

Resistivity anomalies arising from spin correlations in magnetic metals

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Some magnetic materials exhibit a pronounced peak or upturn in resistivity upon cooling through the ordering transition, while others do not. To explain and quantify this phenomenon, we develop a simple model in which the carrier lifetime is calculated from the scattering of conduction electrons by local magnetic moments. Within this framework, we derive a general formula for the electron scattering rate, applicable to arbitrary energy dispersions and magnetic structure factors. Using ferromagnets as an example, we show that a resistivity anomaly occurs at T_c , arising from the rapid increase in the magnetic correlation length ξ . We find that this behavior is robust across dimensionality d and persists in anisotropic systems. The maximum resistivity is reached when $1/\xi$ is comparable to the Fermi momentum k_F , and the peak height scales inversely with k_F^{d+2} . As a result, the effect is most pronounced for smaller Fermi surfaces, where the anomaly can dominate over other resistivity contributions such as electron-phonon or electron-impurity scattering.

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I. INTRODUCTION

Electrical resistivity is a readily accessible observable and is often among the first measurements used to probe a material's electronic structure. Interpreting its temperature and magnetic field dependence remains challenging because multiple scattering mechanisms contribute. While the ubiquitous electron-phonon scattering is well understood, the presence of magnetic moments introduces additional complexity. Many magnetic materials exhibit resistivity anomalies near their magnetic transition temperatures, including rare-earth metals (Gd, Ho, Er, and Tm), intermetallic compounds (DyNi₅, TbNi₅), doped manganites, and even the parent compounds of cuprate superconductors [1–4]. In some cases, these anomalies appear as pronounced peaks, such as those observed in Gd₂PdSi₃, RCuAs₂, and RInCu₄ ($R = \text{Gd and Dy}$) [5–7]. These peaks may arise from either the Kondo effect [8–11] or metal-insulator transitions in which an energy gap opens under certain conditions [12–14]. There are also materials with d - or f -orbital character that show peaks near the magnetic phase transition in the absence of an energy gap [15–18]. Many of these materials show suppression of the resistivity peak in an external magnetic field, which indicates a magnetic scattering origin and leads to strong magnetoresistance [19–21]. Despite these diverse and complex phenomena, there exist relatively few theoretical models capable of quantifying spin-driven scattering effects [8,22].

We present a model in which conduction electrons scatter from the potential created by spatially correlated local spins. The necessary inputs to this formalism are the electronic band structure near the Fermi level and the temperature-dependent spin correlations in the form of the magnetic structure factor $S(\mathbf{q}, T)$, which governs the momentum transfer and thereby the transport lifetime. We demonstrate the application of the formalism by focusing on ferromagnetic correlations to explain resistivity anomalies in metals, primarily in the regime above the magnetic phase transition temperature, where the electronic bands are not spin-split. We parametrize $S(\mathbf{q}, T)$ with a temperature-dependent correlation length $\xi(T)$, which diverges as the system approaches the magnetic phase transition temperature T_c from above. In the present work, this parametrization is used as a convenient way to generate model structure factors $S(\mathbf{q}, T)$, but the formalism itself does not rely on a specific $\xi(T)$ and can equally use $S(\mathbf{q}, T)$ obtained directly from experiment or from independent microscopic calculations. To illustrate the roles of Fermi surface shape and magnetic correlation length, we compute the electrical resistivity upon approaching ferromagnetic ordering in both isotropic and anisotropic 1D, 2D, and 3D systems.

Within this framework, we derive a general transport lifetime that can be used for arbitrary electronic dispersion and magnetic structure factor $S(\mathbf{q}, T)$. The resulting formula provides a practical bridge between band structure and spin correlations on the one hand and the magnetic contribution to the resistivity on the other, and can be used as a general tool once $\varepsilon(\mathbf{k})$ and $S(\mathbf{q})$ are specified. In contrast to previous approaches that tie the anomaly directly to the critical divergence of spin fluctuations and therefore predict a maximum at T_c [10,22], we show that the peak in the magnetic contribution to the resistivity is controlled by a simple geometric matching condition between the Fermi surface and the main weight of $S(\mathbf{q})$. This condition leads to a maximum slightly above T_c when combined with a standard $\xi(T)$ that diverges at the

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transition. It therefore provides a practical route to understand magnetic metals where pronounced resistivity peaks appear above the ordering temperature without invoking a true gap or carrier localization, and it explains why such peaks are especially strong in small Fermi surface ferromagnets.

II. RESISTIVITY FROM ELECTRON SCATTERING ON THE LOCAL SPINS

Using the relaxation time approximation, we derive an analytical expression for the scattering time:

$$\frac{1}{\tau(\mathbf{k})} = \int \frac{d^3\mathbf{k}'}{(2\pi)^3} W(\mathbf{k}, \mathbf{k}') \left[1 - \frac{\mathbf{E} \cdot \mathbf{v}(\mathbf{k}')}{\mathbf{E} \cdot \mathbf{v}(\mathbf{k})} \right], \quad (1)$$

where $W(\mathbf{k}, \mathbf{k}')$ is the transition rate from the initial state $\mathbf{k}$ to the final state $\mathbf{k}'$. Here $\mathbf{E}$ is the electric field, and $\mathbf{v}(\mathbf{k}) = \partial \varepsilon(\mathbf{k}) / (\hbar \partial \mathbf{k})$ is the electron band velocity corresponding to the dispersion $\varepsilon(\mathbf{k})$. The general transport scattering time formula provided in Eq. (1) is applicable for arbitrary electron energy dispersion and scattering source. The derivation is provided in Appendix A. For a given scattering mechanism i (localized spins, phonons, impurities, electrons), the corresponding contribution to the scattering time is obtained by inserting the transition rate $W_i(\mathbf{k}, \mathbf{k}')$ into Eq. (1), which we denote by $1/\tau_i(\mathbf{k}, T)$. When several mechanisms are present, we adopt Matthiessen's rule so that $1/\tau_{\text{tot}}(\mathbf{k}, T) = \sum_i 1/\tau_i(\mathbf{k}, T)$.

Further, we consider the case in which $W(\mathbf{k}, \mathbf{k}')$ arises from scattering of conduction electrons by localized spins. A detailed derivation of the matrix element from the real-space spin-dependent potential and its connection to the scattering probability is presented in Appendixes B and C. Following the formalism developed by Kondo, Kasuya, and Kataoka [8,22,23], the scattering probability is expressed in terms of spin-spin correlations. In general, scattering from spin fluctuations includes both elastic and inelastic processes, and is described by the dynamical structure factor $S(\mathbf{q}, \omega)$, where $\mathbf{q} = \mathbf{k} - \mathbf{k}'$ is the momentum transfer, and $\omega = (E_{\mathbf{k}} - E_{\mathbf{k}'})/\hbar$ is the corresponding energy transfer. In this work, we approximate the full dynamical structure factor $S(\mathbf{q}, \omega)$ by its energy integral $S(\mathbf{q})$, because near the magnetic transition temperature, most of the spectral weight lies at lower energies (see Appendix C for details). $S(\mathbf{q})$ is the magnetic structure factor, which is defined as the sum of the diagonal components of the spin-spin correlation function in $\mathbf{q}$ space. The diagonal component of the structure factor is the Fourier transform of the real space spin-spin correlation function:

$$S_{\alpha\alpha}(\mathbf{q}) = \frac{1}{N^2} \sum_{\mathbf{r}_m, \mathbf{r}_n} \langle S_{\alpha}(\mathbf{r}_m) S_{\alpha}(\mathbf{r}_n) \rangle e^{-i\mathbf{q} \cdot (\mathbf{r}_n - \mathbf{r}_m)}. \quad (2)$$

Here, N is the number of spins in the crystal, and $S_{\alpha}(\mathbf{r}_m)$ denotes the α -direction component of the spin at the position $\mathbf{r}_m$. In this study, we focus on temperatures near and above the ferromagnetic transition, where the diagonal components are identical ($S_{xx} = S_{yy} = S_{zz}$). Accordingly, the total structure factor used in the scattering calculations is defined as $S(\mathbf{q}) = S_{xx} + S_{yy} + S_{zz}$. In the quasielastic limit with spin isotropy, the

scattering probability becomes

$$W(\mathbf{k}, \mathbf{k}') = \frac{2\pi}{\hbar} J^2 S(\mathbf{q}) \delta[\varepsilon(\mathbf{k}) - \varepsilon(\mathbf{k}')], \quad (3)$$

where J is the exchange interaction between a conduction electron and a local spin. Equation (3) is general for arbitrary magnetic structures.

The diagonal components of the conductivity tensor are expressed as integrals over all momentum space:

$$\sigma_{\alpha} = 2e^2 \int \frac{d^3\mathbf{k}}{(2\pi)^3} v_{\alpha}^2(\mathbf{k}) \tau(\mathbf{k}) \left(-\frac{\partial f_0(\mathbf{k})}{\partial \varepsilon(\mathbf{k})} \right), \quad (4)$$

where $v_{\alpha}(\mathbf{k})$ denotes the α component of the band velocity vector, and $f_0(\mathbf{k})$ is the Fermi-Dirac distribution.

The temperature-dependent conductivity can be obtained by substituting Eqs. (3) and (1) into (4). These equations are general and can be directly applied when the electronic dispersion and temperature dependence of the magnetic structure factor $S(\mathbf{q}, T)$ are known, either from experiments, from simulations or from analytical solutions. In practice, $\varepsilon(\mathbf{k})$ can be taken as the renormalized quasiparticle dispersion determined from experiment or many-body calculations, so electron-electron interactions that preserve a Fermi-liquid description are already included in the band structure input.

In the present work, we restrict ourselves to a single spin-degenerate conduction band. This is an appropriate description for materials and temperature ranges where one band dominates the dc conductivity and the exchange splitting of the bands is small compared with the overall electronic bandwidth. The formal expressions for the scattering rate and the conductivity are not limited to this case in principle. They can be generalized to multiband and spin-split systems by introducing band indices in the dispersion and in the scattering matrix elements and by summing the conductivity over the different Fermi surface sheets. Here we focus on the single-band limit in order to isolate the mechanism that links the Fermi surface and the magnetic structure factor $S(\mathbf{q})$ and to obtain transport scaling relations for the resistivity peak in 1D, 2D, and 3D.

III. COMPUTATIONAL METHOD

For the numerical calculations, we apply a zone-folding scheme in which the initial wave vector $\mathbf{k}$ is restricted to the first Brillouin zone. The scattering is evaluated based on the momentum transfer $\mathbf{q} = \mathbf{k} - \mathbf{k}'$, which is folded back into the first Brillouin zone. This approach captures a subset of Umklapp processes, as final state $\mathbf{k}'$ may lie outside the first Brillouin zone, but the resulting $\mathbf{q}$ falls within the folded zone. While such treatment includes the dominant contributions near small Fermi surfaces, it may underestimate the full Umklapp contribution for larger Fermi surfaces or at higher temperatures, where multiple-zone Umklapp processes contribute appreciably.

To maintain consistency with the limited treatment of Umklapp processes described above, we normalize the structure factor $S(\mathbf{k} - \mathbf{k}')$ within the first Brillouin zone, as detailed in Appendix C. To facilitate numerical integration, we approximate the delta function in Eq. (3) by a Gaussian with a finite width. The standard deviation of the Gaussian is

determined by the effective energy resolution imposed by the finite $\mathbf{k}$ -space grid, estimated as $\Delta\varepsilon \sim \frac{d\varepsilon}{dk}|_{\mathbf{k}=\mathbf{k}_F} \Delta k_F$. We include only those $\mathbf{k}'$ states satisfying $|\varepsilon(\mathbf{k}')| < 5\Delta\varepsilon$, as contributions beyond this range are strongly suppressed by the Gaussian and have a negligible effect. The scattering time is therefore computed only for $\mathbf{k}$ points within the same energy window. The derivative of the Fermi distribution in Eq. (4) is evaluated analytically as a function of energy, with the effective temperature chosen as $k_B T = \Delta\varepsilon$ to match the energy resolution used in the scattering calculation. We also tested broader energy width ($k_B T = 3\Delta\varepsilon$ and $5\Delta\varepsilon$) and found that the resulting resistivity curves differ only in overall amplitude, with their peak positions and ξ dependence unchanged. The electrical conductivity along the principal axes was then obtained by summing the integrand over the same set of $\mathbf{k}$ points, ensuring that all thermally active states near the Fermi level are captured.

To compute the structure factor $S(\mathbf{q})$, we model the real-space correlation function as exponentially decaying [10,24]:

$$\langle S_\sigma(\mathbf{r}_0) S_\sigma(\mathbf{r}_0 + \mathbf{r}) \rangle = \langle S_\sigma^2 \rangle \exp(-|\mathbf{r}|/\xi(T)). \quad (5)$$

Equation (5) is a standard Ornstein-Zernike type approximation for systems with a finite correlation length, and it primarily sets the width of the peak in $S(\mathbf{q})$ through the parameter $\xi(T)$. The central input to our transport calculation is the full magnetic structure factor $S(\mathbf{q}, \xi(T))$, and the key control parameter for the resistivity peak is the combination $k_F \xi$ rather than the detailed shape of the real-space correlator. Other short-ranged correlation functions that produce a peak in $S(\mathbf{q})$ at $\mathbf{q} = 0$ with a width of order $1/\xi$ are therefore expected to lead to the same qualitative behavior. This form is inserted into Eq. (2) and the Fourier transform is performed analytically to obtain a closed-form expression for $S(\mathbf{q})$. Throughout this work, Fourier transforms are carried out analytically whenever possible; otherwise, numerical calculation is used. The resulting factor is peaked at $\mathbf{q} = 0$, with a width in the order of $1/\xi$, and governs the momentum dependence of magnetic scattering in our model.

To introduce anisotropy in the simulations, we assume an anisotropic effective mass (different k_{F_x} and k_{F_y}) and likewise use different correlation lengths ξ_x and ξ_y in the structure factor $S(\mathbf{q})$. The explicit form of $S(\mathbf{q})$ is provided in Appendix E. Finally, we verify that varying technical parameters, including the $\mathbf{k}$ -point mesh density, does not affect the peak behavior or our conclusions.

IV. RESULTS AND DISCUSSION

A. Isotropic case

In the isotropic case, where both initial $\mathbf{k}$ and final $\mathbf{k}'$ lie on a spherical Fermi surface, the ratio $\mathbf{E} \cdot \mathbf{v}(\mathbf{k}')/\mathbf{E} \cdot \mathbf{v}(\mathbf{k})$ reduces to $\cos\theta_{\mathbf{k},\mathbf{k}'}$, where $\theta_{\mathbf{k},\mathbf{k}'}$ is the angle between the two momenta. Fig. 1(a) shows the magnetic structure factor $S(\mathbf{q})$ plotted along q_x with $q_y = 0$, for three representative correlation lengths. The value at $\mathbf{q} = 2\mathbf{k}_F$, corresponding to backscattering, increases and then decreases as ξ increases. Although $\mathbf{q} = 2\mathbf{k}_F$ captures one relevant momentum transfer, resistivity involves contributions from a range of momentum

transfers. The behavior at this specific q value may not directly explain the resistivity peak, but it illustrates how parts of the structure factor evolve with correlation length.

To understand the angular contributions to resistivity and the origin of the peak, we examine the components of the scattering rate integration and their impact in 2D as shown in Fig. 1 panels (b)–(d). The left panels show structure factor $S(\mathbf{k} - \mathbf{k}', \xi)$ (red) and the geometric factor $1 - \cos\theta_{\mathbf{k},\mathbf{k}'}$ (green) for $\xi/a = 0.2$, $\xi/a = 1.7$, and $\xi/a = 20$ with the same vertical scale. The right panels in (b), (c), and (d) show the product $S(\mathbf{q}, \xi)(1 - \cos\theta_{\mathbf{k},\mathbf{k}'})$ in gray, also on a common scale. We plot these quantities specifically for $\mathbf{k}'$ points on the Fermi surface with the fixed $\mathbf{k}$ point.

The origin of the peak can be understood by analyzing the integrand of the scattering time [see Eq. (1) and Appendix A], which consists of the magnetic structure factor $S(\mathbf{q}, \xi)$ multiplied by the geometric factor $1 - \cos\theta_{\mathbf{k},\mathbf{k}'}$. For a small ξ corresponding to high temperatures illustrated by the $\xi/a = 0.2$ case, the structure factor is broadly distributed in momentum space, allowing scattering over a wide range of angles. However, the value of the structure factor is low for any individual momentum transfer, resulting in a low scattering probability at each $\mathbf{k}'$ point. Consequently, the contribution of spin fluctuations to resistivity is limited as shown in the right panel (b) for the $\xi/a = 0.2$ case. For large ξ corresponding to low temperatures, $S(\mathbf{q}, \xi)$ is sharply peaked as shown in the left panel (d). This indicates that scattering predominantly occurs at small angles (forward scattering). While the total scattering probability is high in this regime, such forward scattering events do not significantly degrade current flow, because the geometric factor $1 - \cos\theta_{\mathbf{k},\mathbf{k}'}$ suppresses their contributions to resistivity. As a result, the effective scattering rate that enters the transport lifetime remains low, as shown in the right panel (d) for the $\xi/a = 20$ case. The maximum resistivity occurs at intermediate ξ , as shown in panel (c), where $S(\mathbf{q}, \xi)$ is broad enough to enable large-angle scattering, while the amplitude of the structure factor remains sufficient to produce many scattering events.

Figure 1(e) shows the computed resistivity in 2D for a Fermi wave vector $k_F a = 0.5$, chosen to enhance the visibility of the resistivity peak by using a relatively small Fermi surface. The resistivity exhibits a pronounced peak at an intermediate correlation length, while both shorter and longer ξ lead to lower values. This nonmonotonic behavior reflects the enhanced momentum-selective scattering that occurs when the width of the structure factor $S(\mathbf{q})$ becomes comparable to the Fermi surface diameter. The same mechanism responsible for the peak near $k_F \xi \sim 1$ also operates in other dimensions. Figure 2 examines the effect of Fermi momentum k_F on resistivity $\rho(\xi)$ in different dimensions. Panels (a.1), (a.2), and (a.3) show the computed resistivity as a function of the correlation length ξ for several Fermi momenta in 3D, 2D, and 1D isotropic systems, respectively. In all three cases, the peak becomes more pronounced for smaller Fermi surfaces.

For the isotropic 1D and 3D cases, we could derive analytical expressions for the resistivity, providing a direct description of how the resistivity depends on ξ and k_F (see Appendix D for details). For the isotropic 2D case, the momentum integral cannot be reduced to a simple closed form.

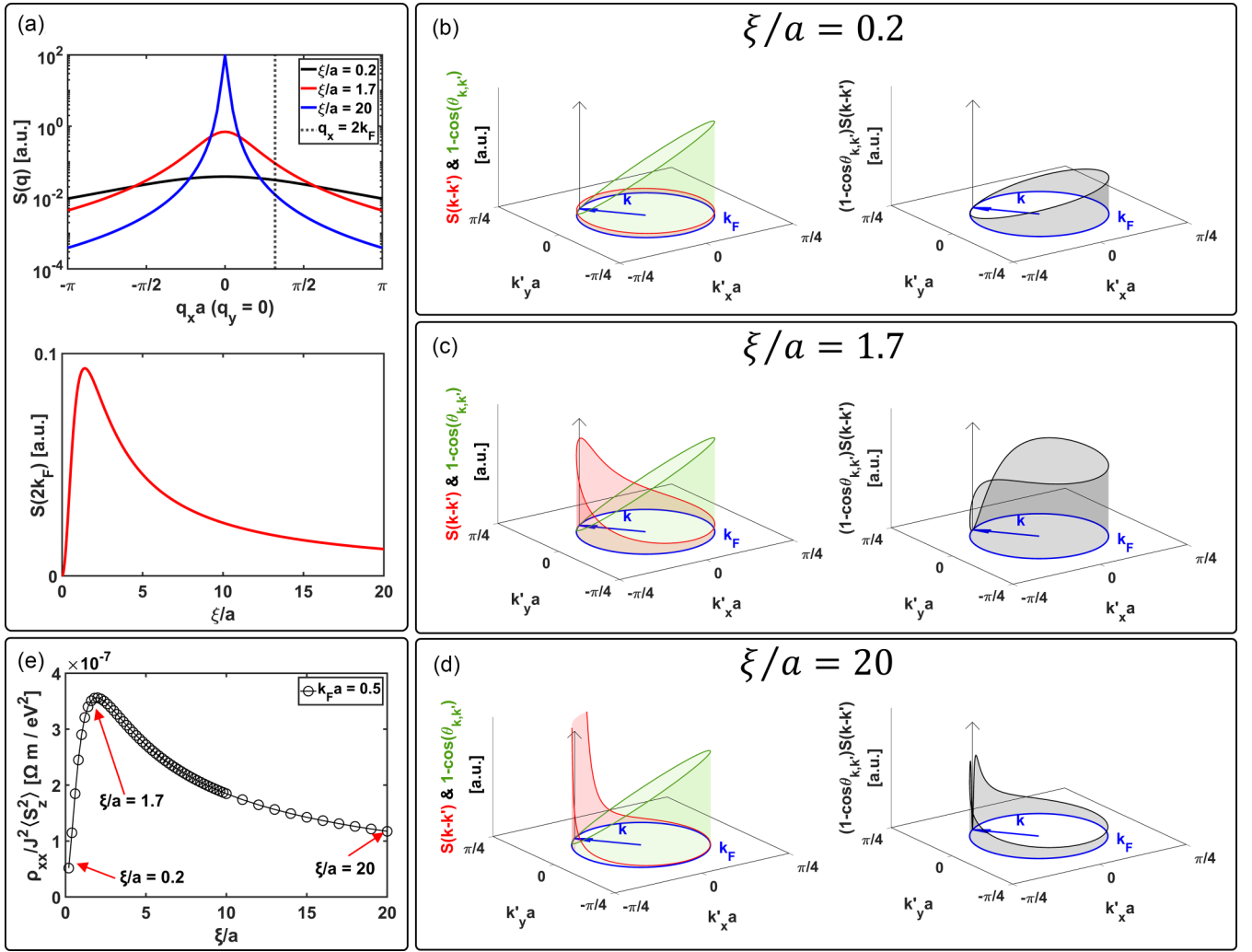


FIG. 1. Origin of the integral describing the scattering rate in 2D isotropic system with $k_F a = 0.5$, where a is the lattice constant for $\xi/a = 0.2$, $\xi/a = 1.7$, and $\xi/a = 20$. (a) Magnetic structure factor $S(\mathbf{q})$ plotted along q_x with $q_y = 0$ for three correlation lengths $\xi/a = 0.2$, 1.7 , and 20 . The bottom panel shows $S(\mathbf{q} = 2\mathbf{k}_F)$ as a function of ξ/a . (b)–(d) Angular contributions to the scattering integral for a fixed initial state $\mathbf{k}a = 0.3\hat{k}_x + 0.4\hat{k}_y$, on the Fermi surface. Left panels show $S(\mathbf{k} - \mathbf{k}', \xi)$ (red) and the geometric factor $1 - \cos \theta_{\mathbf{k}, \mathbf{k}'}$ (green), plotted on a common vertical scale across [(b)–(d)]. Right panels show the product $S(\mathbf{k} - \mathbf{k}', \xi)(1 - \cos \theta_{\mathbf{k}, \mathbf{k}'})$ in gray, representing the angular contribution to resistivity, also plotted on a shared vertical scale. (e) Computed resistivity as a function of correlation length ξ/a , exhibiting a pronounced peak at intermediate ξ/a .

We therefore evaluate the 2D resistivity only numerically. The resulting expressions for the 3D and 1D cases are

$$\rho_{3D} = \frac{3\pi^2 m^* J^2 \langle S_z^2 \rangle}{e^2 \hbar^3 k_F^5} \times \frac{1}{k_F \xi} \left[\frac{1}{1 + 4(k_F \xi)^2} + \ln[1 + 4(k_F \xi)^2] - 1 \right], \quad (6)$$

$$\rho_{1D} = \frac{2\pi^3 m^* J^2 \langle S_z^2 \rangle}{e \hbar^3 k_F^3} \frac{k_F \xi}{1 + 4(k_F \xi)^2}. \quad (7)$$

These expressions contain two main components: the amplitude and a dimensionless function that depends only on the product $k_F \xi$.

The amplitude inversely scales as $k_F^{(d+2)}$, where the k_F^d factor arises from carrier density and the k_F^2 from the magnetic

scattering. The $k_F \xi$ -dependent term has a fixed functional form and gives rise to the non-monotonic behavior, producing a peak.

For small k_F , electrons predominantly scatter via small $\mathbf{q}$, where $S(\mathbf{q})$ is largest. In contrast, for large k_F , the required momentum transfers cover regions of $\mathbf{q}$ where the structure factor is smaller. Although a peak in $\rho(\xi)$ appears for all values of k_F , its prominence diminishes for larger Fermi surfaces, where magnetic scattering becomes less effective and other scattering mechanisms dominate. Nickel, for example, exhibits only a small resistivity bump near the ferromagnetic transition, consistent with its large Fermi surface, which practically covers the entire Brillouin zone [25].

Figure 2 panels (b.1), (b.2), and (b.3) test this scaling relation. Here, the numerically obtained resistivity is multiplied by $k_F^{(d+2)}$ and plotted as a function of $k_F \xi$. This removes the trivial $k_F^{(d+2)}$ amplitude scaling from the analytical expression

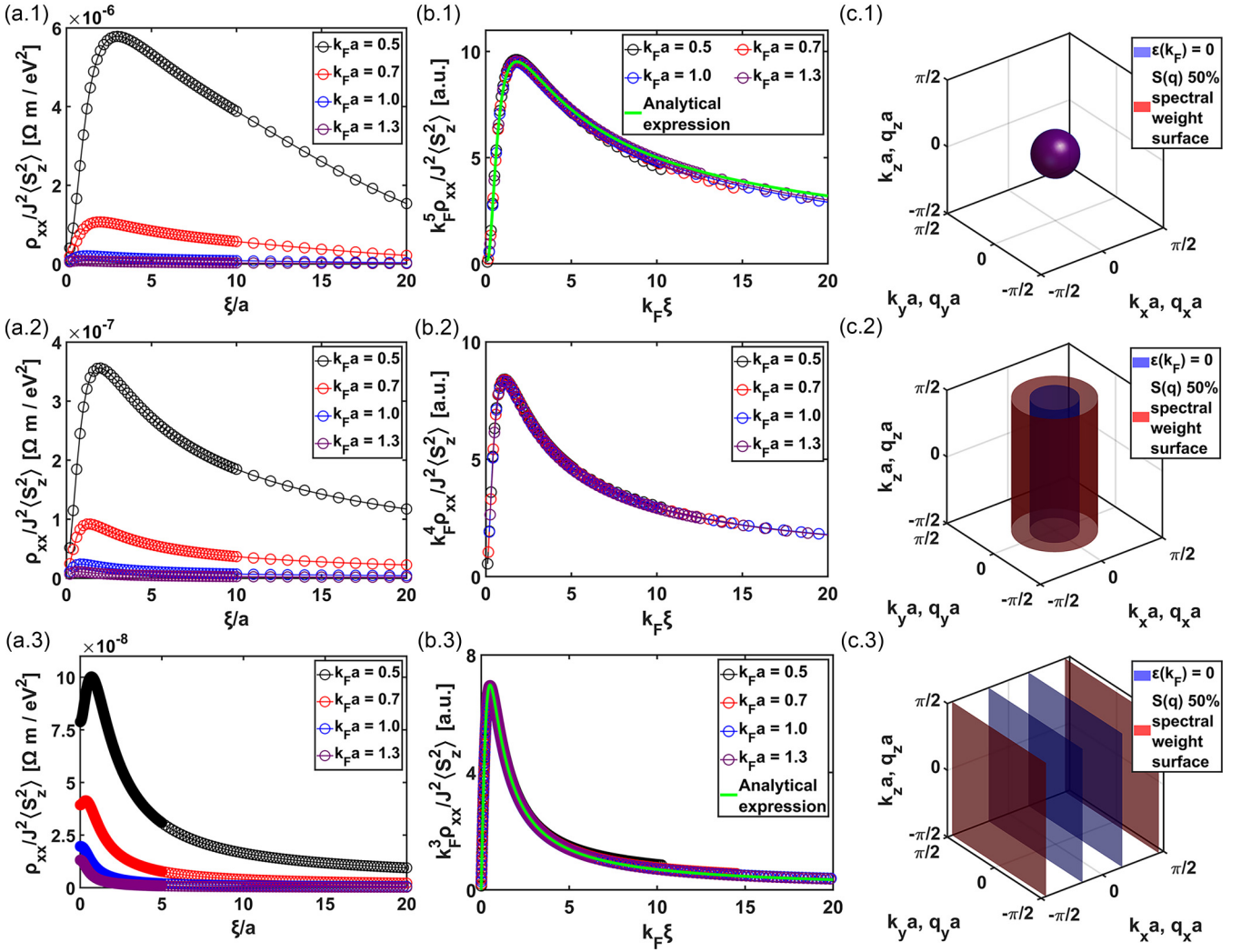


FIG. 2. Scaling behavior of resistivity and its relation to the Fermi surface and structure factor. [(a.1)–(a.3)] Computed resistivity for several values of Fermi momenta k_F in 3D (a.1), 2D (a.2), and 1D (a.3) systems. [(b.1)–(b.3)] Same data as in (a.1)–(a.3), but scaled by $k_F^{(d+2)}$ and plotted as a function of dimensionless product $k_F \xi$, where d is the dimensionality. The green curves in (b.1) and (b.3) represent the analytical results for 3D and 1D, respectively. [(c.1)–(c.3)] Fermi surfaces (blue) and the surface enclosing 50% of the total weight of $S(\mathbf{q})$ (red), at the correlation length corresponding to the resistivity peak for each k_F .

and isolates the dimensionless function that governs the shape of the curve. All curves collapse onto a single universal curve in each dimension, confirming the scaling behavior. In 1D and 3D [panels (b.1) and (b.3)], the numerical data match well with the analytical expressions as shown in green solid lines. The value of $k_F \xi$ at which peak occurs differs by dimensions: $k_F \xi \sim 1.8$ in 3D, ~ 0.9 in 2D, and ~ 0.5 in 1D.

Figure 2 panels (c.1), (c.2), and (c.3) visualize the interplay between the Fermi surface and the magnetic structure factor. The blue surface in each panel represents the Fermi surface for $k_F a = 0.5$. The red surface encloses 50% of the total $S(\mathbf{q}, \xi)$ weight at the ξ value where $\rho(\xi)$ is maximal. This visualization highlights the fact that maximal scattering and hence maximal resistivity occur when the Fermi surface and the main weight of $S(\mathbf{q})$ substantially overlap in momentum space. A more detailed geometric visualization of this overlap for different correlation lengths and effective dimensionalities is presented in Fig. S1 of Ref. [26].

B. Anisotropic cases

Since most real materials are anisotropic, we explore three simple anisotropic scenarios in 3D systems illustrated in Fig. 3: (a) anisotropy in the band dispersion only, (b) anisotropy in the magnetic structure factor only, and (c) anisotropy in both with the same anisotropy. In case (a) shown in panels (a.1), (a.2), and (a.3), anisotropy is introduced only along the k_y direction, with k_{F_x} and k_{F_z} left unchanged. This results in an elliptical Fermi surface, while the magnetic structure factor $S(\mathbf{q}, \xi)$ remains isotropic. In case (b) shown in panels (b.1), (b.2), and (b.3), anisotropy is introduced in the magnetic structure factor by varying the spin correlation length along the y direction, while the electronic dispersion remains isotropic. In case (c), shown in panels (c.1), (c.2), and (c.3), anisotropy is introduced equally in both the electronic band dispersion and magnetic structure factor, with $k_{F_y}/k_{F_x} = \xi_x/\xi_y$, so that the elongation in both matches.

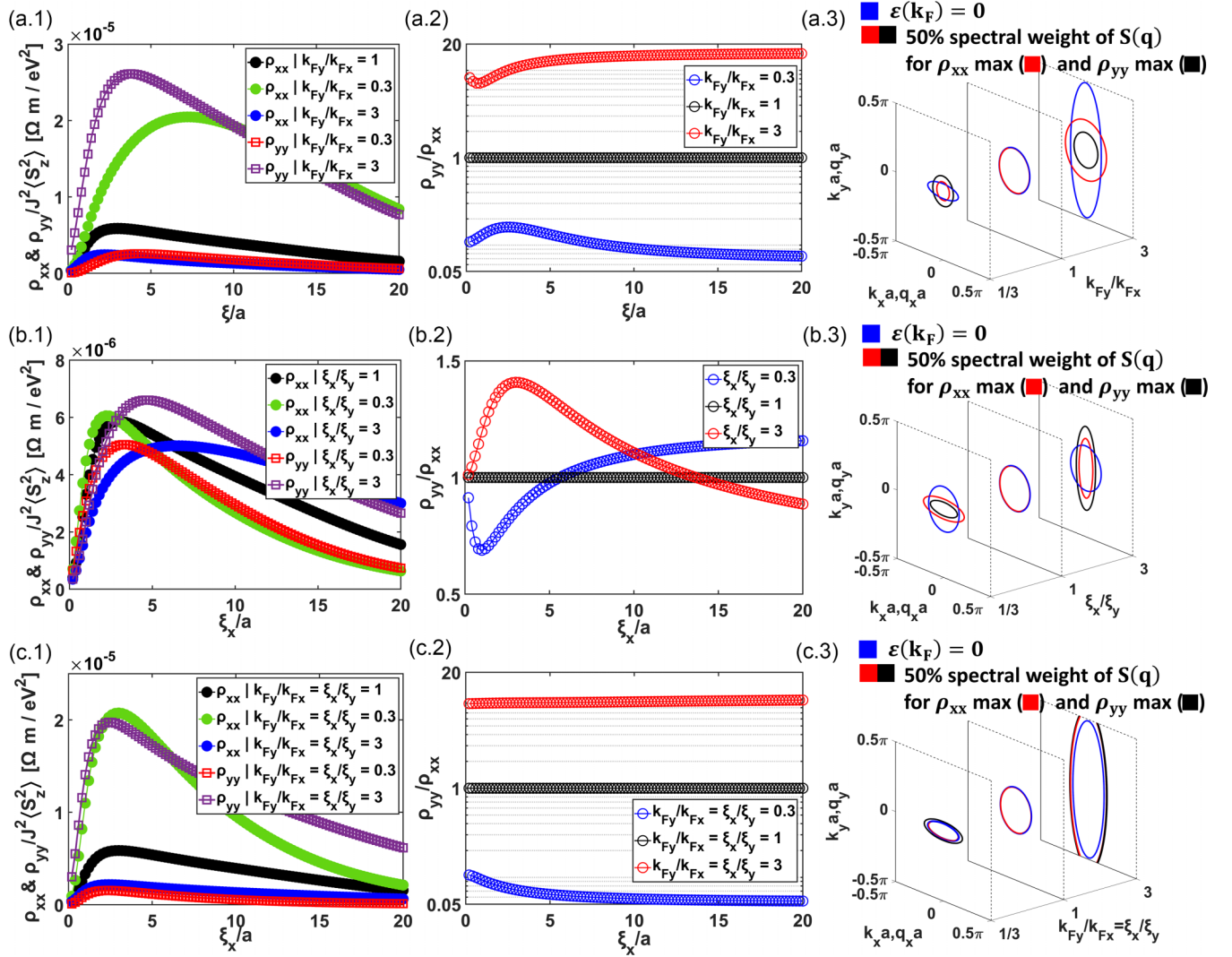


FIG. 3. Calculated resistivity for 3D anisotropic systems. x and z directions remain equivalent, while y can be different. (a.1) Resistivity $\rho_{xx} = \rho_{zz}$ and ρ_{yy} for anisotropic band dispersion, and isotropic structure factor. (b.1) Resistivity $\rho_{xx} = \rho_{zz}$ and ρ_{yy} for isotropic band dispersion, and anisotropic structure factor. (c.1) Resistivity $\rho_{xx} = \rho_{zz}$ and ρ_{yy} for both anisotropic band dispersion and structure factor. [(a.2)–(c.2)] Resistivity anisotropy ρ_{yy}/ρ_{xx} for each case, showing peaks that arise due to the shift in peak positions between ρ_{xx} and ρ_{yy} . [(a.3)–(c.3)] Two-dimensional momentum-space cuts showing the Fermi surface (blue) and the 50% spectral-weight surface of $S(\mathbf{q})$, evaluated at the correlation length where ρ_{xx} (red) and ρ_{yy} (black) peak.

Figure 3 panels (a.1), (b.1), and (c.1) present the calculated resistivity along respective x and y directions for several different combinations of anisotropy. In all cases, a clear resistivity peak appears at an intermediate ξ , consistent with the behavior observed in the isotropic case. Anisotropy in the band dispersion [panel (a.1)] has a stronger effect on the absolute magnitude of resistivity than anisotropy in the magnetic structure factor [panel (b.1)]. The peak position ξ_{peak} is sensitive to the shapes of both the Fermi surface and the magnetic structure factor. In the case where the band and spin anisotropies are aligned and have the same form, as shown in panel (c.1), the resistivity peaks in the x and y directions occur at essentially the same ξ . This suggests that the peak is governed by the degree of alignment between the Fermi surface and the magnetic structure factor.

Figure 3 panels (a.2), (b.2), and (c.2) plot the resistivity anisotropy ratio ρ_{yy}/ρ_{xx} for the three cases. In some

cases, the peak position of ρ_{xx} and ρ_{yy} differ slightly in ξ , which leads the anisotropy ratio ρ_{yy}/ρ_{xx} to develop a peak. This anisotropy peak could serve as an experimentally accessible signature of the underlying mechanism, potentially aiding interpretation of resistivity measurements. Additionally, in systems with anisotropic band structure, the Fermi velocities become direction-dependent, giving rise to significant resistivity anisotropy even away from the magnetic transition. This anisotropy originates from the band structure itself and can be understood within the Drude model, where the conductivity along each direction scales as the square of the corresponding Fermi surface size for parabolic dispersion.

Figure 3 panels (a.3), (b.3), and (c.3) visualize the Fermi surface and magnetic structure factor geometry for each case. The blue contour is the Fermi surface for $k_z = 0$, while the red (for ρ_{xx}) and black (for ρ_{yy}) surfaces enclose 50% of the

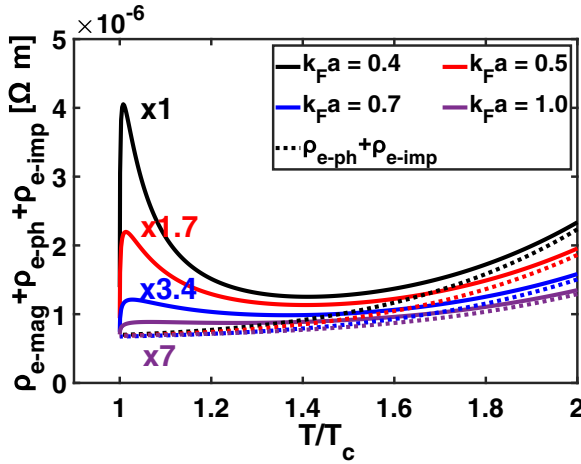


FIG. 4. Temperature dependence of resistivity in an isotropic 3D system, including magnetic scattering and conventional contributions from electron-phonon and electron-impurity processes. The solid curves show $\rho(T)$, where the magnetic scattering contribution is calculated using a temperature-dependent correlation length $\xi(T)/a = 1/\sqrt{6}(T/T_c - 1)^{-0.5}$, based on mean-field theory [27]. The calculations use $J = 0.15$ eV and $\langle S_z \rangle = 7/2$. To enhance the visibility of curves corresponding to larger Fermi surfaces, the magnitudes have been scaled by the factors indicated in the panel. The dashed lines correspond to the background contributions from electron-phonon and electron-impurity scattering for each $k_F a$ value, given by $\rho_{e-ph}(T) = 3 \times 10^{-9}(T/T_c)^5/(k_F a)^3$ [28,29] and $\rho_{e-imp} = 10^{-7}/(k_F a)^2$, respectively. The k_F dependence of ρ_{e-imp} was obtained by taking $S(\mathbf{q}) = S_0$, independent of $\mathbf{q}$. The prefactors are chosen such that all resistivity values are expressed in Ω m.

spectral weight of $S(\mathbf{q})$ at the ξ value corresponding to the respective resistivity peaks.

C. Temperature dependence

It is natural to convert $\rho(\xi)$ into a temperature-dependent resistivity $\rho(T)$, since the correlation length ξ itself depends on temperature. We assume a common form $\xi(T) \sim |T/T_c - 1|^{-0.5}$ [27]. This mean field inspired form is used only as a simple way to map the generic $\rho(\xi)$ curves onto a concrete $\rho(T)$. Within the same framework, one can instead use $\xi(T)$ extracted from experiment or from a more advanced theory, which would change the detailed shape of $\rho(T)$ but not the fact that the maximum occurs when $k_F \xi(T)$ reaches the dimension-dependent constant. Fig. 4 shows the temperature dependence of resistivity $\rho(T)$ for an isotropic 3D system and for several values of k_F , incorporating contributions from magnetic scattering, electron-phonon interactions, and impurity scattering. The dashed curves represent the non-magnetic contributions to resistivity from electron-phonon and electron-impurity scattering. The magnetic scattering contribution produces a distinct resistivity peak near T_c , especially for smaller k_F . In our parametrization, the condition $k_F \xi(T_{\text{peak}}) \approx \text{const}$ is satisfied slightly above T_c , so the maximum in $\rho(T)$ appears at a temperature higher than the transition. For large k_F , the peak becomes less visible because the magnetic contribution is overshadowed by the stronger phonon background.

V. CONCLUSIONS

In summary, we have developed a general framework to compute the resistivity arising from conduction electron scattering off spatially correlated local spins. The model accommodates arbitrary electronic band dispersions and magnetic structure factors. We applied this approach to systems characterized by parabolic electronic dispersions and ferromagnetic spin correlations decaying exponentially with distance. Our results demonstrate that a pronounced peak in resistivity arises near the magnetic phase transition temperature, where the correlation length diverges. This peak emerges when the inverse correlation length becomes comparable to the Fermi momentum $1/\xi \sim k_F$. We find that the peak height scales inversely with k_F^{d+2} , making the effect particularly significant in systems with small Fermi surfaces.

This model explains why pronounced resistivity peaks are often observed in materials with small k_F , such as Eu-based pnictides such as EuTn_2Pn_2 ($Tn = \text{Zn, In, and Cd}$, $Pn = \text{P, As, and Sb}$) [15–17,19,30]. These materials are known to have a relatively small Fermi surface, which likely explains why their peaks are sharp and prominent, consistent with our model's predictions, while the effect appears much weaker in systems like nickel with larger Fermi surfaces. Gadolinium and doped YPrCo_2 also show a smaller resistivity bump at their phase transitions [31,32], presumably for the same reason. In such cases, the magnetic contribution is present but masked by other scattering mechanisms.

Magnetic field effects can also be included by using field-dependent structure factors, either from experiment or simulation. The field introduces a uniform spin component corresponding to a delta function at $\mathbf{q} = 0$, which does not contribute to resistive scattering. Owing to the sum rule on $S(\mathbf{q})$, this delta function component reduces the remaining spectral weight available for scattering, thereby lowering the resistivity. This mechanism may account for the pronounced field-induced resistivity drops observed in Eu-based pnictides [15–17,19].

We note that while our calculations focused on ferromagnetic correlations, the concept of a structure factor derived from spatial correlations is not restricted to ferromagnetic ordering. The same approach could be applied to various systems with antiferromagnetic ordering or even charge density wave transitions, which also often show anomalous resistivity bumps or peaks [33–37]. In antiferromagnets, the dominant scattering processes transfer the ordering wave vector $\mathbf{Q}$, so the important events connect Fermi surface points $\mathbf{k}$ and $\mathbf{k} + \mathbf{Q}$ that both lie close to the Fermi level. A pronounced resistivity anomaly is therefore expected only when $\mathbf{Q}$ efficiently connects current-carrying parts of the Fermi surface, so that there are extended regions where both $\mathbf{k}$ and $\mathbf{k} + \mathbf{Q}$ lie close to the Fermi level and experience strong scattering. If the Fermi surface is very small and centered near the zone center while the magnitude of $\mathbf{Q}$ is large and close to the zone boundary, there are few such pairs of states and the anomaly is correspondingly weak. Conversely, for larger or nested Fermi surfaces that are well connected by $\mathbf{Q}$, the same geometric mechanism can produce a sizable anomaly, in close analogy with the ferromagnetic case studied here. The model developed in this study may therefore provide a

versatile and computationally accessible framework to understand temperature- or field-dependent resistivity anomalies in a broad range of correlated electron systems beyond ferromagnets. Another natural extension would be to incorporate the effect of band splitting, which is essential to describe resistivity behaviors below the ordering temperature.

The present implementation of the framework has a number of deliberate simplifications. We consider a single spin degenerate conduction band and magnetic structure factor $S(\mathbf{q}, \xi(T))$ that we generate from a correlation length $\xi(T)$ with a simple temperature dependence. These choices allow us to isolate the mechanism that links the Fermi surface and the main weight of $S(\mathbf{q})$ and to obtain transparent $k_F \xi$ scaling relations in 1D, 2D, and 3D. Extending the present approach to multiband and spin-split systems and applying it to specific magnetic metals with pronounced resistivity peaks will be a natural next step.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [38].

APPENDIX A: DERIVATION OF SCATTERING TIME

For a given electron distribution $f(\mathbf{k})$, the current density is defined by

$$\mathbf{J} = -2e \int \frac{d^3\mathbf{k}}{(2\pi)^3} \mathbf{v}(\mathbf{k}) f(\mathbf{k}), \quad (\text{A1})$$

where $\mathbf{v}(\mathbf{k}) = \partial \varepsilon(\mathbf{k}) / \hbar \partial \mathbf{k}$ is the band velocity associated with the electronic dispersion $\varepsilon(\mathbf{k})$. To evaluate the current response, it is necessary to determine the form of $f(\mathbf{k})$ under nonequilibrium conditions induced by the electric field $\mathbf{E}$. In the steady state and assuming spatial uniformity, the Boltzmann transport equation simplifies to a balance between the drift term and the scattering term:

$$-e\mathbf{E} \cdot \nabla_{\mathbf{k}} f(\mathbf{k}) = \left(\frac{\partial f}{\partial t} \right)_{\text{collision}}. \quad (\text{A2})$$

Within the relaxation time approximation, the collision term is modeled as

$$\left(\frac{\partial f}{\partial t} \right)_{\text{collision}} = -\frac{f(\mathbf{k}) - f_0(\mathbf{k})}{\tau(\mathbf{k})}, \quad (\text{A3})$$

where $f_0(\mathbf{k})$ is the equilibrium Fermi-Dirac distribution and $\tau(\mathbf{k})$ is the momentum-dependent relaxation time.

Assuming the applied field induces only a small deviation from equilibrium, the distribution function can be written as $f(\mathbf{k}) = f_0(\mathbf{k}) + f_1(\mathbf{k})$, with $f_1 \ll f_0$. In linear response, we retain only first-order terms and note that $f_1(\mathbf{k})$ is much smaller than $f_0(\mathbf{k})$. Since f_1 is smooth and slowly varying compared to the sharp variation of f_0 near the Fermi energy,

the term $\nabla_{\mathbf{k}} f_1$ can be neglected, and we approximate $\nabla_{\mathbf{k}} f \approx \nabla_{\mathbf{k}} f_0$. Substituting this into the Boltzmann equation gives

$$-\frac{e\mathbf{E}}{\hbar} \cdot \nabla_{\mathbf{k}} f_0(\mathbf{k}) = -\frac{f_1(\mathbf{k})}{\tau(\mathbf{k})}. \quad (\text{A4})$$

Since f_0 depends only on energy, the gradient in $\mathbf{k}$ -space becomes

$$\nabla_{\mathbf{k}} f_0 = \hbar \mathbf{v}(\mathbf{k}) \frac{\partial f_0(\varepsilon(\mathbf{k}))}{\partial \varepsilon(\mathbf{k})}. \quad (\text{A5})$$

Thus the nonequilibrium distribution takes the form

$$f(\mathbf{k}) = f_0(\mathbf{k}) - e\mathbf{E} \cdot \mathbf{v}(\mathbf{k}) \tau(\mathbf{k}) \left(-\frac{\partial f_0(\mathbf{k})}{\partial \varepsilon(\mathbf{k})} \right). \quad (\text{A6})$$

This expression forms the basis for evaluating charge transport in the linear response regime, and allows the current density to be directly related to the electronic structure and scattering properties.

Once the energy dispersion is specified, all quantities in the linearized distribution function can be explicitly determined, except for the scattering time $\tau(\mathbf{k})$. This quantity can be microscopically evaluated using quantum mechanical scattering theory in conjunction with the relaxation time approximation. In this framework, the collision integral in the Boltzmann equation takes the form:

$$-\frac{f(\mathbf{k}) - f_0(\mathbf{k})}{\tau(\mathbf{k})} = \int \frac{d^3\mathbf{k}'}{(2\pi)^3} f(\mathbf{k}') [1 - f(\mathbf{k})] W(\mathbf{k}' \rightarrow \mathbf{k}) - f(\mathbf{k}) [1 - f(\mathbf{k}')] W(\mathbf{k} \rightarrow \mathbf{k}'), \quad (\text{A7})$$

where $W(\mathbf{k}' \rightarrow \mathbf{k})$ and $W(\mathbf{k} \rightarrow \mathbf{k}')$ are the scattering probabilities from $\mathbf{k}'$ to $\mathbf{k}$ states and vice versa, respectively. These scattering probabilities, also called transition rate, are determined from Fermi's golden rule, which incorporates the scattering potential, such as that arising from localized spins, as discussed in the next section.

At equilibrium, substituting $f(\mathbf{k}) = f_0(\mathbf{k})$ into the equation above ensures that the collision integral vanishes. This leads to the detailed balance condition:

$$W(\mathbf{k}' \rightarrow \mathbf{k}) = W(\mathbf{k} \rightarrow \mathbf{k}') \exp\left(\frac{\varepsilon(\mathbf{k}) - \varepsilon(\mathbf{k}')}{k_B T}\right). \quad (\text{A8})$$

This condition reflects the requirement that transitions between two states occur at equal rates in opposite directions under equilibrium. For elastic scattering (where $\varepsilon(\mathbf{k}) = \varepsilon(\mathbf{k}')$), this implies $W(\mathbf{k} \rightarrow \mathbf{k}') = W(\mathbf{k}' \rightarrow \mathbf{k})$. To simplify notation, we use $W(\mathbf{k}, \mathbf{k}') \equiv W(\mathbf{k} \rightarrow \mathbf{k}')$. Substituting both the detailed balance condition (A8) and the linearized distribution (A6) into the collision integral (A7) yields a general expression for the scattering time:

$$\frac{1}{\tau(\mathbf{k})} = \int \frac{d^3\mathbf{k}'}{(2\pi)^3} W(\mathbf{k}, \mathbf{k}') \left[1 - \frac{\mathbf{E} \cdot \mathbf{v}(\mathbf{k}') \tau(\mathbf{k}')}{\mathbf{E} \cdot \mathbf{v}(\mathbf{k}) \tau(\mathbf{k})} \right]. \quad (\text{A9})$$

In standard textbooks [39,40], the scattering time formulas are generally derived for isotropic energy dispersion under elastic scattering. Equation (A9) generalizes this to arbitrary dispersion and reduces to the textbook result in the isotropic limit. Isotropic energy dispersion results in isotropic scattering time, meaning $\tau(\mathbf{k}) = \tau(|\mathbf{k}|)$. In this case, the term $\mathbf{E} \cdot \mathbf{v}(\mathbf{k}') / \mathbf{E} \cdot \mathbf{v}(\mathbf{k})$ simplifies to $\cos \theta_{\mathbf{k}, \mathbf{k}'}$, where $\theta_{\mathbf{k}, \mathbf{k}'}$ is the

angle between the reciprocal vectors $\mathbf{k}$ and $\mathbf{k}'$. Anisotropy introduces a weighting factor for the $\cos \theta_{\mathbf{k},\mathbf{k}'}$ term, making certain scattering directions on the Fermi surface more probable while reducing others. Eq. (A9) provides an improvement over the isotropic relaxation time approximation by incorporating the directional dependence of both the scattering probability and the velocity. The equation applies to arbitrary shapes of energy dispersion and reduces to the textbook result when the energy dispersion is isotropic. Until developing the general scattering time, no specific scattering mechanisms have been considered, it means the formulation applies to any scattering sources.

The scattering time $\tau(\mathbf{k})$ derived here represents the transport relaxation time, which accounts for the momentum-relaxing nature of scattering events. Unlike the total lifetime $1/\tau_{\text{total}}(\mathbf{k}) = \int d^3\mathbf{k}' W(\mathbf{k}, \mathbf{k}')$, the transport time emerges only after inserting the linearized nonequilibrium distribution into the Boltzmann collision integral. Its dependence on velocity and energy ensures that forward scattering contributes less than backscattering, as appropriate for current-carrying states.

The current density is a vector, and the conductivity is a tensor, representing the relationship between the direction of the electric current and the electric field. The scattering time should be indexed by the direction of the applied electric field $[\tau_E(\mathbf{k})]$. In the study, we focus solely on the conductivity along the principal axes, in which the electric field and current directions are aligned, both in isotropic and anisotropic cases. In such geometry, the transverse conductivity is zero.

By substituting Eq. (A6) into Eq. (A1), the current density can be expressed as

$$J_\alpha = 2e^2 E_\alpha \int \frac{d^3\mathbf{k}}{(2\pi)^3} v_\alpha^2(\mathbf{k}) \tau(\mathbf{k}) \left(-\frac{\partial f_0(\mathbf{k})}{\partial \varepsilon(\mathbf{k})} \right). \quad (\text{A10})$$

The electrical resistivity is

$$\frac{1}{\rho_{\alpha\alpha}} = \sigma_{\alpha\alpha} = \frac{J_\alpha}{E_\alpha}, \quad (\text{A11})$$

where $\alpha = x, y, z$ denotes the Cartesian components of the current and electric field directions.

APPENDIX B: SCATTERING ON THE POTENTIAL CORRELATION

For calculating the scattering time from Eq. (A9), the scattering probability $W(\mathbf{k}, \mathbf{k}')$ has not yet been defined. This can be done using the well-known Fermi's golden rule:

$$W(\mathbf{k}, \mathbf{k}') = \frac{2\pi}{\hbar} |\langle \mathbf{k}' | V | \mathbf{k} \rangle|^2 \delta[\varepsilon(\mathbf{k}) - \varepsilon(\mathbf{k}')]. \quad (\text{B1})$$

Here, V is the interaction potential from the Hamiltonian and $\langle \mathbf{k}' | V | \mathbf{k} \rangle$ is the matrix element of the interaction potential in reciprocal space. Strictly speaking, the correct basis is the set of Bloch waves in a crystalline lattice, however, we simplify the calculation by assuming free conduction electrons described by plane-wave states. If the interaction potential V is defined in real space as $V(\mathbf{r})$, one must take its Fourier transform.

For the magnetic structure, the potential should be related to the localized spins. Since spins are not fully aligned at finite temperatures, a statistical representation of spin

configurations and the associated potentials is required. The matrix element of the interaction potential squared can be expressed as the average correlation between two distant potentials:

$$\begin{aligned} |\langle \mathbf{k}' | V | \mathbf{k} \rangle|^2 &= \frac{1}{L^3} \int d^3\mathbf{r}_1 V(\mathbf{r}_1) e^{-i\mathbf{q}\cdot\mathbf{r}_1} \frac{1}{L^3} \int d^3\mathbf{r}_2 V(\mathbf{r}_2) e^{i\mathbf{q}\cdot\mathbf{r}_2} \\ &= \frac{1}{L^3} \int d^3\mathbf{r} e^{i\mathbf{q}\cdot\mathbf{r}} \frac{1}{L^3} \int d^3\mathbf{r}_0 V(\mathbf{r}_0) V(\mathbf{r}_0 + \mathbf{r}) \\ &= \frac{1}{L^3} \int d^3\mathbf{r} e^{i\mathbf{q}\cdot\mathbf{r}} \langle V(\mathbf{r}_0) V(\mathbf{r}_0 + \mathbf{r}) \rangle, \end{aligned} \quad (\text{B2})$$

where the variables $\mathbf{r}_1$ and $\mathbf{r}_2$ in the second term were rewritten as $\mathbf{r}_0$ and $\mathbf{r}_0 + \mathbf{r}$, respectively, in the third term. Equations (B1) and (B2) indicate that the scattering probability can be expressed as the Fourier transform of the real-space correlation function of two distant potentials $\langle V(\mathbf{r}_0) V(\mathbf{r}_0 + \mathbf{r}) \rangle$. Details of the potential arising from localized spins are presented in Appendix C.

APPENDIX C: DERIVATION OF SCATTERING PROBABILITY FROM SPIN FLUCTUATIONS

In this Appendix, we derive the scattering probability and the corresponding scattering rate, which is determined by the magnetic structure factor $S(\mathbf{q})$. In general, the isotropic magnetic structure factor $S(\mathbf{q})$ is defined as the sum of the diagonal spin-spin correlation components:

$$S(\mathbf{q}) = S_{xx}(\mathbf{q}) + S_{yy}(\mathbf{q}) + S_{zz}(\mathbf{q}). \quad (\text{C1})$$

In this work, we assume isotropic spin fluctuations above the critical temperature T_c , which implies $S_{xx}(\mathbf{q}) = S_{yy}(\mathbf{q}) = S_{zz}(\mathbf{q})$. This condition is also stated in Kataoka's formalism [22] above T_c , as no preferred spin direction exists in the paramagnetic phase.

Conduction electrons experience a potential arising from localized spins, whose explicit form depends on the chosen Hamiltonian. The derivation presented here follows the formalism developed by Kondo, Kasuya, and Kataoka [8,22,23], in which the scattering probability is governed by the magnetic structure factor derived from thermal spin fluctuations. The Hamiltonian can be expressed as

$$\begin{aligned} H &= \sum_{\mathbf{k},\sigma} \varepsilon_{\mathbf{k},\sigma} a_{\mathbf{k},\sigma}^\dagger a_{\mathbf{k},\sigma} - \frac{J}{N} \sum_{\mathbf{q},\mathbf{k}} [S_z(\mathbf{q}) (a_{\mathbf{k}+\mathbf{q},\uparrow}^\dagger a_{\mathbf{k},\uparrow} - a_{\mathbf{k}+\mathbf{q},\downarrow}^\dagger a_{\mathbf{k},\downarrow}) \\ &\quad + S_-(\mathbf{q}) a_{\mathbf{k}+\mathbf{q},\uparrow}^\dagger a_{\mathbf{k},\downarrow} + S_+(\mathbf{q}) a_{\mathbf{k}+\mathbf{q},\downarrow}^\dagger a_{\mathbf{k},\uparrow}]. \end{aligned} \quad (\text{C2})$$

Here, $a_{\mathbf{k},\sigma}^\dagger$ and $a_{\mathbf{k},\sigma}$ are the creation and annihilation operators for electrons with $\mathbf{k}$ and spin $\sigma = \uparrow$ or $\downarrow$, respectively. J is the exchange integral between conduction electrons and local spins, and N is the total number of unit cells. $S_z(\mathbf{q})$ is the Fourier transform of the local spins $S_z(\mathbf{r}_n)$ on the n -th magnetic ion by following:

$$S_z(\mathbf{q}) = \sum_{\mathbf{r}_n} S_z(\mathbf{r}_n) e^{-i\mathbf{q}\cdot\mathbf{r}_n}, \quad (\text{C3})$$

where $\mathbf{q} = \mathbf{k} - \mathbf{k}'$ denotes the momentum transfer.

The first term in Eq. (C2) represents the energy of the free conduction electron, while the second term describes the

interaction between the electron spin and the localized magnetic moments. Although this interaction does not explicitly introduce a real-space scattering potential $V(\mathbf{r})$, it plays the same physical role as it mediates momentum transfer between electronic states via coupling to the spin background. The Hamiltonian is formulated in momentum space, with spin operators $S_z(\mathbf{q})$, $S_+(\mathbf{q})$, and $S_-(\mathbf{q})$ entering directly.

Therefore the scattering probability can be directly evaluated from the spin-dependent interaction terms in momentum space, without computing the matrix element of the potential $V(\mathbf{q})$ derived from a real-space potential $V(\mathbf{r})$. The squared matrix element $|\langle \mathbf{k}' | V | \mathbf{k} \rangle|^2$ is effectively equivalent to $J^2 S(\mathbf{q})$, as it appears in Kataoka's formalism. This establishes a link between spin fluctuations and the scattering strength governing transport.

From Eq. (C2), the spin nonflip scattering probability, where the interaction potential matrix element is replaced by the spin-spin correlation function, is given by

$$W(\mathbf{k}, \mathbf{k}') = \frac{2\pi}{\hbar} \frac{J^2}{N} \langle \bar{S}_z(\mathbf{q}) \bar{S}_z(-\mathbf{q}) \rangle \delta[\varepsilon(\mathbf{k}) - \varepsilon(\mathbf{k}')], \quad (\text{C4})$$

where $\langle \bar{S}_z(\mathbf{q}) \bar{S}_z(-\mathbf{q}) \rangle$ is the Fourier transform of the real-space spin correlation function $\langle S_z(\mathbf{r}_0) S_z(\mathbf{r}_0 + \mathbf{r}) \rangle$. $\bar{S}_z(\mathbf{q}) = S_z(\mathbf{q}) - \langle S_z(0) \rangle \delta_{\mathbf{q},0}$, but $\mathbf{q} = 0$ does not contribute to the transport scattering, the second term can be ignored. Parseval's theorem leads to the following relation:

$$\sum_{\mathbf{r}_n} |S_z(\mathbf{r}_n)|^2 = \frac{1}{N} \sum_{\mathbf{q}_m} |S_z(\mathbf{q}_m)|^2 = N \langle S_z^2 \rangle. \quad (\text{C5})$$

This shows the normalization condition for the $\langle \bar{S}_z(\mathbf{q}) \bar{S}_z(-\mathbf{q}) \rangle = |S_z(\mathbf{q}_m)|^2$, where the sum over all $\mathbf{q}$ should equal $N \langle S_z^2 \rangle$ when the spins are fully aligned in the same direction. One factor of N cancels out $1/N$ in the scattering probability of Kataoka's formalism. The z -direction diagonal spin-spin correlation component is defined by the following relation:

$$S_{zz}(\mathbf{q}) = \frac{1}{N} \langle \bar{S}_z(\mathbf{q}) \bar{S}_z(-\mathbf{q}) \rangle, \quad (\text{C6})$$

$$\frac{1}{N} \sum_{\mathbf{q}} S_{zz}(\mathbf{q}) = \frac{1}{N^2} \sum_{\mathbf{q}} \langle \bar{S}_z(\mathbf{q}) \bar{S}_z(-\mathbf{q}) \rangle = \langle S_z^2 \rangle. \quad (\text{C7})$$

Thus the structure factor is normalized to $\langle S_z^2 \rangle$ over the entire reciprocal space. In the numerical calculation, we fix the initial wave vector $\mathbf{k}$ within the first Brillouin zone and restrict the momentum transfer $\mathbf{q} = \mathbf{k} - \mathbf{k}'$ to the range $[-\pi/a]$ along each direction. Consequently, the final wave vector $\mathbf{k}'$ may lie outside the first Brillouin zone. For a large correlation length ξ , the structure factor $S_{zz}(\mathbf{k} - \mathbf{k}', \xi)$ is sharply peaked at $\mathbf{q} = 0$, and contributions from large momentum transfers are negligible. In contrast, for small ξ , the broad $S(\mathbf{q})$ has finite weight near $\mathbf{G}$, allowing scattering to $\mathbf{k} + \mathbf{q}$ outside the first Brillouin zone (folded back by $-\mathbf{G}$), thus Umklapp can matter even when the Fermi surface is small.

To account for the periodicity of reciprocal space, we relate the full sum over all momentum transfers to a sum restricted within the first Brillouin zone. Starting from the total

contribution of the structure factor, we write

$$\sum_{\text{all } \mathbf{q}} S_{zz}(\mathbf{q}) = \sum_{\mathbf{G}} \sum_{\mathbf{q} \in 1\text{st B.Z.}} S_{zz}(\mathbf{q} + \mathbf{G}), \quad (\text{C8})$$

where the sum over n accounts for reciprocal lattice vectors in a 3D axis. By interchanging the order of summations, we obtain

$$\sum_{\text{all } \mathbf{q}} S_{zz}(\mathbf{q}) = \sum_{\mathbf{q} \in 1\text{st B.Z.}} \sum_{\mathbf{G}} S_{zz}(\mathbf{q} + \mathbf{G}). \quad (\text{C9})$$

We define the effective (folded) structure factor as

$$\tilde{S}_{zz}(\mathbf{q}) = \sum_{\mathbf{G}} S_{zz}(\mathbf{q} + \mathbf{G}) \quad (\text{C10})$$

so that the total contribution over reciprocal space becomes

$$\sum_{\text{all } \mathbf{q}} S_{zz}(\mathbf{q}) = \sum_{\mathbf{q} \in 1\text{st B.Z.}} \tilde{S}_{zz}(\mathbf{q}). \quad (\text{C11})$$

This effective structure factor is used in the numerical analysis to approximate the full scattering contribution within the zone-folding scheme. The scattering time, originally expressed as an integral over all $\mathbf{k}'$ (or equivalently over all momentum transfer $\mathbf{q}$), can thus be restricted to $\mathbf{q}$ within the first Brillouin zone.

$$\begin{aligned} \frac{1}{\tau(\mathbf{k})} &= \int_{\text{all } \mathbf{q}} d^3 \mathbf{q} \frac{J^2 S_{zz}(\mathbf{q})}{4\pi^2 \hbar} \left[1 - \frac{\mathbf{E} \cdot \mathbf{v}(\mathbf{k} - \mathbf{q}) \tau(\mathbf{k} - \mathbf{q})}{\mathbf{E} \cdot \mathbf{v}(\mathbf{k}) \tau(\mathbf{k})} \right] \\ &= \int_{\mathbf{q} \in 1\text{st B.Z.}} d^3 \mathbf{q} \frac{J^2 \tilde{S}_{zz}(\mathbf{q})}{4\pi^2 \hbar} \left[1 - \frac{\mathbf{E} \cdot \mathbf{v}(\mathbf{k} - \mathbf{q}) \tau(\mathbf{k} - \mathbf{q})}{\mathbf{E} \cdot \mathbf{v}(\mathbf{k}) \tau(\mathbf{k})} \right]. \end{aligned} \quad (\text{C12})$$

The scattering rate in Eq. (C4) includes the structure factor and energy delta function, representing momentum and energy conservation, respectively. Strictly speaking, the structure factor should depend on both momentum and energy, as it can involve inelastic scattering. This is described by the dynamical structure factor $S_{zz}(\mathbf{q}, \omega)$, which determines the scattering possibilities involving both momentum and energy transfer.

The relation between $S_{zz}(\mathbf{q}, \omega)$ and $S_{zz}(\mathbf{q})$ is the following:

$$S_{zz}(\mathbf{q}) = \int d(\hbar\omega) S_{zz}(\mathbf{q}, \omega). \quad (\text{C13})$$

Equation (C13) shows that the structure factor is not simply $S_{zz}(\mathbf{q}, \omega = 0)$. In this study, we use the energy integrated dynamical structure factor based on the fact that, near the magnetic transition temperature, most of the spectral weight lies at low energies.

In a ferromagnetic system, the spin-spin correlation is often modeled using an exponential decay model as follows:

$$\langle S_z(\mathbf{r}_0) S_z(\mathbf{r}_0 + \mathbf{r}) \rangle = \langle S_z^2 \rangle e^{-\frac{|\mathbf{r}|}{\xi}}. \quad (\text{C14})$$

Taking the Fourier transform of such spin-spin correlation function in 3D, we obtain the structure factor with the normalization constant:

$$S_{zz}(\mathbf{q}) = \langle S_z^2 \rangle A_3 \frac{8\pi \xi^3}{(1 + \mathbf{q}^2 \xi^2)^2}. \quad (\text{C15})$$

Here, A_3 is the normalization constant for the First Brillouin zone, with $A_3 = 1$ indicating normalization over the entire $\mathbf{q}$ space. A_3 significantly differs from 1 only when ξ is very small. This correction does not impact the peaked resistivity behavior, and since $\xi/a < 1$, where the correlation length is smaller than a unit cell, may not be physically meaningful, the adjustment applies only to a limited range.

Similar Fourier transforms can be performed for the exponential decay model in 1D and 2D:

$$S_{zz}(\mathbf{q}) = \langle S_z^2 \rangle A_1 \frac{2\xi}{1 + \mathbf{q}^2 \xi^2}, \quad (\text{C16})$$

$$S_{zz}(\mathbf{q}) = \langle S_z^2 \rangle A_2 \frac{2\pi \xi^2}{(1 + \mathbf{q}^2 \xi^2)^{\frac{3}{2}}}. \quad (\text{C17})$$

The structure factors Eqs. (C15)–(C17) are normalized as follows $\int d^d \mathbf{q} S_{zz}(\mathbf{q}) = (2\pi)^d \langle S_z^2 \rangle A_d$, where d is the dimensionality of the system. The normalization factor A_d is numerically determined so that the sum over $\mathbf{q}$ within the first Brillouin zone in the calculation of $S_{zz}(\mathbf{q})$ satisfies the normalization condition. Any of the structure factors from Eqs. (C15), (C16), or (C17) can be substituted into Eq. (3), with the interaction potential norm provided in Eq. (C5). For the 3D case, this results in the following expression for the scattering probability:

$$W(\mathbf{k}, \mathbf{k}') = \frac{2\pi}{\hbar} J^2 \langle S_z^2 \rangle A_{3d} \frac{8\pi \xi^3}{(1 + \mathbf{q}^2 \xi^2)^2} \delta[\varepsilon(\mathbf{k}) - \varepsilon(\mathbf{k}')]. \quad (\text{C18})$$

Equation (C18) describes the scattering probability due to the potentials created by local spins, with the degree of spin correlation parameterized by the correlation length ξ . This correlation length depends on both temperature and magnetic field. The scattering time, which is essential for calculating electrical resistivity, can be obtained by substituting Eq. (C18) into Eq. (A9).

APPENDIX D: DERIVATION OF THE SCALING BEHAVIOR OF RESISTIVITY WITH $k_F \xi$

For an isotropic system in 3D, the scattering time and electrical resistivity can be derived in analytical form. The scattering time is expressed as follows:

$$\frac{1}{\tau(\mathbf{k})} = \int \frac{d^3 \mathbf{k}'}{(2\pi)^3} (1 - \cos \theta_{\mathbf{k}, \mathbf{k}'}) \frac{2\pi}{\hbar} J^2 S_{zz}(\mathbf{k} - \mathbf{k}') \delta[\varepsilon(\mathbf{k}) - \varepsilon(\mathbf{k}')]. \quad (\text{D1})$$

By substituting the structure factor from Eq. (C15), derived from the exponential decaying correlation model, the scattering time Eq. (D1) becomes

$$\frac{1}{\tau(k_F)} = \frac{2\pi}{\hbar} J^2 \langle S_z^2 \rangle A_{3d} \int_{\mathbf{k}' \in \text{F.S.}} \frac{d^2 \mathbf{k}'}{(2\pi)^3} (1 - \cos \theta_{\mathbf{k}, \mathbf{k}'}) \times \frac{8\pi \xi^3}{[1 + 2k_F^2 \xi^2 (1 - \cos \theta_{\mathbf{k}, \mathbf{k}'})]^2} \frac{m^*}{\hbar^2 k_F}. \quad (\text{D2})$$

Here, the delta function in Eq. (D1) is simplified by converting one integral over $\mathbf{k}'$ into energy space using the Jacobian

$m^*/\hbar^2 k_F$, and performing the integration explicitly over the Fermi surface. Converting the Fermi surface integral to an angular integral results in the following:

$$\frac{1}{\tau(k_F)} = \frac{2\pi}{\hbar} \frac{J^2 \langle S_z^2 \rangle A_{3d}}{(2\pi)^3} \int_0^{2\pi} d\phi \int_0^\pi d\theta k_F^2 \sin \theta (1 - \cos \theta) \times \frac{8\pi \xi^3}{[1 + 2k_F^2 \xi^2 (1 - \cos \theta)]^2} \frac{m^*}{\hbar^2 k_F}. \quad (\text{D3})$$

Evaluating this angle yields the result:

$$\frac{1}{\tau(k_F)} = \frac{m^* J^2 \langle S_z^2 \rangle A_{3d}}{\hbar^3} \frac{1}{k_F \xi} \times \left[\frac{1}{1 + 4(k_F \xi)^2} + \ln [1 + 4(k_F \xi)^2] - 1 \right]. \quad (\text{D4})$$

This scattering time can then be used in the calculation of the electron distribution and current density, leading to the following expression for resistivity, which is consistent with the Drude model for an isotropic system:

$$\rho = \frac{3\pi^2 m^*}{e^2 k_F^3} \frac{1}{\tau(k_F)}. \quad (\text{D5})$$

This resistivity reaches a peak at approximately $k_F \xi \sim 1.8$ from $d\rho/d(k_F \xi) = 0$. The expression can be rewritten Eq. (D5) in terms of scaled variables as

$$\frac{k_F^5 \rho}{A_{3d}} = \frac{3\pi^2 m^{*2}}{e^2 \hbar^3} J^2 \langle S_z^2 \rangle \frac{1}{k_F \xi} \times \left[\frac{1}{1 + 4(k_F \xi)^2} + \ln [1 + 4(k_F \xi)^2] - 1 \right]. \quad (\text{D6})$$

The right-hand side of Eq. (D6) is a function of the product $k_F \xi$, illustrating the scaling of resistivity curves at different k_F when multiplied by k_F^5 and plotted as a function of $k_F \xi$.

For the 1D case, the scattering time at k_F is given by

$$\frac{1}{\tau(k_F)} = \frac{1}{2\pi} \sum_{\theta=0, \pi} (1 - \cos \theta) \frac{2\pi}{\hbar} J^2 \langle S_z^2 \rangle A_{1d} \times \frac{2\xi}{1 + 2k_F^2 \xi^2 (1 - \cos \theta)} \frac{m^*}{\hbar^2 k_F} = \frac{4m^* J^2 \langle S_z^2 \rangle A_{1d}}{\hbar^3} \frac{1}{k_F} \frac{\xi}{1 + 4(k_F \xi)^2}. \quad (\text{D7})$$

The analytical solution was derived in 3D and 1D due to their relative simplicity, and it confirms the numerical results. Only the term $\theta = \pi$ contributes since forward scattering does not affect transport due to the $1 - \cos \theta$ term. This gives a peak at $k_F \xi = 0.5$.

APPENDIX E: ANISOTROPY OF THE BAND DISPERSION AND STRUCTURE FACTOR

To introduce anisotropy in the electronic dispersion, we assign different effective masses such that the Fermi momenta satisfy $k_{F_x} = k_{F_z} \neq k_{F_y}$, i.e., the anisotropy is along the y direction. The corresponding energy dispersion relation is given

below:

$$\varepsilon(\mathbf{k}) = \frac{\hbar^2}{2m^*} \left[k_x^2 + \left(\frac{k_{Fx}}{k_{Fy}} \right)^2 k_y^2 + k_z^2 \right]. \quad (\text{E1})$$

Anisotropy is also introduced in the magnetic structure by using different correlation lengths in each direction. Specifically,

we set $\xi_x = \xi_z \neq \xi_y$ to reflect anisotropic correlations along y . The explicit form of the structure factor $S(\mathbf{q})$ is provided below:

$$S(\mathbf{q}) = \langle S_z^2 \rangle A_3 \frac{8\pi \xi_x \xi_y \xi_z}{(1 + q_x^2 \xi_x^2 + q_y^2 \xi_y^2 + q_z^2 \xi_z^2)^2}. \quad (\text{E2})$$

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