The time evolutions are obtained by numerically integrating Kadanoff-Baym equations of large- N saddle point equations, without introducing additional approximations. We analyzed spectral functions, effective distribution functions, effective temperatures, occupation densities, and other observables. Additionally, we derived simplified forms of the

Kadanoff-Baym equations to obtain analytical insights into the relaxation behaviors.

Interestingly, as shown in Fig. 7, quenching the coupling v to zero with small amplitude reveals the emergence of two distinct transient temperatures associated with the bosons and fermions, respectively. For all considered values of $q = 2, 4, 6$ and initial temperatures $T_i = 1/40, 1/60, 1/80$, the inverse temperature relaxation rates Γ exhibit a linear dependence on the final temperature T_f , with fermions and bosons relaxing at different rates. This behavior bears similarities to that observed in SYK models [16], but differs due to the interplay between bosonic and fermionic degrees of freedom.

The observed linear-in- T relaxation rates reflect the absence of quasiparticles and are indicative of Planckian transport. In such systems, the relaxation rate Γ , defined as the inverse relaxation time τ_{relax} , scales linearly with temperature [32,49]:

$$\Gamma = \frac{1}{\tau_{\text{relax}}} = f \frac{k_B T}{\hbar}, \quad (47)$$

where the dimensionless prefactor f is expected to be of order unity, $f \sim O(1)$. In the YSYK model, we find $f_F \sim 4.1$ for fermions and $f_B \sim 1.9$ for bosons, consistent with previous results for extended SYK models, where $f \simeq 4.8$ [49], and for the SYK₄ model, where $f \simeq 2$ [16].

In summary, we have identified clear signatures of Planckian behavior in the q -fermion Yukawa-Sachdev-Ye-Kitaev model with the random q -point hopping term by investigating its nonequilibrium dynamics following quenches in the random hoppings. The linear in T_f relaxation spans over approximately a decade in final temperature T_f , with fermions and bosons exhibiting distinct relaxation rates, highlighting the system's intricate dynamics without quasiparticles.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data that support the findings of this article are openly available [50].

APPENDIX A: DISORDER AVERAGE FINITE q YSYK IN REAL TIME

Now we consider real-time equations. Since the $\mathbf{E}[v_{IJ}(\mathbf{x})v_{KL}(\mathbf{x}')]]$ gives delta functions in position space, the Lagrangian in the main text is equivalent to the following Hamiltonian in the sense of large- N disorder averaged

effective action

$$H_{\text{int}} = \left(\frac{1}{\sqrt{N}}\right)^{q-1} \sum_{\sigma} \sum_{i_1 i_2 \dots i_{\frac{q}{2}}} \sum_{j_1 j_2 \dots j_{\frac{q}{2}}} v_{i_1 \dots i_{\frac{q}{2}} j_1 \dots j_{\frac{q}{2}}} \psi_{i_1 \sigma}^{\dagger} \dots \psi_{i_{\frac{q}{2}} \sigma}^{\dagger} \psi_{j_1 \sigma} \dots \psi_{j_{\frac{q}{2}} \sigma} \\ + \left(\frac{1}{\sqrt{N}}\right)^{q-1} \frac{1}{\sqrt{M}} \sum_{\sigma k} \sum_{i_1 i_2 \dots i_{\frac{q}{2}}} \sum_{j_1 j_2 \dots j_{\frac{q}{2}}} g_{i_1 \dots i_{\frac{q}{2}} j_1 \dots j_{\frac{q}{2}} k} \psi_{i_1 \sigma}^{\dagger} \dots \psi_{i_{\frac{q}{2}} \sigma}^{\dagger} \psi_{j_1 \sigma} \dots \psi_{j_{\frac{q}{2}} \sigma} \phi_k. \quad (\text{A1})$$

We may focus on the first line and integrate out v . Note from hermicity we have $v_{i_1 \dots i_{\frac{q}{2}} j_1 \dots j_{\frac{q}{2}}} = v_{j_{\frac{q}{2}} \dots j_1 i_{\frac{q}{2}} \dots i_1}^*$, and we can choose disorder potential to satisfy

$$\mathbf{E}[v_{IJ}^* v_{KL}] = v^2 \frac{2}{q} \delta_{IK} \delta_{JL}, \quad (\text{A2})$$

where we defined collected index $I = i_1 \dots i_{\frac{q}{2}}$. The disorder average can be performed as

$$\mathbf{E}[e^{i v_{IJ} O_{IJ}}] = \exp \left\{ i \left(i v^2 \frac{2}{q} \sum_{IJ} O_{IJ} O_{IJ} \right) \right\}. \quad (\text{A3})$$

The $\bar{I} = i_{\frac{q}{2}} \dots i_1$ is the reversed ordered compact index, the same for $\bar{J}$. Note, we always assume replica symmetry here. From the above considerations, the v coupling part interacting action under a functional integral of fermionic fields ψ and ψ can be written as

$$iS_{\text{int}, v, \sigma} = i^2 \frac{1}{N^{q-1}} v^2 \frac{2}{q} \int_{t_1 t_2} \sum_{IJ} \bar{\psi}_{\bar{J}\sigma}(t_1) \psi_{\bar{I}\sigma}(t_1) \bar{\psi}_{I\sigma}(t_2) \psi_{J\sigma}(t_2), \quad (\text{A4})$$

here $\psi_{I\sigma}(t) = \psi_{i_1 \sigma}(t) \dots \psi_{i_{\frac{q}{2}} \sigma}(t)$, note that if $\frac{q}{2}$ is an odd number we have ψ_I fermionic that gives us a prefactor $(-1)^{\frac{q}{2}}$ after commuting.

$$iS_{\text{int}, v, \sigma} = \frac{i^2 (-1)^{\frac{q}{2}}}{N^{q-1}} v^2 \frac{2}{q} \int_{t_1 t_2} \sum_{IJ} \psi_{\bar{I}\sigma}(t_1) \bar{\psi}_{I\sigma}(t_2) \psi_{J\sigma}(t_2) \bar{\psi}_{J\sigma}(t_1). \quad (\text{A5})$$

Define bilocal field $iG_{\sigma}(t_1, t_2) = \frac{1}{N} \sum_i \psi_{i\sigma}(t_1) \bar{\psi}_{i\sigma}(t_2)$. Note that rearranging $\psi_I \psi_I$ to pairs will not give additional signs. Here we further assume spin space diagonal and symmetric solution $G_{\uparrow} = G_{\downarrow}$, which allows us to drop the spin index, and divide the bosonic part by 2 to get per-spin effective action. The v part now becomes

$$\frac{1}{N} iS_{\text{int}, v} = \int_{t_1 t_2} v^2 \frac{2}{q} [iG(t_1, t_2)]^{\frac{q}{2}} [-iG(t_2, t_1)]^{\frac{q}{2}}, \quad (\text{A6})$$

which agrees with previous studies [51] up to imaginary unit i that from definitions. The boson-fermion part $iS_{\text{int}, g}$ is along the same line and straightforward. Restore the time dependence of the couplings. The effective action per spin is then

$$\frac{1}{N} iS_{\text{eff}} = \text{tr} \ln (G_0^{-1} - \Sigma) - \frac{\lambda}{2} \text{tr} \ln (D_0^{-1} - \Pi) \\ + \int_{t_1, t_2} v(t_1) v(t_2) \frac{2}{q} [G(t_1, t_2)]^{\frac{q}{2}} [G(t_2, t_1)]^{\frac{q}{2}}$$

$$+ i \int_{t_1, t_2} g(t_1) g(t_2) \frac{2}{q} [G(t_1, t_2)]^{\frac{q}{2}} [G(t_2, t_1)]^{\frac{q}{2}} D(t_1, t_2) \\ + \int_{t_1, t_2} \Sigma(t_1, t_2) G(t_2, t_1) - \frac{\lambda}{2} \int_{t_1, t_2} \Pi(t_2, t_1) D(t_1, t_2). \quad (\text{A7})$$

The saddle point equations are

$$G(t_1, t_2) = [G_0^{-1} - \Sigma]^{-1}(t_1, t_2), \\ \Sigma(t_1, t_2) = (v(t_1) v(t_2) + i g(t_1) g(t_2) D(t_1, t_2)) \\ \times [G(t_1, t_2)]^{\frac{q}{2}} [G(t_2, t_1)]^{\frac{q}{2}-1}, \\ D(t_1, t_2) = [D_0^{-1} - \Pi]^{-1}(t_1, t_2), \\ \Pi(t_1, t_2) = \frac{-2i}{\lambda} \frac{2}{q} g(t_1) g(t_2) [G(t_1, t_2)]^{\frac{q}{2}} [G(t_2, t_1)]^{\frac{q}{2}}. \quad (\text{A8})$$

For general q , we may define the q dependent couplings which should be finite when $q \rightarrow \infty$. This can be reached by $g^2 = \mathcal{J}^2 \frac{2^{q-1}}{q}$ and $\lambda = \frac{4}{q^2} \lambda_1$ with fixed $\mathcal{J}$ and λ_1 . We then have the boson coupling $\frac{g^2}{\lambda} \frac{2}{q} = \mathcal{J}^2 \frac{2^{2q}}{q^2} \frac{q^2}{4\lambda_1} = \mathcal{J}^2 2^{2q-2}$ which is uniform in q as well. The v coupling gives a complex SYK $_q$ model. For a given q , the full model is a SYK $_q$ plus YSYK $_q$ model. Note that due to the bosonic propagator, the YSYK $_2$ is closely related to SYK $_4$, which means for $q = 2$ it may share some features of SYK $_2$ -SYK $_4$ models.

APPENDIX B: MORE ON EQUILIBRIUM PROPERTIES

In Fig. 8, we numerically compute the renormalized boson frequency, defined as $\omega_r^2 = \omega_0^2 + \text{Re} \Pi(\omega_n = 0)$ with iterative method as that is used in Fig. 2. This actually reproduces the known result in the literature [5], where the $v = 0$ curve is

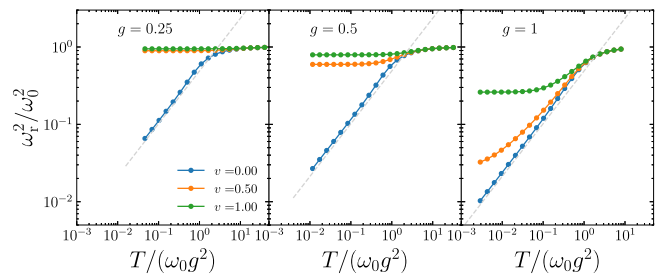


FIG. 8. Equilibrium boson squared renormalized frequency ω_r^2 vs temperature T , $q = 2$. The dashed lines are $\sim T^{4\Delta-1}$, i.e., the expected low-temperature behavior. Here, 10^4 Matsubara frequencies are used for the numerical solutions mainly because of those low target temperatures.

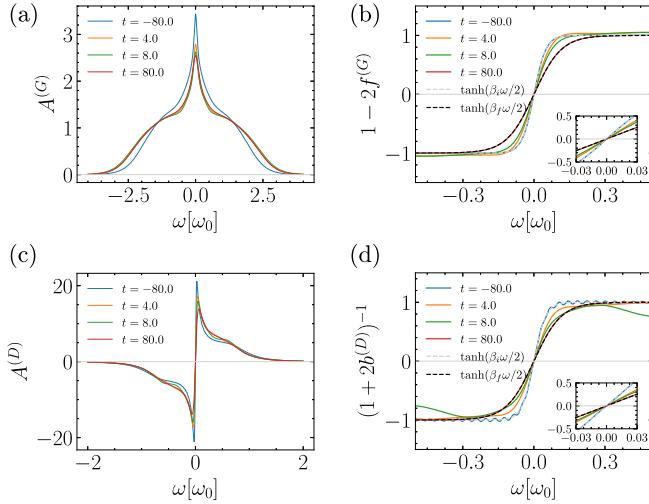


FIG. 9. Spectral functions and effective distributions for fixed. Fixed v quench where $v = 0$, $g_i = 1$, $g_f = 1.1$ and $q = 2$. The insets of (c) and (d) are zoomed-in pictures. Here we can see a sudden width change of the spectral function, which is because the g , which is controlling the width, is quenched.

following the universal scaling $\omega_r^2 \sim T^{4\Delta-1}$. As we have also seen from Fig. 1, for $q = 2$ the effect of increasing the finite v is to introduce a SYK₂ Fermi liquid phase that covers an increasing g interval and pushes the non-Fermi liquid phase to the right, i.e., the larger g . This can be seen from Fig. 8 that for $g = 1$ and $v = 0.5$, the renormalized phonon frequencies are softened considerably by lowering the temperature until the SYK₂ Fermi liquid at very low temperature. This effect can be used to tune the phase from NFL to FL without changing the temperature, and this may be done dynamically through quantum quenches.

APPENDIX C: QUENCH WITH FIXED COUPLING v

Here we consider the Yukawa interaction to quench

$$v \rightarrow v, \quad g_i \rightarrow g_f, \quad (\text{C1})$$

which means we fix the coupling v and quench the YSYK coupling to a different value after the quench time point $t_0 = 0$. The quenches are applied through a theta function $\theta(t_0)$ which enters the $g(t_1)g(t_2)$ terms of the saddle point equations and thus the Kadanoff-Baym equations.

1. Spectral functions and distribution

In Fig. 9, we show the time dependent spectral function $A^{(G)}(\omega, t_a)$, $A^{(D)}(\omega, t_a)$ and effective distributions $f^{(G)}(\omega, t_a)$ and $b^{(D)}(\omega, t_a)$ for fixed $v = 0$ and a sudden quench of g from 1.0 to 1.1. Note that the superscripts on the observables indicate which two-point function they are extracted from. From the plots, one can see that the fermion spectral function has a jump in its bandwidth, which is because of the sudden quench of the g , which determines the bandwidth, especially when $v = 0$. From the $A^{(G)}$ and $A^{(D)}$, we can see that the peak heights of the spectral functions decreased and converged to the final value quickly after the quench, and the overall shape

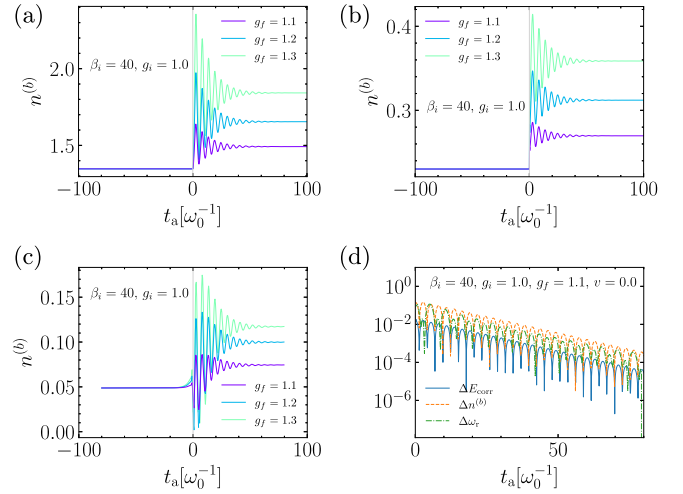


FIG. 10. In (a)–(c), boson occupations, correlation energies, and renormalized frequencies, respectively. Fixed $v = 0$, $q = 2$. Note that ω_r^2 is extracted from Fourier space using the Wigner coordinate two-point function; thus, some precursors occur. In (d), we compare the quantities of (a)–(c) with $g_f = 1.1$ to see if their time dependence, oscillation frequencies, and decay rates are very similar to each other.

remains almost unchanged which suggests a quasiequilibrium process since it is known that the low-frequency peak heights are reflecting the temperatures in equilibrium. Similar behaviors can be found for the effective distributions $f^{(G)}$ and $b^{(D)}$ plots, where the slope around $\omega = 0$ can be regarded as an effective temperature. For instance, at $t = 8$, both the spectral functions and distributions of the fermions and bosons are close to the final value, but still have obvious deviations. This suggests that the boson and fermion observables thermalize on a comparable timescale.

2. Dynamics of boson occupations, renormalized frequencies, and correlation energies

In Fig. 10, we show boson occupations $n^{(b)}(t_a)$, boson renormalized frequencies $\omega_r^{(b)}(t_a)$ and boson correlation energies $E_{\text{corr}}^{(b)}(t_a)$ for the fixed v quenches. The quench dynamics of those quantities give under-damped oscillations.

In Fig. 11, we show the correlation energy extracted from the fermion two-point functions. Use the observation that the kinetic energy purely from bosons is where $E_{\text{kin}}(t) \sim \partial_t^2 \langle \phi(t)\phi(t) \rangle$, which then shares the same frequency with the boson occupation oscillations. Since the total energy is conserved after the quench $E_{\text{kin}}(t) + E_{\text{corr}}(t) = E_{\text{total}}$, the correlation energy should oscillates with the frequency ω_{ϕ^2} as well. The oscillations may be understood from the discussions in Sec. VC in the main text.

3. Time dependent effective temperature

In Fig. 12, we plot the time-dependent effective inverse temperatures $\beta^{(G)}(t_a)$ and $\beta^{(D)}(t_a)$ extracted from boson and fermion propagators through a symmetric numerical differentiation around $\omega = 0$ of $1 - 2f^{(G)}(\omega, t_a)$ and $(1 + 2b^{(D)}(\omega, t_a))^{-1}$. The left column [(a), (c), and (e)] shows

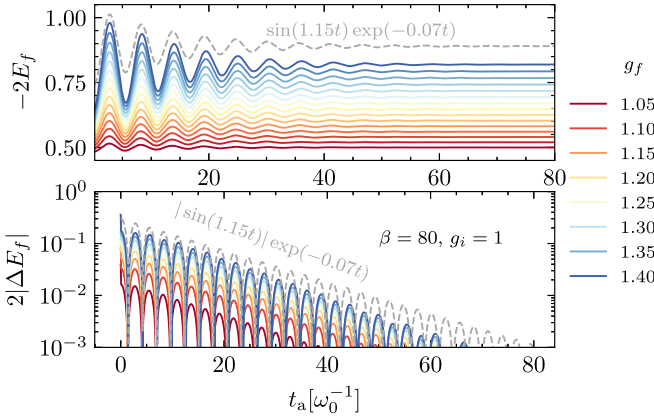


FIG. 11. The real time correlation energies $E_{\text{corr}}^{(f)}(t_a)$ (in plots labeled as E_f) from fermion propagators and self-energies through Galitskii-Migdal formula. We defined $\Delta E^{(f)} = E^{(f)}(t) - E_f^{(f)}$. The early to intermediate time decay rate is about $0.07\omega_0$ and is almost the same for different final coupling strengths. The oscillation frequencies are also insensitive to different g_f and are about $1.15\omega_0$. Here $v = 0$ and $q = 2$. Note here the quench is applied immediately at the initial time $t_0 = t_i$ and we used $N_\tau = 600$ and $h = 0.02$, with initially $\beta = 80$.

$g_i = 0.5$, and the right column [(b), (d), and (f)] shows $g_i = 1.0$ results. The rows have constant $v = 0, 0.5$, and 1.0 , respectively.

From all plots, we can see that the quench affects two subsystems, the bosons and the fermions, differently. For $v = 0$, Figs. 12(a) and 12(b) the fermion inverse temperature $\beta^{(G)}(t_a)$ decays slower than the boson, however, the initial postquench inverse temperature of the fermion is larger than the boson, which means the fermion temperature is initially lower than boson temperature. There are also oscillations in the fermion and boson inverse temperature. However, with different periodicity, those for fermions oscillate more slowly than bosons. For $v = 0.5$, in (c), we can see that since the starting state is closer to the Fermi liquid phase, the equilibration time when the two temperatures equal is later. For $g_f = 0.6$, the effective temperatures of fermions and bosons are not equal to each other even for the latest computed time $t_a = 80$. Also, there are strong oscillations for bosons that signal temperatures are not well defined. For $v = 1.0$, Figs. 12(e) and 12(f), we can see that for $g_i = 0.5$ and $g_f = 0.6$, where the quench is deep inside the Fermi liquid phase, the late time $\beta_{\text{eff}}(t_a)$ for bosons and fermions are almost parallel.

In Fig. 13, the inverse temperature relaxation rates Γ_β are shown, where they are computed by using the same technique as that for Fig. 6. The initial temperature is set to $\beta = 40$. Three different protocols are considered here. They are shown in Fig. 13. In (a), we show the fits for the quench from larger Yukawa coupling to smaller ones $g_i > g_f$. In (b) $g_i < g_f$ and $g_f = 1$. In (c), $g_i < g_f$ and $g_i = 1$. Those protocols are for studying the effect of being quenched from different sides of $g = 1$ and to see if they are universal. From (a) to (c), we find that there are stronger oscillations for the fermion effective inverse temperatures, which make the fit harder than the v quenches. In (d), we summarize the relaxation rates. One can

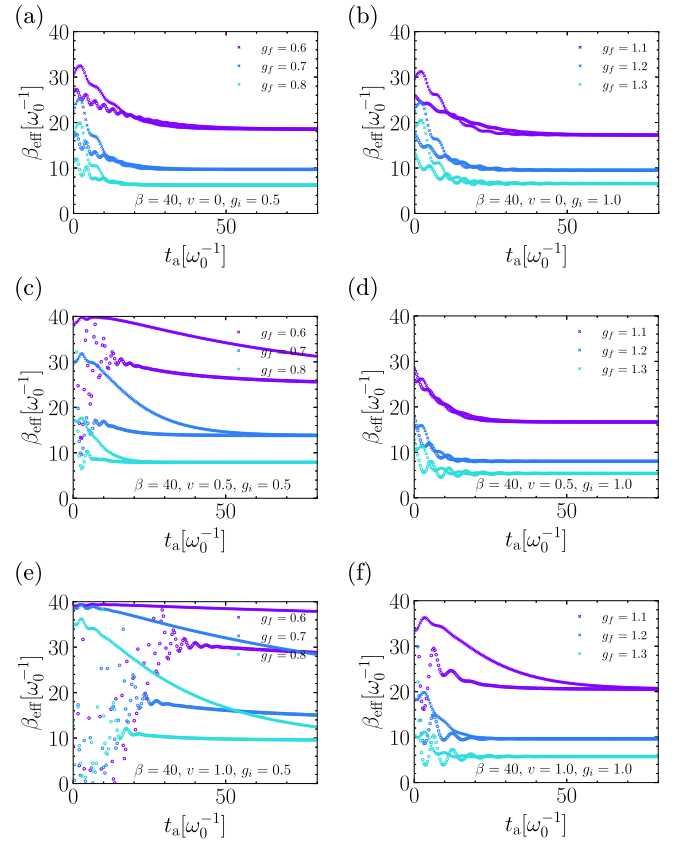


FIG. 12. Time dependence of effective inverse temperatures for $q = 2$. Here, crosses are for fermions and circles for bosons. Left column [(a), (c), and (e)] $g_i = 0.5$, $g_f = 0.6, 0.7, 0.8$ and right column [(b), (d), and (f)] $g_i = 1.0$, $g_f = 1.1, 1.2, 1.3$. The rows are $v = 0, 0.5, 1.0$ respectively. From (a), (b), (d), and (f), we can see similar temperature dynamics as the v quench. From (c) and (e), we can see that we cannot reach the equilibrium timescale because the thermalization between the boson and fermion subsystems is much slower when quenched from the Fermi liquid phase. The distribution functions cannot be described by a single parameter for the latest time we can reach. This can be regarded as prethermalization. Note that the strong oscillations for short times indicate the effective temperatures may not be well-defined.

see the difference compared with the v quenches, especially from the rates of fermions.

4. Quantum quenches for $q \geq 4$ with $M = N/q^2$

We use the scaling

$$g_{q|i,f} = g \sqrt{\frac{2^{q-1}}{q}}, \quad v_{q|i,f} = v \sqrt{\frac{2^{q-1}}{q}}, \quad \lambda_q = \lambda \frac{4}{q^2} \quad (\text{C2})$$

to get a more uniform q dependence.

In Fig. 14, we present the results for $q = 4$ where the v coupling part is an SYK₄. Otherwise, the parameters are similar to those in Fig. 12. If we look at (c) and (e), we can find that the fermions thermalize much faster in the $q = 4$ case, which is expected because the v terms are SYK₄.

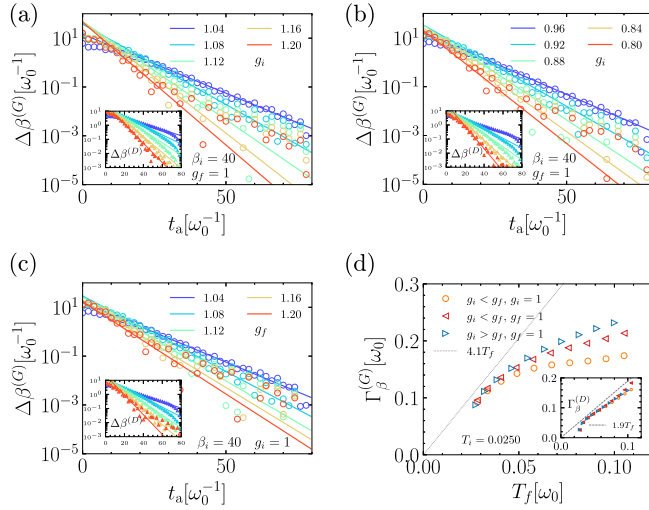


FIG. 13. Temperature relaxations. Fixed $v = 0$, $q = 2$. $\Delta\beta(t) = \beta(t) - \beta_f$. Here, markers are data, and solid lines are fitted curves or guides of the eye. Here (a) $g_i > g_f$, (b) $g_i < g_f$ and $g_f = 1$, (c) $g_i < g_f$ and $g_i = 1$. The relaxation rates for (a)–(c) are summarized in (d), and the inset of (d) shows the results for bosons. One can see that boson relaxations are linear in T_f and close to each other for the three protocols, and also close to the v quench slope $1.9T_f$. Fermions show different dynamics.

APPENDIX D: DETAILS ABOUT THE KINETIC EQUATIONS FOR SCALE SEPARATION

1. Approximate kinetic equations

Use the Keldysh component of the Dyson equation

$$[G^{-1}]^R \circ G^K \circ [G^{-1}]^A = \Sigma^K \quad (\text{D1})$$

and apply the parametrization $G^K = G^R \circ F - F \circ G^A$, we get

$$F \circ G_0^{-1} - G_0^{-1} \circ F = \Sigma^K - (\Sigma^R \circ F - F \circ \Sigma^A), \quad (\text{D2})$$

where we dropped the R, A of the inverse free Green's function since they are different only by a $\pm i0^+$ for causality, which won't affect the equations here. Use $[A \circ B] = A \circ B - B \circ A$, it is

$$-[G_0^{-1} \circ F] = \Sigma^K - (\Sigma^R \circ F - F \circ \Sigma^A). \quad (\text{D3})$$

The boson equation is the same

$$-[D_0^{-1} \circ B] = \Pi^K - (\Pi^R \circ B - B \circ \Pi^A). \quad (\text{D4})$$

Use the Wigner transform identities [44]

$$[A \circ B] \xrightarrow{\text{WT}} i(\partial_t A \partial_\omega B - \partial_\omega A \partial_t B) + \dots, \quad (\text{D5})$$

$$A \circ B \xrightarrow{\text{WT}} AB + \dots, \quad (\text{D6})$$

which gives

$$[(i\partial_t + \mu) \circ F] \xrightarrow{\text{WT}} i\partial_\omega \omega \partial_t F = i\partial_t F, \quad (\text{D7})$$

$$[(-\partial_t^2 - \omega^2) \circ B] \xrightarrow{\text{WT}} i\partial_\omega \omega^2 \partial_t B = 2i\omega \partial_t B, \quad (\text{D8})$$

and thus the equations in Wigner transformed space

$$-i\partial_t F = \Sigma^K - F(\Sigma^R - \Sigma^A), \quad (\text{D9})$$

$$-2i\omega \partial_t B = \Pi^K - B(\Pi^R - \Pi^A). \quad (\text{D10})$$

Note the functions are with variables (ω, t_a) .

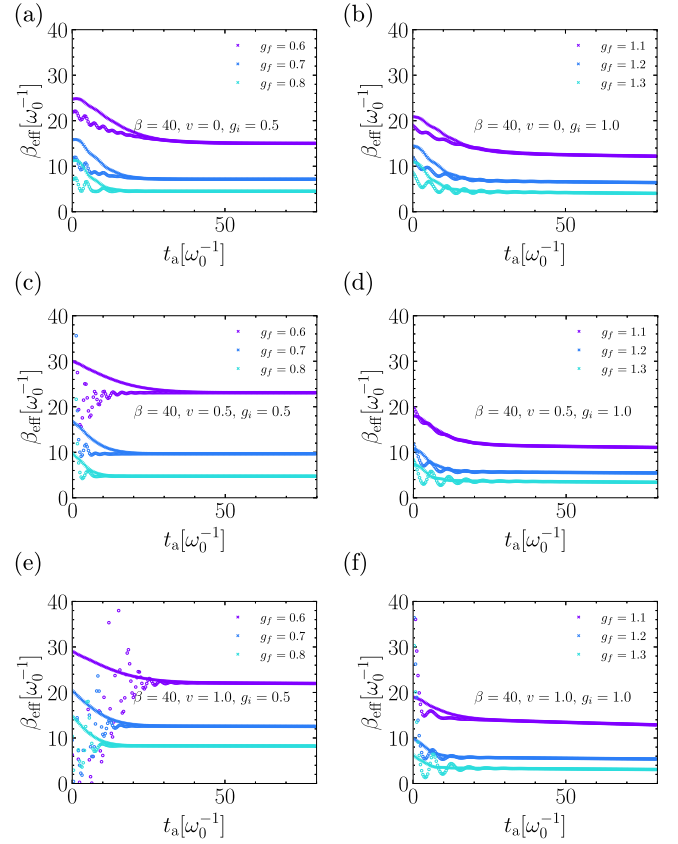


FIG. 14. Time dependence of effective inverse temperatures for $q = 4$. Here, solid lines are for fermions and circles for bosons. Left column [(a), (c), and (e)] $g_i = 0.5$, $g_f = 0.6, 0.7, 0.8$ and right column [(b), (d), and (f)] $g_i = 1.0$, $g_f = 1.1, 1.2, 1.3$. The rows are $v = 0, 0.5, 1.0$ respectively. Compare (c) and (e) with the $q = 2$ results in Figs. 12(c) and 12(e), we can see that for $q = 4$ the temperature relaxations are much faster, which is because of the $q = 4$ SYK from the v coupling term.

2. Details about collision integrals

The collision integrals can be written as

$$I_{\text{col}}^{(f)} = \Sigma^K(\omega, t_a) - F(\omega, t_a)(\Sigma^>(\omega, t_a) - \Sigma^<(\omega, t_a)), \quad (\text{D11})$$

$$I_{\text{col}}^{(b)} = \Pi^K(\omega, t_a) - B(\omega, t_a)(\Pi^>(\omega, t_a) - \Pi^<(\omega, t_a)). \quad (\text{D12})$$

We can use the exact relations $\Sigma^K(\omega, t_a) = \Sigma^>(\omega, t_a) + \Sigma^<(\omega, t_a)$ and $\Pi^K(\omega, t_a) = \Pi^>(\omega, t_a) + \Pi^<(\omega, t_a)$ then only greater and lesser functions are required. Under the scale separation assumption, we can assume Green's functions have the form

$$G^>(\omega, t_a) = -iA^{(G)} \frac{F + 1}{2} = -iA^{(G)}(1 - f), \quad (\text{D13})$$

$$G^<(\omega, t_a) = -iA^{(G)} \frac{F - 1}{2} = iA^{(G)}f, \quad (\text{D14})$$

$$D^>(\omega, t_a) = -iA^{(D)} \frac{B + 1}{2} = -iA^{(D)}(1 + b), \quad (\text{D15})$$

$$D^<(\omega, t_a) = -iA^{(D)} \frac{B - 1}{2} = -iA^{(D)}b, \quad (\text{D16})$$

where $F = 1 - 2f$ and $B = 1 + 2b$.

We then recall that the self-energies are

$$\begin{aligned}\Sigma(t_1, t_2) &= R_{q-2}(t_1, t_2)(v^2 + g^2 i D(t_1, t_2))G(t_1, t_2), \\ \Pi(t_1, t_2) &= \frac{-4ig^2}{\lambda q} R_{q-2}(t_1, t_2)G(t_1, t_2)G(t_2, t_1)\end{aligned}\quad (\text{D17})$$

with $R_{q-2}(t_1, t_2) \equiv [G(t_1, t_2)G(t_2, t_1)]^{\frac{q-2}{2}}$. Since we are considering the quenches, we suppress the time dependence of the parameters. They can be restored whenever it is necessary. We define the frequency convolution

$$[A \bullet B](\omega) = \int_{\omega} A(v - \omega)B(\omega), \quad (\text{D18})$$

where $\int_{\omega} = \int d\omega/2\pi$. For the terms inside the self-energies, we have

$$A(t_1, t_2)B(t_1, t_2) \xrightarrow{\text{WT}} A \bullet B. \quad (\text{D19})$$

One can then write down the Wigner-transformed self-energies

$$\Sigma^{>,<}(\omega, t_a) = R_{q-2}^{>,<} \bullet (v^2 + g^2 i D^{>,<}) \bullet G^{>,<}, \quad (\text{D20})$$

$$\Pi^{>,<}(\omega, t_a) = \frac{-4ig^2}{\lambda q} R_{q-2}^{>,<} \bullet G^{>,<} \bullet G_{-}^{<,>}, \quad (\text{D21})$$

$$R_{q-2}^{>,<}(\omega, t_a) = \bullet^{\frac{q-2}{2}} [G^{>,<} \bullet G_{-}^{<,>}], \quad (\text{D22})$$

where $\bullet^{\frac{q-2}{2}}[A] = A \bullet A \bullet \dots \bullet A$ with total $\frac{q-2}{2}$ terms. where G_{-} is the Wigner half-fourier transformed function $G_{-}(\omega, t_a) = [G(t_2, t_1)](\omega, t_a)$.

We rewrite the Green's functions in terms of the spectral functions and distribution functions

$$\begin{aligned}\Sigma^{>}(\omega, t_a) &= R_{q-2}^{>} \bullet (v^2 + g^2 i D^{>}) \bullet G^{>} \\ &= R_{q-2}^{>} \bullet \left(v^2 + g^2 A^{(D)} \frac{B+1}{2} \right) \bullet \left(-i A^{(G)} \frac{F+1}{2} \right),\end{aligned}\quad (\text{D23})$$

$$\begin{aligned}\Sigma^{<}(\omega, t_a) &= R_{q-2}^{<} \bullet (v^2 + g^2 i D^{<}) \bullet G^{<} \\ &= R_{q-2}^{<} \bullet \left(v^2 + g^2 A^{(D)} \frac{B-1}{2} \right) \bullet \left(-i A^{(G)} \frac{F-1}{2} \right),\end{aligned}\quad (\text{D24})$$

$$\begin{aligned}\Pi^{>}(\omega, t_a) &= \frac{-4ig^2}{\lambda q} R_{q-2}^{>} \bullet G^{>} \bullet G_{-}^{<} \\ &= \frac{-4ig^2}{\lambda q} R_{q-2}^{>} \bullet \left(A^{(G)} \frac{F+1}{2} \right) \bullet \left(A_{-}^{(G)} \frac{1-F_{-}}{2} \right),\end{aligned}\quad (\text{D25})$$

$$\begin{aligned}\Pi^{<}(\omega, t_a) &= \frac{-4ig^2}{\lambda q} R_{q-2}^{<} \bullet G^{<} \bullet G_{-}^{>} \\ &= \frac{-4ig^2}{\lambda q} R_{q-2}^{<} \bullet \left(A^{(G)} \frac{1-F}{2} \right) \bullet \left(A_{-}^{(G)} \frac{F_{-}+1}{2} \right),\end{aligned}\quad (\text{D26})$$

$$\begin{aligned}R_{q-2}^{>}(\omega, t_a) &= \bullet^{\frac{q-2}{2}} [G^{>} \bullet G_{-}^{<}] \\ &= \bullet^{\frac{q-2}{2}} \left[\left(A^{(G)} \frac{F+1}{2} \right) \bullet \left(A_{-}^{(G)} \frac{1-F_{-}}{2} \right) \right],\end{aligned}\quad (\text{D27})$$

$$\begin{aligned}R_{q-2}^{<}(\omega, t_a) &= \bullet^{\frac{q-2}{2}} [G^{<} \bullet G_{-}^{>}] \\ &= \bullet^{\frac{q-2}{2}} \left[\left(A^{(G)} \frac{1-F}{2} \right) \bullet \left(A_{-}^{(G)} \frac{F_{-}+1}{2} \right) \right].\end{aligned}\quad (\text{D28})$$

Note $A_{-} = A(-\omega, t_a)$.

3. $q = 2$

We first consider $q = 2$, which means we can set $R_{q-2} = 1$. The results are listed in (D29) and (D30)

$$\begin{aligned}I_{\text{col}}^{(f)} &= \frac{ig^2}{2} (F((A^{(D)}B) \bullet A^{(G)} + A^{(D)} \bullet (A^{(G)}F)) \\ &\quad - (A^{(D)}B) \bullet (A^{(G)}F) - A^{(D)} \bullet A^{(G)}),\end{aligned}\quad (\text{D29})$$

$$\begin{aligned}I_{\text{col}}^{(b)} &= \frac{ig^2}{\lambda} (F((A^{(G)}F) \bullet A_{-}^{(G)} - A^{(G)} \bullet (A_{-}^{(G)}F_{-})) \\ &\quad + (A^{(G)}F) \bullet (A_{-}^{(G)}F_{-}) - A^{(G)} \bullet A_{-}^{(G)}).\end{aligned}\quad (\text{D30})$$

One can see that there is no v^2 term for the $q = 2$ collision integral, similar to that in Ref. [17]. This can be explained by that the SYK₂ is a free fermion model.

4. $q > 2$ collision integrals

Unlike that of $q = 2$, for $q > 2$ the v^2 terms contribute to the collision integral. The collision integrals for $q > 2$ are lengthier while still straightforward; we list them here in the equations (D31) and (D32). The main difference is that the “rung” function R contributes to the collision integral, where the pure SYK term, the v^2 terms, become collisional. This suggests that the relaxation dynamics of $q = 2$ and $q > 2$ should be very different.

$$\begin{aligned}I_{\text{col}}^{(f)} &= \frac{1}{2} i (F - 1) R_{q-2}^{>} \bullet \left(\frac{1}{2} A^{(D)} (B + 1) g^2 + v^2 \right) \\ &\quad \bullet (A^{(G)}(F + 1)) - \frac{1}{2} i (F + 1) R_{q-2}^{<} \\ &\quad \bullet \left(\frac{1}{2} A^{(D)} (B - 1) g^2 + v^2 \right) \bullet (A^{(G)}(F - 1)),\end{aligned}\quad (\text{D31})$$

$$\begin{aligned}I_{\text{col}}^{(b)} &= \frac{ig^2}{\lambda q} (((F - 1) R_{q-2}^{>} \bullet (A^{(G)}F + A^{(G)}) \\ &\quad \bullet (A_{-}^{(G)} - A_{-}^{(G)}F_{-}) - (F + 1) R_{q-2}^{<} \\ &\quad \bullet (A^{(G)} - A^{(G)}F) \bullet (A_{-}^{(G)}F_{-} + A_{-}^{(G)})).\end{aligned}\quad (\text{D32})$$

5. Equations for spectral functions

Equations for spectral functions can be found by using left and right Dyson equations [17,46] $G_0^{-1} \circ G^{R,A} = \delta + \Sigma^{R,A} \circ G^{R,A}$ and $G^{R,A} \circ G_0^{-1} = \delta + G^{R,A} \circ \Sigma^{R,A}$ where the $\circ$ is the time-space convolution for two-time functions. The δ is the delta function for the time difference. The difference between the left and right Dyson equations gives $[G_0^{-1} \circ G^R] = [G^R \circ \Sigma^R]$ and $[G_0^{-1} \circ G^A] = [G^A \circ \Sigma^A]$, where we used a convolution commutator notation $[A \circ B] = A \circ B - B \circ A$. Apply the Wigner transform and gradient expansion to the lowest order, we get

two equations

$$-i\partial_{t_a} G^{R,A}(\omega, t_a) = 0. \quad (\text{D33})$$

Further difference R and A equations and take $\mu = 0$ gives

$$\partial_{t_a} A^{(G)}(\omega, t_a) = 0. \quad (\text{D34})$$

Apply the same procedures to the boson, we have

$$-2i\omega\partial_{t_a} D^{R,A}(\omega, t_a) = 0, \quad (\text{D35})$$

which means

$$\partial_{t_a} A^{(D)}(\omega, t_a) = 0, \quad \text{if } \omega \neq 0. \quad (\text{D36})$$

For strongly correlated systems, the above equations are not expected to capture the actual spectral function even with the local equilibrium, this is because the spectral functions are sensitive to the distributions. One may assume an instant influence from the distributions like that used in Ref. [48], the spectral functions are then determined by consistent solutions

$$A^{(G,D)}(\omega, t_a) = A^{(G,D)}(\omega, \beta_{\text{eff}}(t_a)), \quad (\text{D37})$$

where the spectral functions are solved from an equilibrium self-consistent problem with the input inverse temperatures $\beta_{\text{eff}}(t_a)$.

APPENDIX E: DETAILS ABOUT THE EQUAL-TIME EQUATIONS

Recall the Dyson equation for bosons on the contour

$$D_0^{-1} \circ_C D = \Pi \circ_C D. \quad (\text{E1})$$

Here $\circ_C$ is the contour convolution $A \circ_C B \equiv \int_{s \in C} A(t_1, s) B(s, t_2)$,

$$D_0^{-1}(t_1, t_2) = (-\partial_{t_1}^2 - \omega_0^2) \delta_C(t_1, t_2), \quad (\text{E2})$$

where the contour delta function behaves like the usual delta under the contour integral $\int_{x \in C} \delta_C(x, y) g(x) = g(y)$. The left and right Kadanoff-Baym equations (KBE) on the three-branch contour can be written down by using Langreth rules [45,52]. Here, we consider the KBE for bosons and the adjoint equation as well

$$\begin{aligned} [-\partial_{t_1}^2 - \omega_0^2] D^<(t_1, t_2) &= \int_0^{t_1} ds \Pi^R(t_1, s) D^<(s, t_2) \\ &+ \int_0^{t_2} ds \Pi^<(t_1, s) D^A(s, t_2) \\ &- i \int_0^\beta d\tau \Pi^1(t_1, \tau) D^\Gamma(\tau, t_2), \quad (\text{E3}) \\ [-\partial_{t_2}^2 - \omega_0^2] D^<(t_1, t_2) &= \int_0^{t_1} ds D^R(t_1, s) \Pi^<(s, t_2) \\ &+ \int_0^{t_2} ds D^<(t_1, s) \Pi^A(s, t_2) \\ &- i \int_0^\beta d\tau D^\Gamma(t_1, \tau) \Pi^\Gamma(\tau, t_2). \quad (\text{E4}) \end{aligned}$$

Note that the scalar boson KBE has an important difference compared with the fermion (or complex boson) KBE, where for fermions the time derivatives are $i\partial_{t_1} G(t_1, t_2)$ and $-i\partial_{t_2} G(t_1, t_2)$. The fermion KBE can be found by replacing

the two-point function with those for fermions in the above equations, as well as the free Green's functions.

The second derivative in the boson KBE here seems invalid in the usual subtraction trick [53,54], which needs the first-time derivatives with a relative minus sign as those presented in fermion equations. Here, we may use a workaround that splits the free boson propagator into first derivatives and moves one of them to the right-hand side. We rewrite the boson KBE as

$$[i\partial_{t_1}^+ + \omega_0][i\partial_{t_1}^+ - \omega_0] D^<(t_1, t_2) = K_1(t_1, t_2), \quad (\text{E5})$$

$$[-i\partial_{t_2}^- + \omega_0][i\partial_{t_2}^- - \omega_0] D^<(t_1, t_2) = K_2(t_1, t_2), \quad (\text{E6})$$

where the K_1 and K_2 are corresponding terms in (E4) and (E3). Here we also marked $\partial_t^\pm = \partial_t \pm i0^+$ to remind ourselves they are for retarded (advanced) functions. The above equations can then be rewritten formally as

$$P_+^{-1} \circ P_-^{-1} \circ D^< = K_1, \quad (\text{E7})$$

$$D^< \circ M_-^{-1} \circ M_+^{-1} = K_2, \quad (\text{E8})$$

where $P_+^{-1} \equiv (i\partial_{t_1}^+ + \omega_0)\delta(t_1, t_2)$, $P_-^{-1} \equiv (i\partial_{t_1}^+ - \omega_0)\delta(t_1, t_2)$, $M_+^{-1} \equiv (i\partial_{t_1}^- + \omega_0)\delta(t_1, t_2)$, and $M_-^{-1} \equiv (i\partial_{t_1}^- - \omega_0)\delta(t_1, t_2)$. Then we can simply rewrite them by assuming $P_\pm^{-1}$ and $M_\pm^{-1}$ are invertible in functional sense

$$(i\partial_{t_1}^+ - \omega_0) \circ D^< = P_+ \circ K_1, \quad (\text{E9})$$

$$(-i\partial_{t_2}^- - \omega_0) \circ D^< = K_2 \circ M_+. \quad (\text{E10})$$

Note, we did the integral by parts to get

$$\begin{aligned} D(t_1, t_2) \circ M_-^{-1} &= D(t_1, t_2) \circ (i\partial_{t_1}^- - \omega_0)\delta(t_1, t_2) \\ &= (-i\partial_{t_2}^- - \omega_0) D(t_1, t_2). \quad (\text{E11}) \end{aligned}$$

Now we can subtract (E9) from (E10) then let $t_1 = t_2 = t$ resulting a equation looks like master equation [53,54]. Notice that $[\omega_0^2, n^{(b)}(t)] = 0$ and use $n^{(b)}(t, t) = iD^<(t, t)$, and keep in mind the sign difference with fermions, we get

$$-\partial_t n^{(b)}(t) = C_{\text{col}}(t), \quad (\text{E12})$$

where we used $[\partial_{t_1} D(t_1, t_2) + \partial_{t_2} D(t_1, t_2)]_{t_1=t_2=t} = \partial_t D(t, t)$. The $C_{\text{col}}(t)$ is

$$C_{\text{col}}(t) = [P_+ \circ K_1](t, t) - [K_2 \circ M_+](t, t). \quad (\text{E13})$$

The P_+ is retarded function and M_+ is advanced the solutions are

$$P_\pm(t_1, t_2) = -i\theta(t_1 - t_2)e^{\pm i\omega_0(t_1 - t_2)}, \quad (\text{E14})$$

$$M_\pm(t_1, t_2) = i\theta(t_2 - t_1)e^{\pm i\omega_0(t_1 - t_2)}. \quad (\text{E15})$$

One can verify these by $P_\pm^{-1} \circ P_\pm = \delta(t_1, t_2)$ and the same for $M_\pm$.

1. Generalized Kadanoff-Baym ansatz

A way to approximate the equal time equations is to use the so-called generalized Kadanoff-Baym ansatz [53–55], which says one may approximate the lesser function as

$$G^<(t_1, t_2) = -G^R(t_1, t_2)\rho(t_2) + \rho(t_1)G^A(t_1, t_2), \quad (\text{E16})$$

$$G^>(t_1, t_2) = G^R(t_1, t_2)(1 - \rho(t_2)) - (1 - \rho(t_1))G^A(t_1, t_2), \quad (\text{E17})$$

where ρ is a single particle density matrix defined for fermions $\rho(t) \equiv -iG^<(t, t)$. Note that the generalized Kadanoff-Bay ansatz usually also approximates the retarded Green's functions to their Hartree-Fock values. However, this approximation is not expected to work well for the strongly correlated phases, both in and out of equilibrium, so we will not apply it to the YSYK here.

APPENDIX F: FINITE- q YSYK IMAGINARY TIME

In equilibrium, we can use the imaginary time formalism, which greatly simplifies many computations. We will focus on half-filling $\mu = 0$ here. The v contributions are switched off, i.e., $v = 0$.

1. Imaginary time effective action

For the initial condition, we need an imaginary time action which should be adapted to our convention [45] $G(\tau_1, \tau_2) = -\langle T_\tau c(\tau_1)c^\dagger(\tau_2) \rangle$

$$\begin{aligned} 1 &= \int d[G] \prod_{\tau_1 \tau_2} \delta \left(NG_{\sigma'\sigma}(\tau_2, \tau_1) + \sum_i c_{i\sigma'}(\tau_2)c_{i\sigma}^\dagger(\tau_1) \right) \\ &= \int d[\Sigma, G] \exp \left\{ \sum_{\sigma\sigma'} \int_{\tau_1 \tau_2} \Sigma_{\sigma\sigma'}(\tau_1, \tau_2) \right. \\ &\quad \times \left. \left(NG_{\sigma'\sigma}(\tau_2, \tau_1) + \sum_i c_{i\sigma'}(\tau_2)c_{i\sigma}^\dagger(\tau_1) \right) \right\} \quad (\text{F1}) \end{aligned}$$

and the same for D and Π but without the spins. Wick rotation $\tau = it$. The imaginary time-effective action can be written as [2]

$$\begin{aligned} -\frac{1}{N} S_{\text{eff,im}} &= \text{tr} \ln (G_0^{-1} - \Sigma) - \frac{\lambda}{2} \text{tr} \ln (D_0^{-1} - \Pi) \\ &\quad + \int_{\tau_1 \tau_2} G(\tau_2, \tau_1) \Sigma(\tau_1, \tau_2) \\ &\quad - \frac{\lambda}{2} \int_{\tau_1 \tau_2} D(\tau_2, \tau_1) \Pi(\tau_1, \tau_2) \\ &\quad + g^2 \frac{2}{q} \int_{\tau_1 \tau_2} [G(\tau_1, \tau_2)]^{\frac{q}{2}} [-G(\tau_2, \tau_1)]^{\frac{q}{2}} D(\tau_1, \tau_2). \quad (\text{F2}) \end{aligned}$$

Used q is an even number, and the identities $G_0^{-1} = (i\partial_{-\tau} + \mu) = -\partial_\tau + \mu$ and $D_0^{-1} = -\partial_t^2 - \omega_0^2 = -\partial_{-\tau}^2 - \omega_0^2 = \partial_\tau^2 - \omega_0^2$. The saddle point equations become

$$\Sigma = -g^2 R_{q-2}(\tau_1, \tau_2) G(\tau_1, \tau_2) D(\tau_1, \tau_2), \quad (\text{F3})$$

$$\Pi = 2\lambda^{-1} \frac{2}{q} g^2 R_{q-2}(\tau_1, \tau_2) G(\tau_1, \tau_2) G(\tau_2, \tau_1), \quad (\text{F4})$$

where we defined for q an even integer

$$R_{q-2}(\tau_1, \tau_2) = (-1)^{q-2} (-G(\tau_1, \tau_2) G(\tau_2, \tau_1))^{\frac{q}{2}-1}. \quad (\text{F5})$$

Together with the Dyson equations $G^{-1} = G_0^{-1} - \Sigma$ and $D^{-1} = D_0^{-1} - \Pi$ we can solve saddle point equations. This is most convenient by using the imaginary time translational invariance $G(\tau_1, \tau_2) = G(\tau_1 - \tau_2)$ and $D(\tau_1, \tau_2) = D(\tau_1 - \tau_2)$ with imaginary time Fourier transforms $G(\tau) = T \sum_n e^{-i\omega_n \tau} G(i\omega_n)$ and $G(i\omega_n) = \int_0^\beta d\tau e^{i\omega_n \tau} G(\tau)$, which gives $G_0^{-1}(i\omega_n) = i\omega_n + \mu$, $D_0^{-1}(i\nu_n) = (i\nu_n)^2 - \omega_0^2$. In numerics, we discretize the imaginary time in $\tau \in [0, \beta]$, and fast Fourier transform the imaginary time to Matsubara frequencies. For the $G(-\tau)$, we use the antiperiodicity of the fermion Green's function $G(-\tau) = -G(\beta - \tau)$. We can verify that $R_{q-2} \geq 0$ by $R_{q-2}(\tau) = [G(\tau)]^{\frac{q}{2}-1} [-G(-\tau)]^{\frac{q}{2}-1} = [G(\tau)G(\beta - \tau)]^{\frac{q}{2}-1} \geq 0$, where $G(\tau) \leq 0$ if $\tau \in [0, \beta]$.

2. Compute q -dependent exponents for YSYK $_q$

Here we follow [2] to compute low-frequency exponents of propagators from the self-consistent saddle point equations and introduce the additional parameter q as a variable. Note, we will denote zero temperature Matsubara frequencies as ω since they are continuous.

a Counting the exponent

We first omit the prefactor and focus only on exponents for a schematic analysis. We assume a q dependent ansatz for the imaginary time fermion propagator $G(\tau) \sim a|\tau|^{-\frac{4\Delta}{q}}$ which is similar to the usual SYK [15]. The boson self-energy is then $\Pi(\tau) \sim \frac{g^2}{\lambda} G(\tau)^{\frac{q}{2}} G(-\tau)^{\frac{q}{2}} \simeq \frac{a^q g^2}{\lambda} |\tau|^{-4\Delta}$, we keep the prefactor for couplings g and λ and omit others. Fourier transform it to frequency space and keep only the finite contribution $\Pi(\omega) \sim \frac{a^q g^2}{\lambda} |\omega|^{4\Delta-1}$. For low frequencies $D(\omega) \sim \frac{1}{\Pi(\omega)}$, thus $D(\tau) \sim \frac{\lambda}{a^q g^2} |\tau|^{4\Delta-2}$. The fermion self-energy then follows $\Sigma(\tau) \sim g^2 G(\tau)^{\frac{q}{2}} G(-\tau)^{\frac{q}{2}-1} D(\tau) \sim \frac{\lambda}{a^q} |\tau|^{\frac{4\Delta}{q}-2}$. Fourier again $\Sigma(\omega) \sim \frac{\lambda}{a} |\omega|^{1-\frac{4\Delta}{q}}$. $G(\omega) \sim \frac{1}{\Sigma(\omega)} \sim \frac{a}{\lambda} |\omega|^{\frac{4\Delta}{q}-1}$, which means $G(\tau) \sim \frac{a}{\lambda} |\tau|^{-\frac{4\Delta}{q}}$ and we complete one consistent loop.

b Compute the exponent

The numerical value of the exponent Δ can be fixed by taking into account the exact prefactor [1,2,15]. Note we use Fourier convention $1 \xrightarrow{\text{F.T.}} 2\pi\delta(\omega)$. The useful Fourier transform identities are

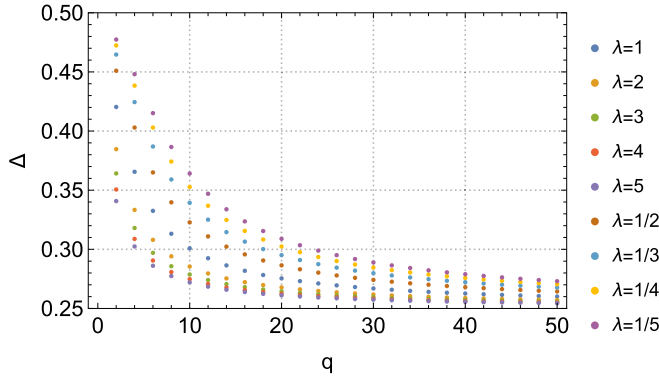
$$\int dt e^{-i\omega t} \text{sgn}(t) |t|^{x-1} = -2i \sin\left(\frac{\pi x}{2}\right) \Gamma(x) \text{sgn}(\omega) |\omega|^{-x}, \quad (\text{F6})$$

$$\int \frac{d\omega}{2\pi} e^{i\omega t} \text{sgn}(\omega) |\omega|^{x-1} = \frac{i}{\pi} \sin\left(\frac{\pi x}{2}\right) \Gamma(x) \text{sgn}(t) |t|^{-x}. \quad (\text{F7})$$

Here x is a real number, and the convergence of Fourier transforms requires $0 < x < 2$.

With the help of the above identities, we can compute the Δ by going through one self-consistent loop. Starting from the ansatz

$$G(\tau) = a \text{sgn}(\tau) |\tau|^{-\frac{4\Delta}{q}}, \quad (\text{F8})$$

FIG. 15. Exponent Δ by numerically solve the Eq. (F15).

where q is an even integer and Δ is a real number. Note the prefactor a is a nonuniversal factor and its exact value depends on UV details [1]. Approximations can be made for the a from scaling solutions, see [2,4]. The finite part of the boson self-energy can be written down by plugging in (F8)

$$\begin{aligned}\Pi(\tau) &= \frac{2g^2}{\lambda} \frac{2}{q} (-1)^{q-2} G(\tau)^{\frac{q}{2}} G(-\tau)^{\frac{q}{2}} \\ &= \frac{2g^2}{\lambda} (a \operatorname{sgn}(\tau) |\tau|^{-\frac{4\Delta}{q}})^q.\end{aligned}\quad (\text{F9})$$

Use q an even number, we have $(-1)^{q-2} = 1$ and $\operatorname{sgn}(\tau)^q = 1$. Then the Fourier of $\Pi(\tau)$ is

$$\Pi(i\omega) = \frac{4g^2}{\lambda} \frac{2}{q} a^q \sin(2\pi\Delta) \Gamma(1-4\Delta) |\omega|^{4\Delta-1} \quad (\text{F10})$$

with $4\Delta - 1 > 0$ for a well-defined Fourier transform. Use $D(i\omega) = \frac{1}{-\Pi(i\omega)}$ and Fourier, the time-space boson propagator reads

$$D(\tau) = \frac{\lambda}{4\pi g^2} \frac{q}{2} (1-4\Delta) a^{-q} \cot(2\pi\Delta) |\tau|^{-(2-4\Delta)} \quad (\text{F11})$$

Here we require $2-4\Delta > 0$ for a convergent Fourier transform. The fermionic self-energy is then

$$\begin{aligned}\Sigma(\tau) &= -g^2 (-1)^{q-2} G(\tau)^{\frac{q}{2}} G(-\tau)^{\frac{q}{2}-1} D(\tau) \\ &= \frac{\lambda}{4\pi} \frac{q}{2} (1-4\Delta) a^{-1} \cot(2\pi\Delta) \operatorname{sgn}(\tau) |\tau|^{\frac{4\Delta}{q}-2}.\end{aligned}\quad (\text{F12})$$

Fourier transform $\Sigma(\tau)$ back to frequency space

$$\begin{aligned}\Sigma(i\omega) &= \frac{i\lambda}{2\pi a} \frac{q}{2} (1-4\Delta) \cot(2\pi\Delta) \cos\left(\frac{2\pi\Delta}{q}\right) \\ &\quad \times \Gamma\left(\frac{4\Delta}{q} - 1\right) \operatorname{sgn}(\omega) |\omega|^{1-\frac{4\Delta}{q}}.\end{aligned}\quad (\text{F13})$$

Use $G_1(i\omega) = \frac{1}{-\Sigma(i\omega)}$, we get

$$\begin{aligned}G_1(\tau) &= \frac{-4a \tan(2\pi\Delta)(q-4\Delta) \tan\left(\frac{2\pi\Delta}{q}\right)}{(4\Delta-1)\lambda q^2} \\ &\quad \times \operatorname{sgn}(\tau) |\tau|^{-\frac{4\Delta}{q}}.\end{aligned}\quad (\text{F14})$$

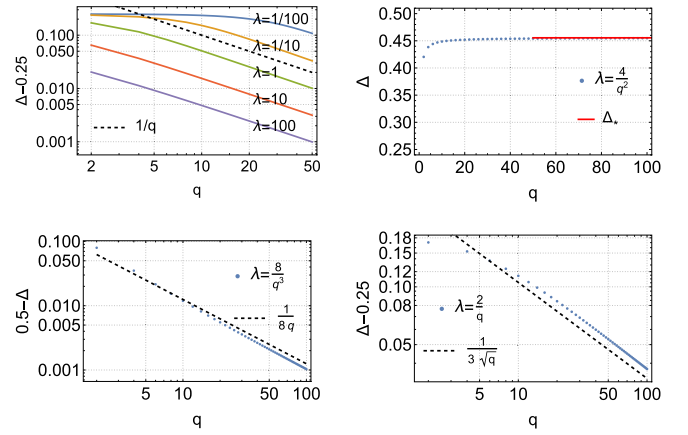


FIG. 16. Numerical solutions of the exponent Δ . (Top) The log-log plot $(\Delta - \frac{1}{4})$ vs q with different λ . The points are joined for better visibility. (Bottom) Choosing a q dependent $\lambda = \frac{4}{q^2}$. This gives an exponent other than $\frac{1}{2}$ or $\frac{1}{4}$ for large q . The red line marks the λ - q double scaling limit $\frac{1}{q} \rightarrow 0$ with $\lambda = \frac{4}{q^2}$, the value is $\Delta_* \simeq 0.45548817958394894$. (Bottom left) The log-log plot $(\frac{1}{2} - \Delta)$ vs q with $\lambda = \frac{8}{q^3}$. One can see that Δ approaching $1/2$ as a linear function of $\frac{1}{q}$. (Bottom right) The log-log plot $(\Delta - \frac{1}{4})$ vs q with $\lambda = \frac{2}{q}$, which can cancel the explicit prefactor in front of the boson self-energy. In this case, Δ is approaching $\frac{1}{4}$ algebraically as a linear function of $\frac{1}{\sqrt{q}}$.

Apply the consistency condition $G(\tau) \stackrel{!}{=} G_1(\tau)$, which implies

$$1 \stackrel{!}{=} -\frac{4 \tan(2\pi\Delta)(q-4\Delta) \tan\left(\frac{2\pi\Delta}{q}\right)}{(4\Delta-1)\lambda q^2}. \quad (\text{F15})$$

Here the Δ should satisfy $\frac{1}{4} < \Delta < \frac{1}{2}$ for a well-defined Fourier transform. We can compare this result with the known exponent [2] for $q=2$ and $\lambda=1$, and they agree.

Several limits can be taken to simplify Eq. (F15). If we assume Δ is not q dependent, then for $q \gg 1$, we can expand around $\frac{1}{q} \rightarrow 0$ where we have up to first order

$$1 \stackrel{!}{=} -\frac{2(\pi\Delta \tan(2\pi\Delta))}{(4\Delta-1)} \frac{1}{\lambda_1} + O\left(\frac{1}{q^3}\right), \quad (\text{F16})$$

where we defined

$$\lambda = \frac{4\lambda_1}{q^2}. \quad (\text{F17})$$

The factor 4 is to cancel q^2 when $q=2$. We can solve Δ for $q=\infty$ and $\lambda_1=1$, the solution is $\Delta \simeq 0.455488$, which is not saturating to $\frac{1}{2}$ or $\frac{1}{4}$. For large (small) λ_1 , the Δ goes to its lower (upper) bounds $\lambda_1 \rightarrow 0$, $\Delta = \frac{1}{2}$ and $\lambda_1 \rightarrow \infty$, $\Delta = \frac{1}{4}$, which we also checked numerically.

We show numerical solutions of Eq. (F15) in Figs. 15 and 16. Several values of Δ are summarized in Table I for fixed λ and can be compared to numerical solutions of the saddle point equations.

3. The q dependent propagators at finite temperature

The finite temperature fermion and boson propagators in the scaling limit can be written down now from previous

computations. The time reparameterization symmetry allows us to map a zero temperature solution to an imaginary time circle [14,15,23]. Here we extend those results for general q but restrict ourselves to spectral symmetry, i.e., half-filling. The results can be obtained straightforwardly by noticing that $G(\tau) \sim |\tau|^{-\frac{4\Delta}{q}}$, and the formal boson scaling dimension is not changed $D(\tau) \sim |\tau|^{2-4\Delta}$. The two-point function has the symmetry

$$\begin{aligned} G(\tau, \tau') &\rightarrow [f'(\tau)f'(\tau')]^{\frac{4\Delta}{q}} \frac{l(\tau)}{l(\tau')} G(f(\tau), f(\tau')), \\ \Sigma(\tau, \tau') &\rightarrow [f'(\tau)f'(\tau')]^{1-\frac{4\Delta}{q}} \frac{l(\tau)}{l(\tau')} \Sigma(f(\tau), f(\tau')), \\ D(\tau, \tau') &\rightarrow [f'(\tau)f'(\tau')]^{1-2\Delta} D(f(\tau), f(\tau')), \\ \Pi(\tau, \tau') &\rightarrow [f'(\tau)f'(\tau')]^{2\Delta} \Pi(f(\tau), f(\tau')), \end{aligned} \quad (\text{F18})$$

where $l(\tau)$ is a arbitrary function from the emergent $U(1)$ symmetry of complex fermions [14] and at half-filling it can be chosen trivially $l(\tau) = 1$. Likewise f is arbitrary and $f'(x) \equiv df(x)/dx$. To map the time to the finite temperature circle, we can choose $f(\tau) = \tan(\frac{\pi\tau}{\beta})$ which maps $\frac{1}{|\tau|} \rightarrow \frac{\pi}{\beta|\sin \frac{\pi\tau}{\beta}|}$. Use $D(\tau_1, \tau_2) = D(\tau_1 - \tau_2)$ and same for G , we get

$$G(\tau) = A_G \text{sgn}(\tau) \left(\frac{\pi}{\beta|\sin \frac{\pi\tau}{\beta}|} \right)^{\frac{4\Delta}{q}} \quad (\text{F19})$$

and

$$D(\tau) = A_D \left(\frac{\pi}{\beta|\sin \frac{\pi\tau}{\beta}|} \right)^{2-4\Delta}. \quad (\text{F20})$$

A_G and A_D are overall constants.

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